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- Comprehensive analysis of return period and reliability under nonstationarity
- Trends change the shape of the distribution of the return period
- Reliability over project life is a sensible design metric under nonstationarity

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- Supporting Information S1

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Reliability, return periods, and risk under nonstationarity

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Abstract Water resources design has widely used the average return period as a concept to inform management and communication of the risk of experiencing an exceedance event within a planning horizon. Even though nonstationarity is often apparent, in practice hydrologic design often mistakenly assumes that the probability of exceedance, p, is constant from year to year which leads to an average return period T_o equal to $1/p$; this expression is far more complex under nonstationarity. Even for stationary processes, the common application of an average return period is problematic: it does not account for planning horizon, is an average value that may not be representative of the time to the next flood, and is generally not applied in other areas of water planning. We combine existing theoretical and empirical results from the literature to provide the first general, comprehensive description of the probabilistic behavior of the return period and reliability under nonstationarity. We show that under nonstationarity, the underlying distribution of the return period exhibits a more complex shape than the exponential distribution under stationary conditions. Using a nonstationary lognormal model, we document the increased complexity and challenges associated with planning for future flood events over a planning horizon. We compare application of the average return period with the more common concept of reliability and recommend replacing the average return period with reliability as a more practical way to communicate event likelihood in both stationary and nonstationary contexts.

1. Introduction

Traditional probabilistic approaches for defining risk, reliability, and return periods under stationary hydrologic conditions assume that extreme events arise from serially independent time series with a probability distribution whose moments and parameters are fixed. *Gumbel* [1941] and *Thomas* [1948] defined a return period of a flood as the interval between flood events, where an event is any streamflow discharge exceeding a known threshold. In some studies, the return period has been defined as the (conditional) interval between two flood events [*Lloyd*, 1970; *Haan*, 1977; *Mays*, 2001], whereas the more common definition of a return period in practice is the unconditional waiting time until an exceedance event [*Fuller*, 1914; *Gumbel*, 1941; *Fernández and Salas*, 1999]. The unconditional return period does not assume a flood has occurred in year 1. Though the two definitions are equivalent for stationary conditions, the conditional return period is not sensitive to hydrologic persistence [*Lloyd*, 1970; *Douglas et al.*, 2002], an attractive feature in drought planning because droughts tend to exhibit persistence. The more commonly used unconditional definition of a return period is useful for describing the recurrence of hydrologic events because it does not depend on knowledge of a previous event, yet its value is sensitive to hydrologic persistence.

Consider the case of planning for a random future annual maximum extreme event X, where the design quantile X_p is the threshold of exceedance, and determines whether a flood event with exceedance probability p, occurs in a given year. Assume the hydrologic event X is defined as the annual maximum streamflow which has a stationary probability distribution function (pdf) denoted by $f_x(x)$ and cumulative distribution function (cdf) denoted by $F_x(x)$. In the case where a structure is built to protect against an event with an annual nonexceedance probability, $1 - p = F_x(x)$, the design event for such a structure is computed as simply the inverse of the cdf and equal to the quantile X_p. Under stationary conditions, if we assume that the exceedance probability p, of annual floods is constant and that flood events are independent and identically distributed, then the return period, T, follows a geometric distribution with probability mass function (pmf) given by

$$f(t)=P(T=t)=(1-p)^{t-1}p \tag{1}$$

where p is the annual exceedance probability. Similarly, for the continuous case, the random variable T follows an exponential pdf. In either case, the average return period is

$$E[T]=T_o=\sum_{t=1}^{\infty}t(1-p)^{t-1}p=\frac{1}{p} \tag{2}$$

Similarly the variance of T is

$$Var[T]=E\left[T^2\right]-E[T]^2=\sum_{t=1}^{\infty}t^2\cdot P[T=t]-\frac{1}{p^2}=\frac{1-p}{p^2} \tag{3}$$

In the design of hydrologic infrastructure, the probability of failure over its lifetime or its associated system reliability over a project lifetime is perhaps the most important piece of information an engineer can communicate to planners and the public. Prior to the 1983, Principles and Guidelines [*Water Resources Council* (*WRC*), 1983], standard practice in the United States for designing hydraulic infrastructure had been to select a design event, compute its average return period, and build the lowest cost structure. Such an approach does not consider the risk of failure over the planning horizon as a decision variable, and instead reports this risk of failure as more of a posterior calculation [*Yen*, 1970]. Since then, hydrologists and the Army Corps of Engineers have adopted a probabilistic approach, or risk-based design [*WRC*, 1983], where a level of infrastructure (i.e., protection) is selected based on minimizing the expected annual damage costs from a hazard, i.e., a flood [*United States Army Corps of Engineers*, 1996]. More recently, Risk-Based Decision Making (RBDM) has become a well-established methodology that determines appropriate levels of infrastructure based on the expected damages avoided versus the cost of the infrastructure required [*Tung*, 2005; *National Research Council*, 2000]. RBDM can be used in place of the traditional design storm approach to first select a particular design event (a distinct T_o year event usually specified by regulation), and then select the necessary infrastructure to protect against the flood event with that specified average return period. *Rosner et al*. [2014] document how RBDM can be applied in a nonstationary setting.

We define risk of failure over a planning period ($Risk_n$) here as in most introductory hydrology textbooks [*Bras*, 1990; *Viessman and Lewis*, 2003; *Mays*, 2001] and hydrology handbooks [*Tung*, 1999; *Stedinger et al*., 1993; *Interagency Committee on Water Data (IAWCD)*, 1982; *Chow*, 1964], as the likelihood of experiencing at least one event exceeding the design event over a given project life of n years:

$$Risk_n=1-(1-p)^n \tag{4}$$

Note that the risk of failure can be directly computed from the stationary average return period by replacing $p=1/T_o$ from (2) into (4). *Yen* [1970] points out that when the project lifetime (n years) is equal to the average return period (T_o) in equation (4), the project risk approaches a value of 0.63 as the project life nears infinity. This result emphasizes an important link that exists between project life and its risk of failure under stationary conditions.

A more modern definition of risk used in environmental and water resource planning involves both the magnitude and frequency of the event [*Krimsky and Golding*, 1996], whereas the definition in (4) is only indicative of the probability of failure over an n year period. For this reason, we recommend no longer using the term risk when discussing the concept defined in equation (4). Instead, we recommend the term "reliability" over a planning period ($Reliability_n$), which is defined as the probability that a system will remain in a satisfactory state [*Hashimoto et al*., 1982; *Salas and Obeysekera*, 2014] during its lifetime, i.e., that an exceedance event will not occur within a project life of n years:

$$Reliability_n=(1-p)^n \tag{5}$$

The concept of reliability is not new to hydrology and is widely used in water supply planning [*Hirsch*, 1979; *Vogel*, 1987; *Harberg*, 1997; *Loucks et al*., 2005] and many other engineering fields [*Kottegoda and Rosso*, 2008; *Tung et al*., 2006]. For stationary systems, the relationship between the reliability and average return periods for $n=25$, 50, and 100 year planning horizons is illustrated in Figure 1 by simply substituting $p=1/T_o$ into (5).

Figure 1 illustrates that to achieve a reliability commensurate with other areas of design (i.e., Reliability >0.9), over typical project lifetimes, the average return period of the design event must be in the

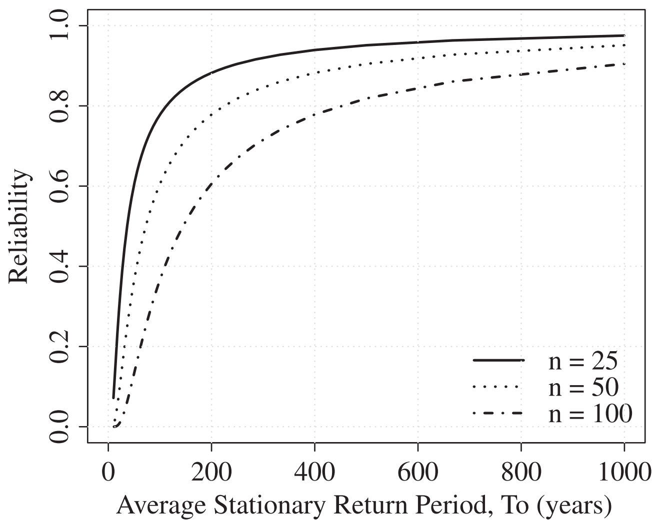

Figure 1. Reliability of a stationary system which is designed on the basis of an average return period T_o, corresponding to $n = 25$, 50, and 100 years.

hundreds of years. Most importantly, Figure 1 illustrates that knowledge of the average return period alone gives little guidance regarding the likelihood that a given project will perform as expected. Under nonstationary conditions, the exceedance probability associated with the design event is likely to change over time, creating additional challenges in selecting a suitable design event. Furthermore, a prerequisite to the use of RBDM under nonstationary conditions is that we develop a complete understanding of the impact of nonstationarity on traditional design metrics such as the expected return period as well as the reliability of a system over its planning horizon.

