Department of Mechanical & Industrial Engineering Concordia University

# MECH 370: Modeling, Simulation and Control Systems

(Formerly: Modeling, Simulation and Analysis of Physical Systems)

Laboratory Manual Supplement

This is the MECH 370 supplemental Lab manual only. To fulfill the MECH 370 laboratory, student needs both ELEC 370 Lab Manual and this MECH 370 Lab Manual. ELEC 370 Lab Manual is available at Bookstore of Concordia University.

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# **General Laboratory Safety Rules**

#### **Follow Relevant Instructions**

- Before attempting to install, commission or operate equipment, all relevant suppliers'/manufacturers' instructions and local regulations should be understood and implemented.
- It is irresponsible and dangerous to misuse equipment or ignore instructions, regulations or warnings.
- Do not exceed specified maximum operating conditions (e.g. temperature, pressure, speed etc.).

#### **Installation/Commissioning**

- Use lifting table where possible to install heavy equipment. Where manual lifting is necessary beware of strained backs and crushed toes. Get help from an assistant if necessary. Wear safety shoes appropriate.
- Extreme care should be exercised to avoid damage to the equipment during handling and unpacking. When using slings to lift equipment, ensure that the slings are attached to structural framework and do not foul adjacent pipe work, glassware etc.
- Locate heavy equipment at low level.
- Equipment involving inflammable or corrosive liquids should be sited in a containment area or bund with a capacity 50% greater than the maximum equipment contents.
- Ensure that all services are compatible with equipment and that independent isolators are always provided and labeled. Use reliable connections in all instances, do not improvise.
- Ensure that all equipment is reliably grounded and connected to an electrical supply at the correct voltage.
- Potential hazards should always be the first consideration when deciding on a suitable location for equipment. Leave sufficient space between equipment and between walls and equipment.
- Ensure that equipment is commissioned and checked by a competent member of staff permitting students to operate it.

#### **Operation**

- Ensure the students are fully aware of the potential hazards when operating equipment.
- Students should be supervised by a competent member of staff at all times when in the laboratory. No one should operate equipment alone. Do not leave equipment running unattended.
- Do not allow students to derive their own experimental procedures unless they are competent to do so.

#### Maintenance

- Badly maintained equipment is a potential hazard. Ensure that a competent member of staff is responsible for organizing maintenance and repairs on a planned basis.
- Do not permit faulty equipment to be operated. Ensure that repairs are carried out competently and checked before students are permitted to operate the equipment.

#### **Electricity**

- Electricity is the most common cause of accidents in the laboratory. Ensure that all members of staff and students respect it.
- Ensure that the electrical supply has been disconnected from the equipment before attempting repairs or adjustments.
- Water and electricity are not compatible and can cause serious injury if they come into contact.
   Never operate portable electric appliances adjacent to equipment involving water unless some form of constraint or barrier is incorporated to prevent accidental contact.
- Always disconnect equipment from the electrical supply when not in use.

#### **Avoiding Fires or Explosion**

- Ensure that the laboratory is provided with adequate fire extinguishers appropriate to the potential hazards.
- Smoking must be forbidden. Notices should be displayed to enforce this.
- Beware since fine powders or dust can spontaneously ignite under certain conditions. Empty vessels having contained inflammable liquid can contain vapor and explode if ignited.
- Bulk quantities of inflammable liquids should be stored outside the laboratory in accordance with local regulations.
- Storage tanks on equipment should not be overfilled. All spillages should be immediately cleaned up, carefully disposing of any contaminated cloths etc. Beware of slippery floors.
- When liquids giving off inflammable vapors are handled in the laboratory, the area should be properly ventilated.
- Students should not be allowed to prepare mixtures for analysis or other purposes without competent supervision.

#### **Handling Poisons, Corrosive or Toxic Materials**

- Certain liquids essential to the operation of equipment, for example, mercury, are poisonous or can give off poisonous vapors. Wear appropriate protective clothing when handling such substances.
- Do not allow food to be brought into or consumed in the laboratory. Never use chemical beakers as drinking vessels

- Smoking must be forbidden. Notices should be displayed to enforce this.
- Poisons and very toxic materials must be kept in a locked cupboard or store and checked regularly. Use of such substances should be supervised.

#### **Avoid Cuts and Burns**

- Take care when handling sharp edged components. Do not exert undue force on glass or fragile items.
- Hot surfaces cannot, in most cases, be totally shielded and can produce severe burns even when not visibly hot. Use common sense and think which parts of the equipment are likely to be hot.

#### **Eye/Ear Protection**

- Goggles must be worn whenever there is risk to the eyes. Risk may arise from powders, liquid splashes, vapors or splinters. Beware of debris from fast moving air streams.
- Never look directly at a strong source of light such as a laser or Xenon arc lamp. Ensure the equipment using such a source is positioned so that passers-by cannot accidentally view the source or reflected ray.
- Facilities for eye irrigation should always be available.
- Ear protectors must be worn when operating noisy equipment.

