A one-dimensional ice thermodynamics model for the Laurentian Great Lakes

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Abstract. Great Lakes hydrologic research requires the use of continuous-simulation daily ice cover models over long time periods in the absence of field observations. They must be physically based, rather than statistically based, for use under conditions different than those under which they were derived. But they also must match existing conditions for which data exist. A review discloses that existing ice dynamics models do not meet all of these criteria; a new one that does is based here on a prismatic ice pack heat balance, ice growth and temperature constraints, and thermodynamic flux terms from companion water heat balance and storage equations. The prismatic ice model is a good first step to understanding complex geometries and is supportable through the use of lake-averaged energy fluxes. The ice model is integrated into an existing lake thermodynamics and one-dimensional heat storage model, and the resulting combination is calibrated for Laurentian Great Lakes applications. Simulation experiments are used to analyze the model's strengths and limitations and to explore its relevance. Comparisons between model output and existing data allow consideration of the ice climatology of the Great Lakes; the climatology description is extended through use of the new model. Promising potential model extensions include spatial extension, additional parameterizations for wind-ice movement, snow, and albedo, and inclusions of remotely sensed data.

Introduction

The Great Lakes Environmental Research Laboratory (GLERL) developed a lumped-parameter (point) model of evaporation and thermodynamic fluxes [*Croley*, 1989a] and a one-dimensional (vertical) heat storage model for the Great Lakes [*Croley*, 1992]. GLERL integrated the two models for simulation studies of changing climates and for forecasting of net basin supplies. The combination model used regressions of ice cover concentrations on air temperatures to estimate the monthly areal extent of ice. Thus it had limited use for meteorological conditions different from the historical meteorology, under which the regressions were developed. The purpose of this paper is to present a physically based ice model that is usable under different conditions.

The approach taken here is to briefly describe Great Lakes ice conditions, to review existing ice dynamics models for use in the Great Lakes, and to outline the basic assumptions and derivation of a daily ice cover model that is usable in the enveloping context of a large-lake thermodynamics and heat storage model. The ice model is integrated into the thermodynamics model, and the resulting combination is calibrated for Great Lakes applications. Its strengths and limitations are analyzed and its relevance explored through simulation experiments.

Great Lakes Ice Conditions

The Great Lakes do not ordinarily freeze over completely [*Assel et al.*, 1983] because of the combination of their large

heat storage capacity, large surface area, and their location in the mid-latitude winter storm track. Alternating period of mild and cold air temperatures combine with episodic high and low wind stresses at the water surface to produce transitory ice conditions during the winter. Ice cover in midlake regions is often in motion. Lake Erie ice speeds have been observed to average 8 cm s^{-1} with a maximum speed of 46 cm s^{-1} [*Campbell et al.*, 1987]. Ice can form, melt, or be advected toward or from most midlake areas throughout the winter [*Rondy*, 1976]. When ice is advected into areas with existing ice covers, it can underride or override the ice cover, forming rafted rubble 5-10 m thick. The normal seasonal progression of ice formation begins in the shallow shore areas of the Great Lakes in December and January. The deeper midlake areas normally do not form extensive ice covers until February and March. Ice is lost over all lake areas during the last half of March and during April.

Great Lakes Ice Dynamics and Thermodynamics Models

Great Lakes hydrologic research requires the use of continuous-simulation daily ice cover models over long time periods. The models are used in forecasting and simulation settings, including outlooks of lake evaporation and lake levels [*Croley and Hartmann*, 1987] and assessment of climate and climate change scenarios [*Croley*, 1990]. Such models must be usable in the absence of water surface temperature and ice cover observations. They also must be physically based to have application under environmental conditions different than those under which they were derived. This omits many excellent statistical treatments relating ice cover to freezing degree-days [*Assel*, 1991]. However, the models should perform similarly to these statistical

treatments under the same conditions for which the latter were developed. They must be capable of coupling to atmospheric and lake thermodynamics, that is, hydrodynamics models to describe the spatial and temporal variation of ice formation, ice movement, ice cover (the percent of unit surface area covered by ice), ice thickness, and ice loss over the winter season.

Unfortunately, the state of the art for Great Lakes ice dynamics and thermodynamics models has not progressed much since the early 1980s [Bolsenga, 1992]. Thermodynamics and dynamic ice cover models meeting the above specifications were developed and tested on Lake Erie by Wake and Rumer [1979] and by Rumer et al. [1981], respectively. Wake and Rumer's [1979, p. 828] thermodynamics model uses a "two-dimensional convective dispersion equation for the conservation of heat energy in the water body, a two-dimensional depth-integrated version of the equation of motion to represent the hydrodynamic heat transport, and interfacial heat exchange equations to account for the various heat exchange processes at the air-water, air-ice, and ice-water interfaces." The model does not include the effects of ice movement or of snow on the ice. Water temperature is assumed to be vertically well mixed (isothermal) so only the time variation of the average column water temperature at each grid cell is modeled; it appears that observed water temperatures are needed to calibrate the model. Rumer et al.'s [1981] ice dynamics model uses an equation of motion for a continuum ice field based on a balance of forces on the ice from internal ice resistance, wind, water drag, gravity, and the coriolis forces. Ice thermodynamics in that model are parameterized by an empirical coefficient relating ice growth or loss to ice thickness, air temperature, ice cover, and wind velocity. This rudimentary consideration of ice thermodynamics and the lack of direct consideration of lake heat storage are areas where improvement is needed in this model. Without consideration of the temperature-depth distribution in a lake it is not possible to simulate water column turnovers and their characterization under severe environmental conditions. However, this model still represents a major step forward in modeling ice movement on the Great Lakes.

One-dimensional bulk thermodynamic lake ice models have been developed for other lakes, but they may not be appropriate for the Great Lakes since the Great Lakes normally do not freeze over completely. Green and Outcalt [1985] developed a one-dimensional thermodynamics river ice model simulating ice thickness as a function of energy flux at the air-ice and ice-water interface coupled with heat diffusion through the ice sheet. Their model was tested on the St. Lawrence River where it did an adequate job simulating ice growth but produced slower-than-observed decay rates because it did not consider mechanical weakening of the ice sheet. Patterson and Hamblin [1988] linked a two-layer (ice and snow) one-dimensional thermodynamics lake ice model, based on Maykut and Untersteiner's [1971] one-dimensional ocean thermodynamics ice model, to a reservoir mixing model [Patterson et al., 1984] to describe the thermodynamic processes between the bottom of the ice and the lake waters. They accounted for partial ice cover by not permitting ice to thicken to more than 10 cm until the entire lake is frozen over. They assumed partial ice cover is advected to the lake boundary and accumulates from shore to midlake at the 10-cm thickness until the entire lake surface

becomes frozen. Recently, Hostetler et al. [1993] modified Patterson and Hamblin's [1988] lake ice model and coupled it with a regional atmospheric model (the National Center for Atmospheric Research, Pennsylvania State University regional model) for use in Great Lakes regional climate simulation. Preliminary tests of the coupled model system showed that some ice extents and concentrations were overestimated at many Great Lakes locations. However, Hostetler et al. [1993] have a completely different focus with their modeling of the Great Lakes than outlined here. They are attempting simulations of climate effects with atmospheric inputs from a climate model by using untuned, uninitialized, and uncalibrated lake models (S. Hostetler, personal communication, 1993). If calibrated, initialized, and driven by requisite meteorological inputs (if available), their model may have practical ice-modeling potential in the Great Lakes.

Coupled Ice Thermodynamics Model

It is feasible to develop a one-dimensional lake-averaged thermodynamic ice cover model for Great Lakes simulation and forecasting applications, notwithstanding the data and model limitations noted above. Here we formulate a thermodynamic ice cover model, patterned on aspects of the work of Wake and Rumer [1979] and Green and Outcalt [1985]. It is based on the energy balance at the lake's surface [Croley, 1989a, b] and on lake heat storage [Croley, 1992]. Thus ice formation and loss is coupled to models of lake thermodynamics and superposition lake heat storage. The lake-averaged surface energy balance is specified from the change in the daily lake heat storage and the atmospheric heat flux toward and from the lake surface for both water and ice.