Overall, our goal is to initiate a discussion as to how engineers can effectively communicate the risk of failure and reliability of hydrologic design over planning horizons for a range of possible conditions. This goal is in line with a number of recent papers in the hydrologic science literature which discuss the existence and mortality/immortality of stationarity as it relates to hydrologic design and extreme events [*Cohn and Lins*, 2005; *Milly et al*., 2008; *Montanari and Koutsoyiannis*, 2014; *Koutsoyiannis and Montanari*, 2014; *Serinaldi*, 2015; *Serinaldi and Kilsby*, 2015; *Condon et al*., 2015]. In the following sections, we provide detailed reasoning for the need to replace the use of the average return period with the concept of reliability over a planning horizon as a metric for design and more efficient means for communicating risk of failure. We show this to be the case under stationary conditions (Figure 1), and even more dramatically under nonstationary conditions. In addition, we document the general impact of nonstationarity on statements of risk of failure, reliability, and the average return period using a simple, realistic, and representative two-parameter nonstationary lognormal (LN2) flood model. We begin by reviewing the past and current approaches to hydrologic design under stationary and nonstationary conditions. Then we introduce the nonstationary LN2 flood model and apply it to develop general relationships among risk of failure, reliability, and average return periods introduced by previous investigators and demonstrate its use for hydrologists and planners. This investigation is presented in the context of the growing societal interest in the impact of future nonstationarities and the need for guidance in selecting a representative design event over a future planning horizon. Finally, we conclude with recommendations concerning suitable statements of risk of failure, reliability, and average return periods in both stationary and nonstationary settings.

2. Hydrologic Design Under Stationary Conditions

In the water resources literature, the concept of an average return period has many applications, with each definition based on the type of hydrologic event described [see *Fernández and Salas*, 1999, and references therein]. For example, several investigators have considered the behavior of average return periods for hydrologic processes that exhibit persistence (long-range dependence) such as droughts and water supply failures [*Lloyd*, 1970; *Vogel*, 1987; *Fernández and Salas*, 1999; *Douglas et al*., 2002], and taken steps to create stochastic models that can reproduce such persistence [*Efstratiadis et al*., 2014]. We focus our attention on flood events which do not tend to exhibit significant interannual persistence as do droughts and water supply failure.

The risk of failure defined by *Yen* [1970] and our preferred definition of reliability are essential metrics for communicating the likelihood associated with a system failure during a project lifetime. Since society and planners are concerned with knowing whether a system will remain undamaged within a given design period, reliability is a more effective tool than the average return period for directly communicating the

Table 1. Reliabilities and Average Return Periods Associated With Important Events Associated With Several Disciplines Which Seek to Protect Against Extreme Events

Discipline	Reliability for Typical n Year Planning Period	Average Return Period (Years)	Citation
Earthquake (shaking)	98%, $n = 50$	2475	*NEHRP* [2010]
Retirement portfolio planning	95%, $n = 30$	585	*Ameriks et al.* [2001]and *Stout and Mitchell* [2006]
Nuclear power plant accident	95%, $n = 21$	409	*United States National Nuclear Regulatory Commission* [1975]
National Flood Insurance	75%, $n = 30$	105	FEMA
Flood design (infrastructure)	61%, $n = 50$	100	*Yen* [1970]

likelihood that a flood exceeding the design event will occur over a planning horizon. *Serinaldi* [2015] and *Serinaldi and Kilsby* [2015] relay a similar message, pointing out that risk of failure better describes this likelihood over a planning horizon because it summarizes the joint probability instead of the average probability, and does not require computations involving sums to infinity (i.e., to times beyond the design life).

Figure 1 shows that systems designed on the basis of typical average return periods are not nearly as reliable as one might anticipate over a typical project life. According to Figure 1, system reliability is only 78% when the design event is based on the 1% exceedance event (100 year flood) for $n = 25$ years. Considering that the design life of some public structures can be much longer than 25 years, and that reliability decreases as project life increases for a given average return period, careful attention should be given to how the system reliability of such structures is impacted by the planning horizon for structures which have been designed on the basis of an average return period.

Other fields concerned with risk and hazard planning ensure a much higher reliability over typical planning horizons than corresponding reliabilities associated with the 100 year flood so commonly used in hydrologic planning. Table 1 provides several examples, highlighting the differences between average return periods and reliabilities associated with various disciplines concerned with hazard planning. For example, earthquake design regulations suggest protection against a "less than 2% chance of failure (collapse) occur[ring] in a 50-year project life" [*National Earthquake Hazard Reduction Program* (*NEHRP*), 2010]. This level of protection corresponds to an earthquake magnitude with an average return period of 2475 years [obtained by combining equations (2) and (5)]. By comparison, traditional flood frequency analysis which often bases designs on an average return period of $T_o = 100$ years corresponds to reliabilities of 78%, 61%, and 37% over a range of $n = 25$, 50, and 100 years, respectively.

In communicating flood risk, the "100 year flood" has a long history as a regulatory concept endorsed in the 1970s by the National Flood Insurance Program (NFIP) and FEMA in an effort to standardize flood risk. See *Pielke* [1999] for a complete discussion of the many misconceptions regarding the "100 year flood" and its influence on flood risk perception. Today the NFIP communicates flood risk to the public on their website (available at https://www.floodsmart.gov/floodsmart/pages/flooding_flood_risks/defining_flood_risks.jsp) by relating the likelihood of flood damage over a typical 30 year mortgage, and categorizing locations as "high-risk" (residences within the 100 year floodplain) and "moderate-to-low" risk (outside the 100 year floodplain). Interestingly the "moderate-to-low risk" areas, which are not required to purchase flood insurance, receive over one-third the payouts of disaster flood assistance or 20% of claims, begging the question of how the category "low risk" was defined. As *Salas* [2013] notes, the omission of uncertainty from estimating the floodplain is in itself a compounding issue.

2.1. Should We Consider Replacing the Average Return Period With Concept of Reliability?

We are not the first researchers to question the use of the average return period as a design metric for hydrologic purposes and for communication of extreme events [*Fernández and Salas*, 1999; *Pielke*, 1999; *Douglas et al*., 2002; *Cooley*, 2013; *Serinaldi*, 2015; *Serinaldi and Kilsby*, 2015]. A summary of others' points and our own thoughts suggest that even under stationary conditions, using the average return period as a design metric for hydrologic infrastructure and as a conceptual tool for communicating risk of failure is problematic because (1) the shape of the geometric (discrete) and exponential (continuous) distribution associated with the time to failure under stationary conditions have a very long right tail, thus the mean time to failure (average return period) is a poor representation of the most likely time to failure; (2) the average return period is not explicitly tied to a planning horizon and thus is unable to characterize the likelihood of an event occurring during a project lifetime. When hydrologic processes exhibit nonstationarity, the probability of exceedance associated with the design event is changing over time, and thus the traditional formulae in equations (1–4) no longer hold. In this situation, the average return period may become even less representative of the future system reliability than under stationary conditions as is shown in the following sections.

3. Hydrologic Design Under Nonstationary Conditions

This work is in part motivated by the studies of *Olsen et al*. [1998], *Cooley* [2009], *Parey et al*. [2007, 2010], and *Salas and Obeysekera* [2014], who introduced much of the mathematics needed to describe the risk of failure, reliability, and average return periods in a nonstationary context. Interestingly, most of the key developments extending hydrologic design indices to nonstationary conditions have appeared primarily in the statistics and climate change literature, likely as a result of the attention and resources devoted to understanding and characterizing climate change [*Wigley*, 1988; *Katz and Brown*, 1992; *Katz*, 2010; *Olsen et al*., 1998; *Parey et al*., 2007, 2010; *Cooley*, 2009, 2013].