#### **Clothing**

- Suitable clothing should be worn in the laboratory. Loose garments can cause serious injury if caught in rotating machinery. Ties, rings on fingers etc. should be removed in these situations.
- Additional protective clothing should be available for all members of staff and students as appropriate.

#### **Guards and Safety Devices**

- Guards and safety devices are installed on equipment to protect the operator. The equipment must not be operated with such devices removed.
- Safety valves, cut-outs or other safety devices will have been set to protect the equipment. Interference with these devices may create a potential hazard.
- It is not possible to guard the operator against all contingencies. Use commons sense at all times when in the laboratory.
- Before staring a rotating machine, make sure staff are aware how to stop it in an emergency.
- Ensure that speed control devices are always set to zero before starting equipment.

#### First Aid

• If an accident does occur in the laboratory it is essential that first aid equipment is available and that the supervisor knows how to use it.

- A notice giving details of a proficient first-aider should be prominently displayed.
- A short list of the antidotes for the chemicals used in the particular laboratory should be prominently displayed.



In case of an emergency use the internal phone to call security by dialing 811. Security will connect you to the appropriate emergency service and immediately dispatch security personnel.

Or you can use your cellular phone to call 848 3717.

The civic address is:
Concordia University, 1455 De Maisonneuve West, H3G1M8
Room\_\_\_\_\_

The te	ecnnician responsible/ ir	charge of this laboratory	
is:	Mr. Gilles Huard	tel: <b>8798</b>	

Safety Regulations for Students in All Mechanical and Industrial Engineering Laboratories;

#### Standard lab safety must be followed in all laboratories.

- a. First discuss your experiment regarding possible hazards or problems, with the demonstrator, or the MIE technical staff, or your professor.
- b. Do not work alone. Work with another person in a lab that has running machinery, machine tools, conveyors, hydraulics, lifting equipment, voltage hazards, or where chemicals are in use.
- c. Safety glasses must be worn in the vicinity of pneumatics, machine tools grinders, power saws, and drills. Users of lasers need special safety glasses for the particular wavelength of the laser.
- d. No equipment or machine may be operated by anyone unless they have received adequate instruction from a qualified instructor eg. machine tools, hydraulics, chemicals, lasers, running machinery, robots. Undergraduate students may not use any machine or equipment unless a Department technical staff member is present. Graduate students are the responsibility of their immediate academic supervisor.
- e. Workplace Hazardous Material training must be obtained before using chemicals or compressed gasses. Contact Dainius Juras tel: 848 3128 for training.
- f. All appropriate safety accessories (lab coats, safety glasses, gloves, etc..) must be used when handling chemicals. No open toe shoes are permitted in laboratories.

- g. No chemicals to be left unattended or unlabeled according to WHMIS. All chemicals must be stored properly.
- h. Long term unattended tests must be fail safe.
- i. When the university is officially closed, you may not work in a lab unless your supervisor or a technical staff member is present.
- j. No eating in laboratories.
- k. Major accidents and injuries must be reported at once to Security tel: 811, the Safety Officer (tel: 3128), the Professor (Supervisor) or the Department Administrator (tel: 7975) should then be informed.
- I. During working hours all minor accidents should be reported to the Safety Officer (tel: 3128), the Professor (Supervisor) or the Department Administrator (tel: 7975).
- m. An "Incident Report" must be filled out by the person involved, for all accidents and injuries.

#### LABORATORY RULES

Considering the large number of students attending the labs and in order for the lab to operate properly, the students are asked to abide by the following rules:

- 1. No food or drinking is permitted in the laboratory.
- 2. Single person is not allowed to do experiment, At least two persons for each station.
- 3. No equipment is allowed to be exchanged from one bench to another.
- 4. Upon entering and when leaving the laboratory students should check equipment against the list posted at each station.
- 5. All damaged or missing equipment and cables must be reported immediately to the demonstrator. Failure to do so will result in students being charged for damages or losses.
- 6. All data must be recorded in the laboratory paper and must be signed by the demonstrator.
- 7. Laboratory demonstrators are not permitted to admit any students other than those on their class list.
- 8. Any student who is more than 30 minutes late will not be permitted into the laboratory. Furthermore, repeated tardiness will not be tolerated.
- 9. After your laboratory session is completed all components, connecting jumpers, and cables must be returned to their respective places.
- 10. No students are allowed access to parts in the cabinets. Your laboratory demonstrator will provide you with all necessary parts.

# Lab 1: Introduction to Simulink

#### **Objectives**

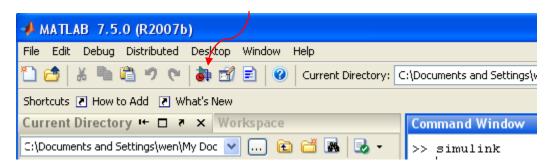
In this lab you will learn how to use Simulink to simulate dynamic systems. Simulink runs under MATLAB and uses block diagrams to represent dynamic systems. Instead of large amounts of code, you can simply drag-and-drop pre-made blocks into a "model" window and connect the blocks' inputs and outputs to create a program or simulation. The progress of the simulation can be monitored while the simulation is running, and the final results can be made available in the MATLAB workspace when the simulation is complete.