The heat added to a lake and the heat added to the ice pack are governed by simple energy balances, energy storage relationships, and boundary conditions on ice growth, water temperature, and ice temperature. The rate of change of heat storage in the lake with time is defined here as

$$\frac{dH}{dt} = A_w(q_i - q_r - q_e + q_h + q_p) + Q_l + Q_I - Q_w \qquad (1)$$

where t is time, dH/dt is the time rate of change of heat storage H in the lake, A_w is the area of the open water (ice-free) surface, q_i is the daily average unit (per unit area) rate of shortwave radiation incident to the Earth's surface, q_r is the average unit reflected shortwave radiation rate from the water surface, q_e is the average unit evaporative (latent and advected) heat transfer rate from the water surface, q_h is the average unit sensible heat transfer rate to the water surface, q_p is the average unit precipitation heat advection rate to the water surface, Q_l is the average net longwave radiation exchange rate between the entire water body and the atmosphere (effects of ice cover on the net longwave exchange are ignored here), Q_I is the daily net heat advection to the lake from overland flow and channel inputs and outputs, and Q_w is the total heat flux between the water body and the ice pack. The rate of change of heat storage in the ice pack with time is defined here as

$$\frac{dH'}{dt} = A(q_i - q_r' - q_e' + q_h' + q_p') + Q_w \qquad (2)$$

where dH'/dt is the time rate of change of heat storage H' in the ice pack, A is the area of the ice surface, q'_r is the average unit reflected shortwave radiation rate from the ice pack, q'_e is the average unit evaporative (latent and advected) heat transfer rate from the ice pack, q'_h is the average unit sensible heat transfer rate to the ice pack, and q'_p is the average unit precipitation heat advection rate to the ice pack.

Water surface temperature is taken here as a superposition of temperature increments associated with past heat additions, each mixed to a different extent depending upon its age (accumulated windage since its inception). By considering both the addition and removal of past daily heat amounts, Croley [1992] shows that this superposition is equivalent to [Croley, 1992, equation (37)]

$$T_k = 3.98°C + \sum_{m=1}^{k} f_{k,m} \left(\min_{m \le n \le k} H_n - \min_{m-1 \le n \le k} H_n \right) \tag{3}$$

where T_k is the water surface temperature, H_k is the heat storage in the lake k days after the last turnover, and $f_{k,m}$ is a "wind-aging" function, defined subsequently, relating heat added on day m to surface temperature rise on day k. Ice pack surface temperature is related to ice pack heat storage here as

$$H'_k = \frac{1}{2}\rho C_i V_k T'_k - \rho \gamma V'_k \tag{4}$$

where T'_k is the ice surface temperature on day k, H'_k is the heat storage in the ice pack on day k, ρ is the density of ice, C_i is the specific heat of ice, γ is the heat of fusion of water, V_k is the volume of the ice pack on day k, and V'_k is the volume of ice formed by freezing or melting on day k. The boundary conditions on water surface temperature and volume of the ice pack for every day (dropping the daily subscript) are

$$V = 0, \qquad T \ge 0°C \tag{5}$$

$$T = 0°C, \qquad V \ge 0 \tag{6}$$

These equations are satisfied by selecting the heat flux between the water and ice, Q_w, appropriately. Q_w, if negative, is yielded as ice forms (to keep water surface temperature from going below freezing) and, if positive, is used in melting ice (to keep water surface temperature at freezing as long as there is ice present). The boundary conditions on ice surface temperature and volume of the ice pack for every day (dropping the daily subscript) are

$$T' = T_a, \qquad V > 0 \text{ and } T_a \le 0°C \tag{7}$$

$$T' = 0°C, \qquad V = 0 \text{ or } T_a > 0°C \tag{8}$$

where T_a is the overice air temperature. The volume of the ice pack, V, and the volume of ice formed by freezing or melting, V', are related:

$$\frac{dV}{dt} = \frac{dV'}{dt} + S - E \tag{9}$$

where S is the volumetric rate of snow falling on the ice, and E is the volumetric rate of evaporation from ice. The "wind-aging" function, $f_{k,m}$, is [Croley, 1992, equation (23)]

$$f_{k,m} = \frac{2 - M_{k,m}/F}{\rho_w C_w M_{k,m}}, \qquad M_{k,m} < \min\left(F, \frac{2V_c}{1 + V_c/F}\right)$$

$$f_{k,m} = \frac{1}{\rho_w C_w M_{k,m}}, \qquad \min\left(F, \frac{2V_c}{1 + V_c/F}\right) \le M_{k,m}$$

$$< \max\left(V_c, \frac{2V_c}{1 + V_c/F}\right)$$

$$f_{k,m} = \frac{1}{\rho_w C_w V_c}, \qquad \max\left(V_c, \frac{2V_c}{1 + V_c/F}\right) \le M_{k,m} \tag{10}$$

where ρ_w is the density of water, C_w is the specific heat of water, V_c is the volume (capacity) of the lake, and $M_{k,m}$ is the mixing volume size in the lake, on day k, of the heat added on day m (a function of accumulated wind movement, W_j, from day m through day k), given by Croley [1992, equation (21)] as

$$M_{k,m} = V_e \left[1 + a \exp\left(-b \sum_{j=m}^{k} W_j\right) \right]^{-1} \tag{11}$$

Also, a, b, F, and V_e are empirical parameters to be determined in a calibration to observed data. V_e is interpreted as the "equilibrium" volume approached as a limit (in a sufficiently deep lake) since the effects of wind mixing at the surface diminish with distance from the surface. F is interpreted as the mixing volume at which a heat addition is fully mixed throughout. Parameters a, b, and F are defined for water temperatures above 3.98°C ("turnover" temperature of water at maximum density) and are replaced by a', b', and F', respectively, for water temperatures below 3.98°C. The flux terms in (1) and (2) and related quantities are given by Croley [1989a, b]. Derivations of (3), (10), and (11) are given by Croley [1992].

In (4), a linear vertical temperature profile is used through the ice pack from T' on the surface to 0°C on the bottom, similar to Green and Outcalt [1985]. Differentiating (4) and ignoring small terms,

$$\frac{dH'}{dt} \cong \frac{1}{2}\rho C_i V \frac{dT'}{dt} - \rho \gamma \frac{dV''}{dt} \tag{12}$$

Thus the heat change is split between a temperature change in the ice pack and a volume change due to melting or freezing. Comparing (2) and (12), note that the temperature change in (12) is taken here as resulting from a portion of the heat added from (or lost to) the atmosphere [$A(q_i - q'_r - q'_e + q'_h + q'_p)$]. The remainder of that heat is identified as Q_a:

$$Q_a = A(q_i - q'_r - q'_e + q'_h + q'_p) - \frac{1}{2}\rho C_i V \frac{dT'}{dt} \tag{13}$$

This heat, Q_a, and all of the heat added from the water body, Q_w, then result in changes to the ice pack volume (freezing or melting); from (2), (9), (12), and (13),

$$Q_a + Q_w = -\rho \gamma \frac{dV''}{dt} = -\rho \gamma \left(\frac{dV}{dt} - S + E \right) \tag{14}$$

Assume a prismatic ice pack with area A, circumference C, and depth (or thickness) D, as in Figure 1. In time

A = area of ice pack
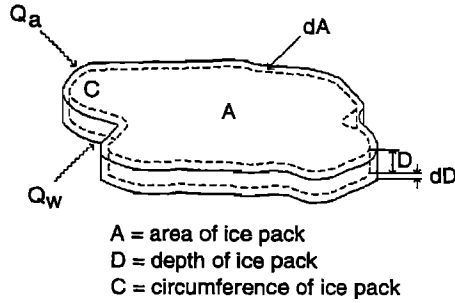D = depth of ice pack
C = circumference of ice pack

Figure 1. Prismatic ice growth.

increment dt, assume prismatic growth so that the ice surface area increases by dA and the (average) ice thickness increases by dD. Assume that the heat exchange between the atmosphere and the ice pack available for freezing or melting, Q_a, results either in melt along the entire atmosphere/ice surface, $A + 0.1DC$ (assuming that ice is nine tenths submerged), or freezing along the entire water/ice surface, $A + 0.9DC$. Along the atmosphere/ice surface, unit changes are partitioned between the vertical and lateral directions in the ratio α_a (i.e., α_a is the ratio of the unit area vertical change to the unit area lateral change). Along the water/ice surface, unit changes are partitioned between the vertical and lateral directions in the ratio α_w. Thus Q_a, if positive, results in changes distributed between the vertical and lateral directions, respectively, as $\alpha_a A/(\alpha_a A + 0.1DC)$ and $0.1DC/(\alpha_a A + 0.1DC)$. If Q_a is negative, changes are distributed between the vertical and lateral directions, respectively, as $\alpha_w A/(\alpha_w A + 0.9DC)$ and $0.9DC/(\alpha_w A + 0.9DC)$.

