Homework 10 in $\LaTeX$

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1 Interpreting Position Velocity

1.1 (a) Starting position, velocity and acceleration

$$q_0 = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \tag{1}$$

1.2 (b) Find the Equation of Motion of the Particle

$$q_f = \begin{bmatrix} 21 \\ 16 \end{bmatrix} \tag{2}$$

2 Trajectory generation for given condition

2.1 (a) Minimal order

For trajectory with initial conditions including position velocity and acceleration we'd need a 4^{th} order polynomial.

2.2 (b) joint position

$$\theta = \tag{3}$$

2.3 (c) plot $\theta(t)$, $\dot{\theta}(t)$ and $\ddot{\theta}(t)$

Plots are in appendix

3 Trajectory Generations with Points

3.1 (a) Find Cubic Polynomials that Fit Points

$$q_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{4}$$

3.2 (b) Plots of the position, velocity and acceleration

plots are located in appendix

4 Symbolic derivation of equations simulations

4.1 (a) Expressions for center of mass

$$r_{c1} = \begin{bmatrix} -n_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

$$r_{c2} = \begin{bmatrix} -n_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

$$r_{c3} = \begin{bmatrix} -n_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

4.2 (b) Location of Joints

$$O_0 = A_0^0 O_0^0 \tag{8}$$

$$O_1 = A_1^0 O_1^1 \tag{9}$$

$$O_2 = A_2^0 O_2^2 \tag{10}$$

$$O_3 = A_3^0 O_3^3 \tag{11}$$

4.3 (c) Translational Jacobians of Points

$$J_{v1} = \begin{bmatrix} R_0^0 \hat{k} \times (r_{c1} - O_0) & 0 & 0 \end{bmatrix} \tag{12}$$

$$J_{v2} = \begin{bmatrix} R_1^0 \hat{k} \times (r_{c2} - O_0) & R_1^0 \hat{k} \times (r_{c2} - O_1) & 0 \end{bmatrix} \tag{13}$$

$$J_{v3} = \begin{bmatrix} R_2^0 \hat{k} \times (r_{c3} - O_0) & R_2^0 \hat{k} \times (r_{c3} - O_1) & R_2^0 \hat{k} \times (r_{c3} - O_2) \end{bmatrix} \tag{14}$$

4.4 (d) Rotational Jacobians of Points

$$J_{\omega 1} = \begin{bmatrix} R_0^0 \hat{k} & 0 & 0 \end{bmatrix} \tag{15}$$

$$J_{\omega 2} = \begin{bmatrix} R_1^0 \hat{k} & R_1^0 \hat{k} & 0 \end{bmatrix} \tag{16}$$

$$J_{\omega 3} = \begin{bmatrix} R_2^0 \hat{k} & R_2^0 \hat{k} & R_2^0 \hat{k} \end{bmatrix} \tag{17}$$

4.5 (e) Expression for the Lagrangian

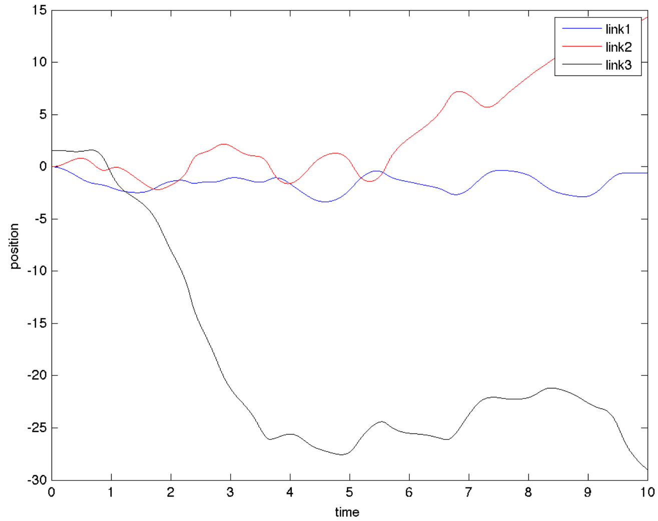
$$K = \frac{1}{2} \dot{q}^T \left[D \right] \dot{q} \tag{18}$$