#### **Getting Started**

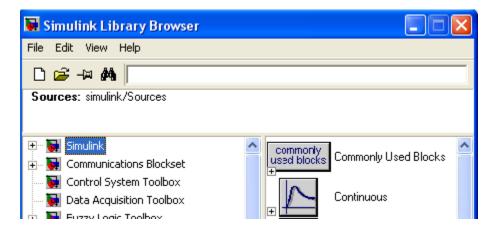
To start a Simulink session, you'd need to bring up Matlab program first. From Matlab command window, enter:

#### >> simulink

Alternately, you may click on the Simulink icon located on the toolbar as shown:

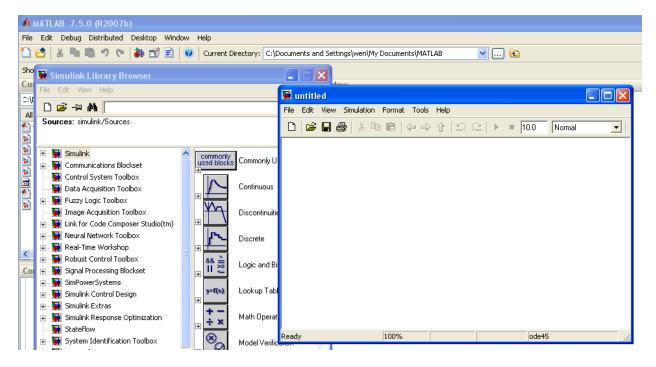


Simulink's library browser window like one shown below will pop up presenting the block set for model construction.



To start a model, click on the "NEW FILE" icon. A new window will appear on the screen. You will be constructing your model (Block Diagram) in this window. Also in this window the

constructed model is simulated. A screenshot of a typical working model (Block Diagram) window is shown below:

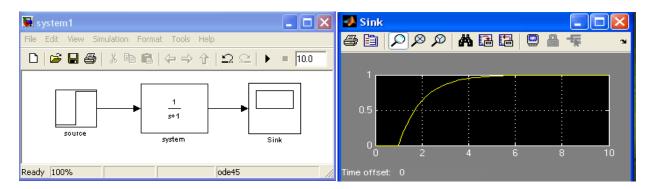


The block diagram in Simulink will have the general form shown in following Figure. They are source, system, and sink.

A simple model is used here to introduce some basic features of Simulink. Please follow the steps below to construct a simple model.

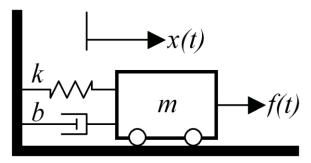
Drag the needed blocks from their library folders to above window. For examples, drag step block from source fold, scope block from sink fold, and transfer function from continuous fold.

You can interconnect them together, save this block diagram as System1 and use the default parameters to run simulink. The block diagram and simulating results are shown below.



#### Simulate a System Using Simulink

As an example for Simulink application, we will obtain the time response of the mass-spring-damper system, as shown below.



The steps required to simulate a system using Simulink are listed below.

- 1. Write the governing equations of the system.
- 2. Create an intermediate block diagram representing the mathematics of the governing equations.
- 3. Use relations between the inputs and outputs of the intermediate block diagram to create a full simulation model (all-integrator model).
- 4. Set up block and simulation parameters.
- 5. Run the simulation.
- 6. Display and analyze the results.

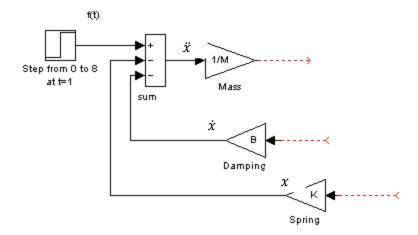
Write the Governing Equations. The first step towards defining the system in Simulink is to determine the governing equations for the system. You should verify that the equation of motion for the above system is

$$M\ddot{x} + B\dot{x} + Kx = f(t)$$
 1-1

**Create an Intermediate Block Diagram.** Rewrite equation (1-1) in the following form. The main purpose of this form is to find an equation representing the highest derivative.

$$\ddot{x} = \frac{1}{M} [f(t) - B\dot{x} - Kx]$$
 1-2

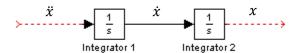
The equation (1-2) can be represented by following block diagram:



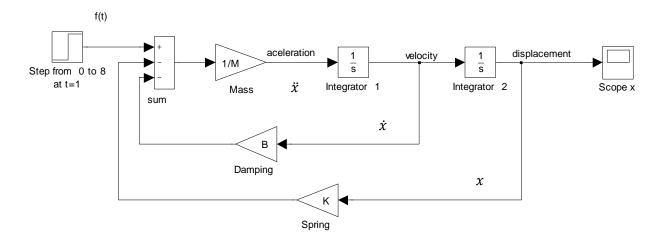
The variables x and  $\dot{x}$  can be considered as inputs to the system. The variable  $\ddot{x}$  can be considered the output of the system.