Assume that the heat exchange between the water body and the ice pack, Q_w, results in changes along only the water/ice surface (either melt or freezing) such that the vertical and lateral splits are again, that is, $\alpha_w A/(\alpha_w A + 0.9DC)$ and $0.9DC/(\alpha_w A + 0.9DC)$, respectively. Note that while actual heat exchanges with the ice pack occur mainly in the vertical direction, the above partitioning can (and does) result in predominantly lateral changes in the ice pack as large areas of thin ice form or melt away. Thus

$$\frac{dD}{dt} = \left\{ -\frac{\alpha_w Q_a}{\alpha_w A + 0.9DC} I_{(-\infty,0]}(Q_a) - \frac{\alpha_a Q_a}{\alpha_a A + 0.1DC} I_{(0,\infty)} \right.$$

$$\cdot (Q_a) - \frac{\alpha_w Q_w}{\alpha_w A + 0.9DC} I_{(-\infty,0]}(Q_w)$$

$$\left. - \frac{\alpha_w Q_w}{\alpha_w A + 0.9DC} I_{(0,\infty)}(Q_w) \right\} \frac{1}{\rho\gamma}$$

$$+ \frac{1}{A} S - \frac{\alpha_a E}{\alpha_a A + 0.1DC} \tag{15}$$

where the indicator function is defined as

$$I_{(-\infty,0]}(x) = 1, \quad x \le 0$$

$$I_{(-\infty,0]}(x) = 0, \quad x > 0$$

$$I_{(0,\infty)}(x) = 1, \quad x > 0$$

$$I_{(0,\infty)}(x) = 0, \quad x \le 0$$

Also,

$$\frac{dA}{dt} = \left\{ -\frac{0.9CQ_a}{\alpha_w A + 0.9DC} I_{(-\infty,0]}(Q_a) - \frac{0.1CQ_a}{\alpha_a A + 0.1DC} I_{(0,\infty)} \right.$$

$$\cdot (Q_a) - \frac{0.9CQ_w}{\alpha_w A + 0.9DC} I_{(-\infty,0]}(Q_w)$$

$$\left. - \frac{0.9CQ_w}{\alpha_w A + 0.9DC} I_{(0,\infty)}(Q_w) \right\} \frac{1}{\rho\gamma}$$

$$- \frac{0.1CE}{\alpha_a A + 0.1DC} \tag{16}$$

and the change in total ice volume is, from (15) and (16),

$$\frac{dV}{dt} = \frac{d(AD)}{dt} = A \frac{dD}{dt} + D \frac{dA}{dt} = \{-Q_a - Q_w\}\frac{1}{\rho\gamma} + S - E \tag{17}$$

Note (14) and (17) agree.

For an unbroken ice pack the circumference can be taken as proportional to the square root of the area for a variety of shapes, $C = \beta A^{1/2}$, where β is a constant dependent on the shape of the ice pack ($\beta = 4$ for a square). Let $\tau_a = 0.1\beta/\alpha_a$ and $\tau_w = 0.9\beta/\alpha_w$; then

$$\rho\gamma A^{1/2} \frac{dD}{dt} = -\frac{Q_a}{A^{1/2} + \tau_w D} I_{(-\infty,0]}(Q_a)$$

$$- \frac{Q_a}{A^{1/2} + \tau_a D} I_{(0,\infty)}(Q_a) - \frac{Q_w}{A^{1/2} + \tau_w D}$$

$$+ \rho\gamma \frac{S}{A^{1/2}} - \rho\gamma \frac{E}{A^{1/2} + \tau_a D} \tag{18}$$

$$\rho\gamma \frac{dA}{dt} = -\frac{\tau_w Q_a}{A^{1/2} + \tau_w D} I_{(-\infty,0]}(Q_a) - \frac{\tau_a Q_a}{A^{1/2} + \tau_a D} I_{(0,\infty)}$$

$$\cdot (Q_a) - \frac{\tau_w Q_w}{A^{1/2} + \tau_w D} - \rho\gamma \frac{\tau_a E}{A^{1/2} + \tau_a D} \tag{19}$$

Equations (1)–(13), (17)–(19), and those for the component fluxes and heat storage in the lake [Croley, 1989a, b, 1992] may be solved simultaneously to determine the heat storage, the water and ice surface temperatures, and the ice pack extents.

Model Determination

Parameters

There are ten parameters to be determined, for application of this model, as in Table 1a (a, b, F, a', b', F', V_e, p, τ_a, and τ_w). The first seven parameters relate to superposition heat storage [Croley, 1992], and the eighth parameter, p, reflects the effect of cloudiness on the atmospheric net longwave radiation exchange [Croley, 1989a, b]. The last two parameters, τ_a and τ_w, reflect ice pack shape, ice buoyancy, and vertical-lateral change ratios along the atmosphere-ice boundary and the water-ice boundary, respectively. Model concepts have been carefully chosen so that the parameters have physical significance; this allows them to be interpreted in terms of the thermodynamics they represent. Initialization of the model corresponds to identi-

Table 1a. Daily Calibration of Ice and Surface Flux Thermodynamics Model

	Superior	Michigan	Huron	Georgian	Erie	Ontario
Surface area, km^2	82,100	57,800	40,640	18,960	25,700	18,960
Volume, km^3	12,100	4,920	2,761	779	484	1,640
Average depth, m	147	85.1	67.9	41.1	18.8	86.5
Calibrated Parameter Values						
a	$6.298 \times 10^{+0}$	$7.290 \times 10^{+0}$	$6.460 \times 10^{+0}$	$1.585 \times 10^{+0}$	$2.820 \times 10^{+0}$	$7.710 \times 10^{+0}$
b, m^{-1} s	3.298×10^{-3}	2.599×10^{-3}	2.810×10^{-3}	5.473×10^{-3}	5.430×10^{-3}	2.800×10^{-3}
F, km^3	$3.273 \times 10^{+3}$	$5.100 \times 10^{+2}$	$4.890 \times 10^{+3}$	$1.101 \times 10^{+3}$	$1.000 \times 10^{+2}$	$2.000 \times 10^{+2}$
a'	$2.019 \times 10^{+0}$	$1.158 \times 10^{+0}$	$3.829 \times 10^{+0}$	$1.471 \times 10^{+0}$	$2.610 \times 10^{+0}$	$4.000 \times 10^{+0}$
b', m^{-1} s	3.795×10^{-3}	2.301×10^{-3}	3.890×10^{-3}	1.103×10^{-2}	5.600×10^{-3}	5.110×10^{-3}
F', km^3	$5.113 \times 10^{+3}$	$4.000 \times 10^{+3}$	$6.789 \times 10^{+3}$	$8.943 \times 10^{+2}$	$1.000 \times 10^{+2}$	$4.600 \times 10^{+2}$
V'_e, km^3	$1.200 \times 10^{+4}$	$5.006 \times 10^{+3}$	$8.010 \times 10^{+3}$	$9.748 \times 10^{+2}$	$8.490 \times 10^{+2}$	$2.000 \times 10^{+3}$
p	$1.299 \times 10^{+0}$	$1.068 \times 10^{+0}$	$1.150 \times 10^{+0}$	$1.223 \times 10^{+0}$	$1.290 \times 10^{+0}$	$1.200 \times 10^{+0}$
τ_a	$9.011 \times 10^{+8}$	$9.001 \times 10^{+8}$	$9.119 \times 10^{+8}$	$9.279 \times 10^{+8}$	$9.988 \times 10^{+8}$	$9.010 \times 10^{+8}$
τ_w	$8.002 \times 10^{+5}$	$2.003 \times 10^{+5}$	$1.080 \times 10^{+6}$	$4.437 \times 10^{+5}$	$9.202 \times 10^{+5}$	$8.001 \times 10^{+4}$
Water Surface Temperature Calibration Statistics[a]						
Means ratio[b]	1.005	1.001	0.998	1.000	1.033	0.987
Variances ratio[c]	1.019	0.908	0.979	1.116	1.157	0.988
Correlation[d]	0.979	0.970	0.968	0.988	0.988	0.981
rms error[e]	1.134	1.588	1.538	1.074	1.569	1.407
Old rms error[f]	1.180	1.558	1.253	1.253	1.510	1.367
Ice Concentration Calibration Statistics[g]						
Means ratio[h]	0.924	0.722	0.698	0.983	1.154	0.385
Variances ratio[i]	1.235	1.016	1.667	1.626	1.090	0.632
Correlation[j]	0.762	0.825	0.725	0.767	0.894	0.537
rms error[k]	23.36	12.41	25.97	21.46	18.99	15.40
Old rms error[l]	30.48	21.01			21.24	12.76