$$\begin{aligned} D = {} & J_{v1}^T M_1 J_{v1} + J_{v2}^T M_2 J_{v2} + J_{v3}^T M_3 J_{v3} \\ & + J_{\omega 1}^T R_1^{b\,T} I_1 R_1^b J_{\omega 1} + J_{\omega 2}^T R_2^{b\,T} I_2 R_2^b J_{\omega 2} + J_{\omega 3}^T R_3^{b\,T} I_3 R_3^b J_{\omega 3} \end{aligned} \tag{19}$$

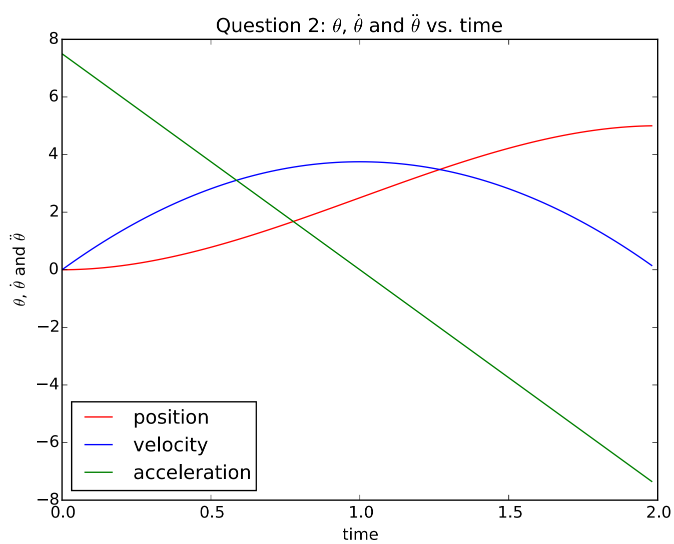
$$P = g^T M_1 r_{c1} + g^T M_2 r_{c2} + g^T M_3 r_{c3} \tag{20}$$

$$L = K - P \tag{21}$$

4.6 (h) Plots of the 3 Link Manipulator



5 Appendix



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#!/usr/bin/env python

import sys
sys.path.append(r"/Users/robertbrothers/Desktop/Fall 2014/Fundamentals_of_Robotics/r
obo_git/python/")
from matplotlib.backends.backend_pdf import PdfPages
import robotics_functions as rf, sympy as sy, numpy as np
from matplotlib import pyplot as plt
pp = PdfPages('homework10plots.pdf')

order = 3

Q = lambda t: np.array([[1, t, t**2, t**3], [0, 1, 2*t, 3*t**2], [0, 0, 2, 6*t]])
a = np.array([10,5,7,-1]).T

time = [0.0,1.0]
Q = [Q(t)[:2] for t in time]
Q = np.vstack((Q[0][0],Q[1][0],Q[0][1],Q[1][1]))

#print np.matrix(Q)*np.matrix(a).T

# Question 2 using the same
ini_con = np.matrix([0.0,5.0,0.0,0.0])
time = [0.0,2.0]
Q = lambda t: np.array([[1, t, t**2, t**3], [0, 1, 2*t, 3*t**2], [0, 0, 2, 6*t]])
Q = [Q(t)[:2] for t in time]
Q = np.matrix(np.vstack((Q[0][0],Q[1][0],Q[0][1],Q[1][1])))
a = np.linalg.solve(Q,ini_con.T)
#print a

time = np.arange(0.0,2.0, 2.0/100)
theta = lambda t: 0+0*t+3.75*t**2-1.25*t**3
thetadot = lambda t: 0+ 1*0.0+ 3.75*2*t - 1.25*3*t**2
thetaddot = lambda t: 0 + 0 + 3.75*2 - 1.25*6*t
plt.plot(time,theta(time[:]),'r', label='position',)
plt.plot(time,thetadot(time[:]),'b',label='velocity',)
plt.plot(time,thetaddot(time[:]),'g',label='acceleration')

