Homework 10 in $\LaTeX$

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1 Interpreting Position Velocity

1.1 (a) Starting position, velocity and acceleration

$$q_0 = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \tag{1}$$

1.2 (b) Find the Equation of Motion of the Particle

$$q_f = \begin{bmatrix} 21 \\ 16 \end{bmatrix} \tag{2}$$

2 Trajectory generation for given condition

2.1 (a) Minimal order

For trajectory with initial conditions including position velocity and acceleration we'd need a 4^{th} order polynomial.

2.2 (b) joint position

$$\theta = \tag{3}$$

2.3 (c) plot $\theta(t)$, $\dot{\theta}(t)$ and $\ddot{\theta}(t)$

Plots are in appendix

3 Trajectory Generations with Points

3.1 (a) Find Cubic Polynomials that Fit Points

$$q_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{4}$$

3.2 (b) Plots of the position, velocity and acceleration

plots are located in appendix

4 Symbolic derivation of equations simulations

4.1 (a) Expressions for center of mass

$$r_{c1} = \begin{bmatrix} -n_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

$$r_{c2} = \begin{bmatrix} -n_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

$$r_{c3} = \begin{bmatrix} -n_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

4.2 (b) Location of Joints

$$O_0 = A_0^0 O_0^0 \tag{8}$$

$$O_1 = A_1^0 O_1^1 \tag{9}$$

$$O_2 = A_2^0 O_2^2 \tag{10}$$

$$O_3 = A_3^0 O_3^3 \tag{11}$$

4.3 (c) Translational Jacobians of Points

$$J_{v1} = \begin{bmatrix} R_0^0 \hat{k} \times (r_{c1} - O_0) & 0 & 0 \end{bmatrix} \tag{12}$$

$$J_{v2} = \begin{bmatrix} R_1^0 \hat{k} \times (r_{c2} - O_0) & R_1^0 \hat{k} \times (r_{c2} - O_1) & 0 \end{bmatrix} \tag{13}$$

$$J_{v3} = \begin{bmatrix} R_2^0 \hat{k} \times (r_{c3} - O_0) & R_2^0 \hat{k} \times (r_{c3} - O_1) & R_2^0 \hat{k} \times (r_{c3} - O_2) \end{bmatrix} \tag{14}$$

4.4 (d) Rotational Jacobians of Points

$$J_{\omega 1} = \begin{bmatrix} R_0^0 \hat{k} & 0 & 0 \end{bmatrix} \tag{15}$$

$$J_{\omega 2} = \begin{bmatrix} R_1^0 \hat{k} & R_1^0 \hat{k} & 0 \end{bmatrix} \tag{16}$$

$$J_{\omega 3} = \begin{bmatrix} R_2^0 \hat{k} & R_2^0 \hat{k} & R_2^0 \hat{k} \end{bmatrix} \tag{17}$$

4.5 (e) Expression for the Lagrangian

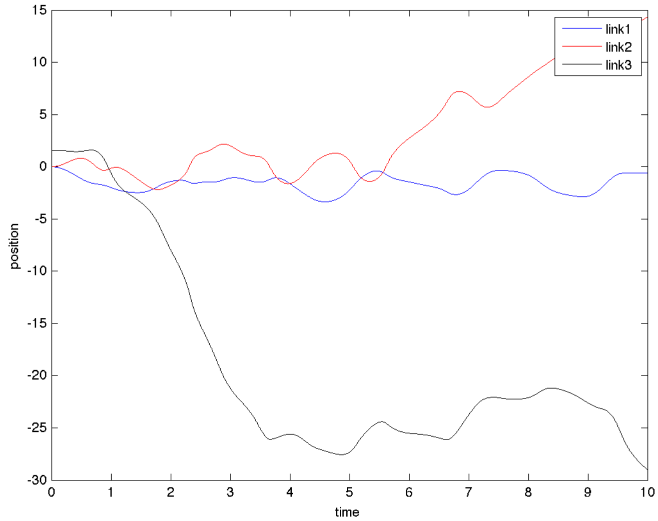
$$K = \frac{1}{2} \dot{q}^T \left[D \right] \dot{q} \tag{18}$$

$$\begin{aligned} D = {} & J_{v1}^T M_1 J_{v1} + J_{v2}^T M_2 J_{v2} + J_{v3}^T M_3 J_{v3} \\ & + J_{\omega 1}^T R_1^{b\,T} I_1 R_1^b J_{\omega 1} + J_{\omega 2}^T R_2^{b\,T} I_2 R_2^b J_{\omega 2} + J_{\omega 3}^T R_3^{b\,T} I_3 R_3^b J_{\omega 3} \end{aligned} \tag{19}$$

$$P = g^T M_1 r_{c1} + g^T M_2 r_{c2} + g^T M_3 r_{c3} \tag{20}$$

$$L = K - P \tag{21}$$

4.6 (h) Plots of the 3 Link Manipulator



5 Appendix