[a]Data between January 1, 1980, and August 31, 1988, for all lakes except Michigan and between January 1, 1981, and August 31, 1988, for Lake Michigan, with an initialization period for all lakes except Georgian Bay starting January 1, 1948, and January 1, 1953, for Georgian Bay.
[b]Ratio of mean model surface temperature to data mean.
[c]Ratio of variance of model surface temperature to data variance.
[d]Correlation between model and data surface temperature.
[e]Root-mean-square error between model and data surface temperatures in degrees Celsius.
[f]Root-mean-square error between previous thermodynamics model (using monthly ice regressions) and actual surface temperatures in degrees Celsius. Note that old model calibrations used different data periods on Lake Michigan (January 1, 1981, through November 30, 1985) and Lake Huron and Georgian Bay areas, and data sets were combined in one calibration.
[g]Data between January 1, 1960, and August 31, 1988, for all Great Lakes except Superior and between March 1, 1963, and August 31, 1988, for Lake Superior, with an initialization period for all lakes starting January 1, 1958.
[h]Ratio of mean model ice concentraiton to data mean.
[i]Ratio of variance of model ice concentration to data variance.
[j]Correlation between model and data ice concentration.
[k]Root-mean-square error between model and data ice concentrations in percent.
[l]Root-mean-square error between old ice regression model and data ice concentrations in percent.

fying values from field conditions which may be measured; interpretations of a lake's thermodynamics then can aid in setting both initial and boundary conditions.

Initialization

Prior to calibration or model use, the (spatial) average temperature-depth profile in the lake and the ice cover must be initialized. While the ice cover is easy to determine as zero during major portions of the year, the average temperature-depth profile in the lake is generally difficult to determine. If the model is to be used in forecasting or for short simulations, then it is important to determine these variables accurately prior to use of the model. If the model is to be used for calibration or for long simulations, then the initial values are generally unimportant. The effect of the initial values diminishes with the length of the simulation, and after 2–3 years of simulation the effects are nil from a practical point of view.

Data Preparation

We used independent data of lake-averaged daily surface temperatures for finding the first eight parameters and lake-averaged daily ice concentrations for finding the last two parameters. Meteorology data for 1948–1985 and water surface temperature data on each of the Great Lakes, except Lake Michigan, were taken and prepared as described by Croley [1989a, b] and extended through August 1988. Water surface temperature data for Lake Michigan from 1981 through 1985 were gleaned from areal maps prepared at the National Weather Service's Marine Predictions Branch (B. Newell, personal communication, 1990) and extended through August 1988 also. Lake-averaged ice cover was calculated from GLERL's digital ice cover data base [Assel, 1983a]. In most cases, less than 100% of a lake was observed on any given date. If less than 70% of the Lake Superior

Table 1b. Daily Comparison Statistics Between "Old" and "New" Ice Thermodynamic Models

	Evaporation, mm	T, °C	Q_i, w m^{-2}	Q_r, w m^{-2}	Q_l, w m^{-2}	Q_e, w m^{-2}	Q_h, w m^{-2}	T', °C	A, %	D, cm
				Superior						
Old Mean	1.5	5.3	158	−19	−58	−43	−34	−3.2	7	29
New Mean	1.5	5.4	158	−19	−58	−43	−34	−3.4	7	10
Old standard deviation	1.7	4.8	90	12	40	50	58	5.8	14	191
New standard deviation	1.8	4.9	90	14	40	51	61	5.8	20	34
rms error	0.3	0.6	0	7	1	9	13	1.3	13	193
Correlation	0.98	0.99	1.00	0.86	1.00	0.98	0.98	0.98	0.77	0.07
Bias	0.0	−0.1	0	0	0	0	0	0.2	0	19
				Michigan						
Old Mean	1.8	8.5	172	−18	−81	−51	−21	−1.3	2	41
New Mean	1.7	8.4	172	−19	−81	−50	−21	−1.4	4	17
Old standard deviation	2.0	7.4	78	8	28	57	45	3.2	4	122
New standard deviation	2.0	7.4	78	10	28	57	45	3.2	11	49
rms error	0.2	0.2	0	5	0	4	5	0.5	9	116
Correlation	1.00	1.00	1.00	0.85	1.00	1.00	0.99	0.99	0.71	0.36
Bias	0.0	0.1	0	1	0	−1	−1	0.0	−2	24
				Huron						
Old Mean	1.6	7.9	163	−21	−67	−47	−24	−1.8	10	20
New Mean	1.7	7.9	163	−20	−70	−48	−23	−1.8	7	8
Old standard deviation	1.9	6.4	96	14	31	54	44	3.8	17	39
New standard deviation	1.9	6.2	96	14	31	53	44	3.8	18	59
rms error	0.3	0.4	5	8	3	8	8	0.5	14	65
Correlation	0.99	1.00	1.00	0.86	1.00	0.99	0.98	0.99	0.72	0.21
Bias	0.0	0.0	−1	−1	3	1	0	0.0	4	11
				Erie						
Old Mean	2.4	10.9	147	−19	−39	−70	−16	−0.9	12	41
New Mean	2.5	11.0	147	−20	−39	−70	−16	−0.9	15	10
Old standard deviation	2.7	8.9	83	10	36	77	34	2.6	23	173
New standard deviation	2.8	9.0	83	13	36	80	36	2.6	32	21
rms error	0.3	0.4	0	6	1	9	10	0.3	17	172
Correlation	0.99	1.00	1.00	0.89	1.00	0.99	0.96	0.99	0.87	0.23
Bias	0.0	0.0	0	1	0	0	−1	0.0	−3	31
				Ontario						
Old Mean	1.8	8.9	147	−16	−55	−51	−22	−1.2	2	245
New Mean	1.7	8.7	147	−15	−58	−49	−22	−1.3	1	8
Old standard deviation	2.0	6.7	82	8	34	58	47	3.1	5	4779
New standard deviation	2.0	6.7	82	8	33	57	48	3.1	5	35
rms error	0.1	0.4	0	2	3	4	4	0.3	5	4785
Correlation	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	0.59	−0.01
Bias	0.1	0.3	0	−1	2	−2	0	0.0	1	237

surface was observed, the ice cover for that date was not included in the model calibration. A subjective estimate of lake-averaged ice cover was made for the other Great Lakes if the data were insufficient; this occurred approximately 50% of the time.