Recognize that the variables  $\dot{x}$ , x can be derived from the output, acceleration, by

 $\dot{x} = \int \ddot{x} dt$  and  $x = \int \dot{x} dt$  Thus, you can find the variables  $\dot{x}$  and x through the integration of  $\ddot{x}$ .



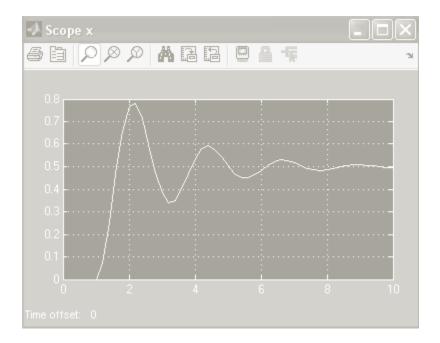
**Create Simulink Block Diagram** Now you can combine above two block diagrams to get system Simulink block diagram as following:



The data representing x and  $\dot{x}$  are fed back from the integration into the respective constants (gains). A scope is attached to the line representing x to show the position with time. In an actual block simulation, the parameters B, K, and M would be given by their respective values.

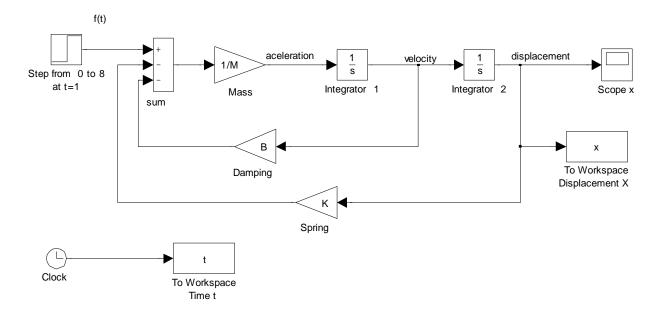
**Set Block Parameters and Simulation Routine.** After all of the connections are made, you will need to set the gains of the gain blocks to the proper values and set the initial conditions of the integrator. To set the value of the B gain block, double-click on the gain block to open the dialog box and enter the appropriate value. In the same manner, set the value of the K and M gain block. The source block labeled step must be set up: step time: 1, initial value: 0, final value: 8. Now the integration routine parameters must be set. To do so, choose menu item *Simulation* and select *Configuration Parameters*. You can choose default setting or change them as you need.

**Simulate the System.** In order to view the output as the simulation is running, open the block titled *Scope* and place its window to the side of the current untitled window so that it can be seen as the simulation is running. To run the simulation, you can select menu item *Simulation* and choose *Start*, or from *start simulation* button. You should see the output in the *Scope* window as the result is generated. You may need to hit the auto-zoom (binoculars) button to see it. The following is the simulation result with B=2, M=2, K=16, and ft change from 0 to 8 at t=1.



#### Display and analyze the results

In order to plot and analyze the system with different damp ratio B, you can modify the above block diagram by adding clock and to workspaces block as following: the *clock* can be found in the *Sources* and *To Workspace* can be found in the *Sinks*. You can double click *To Workspace Displacement* block to change the Variable Name: *x*, *Save format*: Array. You can do the same for *To Workspace Time t*. The model can be saved as spring\_damp.

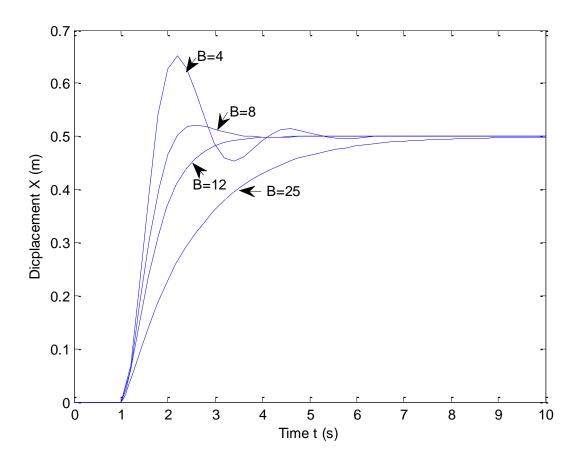


You can write Matlab M-file to plot simulation result with different damp ratio B as following;

```
%this is spring-damp system parameter
M=2; %kg
K=16; %N/m
B=4; %Ns/m
fafinal = 8; %N

sim('spring_damp')
plot(t,x);
hold on
B=8; sim('spring_damp');plot(t,x)
B=12; sim('spring_damp');plot(t,x)
B=25; sim('spring_damp');plot(t,x)
hold off
```

the above M-file can be saved as: $s_d_{plot.m}$ . If you run  $s_d_{plot}$  at Matlab commend window or click button at Editor:  $s_d_{plot}$  window. The plot is as following:



Referring to Mass- Damp-spring block diagram, we can get Mass- Damp-Spring system transfer function by Matlab script as following:

```
%spring-damp system parameter
```

M=2; %kg K=16; %N/m B=4; %Ns/m

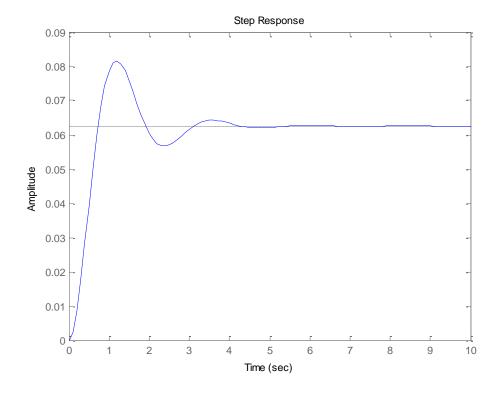
nuM=1; deM=[M 0];sysM=tf(nuM, deM) % mass and integrator 1

nuI=1; deI=[1 0];sysI=tf(nuI,deI) % integrator 2

sysMD=feedback(sysM,B) % inner loop feedback

sysMDS=feedback(series(sysMD,sysI),K) % Mass, Damp, spring system

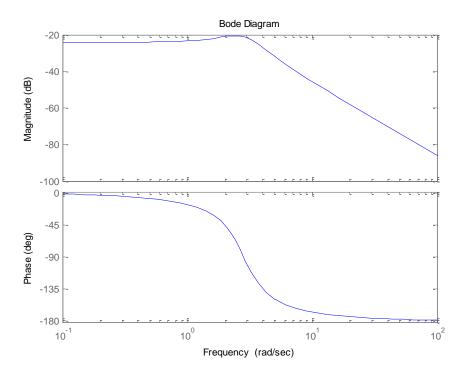
step(sysMDS,10) % step response



The simulation start time at 0, end at 10 sec. The final step fa is 1 at 0 sec.

# We can get Bode diagram by:

bode(sysMDS) % plot Bode diagram, frequence response



#### **Exercise:**

- 1. First create a Simulink model of the mass-spring-damper system. Use the following data: M = 2, B = 10, and K = 10. Perform the following tests on your model. Plot system output graphs showing the mass's position, velocity, and acceleration on one page for each step.
  - (a) Look at what happens to the system when the external force is 0 and the mass is initially defected to the left of its equilibrium position by 2 (the defection is an initial condition for an integrator). Make sure your units are right; graph scales should be in m. The system should start out at -2 m. Check that the initial position is where it should be.
  - (b) Change the damping coefficient B to 1 and look at the response to the same initial defection. Leave B at this lower value for the next exercise.
  - (c) Set all initial conditions to 0. Now apply a step input force of 1 at time 1 second to the system and print the response.
- 2. Assuming M=0, B=10, K=10. Create a Simulink model of Spring-Damp system. Perform the following tests on your model. Plot system output graphs showing the mass's position, and velocity on one page for each step.
  - (a) Look at what happens to the system when the external force is 0 and the mass is initially defected to the left of its equilibrium position by 2. The system should start out at -2 m. Check that the initial position is where it should be.
  - (b) Change the damping coefficient B to 1 and look at the response to the same initial defection.
  - (c) Set initial conditions to 0. Now apply a step input force of 6 at time 1 second to the system and print the response.

# Lab 2: Time Response of a First-order DC Motor System

## **Objectives**

To study the time response of a first-order DC motor system by observing its natural response, verify the motor specification, and compare the experimental response with computer simulation response.

### **Preliminary**

The schematic circuit of an armature-controlled DC motor is shown in Figure 2-1.

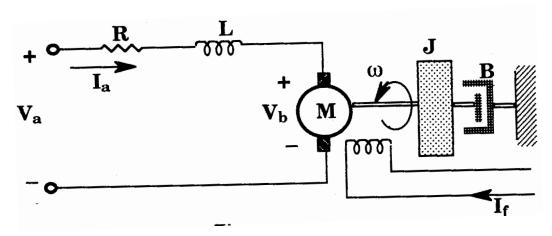


Figure 2-1

In the armature-controlled DC motor, the field current  $I_f$  is kept constant. R and L are the resistance and inductance, respectively, of the armature winding, and  $V_a$  is the input voltage applied to the armature. Since the conductors in the rotating armature cut across a magnetic field, a voltage  $V_b$  is induced in the armature according to Faraday's law of induction. The voltage  $V_b$  has a polarity which opposes the motion due to the applied control voltage and is hence called the "back-EMF". The voltage  $V_b$  is proportional to the angular velocity  $\omega$  of the motor shaft, the relationship being expressed by  $V_b = K_b \omega$ , where  $K_b$  is called the "back-emf constant". If no excessively long shafts (which will introduce the mechanical 'stiffness' element) are involved, the motor and its mechanical load may be considered as being composed of a combined inertia  $K_b$  in parallel with a viscous damping  $K_b$ . The motor develops a torque  $K_b$  (which is also the load torque  $K_b$ ) proportional to the armature current  $K_b$ , and the relationship is expressed by  $K_b$  is called the "torque constant".