When a trend exists in annual maximum streamflow, the expressions for $Risk_n$, $Reliability_n$, and average return period T_o presented in equations (1–4) are no longer correct because the probability of experiencing a flood which exceeds a fixed design threshold is increasing (positive trend)/decreasing (negative trend). Under stationary conditions, the return period follows a homogeneous geometric distribution as described in equations (1) and (2), whereas under nonstationary conditions, the exceedance probability p_t associated with a particular annual maximum flood discharge changes every year. Under nonstationary conditions, the average return period is no longer a sufficient statistic of the distribution of return periods. For example, p, or the average return period $1/p$, are both sufficient statistics for the geometric distribution in (1) because each one is all that is needed (sufficient) to describe the pdf of return periods in a stationary context. However, under nonstationary conditions, knowledge of the average return period is no longer the only piece of information needed to specify the complete distribution of the return period [*Rootzén and Katz*, 2013]. *Olsen et al*. [1998] and *Salas and Obeysekera* [2014] introduce expressions for the average return period for the case where the exceedance probabilities of extreme events are increasing, such that p continuously increases until reaching unity at some future time, $t=t_{max}$. From *Cooley* [2013] and others, the pmf of a return period or waiting time distribution, for a nonstationary process, is given by

$$f(t)=p_t\prod_{i=1}^{t-1}(1-p_i) \quad t=1,\ 2,\ldots t_{max} \tag{6}$$

where t is the time until the first flood that exceeds the design event. An expression for the probability of failure in year t, p_t, can be derived for any distribution depending on the random variable of interest X. Note that as expected for a stationary hydrologic process, equation (6) reduces to equation (1) because p_t is constant in every year. *Cooley* [2013] and *Salas and Obeysekera* [2014] show that the expected value of the return period, $T_1=\mathrm{E}[T]$ under nonstationary conditions is given by

$$T_1=E[T]=\sum_{t=1}^{t_{\max}}tf(t)=\sum_{t=1}^{t_{\max}}tp_t\prod_{i=1}^{t-1}(1-p_i) \tag{7}$$

Here we denote the average return period under nonstationary conditions using the notation T_1 to distinguish it from the average return period T_o under stationary conditions given in equation (2). Equation (7) is a general form to compute T_1; in cases with increasing exceedance probabilities, one hopes that the maximum time (t_{max}) is very far into the future, as it corresponds to experiencing annual floods in excess of some important design threshold, with certainty (i.e., exceedance probability of unity). In the case of decreasing exceedance probabilities (downward trends), t_{max} and the expected return period itself may be both infinite. These are additional challenges for the practical application of applying a nonstationary return period for future planning purposes. An equivalent numerically simplified formula from *Cooley* [2013] is

$$T_1=E[T]=1+\sum_{t=1}^{t_{\max}}\prod_{i=1}^{t}(1-p_i) \tag{8}$$

Parey et al. [2007, 2010] define a nonstationary return period as the number of years, T_2, for which the expected number of exceedance events is equal to 1, which can be solved numerically by computing the upper limit of the summation in

$$1=\sum_{t=1}^{T_2}(1-p_t) \tag{9}$$

Cooley [2013] shows why (9) can be interpreted as the number of years in which the expected number of exceedances is equal to unity. It is interesting to note that under stationary conditions, all three measures of the expected time to failure are equal so that, $T_o = T_1 = T_2$; however, this is not the case under nonstationary conditions. Equations (6–9) can be applied to any variable or process which leads to a system failure defined as the occurrence of an annual maximum streamflow (flood) above some level. All that is needed is the nonstationary cdf of the variable of interest to compute its average return period using (6)–(9). In the sections which follow, we provide examples of the application of (6)–(9) for the nonstationary lognormal model summarized by *Vogel et al.* [2011], *Prosdocimi et al.* [2014], and others. We begin by using this model because it offers the possibility to generalize the probabilistic behavior of floods under nonstationary conditions, and because it is a parsimonious nonstationary model that reproduces the behavior of floods for a large percentage of U.S. and U.K. river systems as shown by *Vogel et al.* [2011] and *Prosdocimi et al.* [2014], respectively.

From *Salas and Obeysekera* [2014], the system reliability over a planning period (n) under nonstationary conditions is given by

$$\mathrm{Re}liability_n=\prod_{i=1}^{n}(1-p_i) \tag{10}$$

Another metric defined by *Stedinger and Crainiceanu* [2000] is the average annual risk of failure, which is simply the average of the annual exceedance probabilities over a planning period (n years):

$$AAR(n)=\frac{1}{n}\sum_{i=1}^{n}p_i \tag{11}$$

Stedinger and Crainiceanu [2000] use (11) along with a damage function and discount rate, to compute the discounted equivalent risks for four flood forecast models. Following *Stedinger and Crainiceanu* [2000], we elect to introduce another measure of reliability which we term the annual average reliability:

$$\mathrm{Re}liability_a=\frac{1}{n}\sum_{i=1}^{n}(1-p_i) \tag{12}$$

The above review of average return periods, risk of failure, reliability over a planning horizon, and average reliability over a planning horizon under both stationary and nonstationary conditions led us to question which among the various indices would be most useful for communicating risk of failure under both stationary and nonstationary conditions. The following sections present our investigation of the average return period and reliability under nonstationarity in the context of design, planning, and communicating risk of floods.

4. Generalized Probabilistic Behavior of LN2 Model of Flood Hazard

4.1. LN2 Stationary Flood Hazard Model

Vogel and Wilson [1996] reviewed the pdfs which were commonly used in flood frequency analysis prior to that date. While the generalized extreme value (GEV), log Pearson type III (LP3) and three-parameter lognormal (LN3) are perhaps the most commonly used pdfs used for flood frequency analysis, the two-parameter lognormal (LN2) model was found by *Beard* [1974] and others to provide an excellent parsimonious alternative to those models and has since been commonly applied in flood studies [*Stedinger and Crainiceanu*, 2000; *Lund*, 2002]. Note also that an LN2 model is a special case of the log Pearson type III distribution, the mandated distribution for use in U.S. federal flood studies by Bulletin 17B [*Interagency Committee on Water Data (IAWCD)*, 1982]. *Vogel et al.* [2011] and *Prosdocimi et al.* [2014] document that a simple nonstationary

LN2 model can provide an approximate yet general representation of the behavior of floods for rivers across the United States and the United Kingdom, respectively, thus we employ that model here. Since our goal is to explore the behavior of flood risk under nonstationary conditions, we begin with a two-parameter pdf, which enables more general comparisons. Future studies should consider extending our results to nonstationary models corresponding to other two-parameter pdfs such as the Gumbel distribution as well as some of the more common three-parameter models such as the GEV, LN3, and LP3 distributions.

Consider a stationary series of annual minima or maxima, X, which follows a two-parameter lognormal (LN2) distribution where $Y = \ln(X)$, and the mean and standard deviation of y are given by

$$\mu_y = \ln\left(\frac{\mu_x}{\sqrt{1+C_x^2}}\right) \tag{13a}$$

$$\sigma_y = \sqrt{\ln(1+C_x^2)} \tag{13b}$$

where μ_x is the mean of X, C_x is the coefficient of variation in real space $C_x = \sigma_x/\mu_x$, and σ_x is the standard deviation of X. The design event corresponding to an LN2 pdf is given by the quantile function

$$X_p = \exp\left(\mu_y + Z_p \sigma_y\right) \tag{14}$$

where Z_p is a standard normal variable with a corresponding exceedance probability p.

Under stationary conditions, the quantile function X_p can be evaluated for the event which has an average return period T_o, by computing the fixed exceedance probability using (2) to obtain $p_o = 1/T_o$. Under stationary conditions, all moments of both X and Y are assumed to be fixed over the planning horizon n.

4.2. Nonstationary LN2 Flood Model

We follow *Vogel et al*. [2011] and *Prosdocimi et al*. [2014] who show that a simple exponential model of X versus time t captures the behavior of annual maximum floods at rivers in the United States and the United Kingdom, respectively. Such a model is easily fit using ordinary least squares (OLS) regression to fit the simple log linear trend model to describe the relationship between y over time t

$$y_t = \ln(x_t) = \alpha + \beta t + \varepsilon_t \tag{15}$$

Note that we do not advocate the use of a trend model using time as a covariate, this model is only used here for illustrative purposes. We do advocate use of meaningful covariates to reflect future changes in the mean annual flood levels, such as covariates which reflect changes due to urbanization and/or climate. Regardless of whether or not a significant trend was detected in the observed flood series at thousands of rivers across the conterminous U.S., *Vogel et al*. [2011] found that the residuals in (15) were homoscedastic and well approximated by a normal distribution, both important assumptions needed to perform further statistical inference. Although their evaluations included detailed hypothesis tests concerning the normality of regression model residuals and t tests on model slope coefficients as well as graphical evaluations of homoscedasticity, their evaluations did not include a comprehensive assessment of the stochastic independence of the model residuals or the flow series. *Vogel et al*. [2011] (see their appendix) concentrated their analysis on the approximately 11% of rivers in the study which exhibited (obvious) positive trends. When applying ordinary least squares (OLS) linear regression, the resulting fitted model yields the conditional mean of the dependent variable so that the expectation of (15) yields an estimate of the conditional mean of Y, denoted here as $\mu_{y|t} = \alpha + \beta t$, because the residual term is assumed to have zero mean. Importantly, this nonstationary trend model implies a reduction in σ_y^2 as compared with stationary conditions, with that reduction proportional to the degree of the trend. As shown in the supporting information, the coefficient of variation of the nonstationary flood series is

$$C_{x|t} = \sqrt{\left(C_x{}^2 + 1\right)^{(1-\rho^2)} - 1} \tag{16}$$

Note the two extreme cases of no trend in which case (16) reduces to $C_{x|t} = C_x$ and a perfect trend model with $\rho = 1$, which leads to $C_{x|t} = 0$. Figure S1 (in supporting information) illustrates the relationship in (16) and documents the small reduction in the coefficient of variation of the flood series X under typical nonstationary conditions. Since values of ρ are likely to be quite small in actual situations (i.e., $\rho < 0.5$), we elected to assume $C_{x|t} = C_x$ because we found this effect to be of minor importance to our overall findings.

