The Euler-Lagrangian of the Ein Concept LaTeX

Robert Brothers Mechanical Engineering Student @ UTSA

November 9, 2014

Abstract

Using the Denavit-Hartenberg Parameters the Lagrangian of Ein was Calculated to be L

1 Question: The Lagrangian

1.1 question about the lagrangian

when i take the derivative of the lagrangian w/ respect to q do i take the derivative of each q? and are the dependent on one another? if i differentiate $q_1 q_2$ with respect to q_1 what do i get?

$$M_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix} \tag{1}$$

$$I_{b,i} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \tag{2}$$

$$L = K - P \tag{3}$$

$$K = \frac{M_i V_i^2}{2} = \frac{1}{2}\dot{q}^T \left[\sum_{i=1}^{n} M_i J_{v,i}^T J_{v,i} + J_{\omega,i}^T R_i I_{b,i} R_i^T J_{\omega,i} \right] \dot{q} \tag{4}$$

$$J_i = \begin{bmatrix} J_{v,i} \\ J_{\omega,i} \end{bmatrix} = \begin{bmatrix} J_{v_0} & J_{v_1} & \dots & J_{v_n} \\ J_{\omega_0} & J_{\omega_1} & \dots & J_{\omega_n} \end{bmatrix} \tag{5}$$

$$P = M_i g h = \sum_{i=1}^{n} \vec{g}^T M_i \vec{r}_{c,i} \tag{6}$$

2 Results

2.1 Denavit-Hartenberg Parameters

Link_i	a_i	α_i	d_i	θ_i
1	0	$\frac{\pi}{2}$	L_t	θ_t
2	0	0	L_s	0

2.2 Jacobian

2.2.1 Jacobian of Link One's Center of Mass

$$J_1 = \begin{bmatrix} -0.5l_1 \sin(t_1) & 0 \\ 0.5l_1 \cos(t_1) & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{7}$$

2.2.2 Jacobian of Link Two's Center of Mass

$$J_2 = \begin{bmatrix} -0.5a_2 \sin(t_1 + t_2) - 1.0l_1 \sin(t_1) & -0.5a_2 \sin(t_1 + t_2) \\ 0.5a_2 \cos(t_1 + t_2) + 1.0l_1 \cos(t_1) & 0.5a_2 \cos(t_1 + t_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1.0 \end{bmatrix} \tag{8}$$

2.3 Lagrangian Results

$$L = 0.125a_2^2m_2q_1^2 + 0.25a_2^2m_2q_1q_2 +$$
$$0.125a_2^2m_2q_2^2 - 0.5a_2gm_2\sin(t_1+t_2) +$$
$$0.5a_2l_1m_2q_1^2\cos(t_2) + 0.5a_2l_1m_2q_1q_2\cos(t_2) -$$
$$0.5gl_1m_1\sin(t_1) - 1.0gl_1m_2\sin(t_1) +$$
$$0.291666666666667l_1^2m_1q_1^2 + 0.5l_1^2m_2q_1^2 +$$
$$0.523598775598567m_2q_1^2r_2^2 + 1.04719755119713m_2q_1q_2r_2^2 +$$
$$0.523598775598567m_2q_2^2r_2^2$$

2.4 Torque: Derived from the Lagrangian

$$\tau_k = 0.5a_2^2m_2\ddot{q}_1 + 0.5a_2^2m_2\ddot{q}_2 + 1.0a_2gm_2\cos(q_1+q_2)$$
$$+ 1.5a_2l_1m_2\ddot{q}_1\cos(q_2) + 0.5a_2l_1m_2\ddot{q}_2\cos(q_2)$$
$$+ 0.5a_2l_1m_2\dot{q}_1^2\sin(q_2) - 1.0a_2l_1m_2\dot{q}_1\dot{q}_2\sin(q_2)$$
$$- 0.5a_2l_1m_2\dot{q}_2^2\sin(q_2) + 0.5gl_1m_1\cos(q_1)$$
$$+ 1.0gl_1m_2\cos(q_1) + 0.583333333333333l_1^2m_1\ddot{q}_1$$
$$+ 1.0l_1^2m_2\ddot{q}_1 + 2.09439510239427m_2\ddot{q}_1r_2^2$$
$$+ 2.09439510239427m_2\ddot{q}_2r_2^2$$