The armature circuit equation is thus:

$$L\frac{dI_a}{dt} + RI_a + V_b = V_a \tag{2-1}$$

The mechanical equation for the motor is:

$$J\frac{d\omega}{dt} + B\omega = T_m \tag{2-2}$$

Equation (2-1) can be rewritten:

$$\frac{L}{R}\frac{dI_a}{dt} + I_a = \frac{V_a - V_b}{R} \tag{2-3}$$

where  $L/R = \tau_{elec}$  is an electrical time constant, The electrical time-constant  $\tau_{elec}$  is negligibly small, equation (2-3) becomes:

$$I_a = \frac{V_a - V_b}{R} \tag{2-4}$$

Equation (2-2) can be rewritten:

$$J_{\frac{d\omega}{dt}}^{\frac{d\omega}{dt}} + B\omega = K_T I_a \tag{2-5}$$

Substitute (2-4) into (5), notice that  $V_b = K_b \omega$ 

$$\frac{RJ}{BR + K_T K_b} \frac{d\omega}{dt} + \omega = \frac{K_T}{BR + K_T K_b} V_a \tag{2-6}$$

The above can be written as:

$$\tau_m \frac{d\omega}{dt} + \omega = K_{motor} V_a \tag{2-7}$$

where:  $au_m = \frac{RJ}{BR + K_T K_h}$ ,  $K_{motor} = \frac{K_T}{BR + K_T K_h}$ 

$$K_{motor} = \frac{K_T}{BR + K_T K_b} \tag{2-8}$$

Equation (2-7) can be solved for time response if  $V_a$  is a constant when t>0:

$$\omega(t) = V_a K_{motor} (1 - e^{-\frac{t}{\tau_m}}) \tag{2-9}$$

The Angular velocity response of this system to a unit step voltage is shown in Fig. 2-2.

We noticed that the angular velocity  $\omega$  reaches a steady state value  $V_aK_{motor}$  when  $t \to \infty$ . This steady state behavior is important in that if one were to ignore the transient response of the system, the motor can be viewed as a purely static system. The motor response in Figure 2-2 involves a transient effect best described in terms of time constant  $\tau_m$ . It is defined as the time it takes for the response to reach 63.2% of its steady state, or final value.

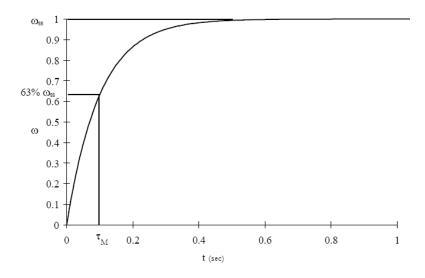


Figure 2-2

## **Experiment Outline**

In this experiment we will be examining DC motors with tachometers. As shown in Figure 2-3, DC motor has two power supply wires at the base of the motor. The speed of the motor can be varied by applying an input voltage between 0 and 12V DC (although the actual turn-on voltage is higher than 0V, which is a dead band). The rotation of a magnetic field will induce current in the windings of wires in motor (back EMF). In a similar manner, the rotation of a magnetic field will induce current in the second windings of wires in motor (tachometer), The tachometer outputs a voltage which is proportional to the angular velocity of the motor. We will check the angular velocity  $\omega$  output response with time when a step voltage input applied.

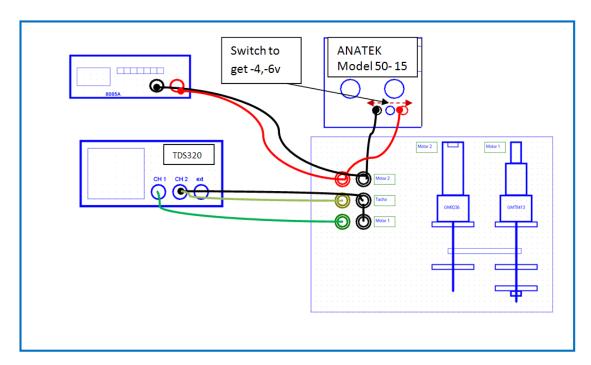


Figure 2-3

#### **Procedure:**

#### 1. DC Motor Winding Resistance

Use DMM 87 IV meter to measure the resistance of DC motor- the resistance between the motor 1's green and black leads. Make measurement with the shaft NOT moving. Observe that the resistance values will vary with motor position due to variation in the internal resistance of the brush commutator. Record the lowest measured resistance value.