Calibration Procedure

Parameters are determined by minimizing the root-mean-square error (RMSE) between model-simulated output and actual data. Two calibrations are actually involved. The first determines the first eight parameters $(a, b, F, a', b', F', V_e,$ and $p)$ by minimizing RMSE between model and actual daily water surface temperatures. The second determines the last two parameters $(\tau_a$ and $\tau_w)$ by minimizing the RMSE between model and actual daily ice concentrations. In both of these calibrations, RMSE is minimized by systematically searching the parameter space [Croley and Hartmann, 1984]. Each

parameter, selected in rotation, is searched until all parameter values converge to four digits, instead of searching only until the RMSE stabilizes. This simple search algorithm does not give unique optima for calibrated parameter sets because of synergistic relationships between parameters which allow parameter compensations to occur.

Parameters were found in an iterative process that used the two calibrations in repeated rotation. First we minimized the RMSE of daily water surface temperature by calibrating with the first eight parameters and holding the last two parameters constant. We then held the first eight parameters constant and minimized the RMSE of daily ice cover by calibrating with the last two parameters. Then we repeated the process until the RMSEs for both calibration variables (water temperature and ice cover) were not significantly reduced from the previous iteration.

Figure 2 shows a typical trial process for Georgian Bay.

When moving from one calibration to the next, the parameter search for the second calibration used the parameters of the first but sometimes used arbitrary starting values for the second-calibration parameters. Thus the RMSEs depicted in Figure 2 deviate more during the joint calibrations than would occur if all parameters are preserved between calibrations. The selection of the starting parameter sets is important, as there are nonunique optimum parameter sets, and the selected suboptimum depends in large part on where the procedure starts. We selected several through trial and error and present the best.

Parameter Values

The results of these calibrations for all lakes are contained in Table 1a. Numbers in Figure 2 do not agree exactly with Table 1a because Table 1a contains RMSEs for water surface temperatures that were calculated by using the entire period subsequent to the data as an initialization period (1948–1979); Figure 2 used only 2 years for initialization (1978–1979). Root-mean-square errors of calibration are comparable to earlier calibrations for water surface temperatures and ice cover [Croley, 1989a, 1992]. Old RMSEs from these references are also given in Table 1a for comparison. Water surface temperature RMSEs are now generally slightly higher than before with the exception of the Lake Superior and Georgian Bay applications. Ice cover RMSEs generally have improved significantly except for Lake Ontario.

With no loss of representation in water surface temperatures we now have a model that does better at estimating ice cover than before [Croley, 1989a, 1992] and is physically based. As mentioned in the introduction, the physically based model is superior because it allows use in extrapolative situations (e.g., under different hypothetical climates) with more confidence than the old ice regression statistical model (which was based on historical conditions).

Calibration Issues

There were several problems in calibrating the model. First, it appears that the models are close to being overspecified in terms of the number of parameters used; that is, there appear to be almost too many degrees of freedom allowed for the data sets used in the calibrations. The result is that the optimums are not unique, and it is not possible to determine meaningful values of any additional parameters. Parameter compensation exists so that changes in one parameter can be offset by changes in other parameters with little change in the RMSE of the calibration. This made it difficult to determine an ice breakup model which had an additional three parameters.

Second, optimizing parameters with regard to two objectives (minimizing RMSEs associated with water surface temperatures and ice cover) does not produce the same parameter sets. There seems to be a trade-off between the two objectives at times. This is seen in some of the changes in Figure 2 where RMSE of water temperatures decreases at the expense of ice cover RMSE and vice-versa.

Parameter Interpretation

There are some similarities in the parameters in Table 1a. For example, in general the mixing volume F or F', at which a heat addition is fully mixed throughout, is larger after the fall turnover (water temperatures are below 3.98°C) than after the spring turnover (water temperatures are above 3.98°C), $F' > F$. Since thermal mixing is less intense during the winter than during the summer (density-depth gradients are smaller), the mixing volume would reach greater depths before full mixing is achieved. This is true for all lakes in Table 1a (except Georgian Bay) and agrees with available observations [Assel, 1983b, 1985; Bennett, 1978] and earlier model results [Croley, 1992].

The mixing coefficients in (11) are also consistent with an increased dependence, during the winter, of mixing on wind rather than on temperature gradients. In Table 1a, $a' < a$ in all cases and $b' > b$ in all cases except Lake Michigan. (Small values of a or a' and large values of b or b' result in more dependence of mixing volume $M_{k,m}$ on accumulated wind speed in (11)). The equilibrium volume V_e is, in general, the same order of magnitude as the capacity of the lake, V_c.

The values of τ_a and τ_w (see (18)–(19)) are large, indicating that lateral changes in ice cover areal extent are many orders of magnitude greater than vertical changes in ice pack depth, as expected. Furthermore, $\tau_a > \tau_w$ in Table 1a for all lakes, indicating that atmospheric heat is distributed in changes that are more predominantly lateral than is true for heat from the water body. The difference between τ_a and τ_w arises because we allow ice growth from freezing only at the water-ice boundary of the ice pack.

Model Evaluation

Comparison With Model of Croley [1992]

A comparison of root-mean-square errors between the (old) previous model [Croley, 1989a, 1992] and the (new) current model, summarized partially in Table 1a, is expanded here in Table 1b with statistics on daily inputs and outputs for both models over the period January 1, 1950, through August 31, 1988. The old and new models are very similar, as indicated by the high correlations and low bias and RMSE, in Table 1b, for all variables unrelated to ice. Evaporation, water temperatures, and heat fluxes are practically unchanged from the previous model. This is expected since the nonice parameters have changed only impercep-
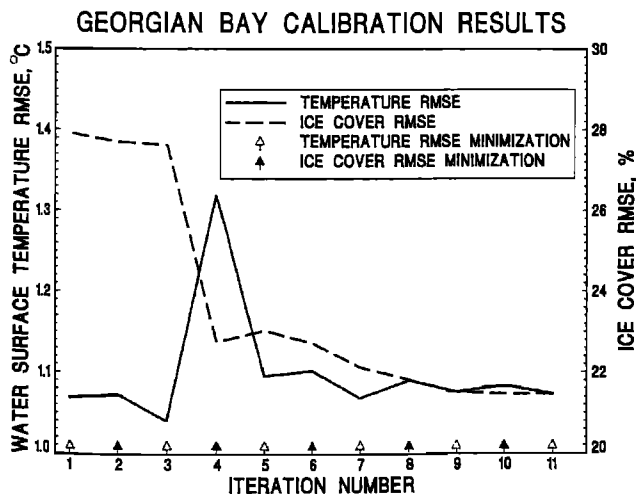

Figure 2. Georgian Bay calibration results.

Table 2a. Average Water Surface Temperature Error (Modeled − Observed) for 1948–1988 Winters

Lake	Half-Month Period[a]							
	Dec1	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2
Superior	0.99	0.09	−0.95	−0.72	−1.07	−0.58	−0.23	−0.38
Michigan	...	...	...	...	...	−1.43	...	...
Huron	1.39	1.62	1.80	0.37	−0.52	−0.41	0.20	0.22
Georgian Bay	1.82	0.13	...	−0.74	...	−0.58	−0.16	−0.19
Erie	1.41	0.54	−0.24	−0.52	−0.53	−0.34	−0.50	−1.00
Ontario	1.47	1.18	1.90	0.38	−0.67	−0.54	−0.29	−0.27

Temperature is in degrees Celsius. No entry indicates one observation or less.
[a]Dec1 corresponds to December 1–15, Dec2 to December 16–31, etc.

tibly between both models. The major difference appears in the way ice behaves.

In the old model, ice formation was derived as an empirical relationship between current month and previous month air temperatures and ice cover extent. The old model allowed more frequent ice formation in December and January, compared to the new model, because of the surface water temperature constraint for ice formation in the new model (see (5) and (6)).