plt.xlabel("time")
plt.ylabel(r"$\theta$, $\dot{\theta}$ and $\ddot{\theta}$")

plt.title(r"Question 2: $\theta$, $\dot{\theta}$ and $\ddot{\theta}$ vs. time")
plt.legend(loc=3)
plt.savefig(pp, format='pdf')
plt.show()


#### question 3
qi = np.array([ 0,1,1,0, 0,0,0,0 ])
ti = [0,1.0,1.0,3.0]
q01 = lambda t: np.array([1,t,t**2,t**3,  0,0,0,0])
qd01 = lambda t: np.array([0,1,2*t,3*t**2, 0,0,0,0])

q12 = lambda t: np.array([0,0,0,0, 1,t,t**2,t**3])
qd12 = lambda t: np.array([ 0,0,0,0, 0,1,2*t,3*t**2])

Q = np.array([
  q01(ti[0]),
  q01(ti[1]),
  q12(ti[2]),
  q12(ti[3]),
  qd01(ti[0]),
  qd01(ti[1]),
  qd12(ti[2]),
  qd12(ti[3]),
  ])

a = np.linalg.solve(Q, qi.T)
print a
[a10,a11,a12,a13,   a20,a21,a22,a23 ] = a
#

qi    = lambda a,t: a[0] + a[1]*t + a[2]*t**2 + a[3]*t**3
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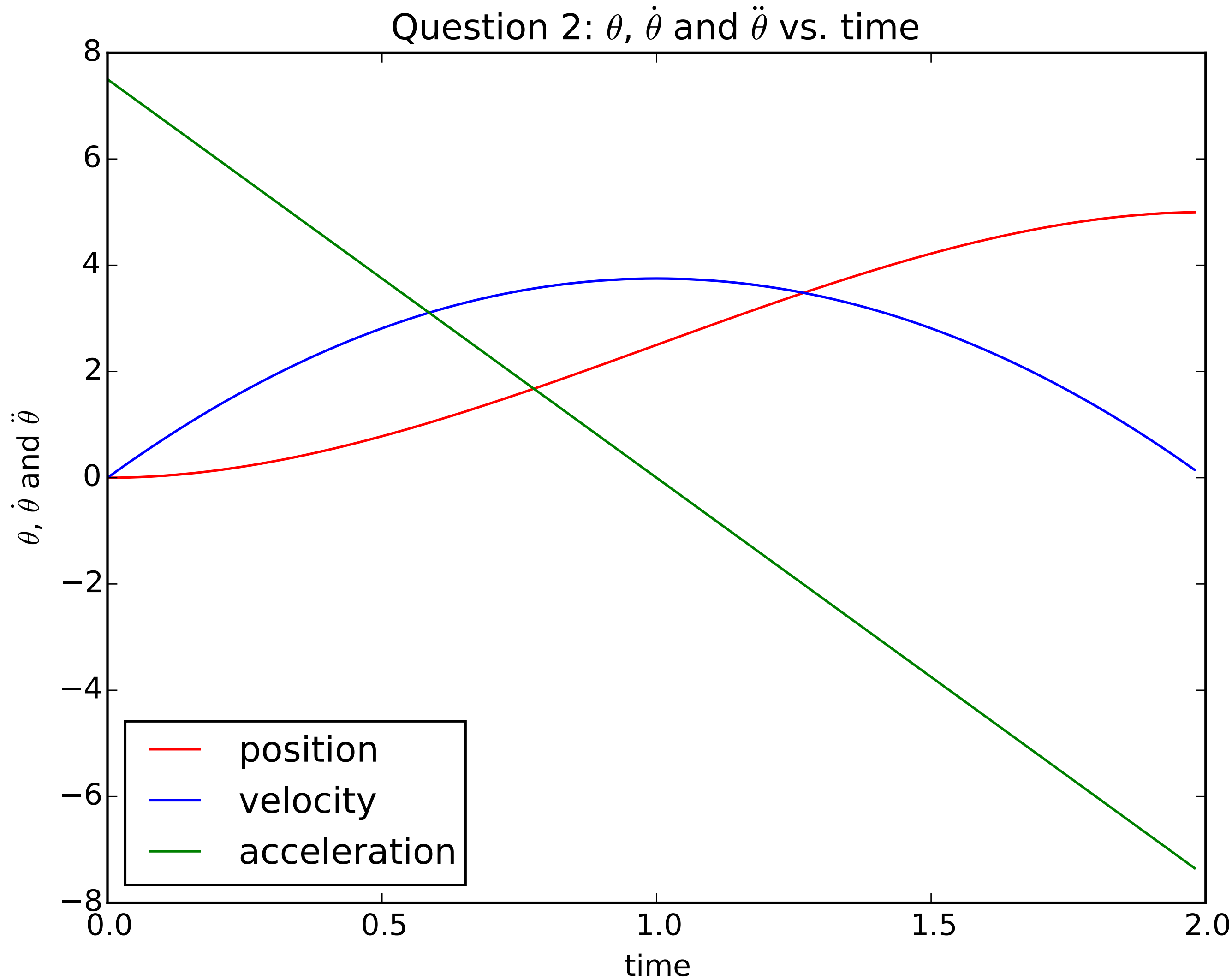
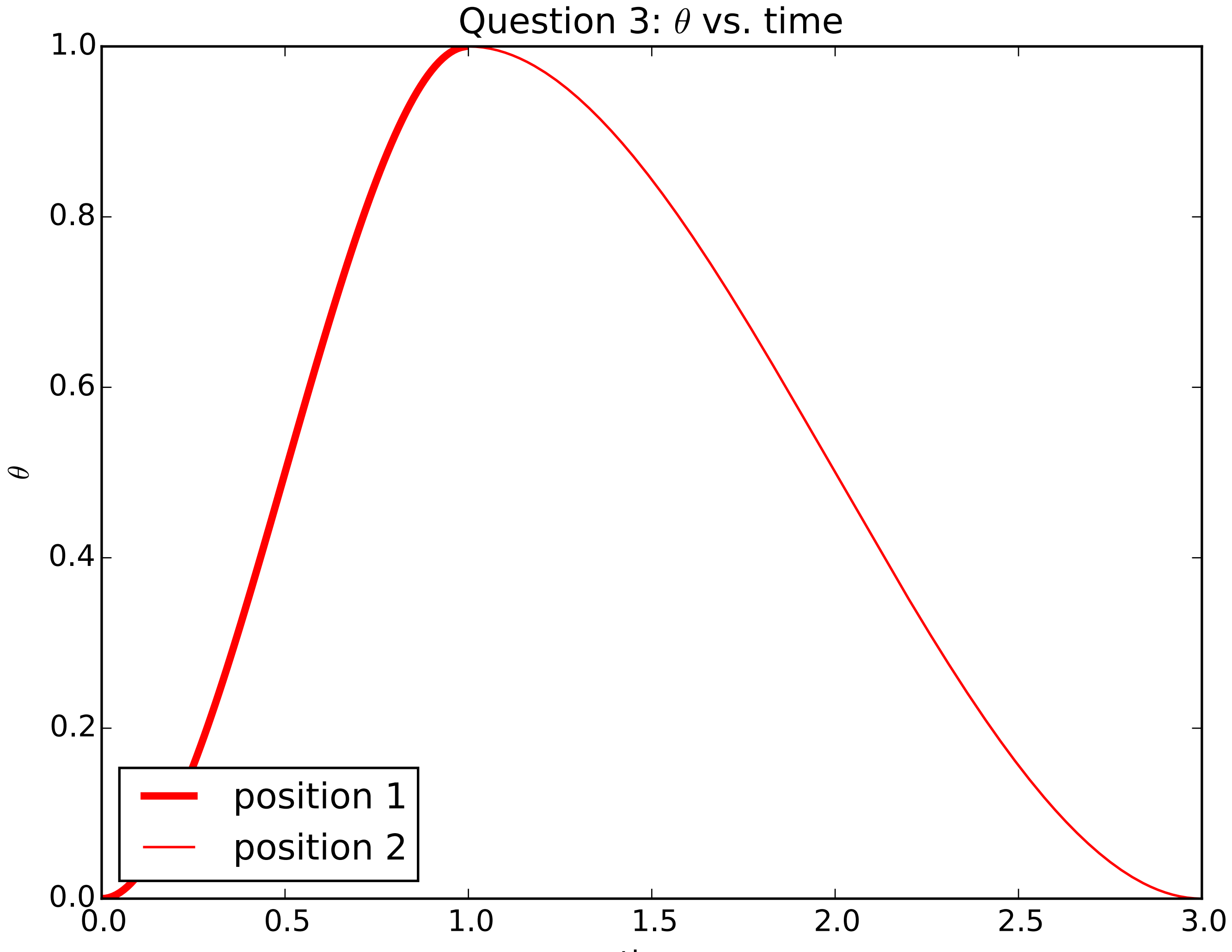
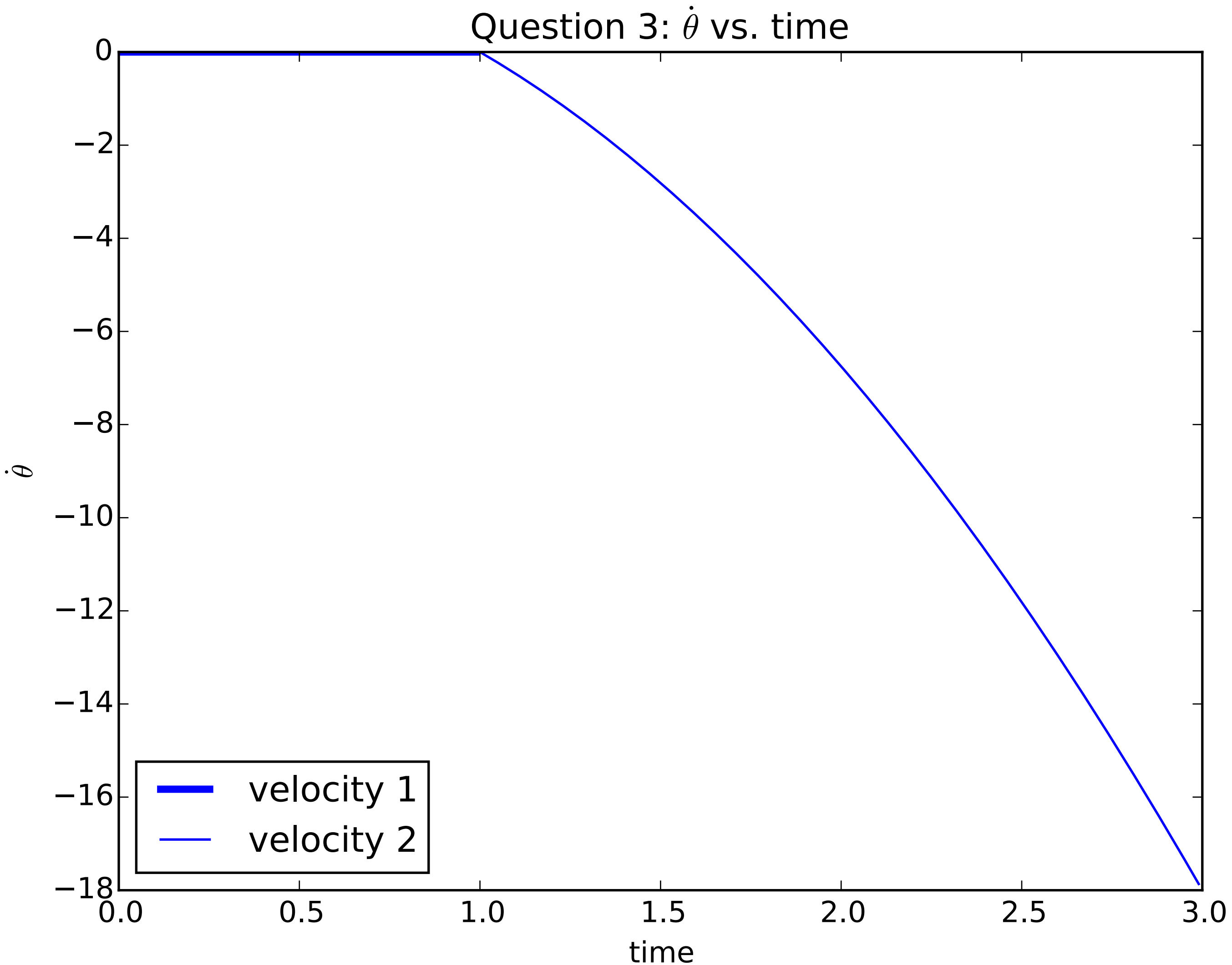
