Homework 9 in LaTeX

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1 Question 1: The Lagrangian and Equations of Motion

1.1 (a) Write an Expression for the Lagrangian of the Particle

$$\begin{aligned} L = &-1.0Rgm_1 \sin(q_1) \\ &+ 0.5\dot{q}_1^2 \left(I_1 + R^2 m_1\right) \end{aligned}$$

1.2 (b) Find the Equation of Motion of the Particle

$$\begin{aligned} \tau_k = &\ 1.0I_1\ddot{q}_1 \\ &+ 1.0R^2 m_1 \ddot{q}_1 \\ &+ 1.0Rgm_1 \cos(q_1) \end{aligned}$$

2 Question 2: Equations of Motion of a Two Link Manipulator

2.1 (a) Find the Lagrangian

$$\begin{aligned} L = &\ 0.5I_1\dot{q}_1^2 + 2.0gm_2q_2\cos(q_1) \\ &+ 0.5m_2\dot{q}_1^2 + 6.12323399573677 \cdot 10^{-17} m_2\dot{q}_1\dot{q}_2 \\ &+ 0.5m_2\dot{q}_2^2 \end{aligned}$$

2.2 (b) Find Equation of Motion

$$\tau_k = 2.0gm_2q_2\sin(q_1) - 2.0gm_2\cos(q_1)$$
$$+ 1.0m_2\ddot{q}_2 + 1.0\ddot{q}_1(I_1 + m_2)$$

3 Question 3: Equations of Motion of a Pendulum

3.1 (a) Find the Lagrangian of the Pendulum

$$L = l_1m_2\left(-1.0g\sin(q_1) + 0.5l_1\dot{q}_1^2\right) \tag{1}$$

3.2 (b) Find Equation of Motion

$$\tau_k = 1.0l_1m_2(g\cos(q_1) + l_1\ddot{q}_1) \tag{2}$$

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\documentclass[10pt]{article}
\usepackage{setspace}
\usepackage{amsmath,amsfonts,amsthm,amssymb}
\usepackage{color}
\usepackage{fancyhdr}
\usepackage{chngpage}
\usepackage{enumerate}
\usepackage{graphicx}
\usepackage{boxedminipage}

\title{Homework 9  in \LaTeX}
\author{Robert Brothers Mechanical Engineering Student @ UTSA}
\begin{document}
\doublespacing
\footnotesize
\maketitle
\date

\section{Question 1: The Lagrangian and Equations of Motion}
\subsection{(a) Write an Expression for the Lagrangian of the Particle}
\begin{equation}
\begin{align}
L  &= - 1.0 R g m_{1} \sin{\left (q_{1} \right )}\nonumber\\
&\qquad + 0.5 \dot{q}_{1}^{2} \left(I_{1} + R^{2} m_{1}\right)
\end{align}
\end{equation}

\subsection{(b) Find the Equation of Motion of the Particle}
\begin{equation}
\begin{align}
\tau_{k} &= 1.0 I_{1} \ddot{q}_{1}\nonumber\\
&\qquad + 1.0 R^{2} m_{1} \ddot{q}_{1}\nonumber\\
&\qquad + 1.0 R g m_{1} \cos{\left (q_{1} \right )}
\end{align}
\end{equation}

\section{Question 2: Equations of Motion of a Two Link Manipulator}
\subsection{(a) Find the Lagrangian}
\begin{equation}
\begin{align}
L  &= 0.5 I_{1} \dot{q}_{1}^{2} + 2.0 g m_{2} q_{2} \cos{\left (q_{1} \right )}\nonu
mber\\
&\qquad + 0.5 m_{2} \dot{q}_{1}^{2} + 6.12323399573677 \cdot 10^{-17} m_{2} \dot{q}_
{1} \dot{q}_{2}\nonumber\\
&\qquad + 0.5 m_{2} \dot{q}_{2}^{2}
\end{align}
\end{equation}

\subsection{(b) Find Equation of Motion}
\begin{equation}
\begin{align}
\tau_{k} &= 2.0 g m_{2} q_{2} \sin{\left (q_{1} \right )} - 2.0 g m_{2} \cos{\left (
q_{1} \right )}\nonumber\\
&\qquad + 1.0 m_{2} \ddot{q}_{2} + 1.0 \ddot{q}_{1} \left(I_{1} + m_{2}\right)
\end{align}
\end{equation}

\section{Question 3: Equations of Motion of a Pendulum}
\subsection{(a) Find the Lagrangian of the Pendulum}
\begin{equation}
\begin{align}
L  &= l_{1} m_{2} \left(- 1.0 g \sin{\left (q_{1} \right )} + 0.5 l_{1} \dot{q}_{1}^
{2}\right)
\end{align}
\end{equation}

\subsection{(b) Find Equation of Motion}
\begin{equation}
\begin{align}
\tau_{k} &= 1.0 l_{1} m_{2} \left(g \cos{\left (q_{1} \right )} + l_{1} \ddot{q}_{1}
\right)
\end{align}
\end{equation}
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\end{document}
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```python
#!/usr/bin/env python

import sys
sys.path.append(r"/Users/robertbrothers/Desktop/Fall 2014/Fundamentals_of_Robotics/r
obo_git/python/")
import robotics_functions as rf, sympy as sy, numpy as np