The correlation of mean ice thickness was low between the old and new ice models. The old ice model has greater average ice thickness on all lakes. The old average of 2.45 m for Lake Ontario is unrealistic, but the average for the other lakes is reasonable; however, it is difficult to say, in a lumped seasonal average such as this, which mean ice thickness is more realistic. The old ice model was expected to simulate greater seasonal average ice concentrations for all lakes because it does not account for summer lake heat storage and the effects of winds on heat transport to the lake's surface from within the lake during autumn and winter. The greater ice cover for the new model for Lake Erie, however, can be explained perhaps by that lake's shallowness, which results in more rapid temperature declines in autumn and early winter and less spatial variation in its surface water temperature. The reason for Michigan's greater ice cover with the new model is less apparent.

Model Bias Associated With Water Temperatures

The model was run on each of the Great Lakes for the 41 winter seasons, 1948–1988, to further compare simulated water surface temperatures and ice covers to available observations (besides the RMSE computed in the calibrations). The model does not permit ice formation until the water surface temperature of the lake reaches 0°C. Water surface temperature errors are averaged over half-month periods from December through March in Table 2a. The model appears biased toward overestimation of the December water surface temperature on all Great Lakes and of the January temperature on Lakes Huron and Ontario. This results in a large number of winters, in the model, with no ice formation on Lakes Huron and Ontario and, to a lesser extent, on Lakes Superior and Michigan; see Table 2b. Compounding the problem is the fact that ice actually forms on the Great Lakes prior to the average water surface temperature reaching 0°C. Ice forms first along the shallow areas in December and January and then in the deeper, more exposed areas of a lake starting the last half of January.

Model Bias for Warm and Cold Winters

Accumulated freezing degree-days is the air temperature difference from 0°C, integrated over time; the annual maximum is used to classify winter severity in five classes [Assel et al., 1983]: mild, milder than normal, normal, more severe than normal, and severe. The average ice cover errors are presented for "warm" winters (mild and milder than normal) of the calibration period (1973 and 1975) in Table 3a and for "cold" winters (more severe than normal and severe) of the calibration period (1963, 1970, 1977, 1978, and 1979) in Table 3b. For warm winters and for the calibration period taken as a whole, the model appeared biased toward underestimation of ice cover. During warm winters the bias was particularly large during February and March for Lake Huron and during January and February for Georgian Bay. The ice model predicted ice-free conditions both during 1973 and 1975 for Lakes Huron and Ontario (see Table 2b) and ice-free conditions on Lake Michigan for the winter of 1975 as well.

Table 2b. Distribution of Model Ice-Free Winters, 1948–1988

Winter	Lake					Total
	Superior	Michigan	Huron	Ontario	Erie	
1949	X	X	X	X		4
1950				X		1
1951			X	X		2
1952	X	X	X	X		4
1953	X	X	X	X	X	5
1954	X	X	X	X		4
1955			X	X		2
1956		X	X	X		3
1957			X	X		2
1964	X		X	X		3
1966			X			1
1969			X	X		2
1973			X	X		2
1974		X	X	X		3
1975		X	X	X		3
1976			X			1
1983	X	X	X	X		4
1985				X		1
1987	X	X	X	X		4
1988				X		1
Total	7	9	18	17	1	52
Percent	17	22	44	41	2	100

Table 3a. Warm Winter Average Ice Concentration Error (Modeled −
Observed) for 1973 and 1975 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	−1.0	−1.0	−13.0	···	1.4	6.9	6.2	···	···
Michigan	−1.9	−15.0	−8.0	−14.9	−14.1	−11.3	−3.9	−8.2	···
Huron	−0.5	−11.5	−10.6	−31.7	−45.8	−24.8	−7.0	−7.1	···
Georgian Bay	−2.0	−14.6	−49.5	−28.7	−14.6	2.4	−14.5	−24.6	···
Erie	−1.5	−5.0	5.2	−21.7	−9.8	4.2	3.2	···	···
Ontario	−3.1	−10.6	−4.0	−29.6	−24.3	−12.2	0.0	···	···

Concentration in percent. No entry indicates one observation or less.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

For cold winters the model bias toward underestimation in early winter (December and January) is reversed by the first half of March, Table 3b. This also occurs to a lesser degree for the calibration period as a whole during the second half of March and first half of April. Water temperature is near its annual minimum and ice cover is near its maximum in February and March and model errors are, in general, small during the first half of March during a cold winter. The model bias changes sign during March and the first half of April from its early winter value, and the model overestimates ice cover. This bias reversal is associated with a concurrent bias to underestimate the water surface temperature in February and March; see Table 2a. It is also likely that the overestimate in ice mass in cold winters is exacerbated by the lack of structural weakening and dynamic breakup processes in the model.

Model Application

The model is very fast; on a 33 MHZ 486DX personal computer it required 230 s to simulate 16,000 days of Lake Superior evaporation, ice cover, and heat storage. On an HP Apollo 9000 model 735 workstation it required 20 s. Times were similar for other lakes.

Great Lakes Ice Cover Climatology for 1960–1979

Average ice cover and thickness were calculated for the 20 winters (1960–1979) given in the National Oceanic and Atmospheric Administration (NOAA) Great Lakes Ice Atlas for each Great Lake [Assel et al., 1983]. Normal atlas ice covers were taken as the spatial averages of the 20-year medians for each 5 km by 5 km square in a grid on each lake. These normals used all the data given in GLERLs digital ice concentration data base [Assel, 1983a], while the model was calibrated over only a selected subset of these data, as noted earlier. Thus the ice atlas contains some data not included in the calibration of the ice model and forms a quasi-independent data set. The differences between the model ice cover and observed climatology, presented in Table 4a, show that the model underestimates multiyear averages most of the winter. However, in 65% of the cases in Table 4a the model ice cover is within 10% of the observed climatology; in 79% of the cases it is within 15%.

While there are no long-term average ice thickness statistics for the midlake areas of the Great Lakes, the climatic upper limit of thermodynamic ice growth is approximately 100 cm in the protected bay and harbor sites in the Great Lakes. Bolsenga [1988] found 10-year average maximum ice thicknesses for bay and harbor sites of approximately 50 cm (Lakes Superior, Huron, and Michigan), 30 cm (Lake Erie), and 40 cm (Lake Ontario). Therefore the simulated thicknesses in Table 4b indicate the model overestimates by about 30 cm on Lake Superior, 60 cm on Lake Michigan, 40 cm on Georgian Bay, 30 cm on Lake Erie, and 35 cm on Lake Ontario; it underestimates by about 10 cm on Lake Huron. The model likely underestimates ice thickness in December through mid-January because ice is usually confined to the shallow lake areas, and the model overestimates water temperatures in early winter, see Table 2a. Table 5 shows that the model water temperatures are usually above freezing during December and much of January.

Table 3b. Cold Winter Average Ice Concentration Error (Modeled −
Observed) for 1963, 1970, 1977, 1978, and 1979 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	···	−23.7	−20.9	−6.2	−11.4	−7.2	4.0	16.4	−6.3
Michigan	−15.4	−9.5	−6.6	−28.9	−21.5	1.1	14.4	21.6	−0.9
Huron	−7.6	−29.3	−13.8	−10.7	−0.8	9.1	28.0	20.2	−6.0
Georgian Bay	−27.5	−15.2	−7.8	0.7	7.4	5.4	10.8	24.2	13.0
Erie	−16.5	4.0	8.6	1.9	5.7	9.4	16.6	20.9	0.5
Ontario	−2.3	···	−6.5	11.9	−20.2	0.3	−6.4	−1.9	···

Concentration in percent. No entry indicates one observation or less.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 4a. Average Ice Concentration Difference (Modeled − Atlas) for 1960–1979 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	−2.0	−7.6	−3.5	−0.3	−18.6	−7.4	−2.7	17.8	−2.3
Michigan	· · ·	−17.3	−10.9	−4.1	−18.3	1.5	3.8	3.8	· · ·
Huron	−5.5	−15.8	−16.1	−27.6	−26.6	−6.6	−5.0	0.3	−2.4
Georgian Bay	−11.0	−37.2	−14.1	−14.5	−6.9	0.8	−2.6	2.3	0.3
Erie	−5.2	2.1	3.5	−9.8	−3.7	14.5	27.9	10.5	−1.8
Ontario	−2.0	−7.7	−4.8	−11.2	−12.9	−0.4	−1.2	−0.5	0.1

Difference in percent. No entry indicates one observation or less. NOAA Great Lakes Ice Atlas [*Assel et al.*, 1983].