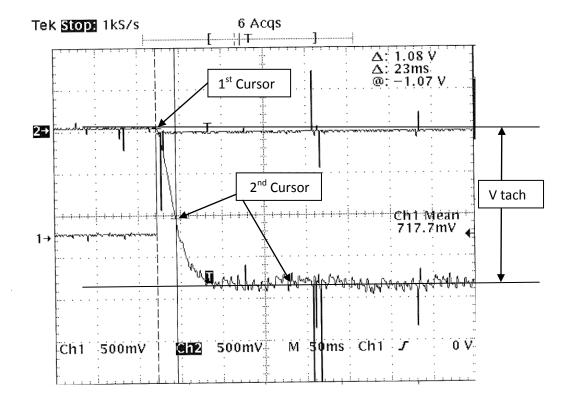
#### 2. Motor Back EMF

- 1) Check motor 1 and motor 2 are linked by belt.
- 2) Connect the motor 1's green and black leads to Scope Ch1, and tachometer's leads (yellow and black for motor 1) to Scope CH2. Turn motor by hand and observe the motor's back-emf voltage displayed on CH1. As shown in Figure 2-3.
- 3) Connect the motor 2's leads (red and black) to DC power supply, and use DMM 8010A meter to measure the supply voltage. Adjust power supply current to maximum, voltage to minimum. Adjust and check voltage slowly by read DMM: 4V, 6V (switch connection to get -4V and -6V). Measure back-emf voltage on Scope CH1, tachometer voltage on Scope CH2 respectively (read **MEAN** from scope), and fill out following table.

Motor 2 Supply voltage (DMM87 IV)	Tachometer voltage (V) CH 2 mean	Motor 1 Angular Velocity (rad/sec)	Motor 1 back-emf voltage (V) CH1 mean
-6 V			
-4 V			
4 V			
6 V			

#### 3. Motor Step Time Response

- 1) Loose motor 2's base screw under plate. Disconnect the belt between motor 1 and motor 2.
- 2) As show in Figure 2-4, with Ch1 of scope connect to Function Generator 50  $\Omega$  out and then connect to Servo-amplifier Ref in #4 and Signal GND #2, Servo-amplifier out (+ Motor, Motor) to Motor 1. Ch2 of Scope connects to Tachometer lead.
- 3) Set Function Generator AMPL to output 1V PK-PK, offset: 0~1v, square wave, frequency: 0.5 Hz, Do not turn on power supply yet. Scope: Ch1 (500mv), Ch2 (500mv), Time (50ms).
- 4) Turn on 20V power supply, a Rising (or falling) waveform will display, press Run/Stop on Scope to freeze waveform as show below. Turn off 20V power.



- 5) From Scope, Press Cursor: Paired: set  $1^{st}$  cursor to original point as show above. Press toggle button to switch to  $2^{nd}$  cursor. Moving along wave ch2 to find final stable value  $V_{tach}$ . We can also use H-bar cursor to check  $V_{tach}$  value as show above.
- 6) Refer to Figure 2.2, find the cursor of 63.2% of  $V_{tach}$ . Move  $2^{nd}$  cursor along ch2 wave, read  $\Delta:1.08V$  (if we assume  $V_{tach}=1.7V$ , 63.2% of 1.7 is 1.08), the  $\Delta:23ms$  is our time constant  $\tau_m$ . Press HARDCOPY to print two copies of the above waveform.
- 7) Repeat from step 3), adjust offset: -1~0v. Obtain time constant of reverse direction.

Function Gen. CH1 Vin	Time Const $ au_m$	Tacho Vtach CH 2	Motor ω (rad/sec)	Motor Const. Kmotor
0~1 V (real)				
-1~0 V				

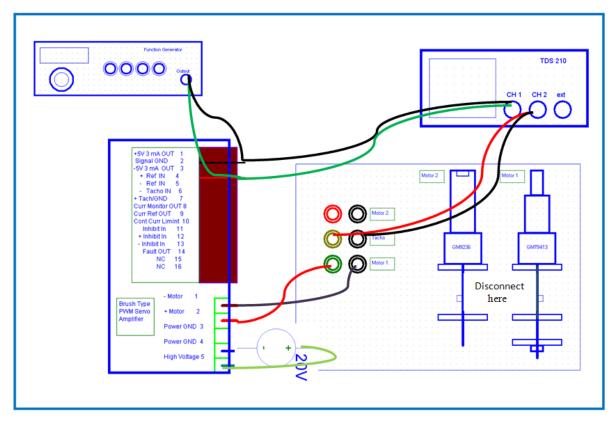


Figure 2-4

## **Experiment Results:**

1. **Motor Winding Resistance**: Determine Pittman's specification for winding resistance. Compare your value to that listed in the Pittman specification sheet for this motor.