Now if a hydrologist has a historic streamflow record of length N represented by x_i for $i = t_1, \ldots, t_N$ years. The trend model $\mu_{y|t} = \alpha + \beta t$ becomes

$$\mu_{y|t} = \bar{y} + \beta\left(t - \left(\frac{t_1 + t_N}{2}\right)\right) \tag{17}$$

where β is the slope of the trend that extends from t_1 to t_N, $\bar{y}$ is simply the mean $\bar{y} = \frac{1}{N}\sum_{t=t_1}^{N} \ln(x_t)$, and the annual maximum floods x_t are measured for N years starting at t_1.

The trend model in (17) can be used to calculate the conditional mean of $Y_t = \ln(X_t)$ for any year in the future, $t > t_N$. We do not advocate the use of (17) for trend extrapolation, unless the user considers the use of physically meaningful covariates in the regression model and/or includes prediction intervals associated with such extrapolations which are known to widen considerably when a model is used in "extrapolation mode." (See *Serinaldi and Kilsby* [2015] for examples and a discussion on the possible implications of such extrapolations for a different model.) The use of OLS regression is quite powerful for trend extrapolation, because when the residuals are approximately independent, homoscedastic, and normally distributed, analytical expressions are readily available for computing both prediction intervals and the likelihood of type I and II errors. Such type I and II probabilities may be readily integrated into a risk-based decision framework [*Rosner et al.*, 2014] as probabilities of overdesign and underdesign, respectively, and have been shown to be particularly important for understanding hydroclimatic change [see *Vogel et al.*, 2013; *Prosdocimi et al.*, 2014].

Since exceedance and nonexceedance values are no longer fixed under nonstationary conditions, the quantile function under nonstationary conditions is obtained by substitution of the nonstationary mean $\mu_{y|t}$, and standard deviation $\sigma_{y|t}$ in (17) and (S2), respectively, into the nonstationary quantile function: $X_{p|t} = \exp(\mu_{y|t} + Z_p \sigma_{y|t})$ which results in

$$X_{p|t} = \exp\left[\bar{y} + \beta(t - \bar{t}) + Z_{p_t}\sigma_y\sqrt{1-\rho^2}\right] \tag{18}$$

See the supporting information for a derivation of the reduction in the variance of Y which is implied by the nonstationary LN2 trend model.

To provide a more physically intuitive understanding of the impact of trends on flood quantiles under nonstationary conditions, we employ the idea of magnification factor introduced by *Vogel et al*. [2011] and also tested by *Prosdocimi et al*. [2014]. *Vogel et al*. [2011] define the magnification factor M as the ratio of the T year flood at some future $t + \Delta t$ period to the T_o year flood at time t. They further show that

$$M = \frac{x_p(t + \Delta t)}{x_p(t)} = \exp[\beta \Delta t] \tag{19}$$

for the nonstationary LN2 model given here.

For a stationary LN2 variable, the *fixed* exceedance probability $p = P(X \leq x)$ associated with a design event X_p, is given as

$$p = 1 - \Phi\left[\frac{\ln(X_p) - \mu_y}{\sigma_y}\right] \tag{20}$$

where the function Φ is the cdf for a standard normal variable. This is easily adapted under nonstationary conditions, to compute the changing values of p_t, the annual exceedance probability associated with experiencing an event greater than the design event X^*:

$$p_t = 1 - \Phi\left[\frac{\ln(X^*) - \mu_{y|t}}{\sigma_{y|t}}\right] \tag{21}$$

Here we denote the design event as X^* to emphasize that it is a fixed value which must be chosen by the design engineer under nonstationary conditions. Under stationary conditions, the design event is denoted as X_p and is given by (14), so that only under stationary conditions is $X_p = X^*$. Note that for the stationary case (21) reduces to (20) and either can be used to compute both the average return period (equation (2)) and reliability (equation (5)) under stationary conditions. Under nonstationary conditions, (21) can be inserted into (7)–(9), and (10) to compute nonstationary average return periods and reliabilities corresponding to a particular design event x_p. Again, the nonstationary mean $\mu_{y|t}$, and standard deviation $\sigma_{y|t}$ in (21) are given in (17) and (S3), respectively. We emphasize that under stationary or nonstationary conditions, the choice of a design event is fixed, denoted here by X^*. Selection and computation of X^* is quite straightforward under stationary conditions, but under nonstationary conditions, with values of p_t changing in every year, its definition and computation are much more complex, as is discussed later in this paper.

5. Results

In the following sections, we explore the general behavior of the nonstationary LN2 model with the goal of improving our understanding of the likelihood of future floods (increasing exceedance probabilities and upward trends) associated with a particular design event X^*, under nonstationary conditions. To enable general comparisons, analyses, and conclusions, we make a number of simplifying assumptions including the following: (1) the stationary coefficient of variation C_x is defined as that value at the end of the period of streamflow record ($t = t_N$), (2) C_x is assumed to be fixed throughout each planning horizon and equal to the nonstationary coefficient of variation, so that $C_{x|t} = C_x$, and (3) the trend in the mean of the annual maximum streamflow series is increasing over time. Now the behavior of floods under nonstationary conditions can be generalized using the nonstationary LN2 model in (18) and (21) along with the magnification factor in (19) and the general relationships between the moments in real and log space given in equation (13).

5.1. Investigation of Return Period Distribution Under Nonstationarity

If the field of flood planning and management is to continue to use the average return period as a tool for communicating risk and informing infrastructure planning and design, it is important to understand the behavior of the probability distribution of the return period under nonstationary conditions. In this section, we examine the behavior of the pdf of the return period under nonstationary conditions for increasing flood magnitudes. We assume a decadal magnification factor ($\Delta t = 10$), where $M = \exp(10\beta)$, and thus a value of $M = 1.02$ implies a 2% increase in flow magnitude every 10 years for all design events, regardless of their probability of exceedance.

We begin by exploring how nonstationarity affects the return period or the waiting time until experiencing an event which exceeds the design event. The pdf of the return period under nonstationary conditions is given in (6)with exceedance probability p_t computed from (21). We assume that a design engineer has chosen a design protection level based on the stationary LN2 flood frequency model using the quantile function in (14) with $p = 0.01$, corresponding to a traditional "100 year" flood.

Of interest here is how a trend in the annual maximum floods impacts the pdf of the return period associated with the traditional 100 year design event. Figure 2 illustrates the pdf of the return period associated with a 100 year ($p_o = 0.01$) design event for the case when $C_x = 1$ for several levels of nonstationarity, as described by increasing values of the magnification factor M. Figure 2 illustrates that what was once an exponential distribution for the return period associated with the 100 year flood under stationary conditions becomes a very different probability distribution as the degree of nonstationarity increases.

Several conclusions can be drawn from Figure 2 concerning our experience of floods which exceed the traditional 100 year flood, under nonstationary conditions. Our experience of the likelihood of the return period until a flood exceeds the design event changes dramatically both in terms of the shape of the pdf of the return period and its expectation. Importantly, the distribution of the return period is no longer exponential as is the case under stationary conditions ($M = 1$). One implication is that if a structure is built for today's $p_o = 0.01$ event and the future is not known with certainty (always the case), we do not know how the return period distribution will evolve, i.e., the shape it will take. We also note from Figure 2 that regardless of the magnitude of a future trend, one can expect the $p_o = 0.01$ event to occur much sooner than 100 years as the magnitude of the trend increases, as evidenced by M.