[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Retrospective Analysis of Great Lakes Ice for 1950–1988

The preceding analysis shows that, within limits, the model can be used to simulate time-averaged lake-averaged ice cover and ice thickness and thus total ice volume. Therefore the model was used to compare ice cover, thickness, and volume for the winters of 1960–1979 (the period of the contemporary NOAA Great Lakes Ice Atlas climatology) with (1) winters of 1950–1959 and (2) winters of 1980–1988. The results are summarized in Tables 6a, 6b, and 6c for ice cover, Tables 4b, 7a, and 7b for thickness, and Tables 8a, 8b, and 8c for volume. These tables indicate that, on the average, ice cover, thickness, and volume during the winters of 1960–1979 were greater than the 10 winters that preceded them and the 9 winters that followed them. It is difficult to quantify how much greater they were. However, if differences in Tables 6b and 6c (representing estimated changes between periods) exceed differences in Table 4a (representing estimated errors of the model), it is likely that the observed changes between the two periods are significant. Tables 4a and 6b imply that ice cover, from the last half of February through the first half of April for 1980–1988, was less than the 20 preceding years, with the exception of Lake Erie. Similarly, Tables 4a and 6c imply that ice cover on Lakes Superior and Erie during January through the first half of April for 1950–1959 was less than the 20 succeeding years; on Lakes Michigan, Huron, and Ontario, it was less during March through the first half of April.

Summary

The one-dimensional ice cover model described here was developed primarily to provide an improved estimate of ice cover for a large-lake bulk-evaporation model. The lake evaporation, water surface temperature, heat storage, and thermodynamic flux estimates appear good on a daily basis for spatial averages. They are largely unchanged from the previous thermodynamics model [*Croley*, 1992], but the model is now capable of simulating time-averaged ice cover, ice thickness, and ice volume consistent with the lake heat balance. By replacing ice regressions with the new one-dimensional ice model we have removed some dependence on past conditions (under which the regressions were developed). The entire model now has application beyond the conditions under which the (former) ice regressions were developed, with no loss in performance. The model can now be used to evaluate ice cover sensitivity to individual energy balance parameters in climate scenarios. This was not possible with the simple ice regression models and so we have some confidence, then, in using the model as a prognostic tool.

The most serious shortcoming of this model relates to the multidimensional nature of the ice formation and loss process, as indicated by the model bias toward overestimation of the number of winters without ice cover and in general toward underestimation of ice cover. The boundary condition of (5) and (6) that prohibits ice growth until the average

Table 4b. Average Model Ice Thickness for 1960–1979 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.4	15.1	52.5	69.4	78.7	76.7	58.2	17.4
Michigan	0.0	3.0	28.4	76.7	105.0	113.1	97.1	61.7	28.3
Huron	0.0	0.0	1.7	40.6	17.8	23.5	18.1	11.6	1.9
Georgian Bay	0.2	5.4	30.4	54.5	71.9	84.3	91.8	91.1	62.2
Erie	1.6	14.2	29.3	42.6	53.6	58.9	53.6	37.3	5.3
Ontario	0.0	2.1	14.7	54.3	75.2	69.0	39.5	15.6	1.3

Thickness in centimeters.

[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 5. Average Model Surface Temperature for 1948–1988 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	3.5	2.1	0.9	0.3	0.2	0.2	0.3	0.6	1.3
Michigan	4.3	2.6	1.0	0.3	0.2	0.3	0.5	1.1	2.0
Huron	5.6	4.2	2.3	0.9	0.5	0.5	0.9	1.6	2.4
Georgian Bay	4.0	1.6	0.2	0.0	0.0	0.0	0.1	0.1	0.7
Erie	3.3	1.1	0.2	0.1	0.1	0.1	0.3	1.2	3.6
Ontario	5.2	3.9	2.3	1.1	0.7	0.8	1.2	2.0	2.9

Temperature in degrees Celsius.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 6a. Average Ice Concentration for 1960–1979 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.4	8.5	37.7	56.4	59.6	48.3	27.8	5.7
Michigan	0.0	0.7	7.1	19.9	26.7	26.5	19.8	9.8	2.8
Huron	0.0	0.0	3.8	15.0	30.4	31.5	18.5	7.1	0.3
Georgian Bay	0.2	6.7	39.8	70.7	85.8	87.0	78.7	57.9	22.4
Erie	3.8	38.1	68.5	80.2	86.3	78.5	53.9	20.5	1.2
Ontario	0.0	0.3	2.2	7.8	11.1	9.6	4.8	1.5	0.1

Concentration in percent.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 6b. Average Ice Concentration Difference (1960–1979 Winters and 1980–1988 Winters)

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.4	3.3	9.8[b]	22.1[b]	24.2[b]	22.4[b]	20.2[b]	4.4[b]
Michigan	0.0	0.7	2.3	2.0	2.8	4.5[b]	7.4[b]	6.4[b]	2.0
Huron	0.0	0.0	3.3	1.3	11.8	12.8[b]	8.0[b]	−0.7[b]	−1.7
Georgian Bay	0.2	0.2	11.2	17.9	19.9[b]	21.3[b]	19.3[b]	24.0[b]	11.9[b]
Erie	−1.0	3.3[b]	2.1	−3.1	4.3[b]	3.7	2.1	5.6	0.5
Ontario	0.0	0.1	0.0	2.9	6.0	5.2[b]	3.3[b]	1.5[b]	0.1

Concentration in percent.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.
[b]Differences are larger than in Table 4a.

Table 6c. Average Ice Concentration Difference (1960–1979 Winters and 1950–1959 Winters)

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.4[b]	7.4[b]	26.9[b]	38.2[b]	38.4[b]	33.7[b]	27.1[b]	5.7[b]
Michigan	0.0	0.7	4.9	10.0	12.6	13.4[b]	11.4[b]	7.4[b]	2.8
Huron	0.0	0.0	2.6	7.4	17.0	17.4[b]	7.7[b]	0.4[b]	−0.8
Georgian Bay	...	...	...	...	...	...	...	...	...
Erie	3.7	30.7[b]	36.5[b]	27.5[b]	33.4[b]	40.1[b]	39.1[b]	19.4[b]	1.2
Ontario	0.0	0.3	2.2	6.8	8.1	7.9[b]	4.5[b]	1.5[b]	0.1

Concentration in percent. No entry indicates one observation or less.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.
[b]Differences are larger than in Table 4a.

Table 7a. Average Ice Thickness Difference (1960–1979 Winters and 1980–1988 Winters)

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.4	9.9	27.4	36.9	39.8	41.3	43.7	13.6
Michigan	0.0	3.0	7.1	0.9	1.7	8.0	19.9	39.5	19.5
Huron	0.0	0.0	1.3	34.1	8.1	9.7	9.4	3.5	−2.2
Georgian Bay	0.2	0.3	9.2	−81.0	−19.3	19.1	22.4	29.1	33.0
Erie	−0.4	1.2	−0.2	−0.2	3.2	4.9	2.7	10.3	2.8
Ontario	0.0	0.6	−3.0	21.5	39.4	33.1	23.8	15.6	1.3

Thickness in centimeters.

[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 7b. Average Ice Thickness Difference (1960–1979 Winters and 1950–1959 Winters)

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.4	13.9	41.2	49.6	53.7	53.0	55.4	17.4
Michigan	0.0	3.0	18.6	37.0	47.9	50.8	51.5	40.8	28.3
Huron	0.0	0.0	0.9	36.9	10.6	14.4	9.2	4.0	−1.3
Georgian Bay	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·	· · ·
Erie	1.5	10.9	15.4	19.3	25.7	31.3	39.2	34.9	5.3
Ontario	0.0	2.1	14.7	45.4	53.1	54.7	34.6	15.6	1.3

Thickness in centimeters. No entry indicates one observation or less.