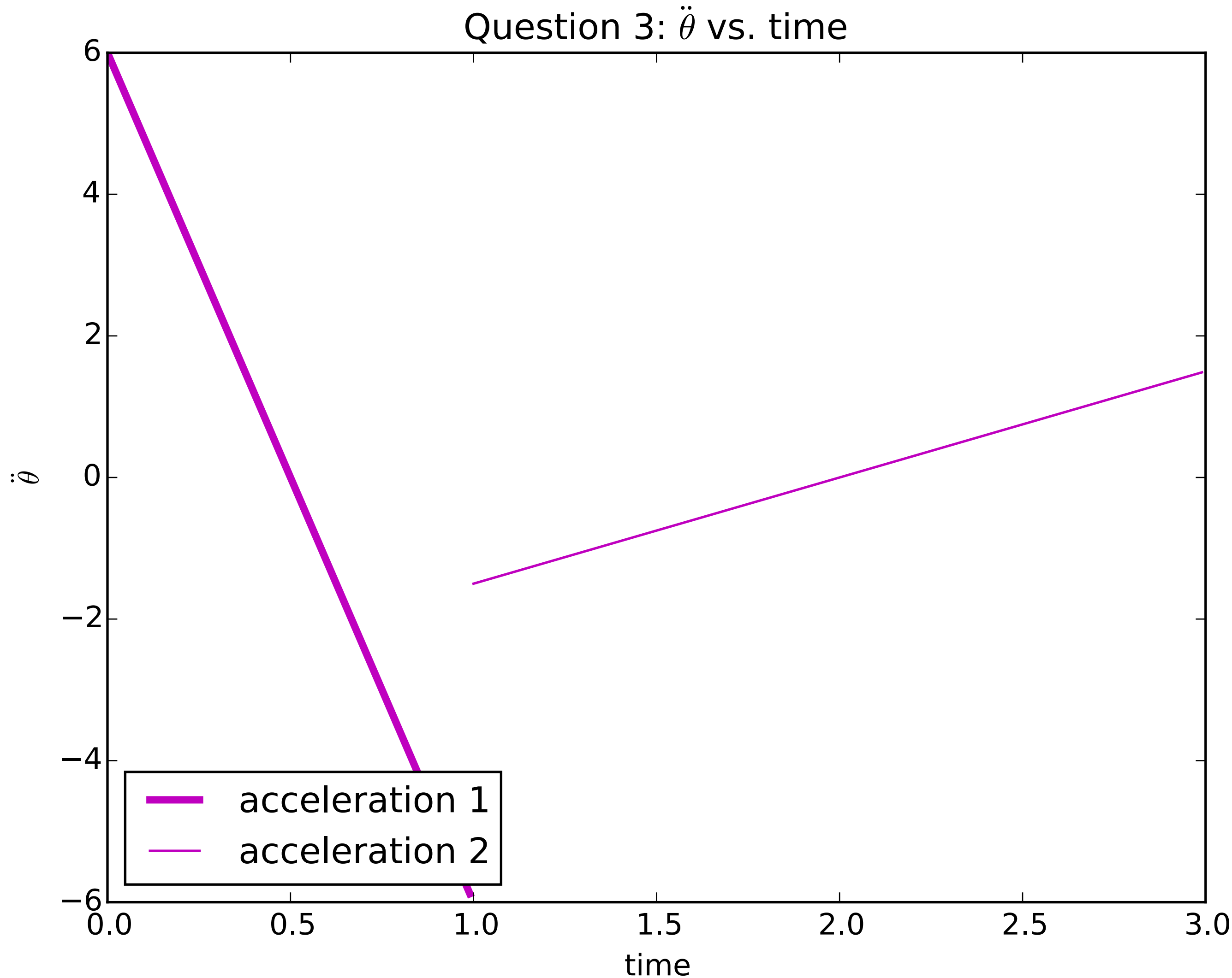
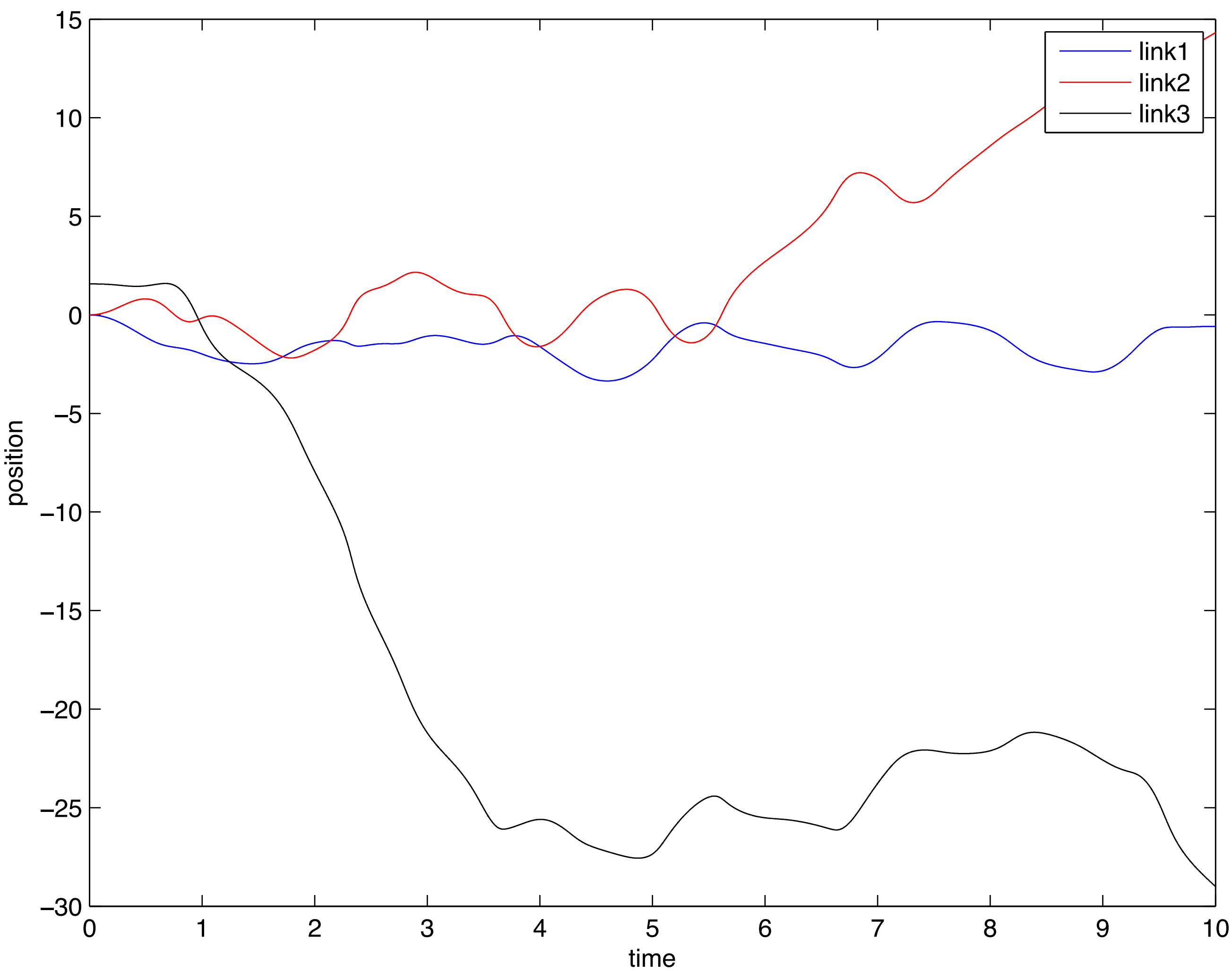
```
qdi   = lambda a,t: a[1] + 2*a[2]*t**2 + 3*a[3]*t**2
qddi  = lambda a,t: 2*a[2] + 6*a[3]*t

time1 = np.arange(0,1.0,1.0/100)
time2 = np.arange(1.0,3.0,1.0/100)

plt.plot(time1,qi( a[0:4], time1[:]),'r', label='position 1',linewidth=3)
plt.plot(time2,qi( a[4:], time2[:]),'r', label='position 2',)

plt.xlabel("time")
plt.ylabel(r"$\theta$")
plt.title(r"Question 3: $\theta$ vs. time")