Of interest is the impact of nonstationarity on our experience of return period associated with design floods of various magnitudes. Figure 3 illustrates the distribution of the return period associated with the traditional $T_o = 10$, 100, and 1000 year design events ($p = 0.1$, 0.01, and 0.001) under nonstationarity conditions described by $M = 1.14$ and $C_x = 1$. Under these nonstationary conditions, the average return period of the

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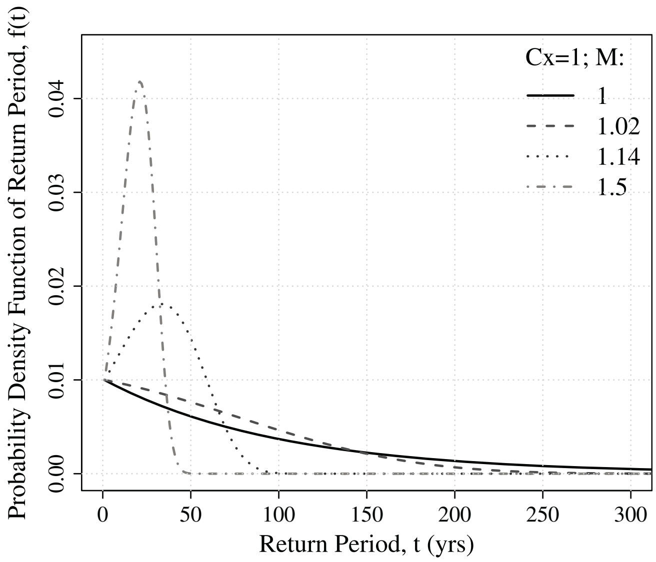

Figure 2. Probability distribution function (pdf) of the return period associated with a traditional 100 year flood for $C_x = 1$ and a range of increasing trends ($M = 1, 1.02, 1.14, 1.5$). Note $M = 1$ corresponds to stationary conditions.

100 year event is shifted to ~32 years, and remarkably, the average return period of the 1000 year flood is reduced to approximately 66 years. Interestingly, the expected return period associated with the 10 year flood is essentially unchanged (~9 years), suggesting that rarer events may be more impacted by nonstationarity than more common floods. We conclude that the evolution of the return period distribution for different levels of nonstationarity (Figure 2) and from common to rare events (Figure 3) exhibits highly nonlinear behavior and is likely to be impractically complicated for purposes of design, planning, management, and risk communication. These findings are consistent with those of *Serinaldi and Kilsby* [2015].

5.2. The Behavior and Choice of the Design Flood Under Nonstationary Conditions

Figures 1–3 assume that the design flood is chosen under the assumption of stationary conditions. Under nonstationary conditions, the design flood should be chosen in such a way as to account for the likelihood of future floods. For example, if we are to estimate a design flood under nonstationary conditions, we must employ equation (7) or (8) to describe the average return period T_1, along with an appropriate nonstationary flood frequency model. To determine the design event X^* which will ensure a value of average return period T_1 under nonstationary conditions defined by M and C_x, one can combine equations (8) and (21) and solve numerically for X^* in the resulting expression:

$$T_1 = E[T] = 1 + \sum_{t=1}^{t_{max}} \prod_{i=1}^{t} \left(\Phi\left[\frac{\ln(X^*) - \mu_{y|i}}{\sigma_{y|i}} \right] \right) \tag{22}$$

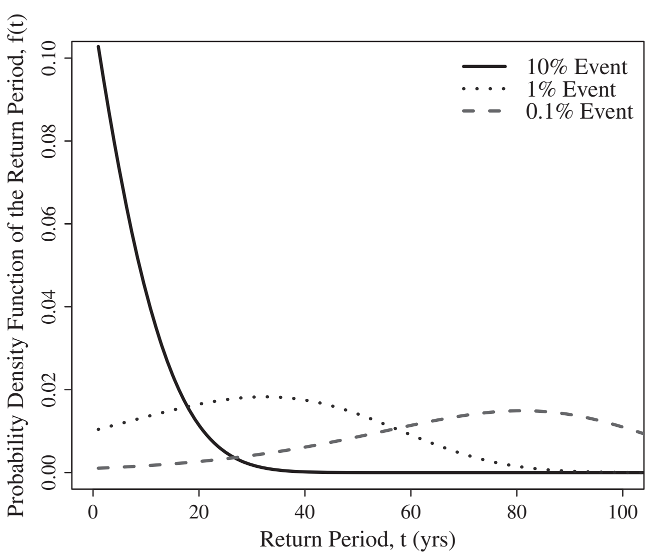

Figure 3. The probability distribution function of the return period for $p_o = 0.1$, 0.01, 0.001 events; $C_x = 1$, $M = 1.14$.

Equation (22) leads to an estimate of the design event under nonstationary conditions which will have an average return period equal to T_1; this is a useful equation for design engineers who need to size infrastructure based on the "new" 100 year flood under nonstationary conditions. Clearly, solving for the design event under nonstationary conditions using (22) is far more complex than under stationary conditions, and it should be noted that in the case of decreasing trends, if t_{max} is infinite, a numerical solution is not even possible. This is clearly problematic for practical use of such a metric for planning. Figure 4 illustrates the behavior of the probability distribution of the return period associated with such a design

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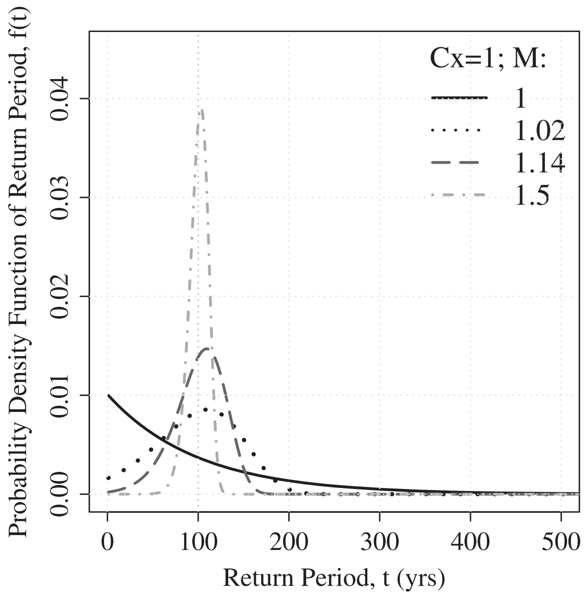

Figure 4. The pdf of the return period associated with a design flood, chosen in such a way as to ensure that the average return period is always 100 years, regardless of the degree of nonstationarity, considering a range of trends ($M = 1$ to $M = 1.5$) given that $C_x = 1$.

event chosen so that average return period $T_1 = 100$ years in all cases, regardless of the increasing degree of nonstationarity as described by increasing values of M. Figure 4 illustrates how the probability distribution of the return period associated with a design event having average return period of $T_1 = 100$ years changes shape from exponential in the stationary case ($M = 1$) to a more normal or symmetrically shaped ($M = 1.5$) distribution. Interestingly, we find that in the presence of an increasing trend, the mean return period is actually more representative of the waiting time distribution and may be more representative of when a failure will occur than under stationary conditions.

For comparison purposes, we examine whether this same behavior in the return period distribution is observed when flood flows arise from a nonstationary Gumbel distribution. Using the same log linear trend model as described in the previous section, we derive p_t for the Gumbel distribution allowing the location parameter to vary with time with fixed C_x (as was described with the LN2 model). As in Figure 4, Figure 5 fixes the average return period at 100 years to illustrate the impact on the return period distribution due to a 5% increase in magnitude of floods over 10 years ($M = 1.05$), $C_x = 0.25$, for both Gumbel and LN2 flood flows. This investigation reveals that regardless of whether the flood flows are LN2 or Gumbel, there is a similar and striking evolution from an exponential to a more symmetric distribution when an exponential trend in the annual maximum flood series is present. Further, Figure 5 indicates that the expected waiting time distribution may differ significantly depending on which probability distribution is selected to represent flood flows, yet another element of complexity added under nonstationarity.

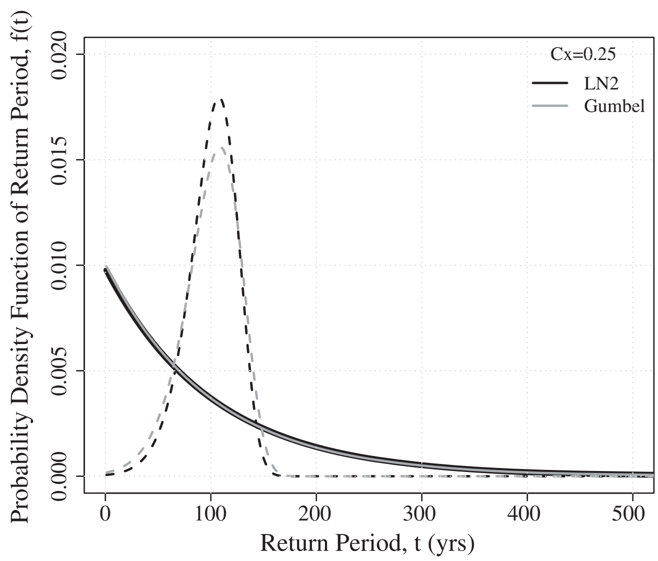

Figure 5. Comparison of the pdf of the return period associated with a design flood, which has a recurrence interval of 100 years, for $M = 1$ stationary (solid lines) and $M = 1.05$ (dashed lines), $C_x = 0.25$ assuming Gumbel flows (grey) and LN2 (black).