[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 8a. Average Ice Volume 1960–1979 Winters

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.1	3.2	18.1	33.4	40.1	35.6	21.0	4.1
Michigan	0.0	0.4	5.1	16.5	24.9	27.1	21.2	10.8	3.0
Huron	0.0	0.0	0.5	2.3	5.3	6.6	4.9	2.2	0.1
Georgian Bay	0.0	0.5	3.6	8.2	12.1	14.3	14.2	10.9	4.3
Erie	0.2	2.7	6.7	10.2	12.7	12.6	9.5	3.8	0.2
Ontario	0.0	0.1	0.7	2.4	3.8	3.5	1.8	0.5	0.0

Volume in cubic kilometers.

[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 8b. Average Ice Volume Difference (1960–1979 Winters and 1980–1988 Winters)

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.1	1.7	6.6	16.3	20.1	19.6	15.9	3.3
Michigan	0.0	0.4	2.3	3.2	5.1	7.4	10.0	7.2	2.3
Huron	0.0	0.0	0.5	0.5	2.2	3.0	2.1	−0.1	−0.5
Georgian Bay	0.0	0.0	0.8	2.0	3.2	4.0	3.9	4.7	2.2
Erie	0.0	0.0	−0.1	−0.2	0.8	0.8	0.7	1.2	0.1
Ontario	0.0	0.1	0.2	1.0	2.3	2.2	1.4	0.5	0.0

Volume in cubic kilometers.

[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

Table 8c. Average Ice Volume Difference (1960–1979 Winters and 1950–1959 Winters)

Lake	Half-Month Period[a]								
	Dec2	Jan1	Jan2	Feb1	Feb2	Mar1	Mar2	Apr1	Apr2
Superior	0.0	0.1	2.9	13.8	24.9	29.3	28.0	20.6	4.1
Michigan	0.0	0.4	3.9	9.0	12.8	16.0	13.7	8.8	3.0
Huron	0.0	0.0	0.4	1.1	2.9	3.6	1.9	0.1	-0.2
Georgian Bay	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
Erie	0.2	2.3	4.5	5.6	7.4	8.5	7.7	3.7	0.2
Ontario	0.0	0.1	0.7	2.2	3.1	3.0	1.7	0.5	0.0

Volume in cubic kilometers. No entry indicates one observation or less.
[a]Dec2 corresponds to December 16–31, Jan1 to January 1–15, etc.

water surface temperature reaches freezing is responsible for this underestimation. Instead, the surface temperature and heat storage in same-depth segments of the lake could be considered in bathymetry-weighted calculations to allow ice formation in the segments as surface temperatures reach freezing. This represents an extension of the existing areal point model in one or more spatial dimensions.

The ice submodel theory also could be improved by including ice breakup and rejoining mechanisms related to wind, melting, and refreezing. However, a trial formulation resulted in an overspecified model for the data sets currently at hand, and the additional parameters were indeterminate. Other improvements include formulation of a snow cover layer on the ice, parameterization of surface (ice and snow) albedo in terms of daily meteorological inputs, and consideration of the effects of solar radiation absorption by ice and snow on ice strength and albedo. It is doubtful if these or other improvements in model theory would add significantly to accuracy in this one-dimensional formulation since extensive data on the spatial and temporal extent of snow on ice are unavailable at present for the Great Lakes. Lateral heating, cooling, and momentum transfer at the lakes surface are not adequately addressed in a one-dimensional model. This tended to be less of a problem on Lake Erie because of its much smaller average depth. However, even on Lake Erie, the effects of winds, currents, and ice movement on lake-averaged ice cover were not adequately addressed.

The model has ten parameters calibrated to match water surface temperatures and ice cover. Seven of them are defined in the superposition heat storage submodel. The number of empirical model parameters could perhaps be reduced by use of other one-dimensional mixed-layer heat storage models [McCormick and Meadows, 1988; Hostetler and Bartlein, 1990]. The model requires 5.3 s of computation per year of daily simulation on a 33 MHZ 486DX personal computer and less than 0.5 s yr^{-1} on an HP Apollo 9000 model 735.

We plan to investigate application of distributed-parameter ice cover models in future studies. The evaluation and improvement of ice models for Great Lakes application have been limited by the lack of high-resolution overlake observations of wind, temperature, humidity, cloud cover, ice thickness, ice albedo, ice concentration, ice temperature, water temperatures, and snow cover. However, recent improved algorithms for extrapolating overlake winds from a limited number of shore locations [Schwab, 1989], observations of spatially distributed lake temperatures from NOAA AVHRR satellites [Schwab et al., 1992; Irbe et al., 1979], and existing ice cover data [Assel, 1983a] are now available. They enable reevaluation of existing two-dimensional ice cover models and development of new thermodynamic and hydrodynamic ice cover models for the Great Lakes.

Notation

a wind parameter, $T > 3.98°C$ (empirical model parameter).

a' wind parameter, $T < 3.98°C$ (empirical model parameter).

A area of the ice surface.

A_w area of the open water (ice-free) lake surface.

α_a ratio of unit area vertical change to unit area lateral change for atmosphere-ice heat fluxes.

α_w ratio of unit area vertical change to unit area lateral change for water-ice heat fluxes.

b wind parameter, $T > 3.98°C$ (empirical model parameter).

b' wind parameter, $T < 3.98°C$ (empirical model parameter).

β shape constant for the ice pack.

C circumference of ice pack.

C_i specific heat of ice.

C_w specific heat of water.

D ice pack depth (thickness).

dH/dt time rate of change of heat storage in the lake.

dH'/dt time rate of change of heat storage in the ice pack.

E volumetric rate of evaporation from ice.

$f_{k,m}$ ratio of surface temperature rise on day k from heat added on day m to that heat addition.

F lake volume at which a heat addition is uniformly fully mixed, $T > 3.98°C$ (empirical model parameter).

F' lake volume at which a heat addition is uniformly fully mixed, $T < 3.98°C$ (empirical model parameter).

γ_f latent heat of fusion.

H heat stored in the lake.

H' heat stored in the ice pack.

$I_{(\ ,\ )}(x)$ indicator function (equal to unity if the quantity in parentheses, x, is within the indicated interval and zero if not).

$M_{k,m}$ mixing volume size on day k of heat added on day m.

p parameter relating cloudiness to atmospheric longwave radiation (empirical model parameter).

q_e daily average unit (per unit area) evaporative (latent and advected) heat transfer rate from water surface.

q'_e daily average unit evaporative (latent and advected) heat transfer rate from ice pack.

q_h daily average unit sensible heat transfer rate from water surface.

q'_h daily average unit sensible heat transfer rate to ice pack.

q_i daily average unit incident shortwave radiation rate.

q_p daily average unit precipitation heat advection rate to water surface.

q'_p daily average unit precipitation heat advection rate to ice pack.

q_r daily average unit reflected shortwave radiation rate from water surface.

q'_r daily average unit reflected shortwave radiation rate to ice pack.

Q_a daily average heat flux between atmosphere and ice pack used for freezing or melting.

Q_l daily average net longwave radiation exchange rate.

Q_w daily average heat flux between the water body and the ice pack.

Q_I daily average net heat advection to the lake from surface flows.

ρ density of ice.

ρ_w density of water.

S volumetric rate of snow falling on ice.

t time.

T water surface temperature.

T' ice surface temperature.

T_a overice air temperature.

τ_a parameter reflecting ice pack shape, vertical-lateral change ratios along atmosphere-ice boundary, and ice buoyancy (empirical model parameter) $(\tau_a = 0.1\beta/\alpha_a)$.

τ_w parameter reflecting ice pack shape, vertical-lateral change ratios along water-ice boundary, and ice buoyancy (empirical model parameter) $(\tau_w = 0.9\beta/\alpha_w)$.

V volume of the ice pack.

V' volume of ice formed only by freezing or melting.

V_c lake volume (capacity).

V_e equilibrium lake volume approached as a limit by mixing (empirical model parameter).

W daily wind movement.

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