plt.legend(loc=3)
plt.savefig(pp, format='pdf')
plt.show()


plt.plot(time1,qdi( a[0:4], time1[:]),'b',label='velocity 1',linewidth=3)
plt.plot(time2,qdi( a[4:], time2[:]),'b',label='velocity 2',)

plt.xlabel("time")
plt.ylabel(r"$\dot{\theta}$")
plt.title(r"Question 3: $\dot{\theta}$ vs. time")
plt.legend(loc=3)
plt.savefig(pp, format='pdf')
plt.show()

plt.plot(time1,qddi( a[0:4], time1[:]),'m',label='acceleration 1',linewidth=3)
plt.plot(time2,qddi( a[4:], time2[:]),'m',label='acceleration 2')

plt.xlabel("time")
plt.ylabel(r"$\ddot{\theta}$")
plt.title(r"Question 3: $\ddot{\theta}$ vs. time")
plt.legend(loc=3)
plt.savefig(pp, format='pdf')
plt.show()
#plt.savefig("question3.png")
pp.close()
```


Question 2: θ, $\dot{\theta}$ and $\ddot{\theta}$ vs. time
θ, $\dot{\theta}$ and $\ddot{\theta}$
time
8
6
4
2
0
−2
−4
−6
−8
0.0
0.5
1.0
1.5
2.0
position
velocity
acceleration


Question 3: θ vs. time
θ
time
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
position 1
position 2


Question 3: $\dot{\theta}$ vs. time
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
$\dot{\theta}$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time
velocity 1
velocity 2


Question 3: $\ddot{\theta}$ vs. time
$\ddot{\theta}$
6
4
2
0
−2
−4
−6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time
acceleration 1
acceleration 2



15
10
5
0
−5
−10
−15
−20
−25
−30
0
1
2
3
4
5
6
7
8
9
10
time
position
link1
link2
link3