We are also interested in how physical hydrology of the river system affects the probability distribution of the return period under nonstationary conditions. Here we use C_x as a proxy for hydrologic variability, where $C_x < 1$ represents a system with relatively low variability as compared with $C_x > 1$. Since rivers with $C_x < 1$ are dominated by changes in the mean, they are most influenced by increasing trends as shown in Figure 6 by the evolution from $C_x = 0.25$ to $C_x = 1.5$ in each plot figure, moving from top left (no trend $M = 1$) to bottom right (large trend $M = 1.5$).

Figures 4 and 5 illustrate that the mean return period is *more* representative of the time to the event

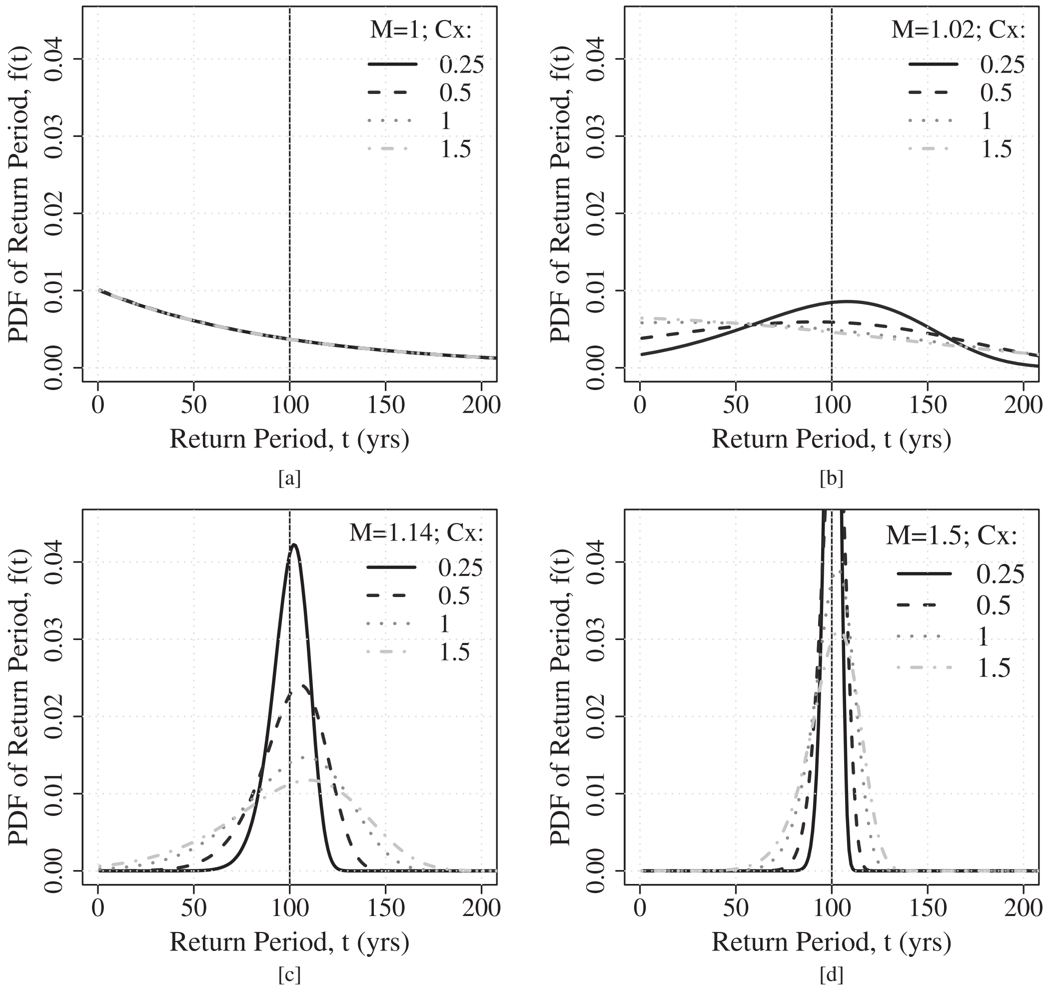

Figure 6. The pdf of the return period associated with a design flood which is chosen to ensure that the average return period is always 100 years, regardless of the degree of nonstationarity or coefficient of variation. Curves show a range of C_x values (0.25–1.5); plot shows trends increasing from (top left) $M = 1$ to (bottom right) $M = 1.02$, 1.14, 1.5.

under nonstationary conditions than under stationary conditions. Figure 6 illustrates that the shape of the pdf of the return period also depends on the hydrologic variability as described by the value of C_x. We conclude from all of these investigations, that the pdf of the return period exhibits extremely complex behavior under nonstationary conditions, with its shape depending on hydrologic variability, the level of nonstationarity, and the magnitude of the design event of interest. Further examination of other commonly used pdfs such as the LP3 and GEV models would likely reveal the same patterns as shown here with LN2 and Gumbel.

5.3. Comparisons of Summary Measures of Average Return Period Under Nonstationarity

Recall that there are two different summary measures of the average return period which have been advanced in the literature. The traditional definition is simply the average of the distribution of the return period T_1 given in (7) and (8). In addition, *Parey et al*. [2007, 2010] and *Cooley* [2013] introduced the return period as the number of years, T_2, for which the expected number of exceedance events is equal to 1, which can be solved numerically using equation (9). In this section, we compare the behavior of these two different summary measures of the time to failure, keeping in mind that under stationary conditions they are equivalent. To accomplish this comparison, we use (14) to compute the design discharge X_p corresponding to stationary conditions for p values corresponding to $T_o = 10$, 100, and 1000 year events. Now each of those X_p values are assumed to be the fixed design event X^* in equation (21), which is then used for a particular C_x and M to determine the corresponding set of p_t values for each design discharge. These values of

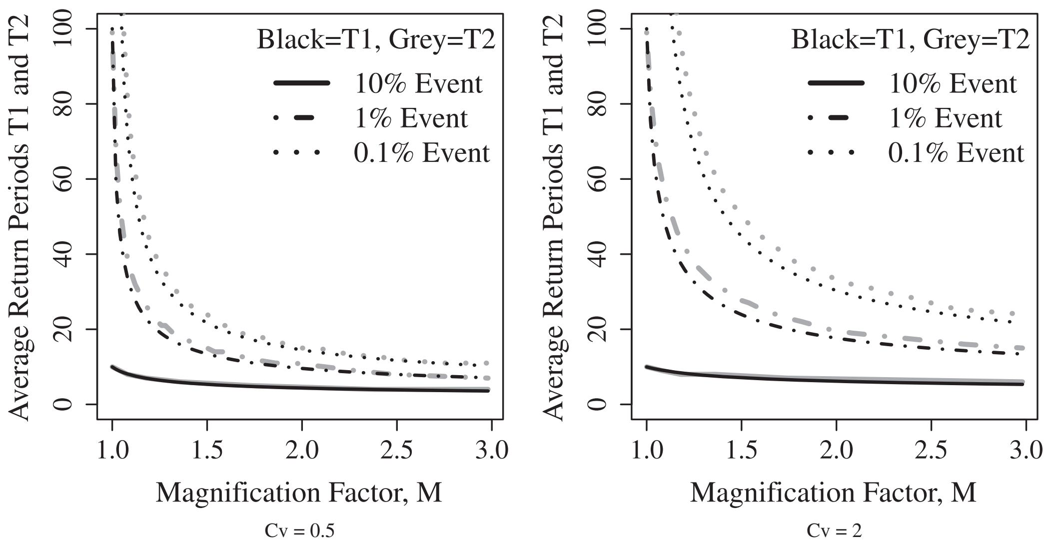

Figure 7. The average return periods T_1 (black) and T_2 (grey) versus the decadal flood magnification factor M, for (left) $C_x = 0.5$ and (right) $C_x = 2.0$; lines are shown for the $T_o = 10$ (solid), 100 (dot-dash), and 1000 year (dashed) design events selected today, t_n.

p_t are in turn used to compute average return periods T_1 using (8) and the return level T_2 from (9). Figure 7 compares estimates of T_1 (black lines) and T_2 (grey lines) to the values of T_o corresponding to stationary conditions and shows how the average return periods reduce as M increases from stationary ($M = 1$) to an extreme case ($M = 3$) for $C_x = 0.5$ (left) and $C_x = 2$ (right). Each plot in Figure 7 illustrates average return periods under stationary conditions: $T_o = 10$ year (solid), 100 year (dot-dash), and 1000 year (dashed) events in the figure legend along with the average return periods T_1 and T_2. The average return periods associated with the T_o year event undergo a dramatic reduction even with a small trend; for example, the original $T_o = 100$ year event, when $C_x = 0.5$ (left plot) and $M = 1.1$, becomes a $T_1 = 30$ year event. From this comparison, we also note that though T_1 and T_2 differ in their assumptions, most significantly that the shift from $T_o \rightarrow T_2$ is more pronounced than $T_o \rightarrow T_1$, the two measures behave similarly for the LN2 nonstationary model over a range of magnification factors and design events characterized by T_o. Thus, our remaining results only concentrate on use of the more common metric, the average return period T_1.