[R, t1, I1, m1, q1, qdot1, qddot1, I2] = sy.symbols("R t1 I1 m1 q1 qdot1 qddot1 I2")

link_list_cm = [[
    [ R, 0, 0, q1],
    ],[
      sy.Matrix(
        [
        [0], [0], [0], [1]
        ])
      ]
    ]

[link_list, ocm] = link_list_cm
A0n = rf.sym_get_A0n(link_list)
J = rf.sym_pt_jacobian(link_list_cm)
qdot = sy.Matrix([
  [qdot1]
  ])

M = [
    sy.Matrix([
      [m1,0,0],
      [0,m1,0],
      [0,0,m1]
      ])
    ]

I = [
    sy.Matrix([
      [I1,0,0],
      [0,I1,0],
      [0,0,I1]
      ])
    ]

q = sy.Matrix([
  [q1],
  ])

tdv_vec = [
    (q1, qdot1),
    (qdot1, qddot1),
    ]
print "A matrices"
for A in A0n:
  sy.pprint(sy.trigsimp(A))
print "A Jacobian"
for j in J:
  sy.pprint(sy.trigsimp(j))
print "a) expression for the Lagrangian of the Particle: \n"
print sy.printing.latex(sy.trigsimp(rf.sym_lagrangian( link_list_cm, M, I, qdot)))
print "\n"
print "b) Find the Equations of Motion of the Particle: \n"
print sy.printing.latex(sy.simplify(sy.trigsimp(rf.sym_torque(link_list_cm, M, I, qd
ot, q, tdv_vec))))

[l1, l2, l3, t1, t2, t3, a1, a2, a3, d1, d2, d3] = sy.symbols("l1 l2 l3 t1 t2 t3 a1
a2 a3 d1 d2 d3")
[q1, q2, qdot1, qdot2, qddot1, qddot2, m1, m2, r1, r2] = sy.symbols("q1 q2 qdot1 qdo
t2 qddot1 qddot2 m1 m2 r1 r2")

link_list_cm = [[
    [0,np.pi/2, 0, q1],
    [0,0,q2,0],
    ],[
      sy.Matrix([
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        [0],[0],[0],[1]]),
      sy.Matrix([
        [0],[0],[q2],[1]])
      ]
    ]

m = np.array([m1, m2])
l = np.array([l1, l2])
r = np.array([r1, r2])


M = [sy.Matrix([
  [m[i],0,0],
  [0,m[i],0],
  [0,0,m[i]]
  ]) for i in range(len(m))]

I = [sy.Matrix([
  [I1,0,0],
  [0,I1, 0],
  [0, 0,I1]
  ]),
  sy.Matrix([
    [I2,0,0],
    [0,I2,0],
    [0,0,I2]
    ])
  ]
q = sy.Matrix([
  [q1],
  [q2]
  ])
qdot = sy.Matrix([
  [qdot1],
  [qdot2]
  ])
tdv_vec = [
    (qdot1,qddot1),
    (qdot2,qddot2),
    (q1, qdot1),
    (q2, qdot2),
    ]
link_list = link_list_cm[0]
A0n = rf.sym_get_A0n(link_list)
J = rf.sym_pt_jacobian(link_list_cm)

print "A matrices"
for A in A0n:
  sy.pprint(sy.trigsimp(A))

print "A Jacobian"
for j in J:
  sy.pprint(sy.trigsimp(j))
print "jacobian"
sy.pprint(rf.sym_pt_jacobian(link_list_cm))
print "\nLagrangian of the Manipulator"
print sy.printing.latex(sy.simplify(sy.trigsimp(rf.sym_lagrangian(link_list_cm, M, I
, qdot)[0])))

print "\nEquations of Motion of the Two Link Manipulator"
print sy.printing.latex(sy.simplify(sy.trigsimp(rf.sym_torque(link_list_cm, M, I, qd
ot, q, tdv_vec))))


link_list_cm = [[
    [l1, 0, 0, q1],
    ],[sy.Matrix([
      [0],[0],[0],[1]])
      ]
    ]

q = sy.Matrix([
  [q1],
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  ])

qdot = sy.Matrix([
  [qdot1],
  ])
tdv_vec = [
    (qdot1,qddot1),
    (q1, qdot1),
    ]

M = [sy.Matrix([
  [m[i],0,0],
  [0,m[i],0],
  [0,0,m[i]]
  ])
  ]

I = [sy.Matrix([
  [0,0,0],
  [0,0, 0],
  [0, 0,0]
  ])
  ]

print "\nLagrangian of the pendulum"
print sy.printing.latex(sy.simplify(sy.trigsimp(rf.sym_lagrangian(link_list_cm, M, I
, qdot)[0])))

print "\nEquations of Motion of the Two Link Manipulator"
print sy.printing.latex(sy.simplify(sy.trigsimp(rf.sym_torque(link_list_cm, M, I, qd
ot, q, tdv_vec))))
```