5.4. The Relationship Between Reliability and Return Periods Under Nonstationary Conditions

Recall that under stationary conditions, there is a unique relationship between reliability, average return period, and planning horizon as was illustrated in Figure 1. Of importance is that the results illustrated in Figure 1 for stationary conditions are invariant to characteristics of the flood frequency model and/or hydrologic characteristics of the river under consideration. In this section, we explore the same relationships shown earlier in Figure 1 under nonstationary conditions by combining the theoretical expressions for average return period and reliability introduced in earlier sections with the nonstationary LN2 model.

We begin by investigating the behavior of the reliability index under nonstationary conditions in Figure 8. Figure 8 illustrates reliabilities over a realistic range of planning horizons for a fixed average return period of $T_1 = 100$ years associated with the design event. Holding the average return period constant at $T_1 = 100$, under nonstationary conditions, is similar to the earlier results shown in Figures 4 and 5 and contrasts with previous results where the design event was chosen using the stationary $T_o = 100$ year event. Each plot in Figure 8 represents a fixed trend ($M = 1$, 1.02, 1.14, 1.5) and uses different lines to represent a range of C_x values (0.25, 0.5, 1, 1.5). We choose to illustrate the impact of nonstationarity due to increasing trends on reliability by fixing the average return period at 100 years in order to highlight that a structure must be built larger (often unrealistically so) to achieve a particular reliability under nonstationary conditions. Such large design events would involve considerable increases in design costs. As expected, regardless of whether conditions are stationary or nonstationary, reliability decreases with planning horizon; and, when a trend is present, in systems with less variability (low C_x), reliability is higher since the time to failure is more predictable (as was shown in Figure 6).

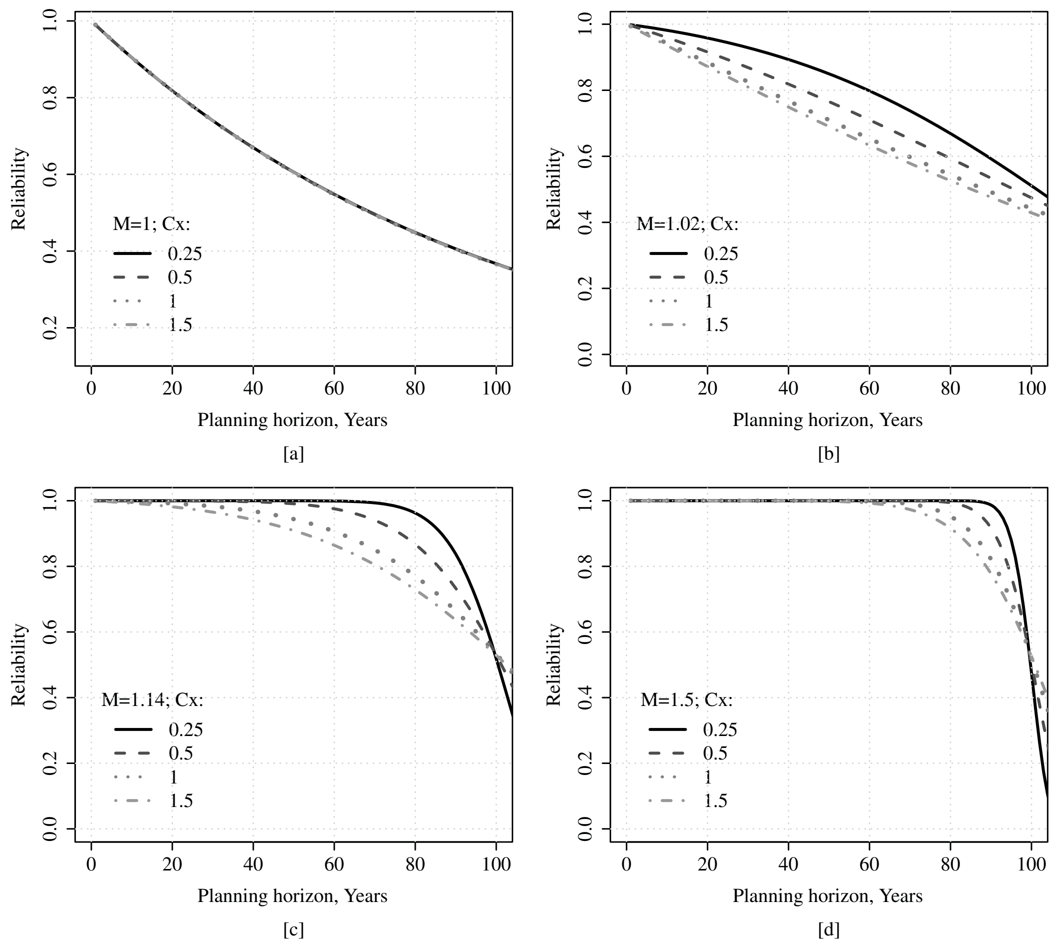

Figure 8. Reliability over planning horizon for a fixed average recurrence interval T_1 of 100 years; curves show a range of $C_x = 0.25, 0.5, 1, 1.5$; each plot considers a fixed trend ($M = 1, 1.02, 1.14, 1.5$).

Of considerable interest are the large increases in reliability associated with design events which are chosen based on the average return period T_1 under nonstationary conditions. One observes that in order to protect against a 100 year flood in a nonstationary setting, the chosen design event X^* results in much higher reliabilities than we are normally accustomed to under stationary conditions. The reason, as we observed earlier, is that under nonstationary conditions the distribution of the time to the event of interest becomes much more symmetric and peaked as M increases, i.e., the average return period becomes a better indicator of the time to the next event as M becomes large. This results in a much higher system reliability for planning horizons less than the design average return period T_1.

In Figures 9a and 9b, we compare reliability to the average return period T_1, as we did in Figure 1, this time considering trends ($M = 1.02, 1.14, 1.5$). Several points arise from Figure 9, namely that the relationship between reliability and average return period is now extremely complicated and can no longer be defined by a single curve for a given n. The relationship between reliability and average return period under stationary conditions is invariant to the flood frequency model, whereas as is shown in Figure 9, the relationship between T_1 and reliability depends critically on the values of both M and C_x. Figure 9 also reiterates that for a given average return period T_1, reliability is higher for larger trends because the design event is larger. For example, in the theoretical case illustrated in Figure 9a, design flows for the $T_1 = 100$ year event are $x = 0.20$ m^3/s for $M = 1$ (stationary), and increase to $x = 0.23$ m^3/s for $M = 1.02$, $x = 0.52$ m^3/s for $M = 1.14$, and $x = 5.45$ m^3/s for $M = 1.5$; thus to design for a structure with a 100 year average return period under nonstationary conditions, the required infrastructure will be much larger than under stationary conditions.

Note also in Figure 9 that knowledge of T_1 alone is insufficient to provide a complete understanding of the likelihood of future flood events, because the reliability associated designs corresponding to a particular

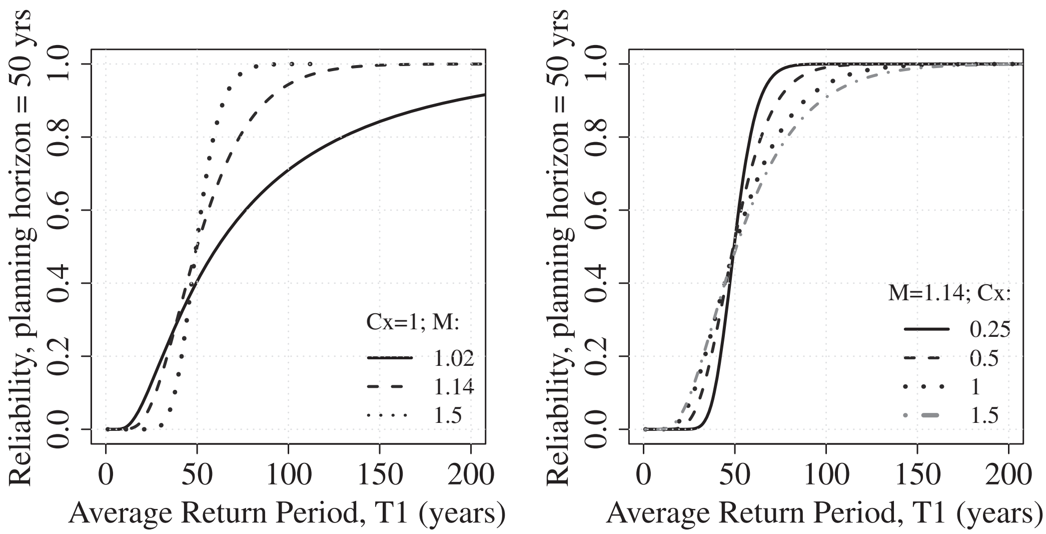

Figure 9. Reliability versus average return period T_1, for $n = 50$, (a) $C_x = 1$; and the set of curves compare the stationary ($M = 1$) case to a set of increasing trends $M = 1.02, 1.14, 1.5$; (b) trend is fixed at $M = 1.14$; set of curves represent different values of $C_x = 0.25, 0.5, 1, 1.5$.

value of T_1 vary dramatically, depending upon both M and C_x. Nevertheless, the relationships shown in Figure 9 can guide hydrologists and design engineers on the likelihood of failure and the expected return period of a flood which will exceed the design flow under a range of nonstationary conditions. Both Figures 8 and 9 indicate that in order to secure high average return periods, we must consider much higher reliabilities, which are likely to be unrealistic in practice. If instead we start with reliability as a way to determine the size of the design event we are willing to protect against, then we can better communicate the likelihood of failure under both stationary and nonstationary conditions.

6. Discussion and Summary

In this paper, we have drawn on existing theory and empirical results from the recent literature to provide the first general, comprehensive analysis of the probabilistic behavior of the return period and reliability under nonstationary conditions. Our results are a summary of old and new reasons for rethinking the use of an average return period as a design and communication metric for flood hazard planning. Under assumptions of stationarity, the average return period does not adequately communicate the likelihood of experiencing a failure over a given project life, which is precisely the concern for engineers designing infrastructure for flood management. We provide a theoretical example using a nonstationary lognormal (LN2) distribution to demonstrate that when evidence of nonstationarity exists in historic data, the time to failure distribution changes shape, the average return period is dramatically impacted, and the relationship between average return period and reliability becomes more complex since reliability now depends on C_x and the magnitude of the trend (M). From studying the time to failure distribution corresponding to a fixed average return period, we note its evolution from a right-skewed exponential tail to a very peaked and nearly symmetric pdf for larger values of M. A similar investigation with the Gumbel distribution reveals the possible generality of this finding.

The illustrations presented here are general and can also be applied to decreasing trends to examine how the behavior of the return period distribution changes and whether these alterations are consistent or inconsistent with results presented here for the case of increasing trends. Further, the relationships here should also be compared with those from other distributions such as the log Pearson type III distribution (LP3) and the generalized extreme value (GEV) [*Salas and Obeysekera*, 2014; *Serinaldi and Kilsby*, 2015].

When assuming stationarity, the relationship between reliability, average return period, and planning horizon is independent of the properties of the river under consideration and the resulting model of flood frequency. However, under nonstationarity, the relationship is far more complex, so that our experience of the reliability of flood management systems as well as the average return period associated with the next flood exceeding a critical design event depend on a number of additional considerations including the characteristics of the underlying nonstationarity, the form of the probability distribution of the annual maximum flood discharges, and the planning horizon. The complexity in the shape and behavior of the distribution of the return period is further confounded by our inability to know and describe both the pdf and the trend in future flood series. Our analysis has only considered a single model of nonstationary flood frequency, so that considering other probabilistic models, as well as other models of trends in the mean, variance, and skewness, would lead to considerable additional complications. In other words, even for one of the simplest possible probabilistic models of nonstationarity, we have shown that our experience of the probability distribution of the return period is considerably more complex and unpredictable than its counterpart under stationary conditions. The assumption of an exponential trend model of annual maximum floods combined with the LN2 distribution, while reasonable, may not apply to all cases (nonstationarities) and thus we recognize that the same patterns may not result from different models. Thus, the complexity of the relationship between the model parameters (M, C_x) and the design metrics (reliability and T_1) grows in complexity as one considers additional uncertainties associated with nonstationary behavior of future floods. *Serinaldi and Kilsby* [2015] discuss how these increases in complexity add uncertainty when developing nonstationary models, and thus suggests careful treatment of uncertainty characterization before employing such models for design purposes.

Interestingly, for the case of increasing trends presented here, we showed that such trends actually improve our ability to know the time to failure, because the distribution of return periods becomes much more peaked and symmetric, so that the average return period becomes a better indicator of the time to an (exceedance) event. Correspondingly, large trends tend to produce very large design events and associated infrastructure costs, which are shown to be more reliable over a planning period than we normally experience under stationary conditions. In order to achieve high reliabilities under nonstationary conditions, we must build structures for these larger design flows or accept a greater risk of failure, a decision that is now further complicated by the uncertainty of future nonstationarities. We show that a unique relationship between reliability and average return period exists for a given nonstationary flood frequency model but without knowing the form of the nonstationary model with certainty, drawing inferences about the reliability from the average return period becomes difficult.

Further, given that the average return period is not intrinsically tied to a planning horizon, one might make a statement similar to the following in order to communicate event likelihood over a certain period of time: "we are 80% sure that this structure, built to withstand today's 1,000-year event, will not experience at least one exceedance event within the next 25 years." From this confusing statement, it is difficult to discern the meaning of the average return period and much more succinct to simply report the 80% reliability of the design over the future 25 year planning horizon. If instead planners designed for a certain desired level of reliability as is done in other fields concerned with planning under risk, then the statement could read: "we are 80% confident that this structure will not fail in the next 25 years." And, the associated average return periods (assuming one would desire reliability $>$ 80%) may be orders of magnitude higher as is illustrated in this study; the need to avoid misrepresenting the risk of failure is one reason we recommend replacing the average return period with the notion of reliability over planning horizon.

The consequences of misinterpreting the expected waiting time to an extreme event do not only impact the physical system but also impact perceptions of individuals interacting with the floodplain. In the literature of flood risk communication, *Lave and Lave* [1991] report that floodplain residents generally expect to be protected for a particular average return period and have little understanding of the true risk of flooding during their lifetimes. Risk communication is itself a complex issue, as empirical research suggests that obstacles to understanding risks from natural hazards often require strategies deeper than presenting facts, such as using tactics to dispel heuristics and preconceived mistaken theories [see *Bier*, 2001, for a review].

7. Conclusions

We have shown how a parsimonious nonstationary lognormal model can be combined with recent research on nonstationary return periods, risk, and reliability to obtain a very general understanding of the risk posed by future floods. The probability distribution of the time to failure of a water resource system under nonstationary conditions no longer follows an exponential distribution as is the case under stationary conditions, with a mean return period equal to the inverse of the exceedance probability $T_o = 1/p$. Our findings raise numerous questions about our ability to understand and communicate the likelihood of future flood events under nonstationary conditions. We recommend replacing the notion of an average return period with reliability over a planning horizon, a metric used in almost all other realms of water resources planning and many other fields that communicate long-term risk. Referring to a system's reliability directly conveys two pieces of information: the likelihood of no failure within a given number of years (i.e., over a planning horizon) and the accepted level of reliability that is implicit in the design. In the context of climate variability and change, and as cities become more urbanized, decisions on how to plan our water resource infrastructure become increasingly complex [*Rootzén and Katz*, 2013; *Obeysekera and Park*, 2013; *Rosner et al*., 2014; *Condon et al*., 2015]. Our evolving perception of design and adaptation is increasingly recognized by those who study "sociohydrology" in the context of floods and propose models and frameworks for the feedbacks/interactions between society and risks from the hydrologic environment [*Di Baldassarre et al*., 2013; *Viglione et al*., 2014]. Future work will investigate how a modern risk-based decision analysis framework can aid selection of an appropriate design event for hydrologic design under nonstationarity and explore new tools for characterizing nonstationary probability distributions.

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