#### 2. Motor Back EMF Constant:

- 1) Plot your data for back-emf (in volts) versus angular velocity in rad/sec.
- 2) From the plot, you can find the experimental value for motor's back-emf constant  $K_b$ , compute values for  $K_b$  in the units of votls/krpm (hint: in the SI units,  $K_b$  and  $K_T$  have identical numerical values). Compare your experimental value with the manufacturer's specification.

#### 3. Motor Step Time Response:

- 1) Determine the time-constant  $\tau$  on the DSO cursors (63.2% method). From the steady state angular velocity, the motor constant can be determined. (note: Va=Vin\*Kserv)
- 2) Using Equation (2-8), compute the time-constant from manufacturer's specification and from all your experimental value respectively. Compare these time-constants with the one obtained from experiment.

- 3) Using Equation (2-7), build block diagrams to simulate the motor with manufacturer's specification and with experimental value respectively. Compare both and compare them with experimental graph.
- 4) Combing Equation (2-1) and (2-5), build a block diagram with manufacturer's specification data to simulate the motor. Compare with step 3).

Pittman DC Motor Specification: Motor 1: GM9413

Motor Data	Symbol	Units	Value
Reference Voltage	$V_a$	V	12
Torque Constants	$K_T$	Oz-in/A (N-m/A)	5.6 (3.95E-02)
Back-EMF constant	$K_b$	V/krpm (V/rad/s)	4.14 (3.95E-02)
Resistance	R	Ω	8.33
Inductance	L	mH	6.17
Rotor Inertia	J	Oz-in-s <sup>2</sup> (kg-m <sup>2</sup> )	3.9E-04 (2.8E-06)
Elect. Time Constant	$ au_E$	ms	0.74
Mech. Time Constant	$ au_M$	ms	14.7
Viscous Damping	В	Oz-in/krpm (N-m-s)	2.8 (1.9E-04)
Reduction Ratio			5.9
Tachometer Sensitivity	$K_t$	V/krpm	2
Serv-Amp Gain	$K_{serv}$		5

# Lab 3: Time Response of a First-order Fluid System

Please follow ELEC 370 lab manual: Experiment #2.

# **Experiment Results:**

- 1-4. Please follow ELEC 370 lab manual.
  - 5. Using Simulink to simulate the system with the element values obtained in Question #3. Compare the result from the simulation with the experimental plots.

# Lab 4: Time Response of a Second-order Mechanical System

Please follow ELEC 370 lab manual: Experiment #3.

#### **Experiment Results:**

- 1-3. Please follow ELEC 370 lab manual.
  - 4. Simulate system using Matlab/Simulink, using ELEC 370 lab manual Equation (3.1), and system constants: m, k, and  $b_{min}$  from Results #3. All initial condition should be same as experimental one. Compare the simulated result with that obtained from the minimum-damping plot.

# Lab 5: Frequency Response of Passive Electrical Filters

Please follow ELEC 370 lab manual: Experiment #4.

# **Experiment Results:**

- 1-4. Please follow ELEC 370 lab manual Experiment #4
  - 5. Using circuit analyzing to get differential input-output equations for ELEC 370 Figure 4-4. Find transfer functions for them and draw Bode diagrams by Matlab M script, Compare the results with the experimental results.
  - 6. Using the transfer function of circuit in ELEC 370 Figure 4-6, draw the Bode diagram by Matlab and compare the result with the experimental one

# Lab 6: Time & Frequency Responses of a Second-Order DC Motor with Unit Feedback

### **Objectives**

To study the time and frequency responses of a second-order DC motor system by observing its responses with respect to time change and frequency sweep respectively. Compare the experimental response with computer simulation response.

### **Preliminary**

Referring to Figure 2-1 and Equation (2-7), we consider the DC motor with unit feedback system. The block diagram is shown in Figure 6-1.

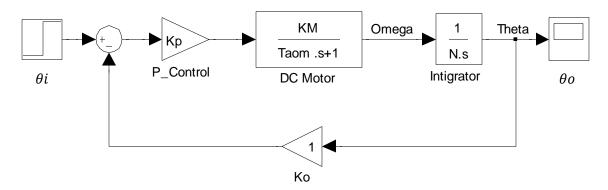


Figure 6-1

From above block diagram, we can get the closed loop transfer function:

$$\frac{\theta o(s)}{\theta i(s)} = \frac{Km}{N\tau_m S^2 + NS + Km} = \frac{\frac{Km}{N\tau_m}}{S^2 + \frac{S}{\tau_m} + \frac{Km}{N\tau_m}} = \frac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$$
where, the Natural Frequency:  $\omega_n = \sqrt{\frac{Km}{N\tau_m}}$ 
and the damping ratio:  $\zeta = \sqrt{\frac{N}{4Km\tau_m}}$ 

The dynamic behavior of the second-order DC motor system can then be described in terms of two parameters  $\zeta$  and  $\omega_n$ . If  $0 < \zeta < 1$ , the system is called under-damped, transient response is oscillatory.  $\zeta = 1$ , the system is called critical damped. Over-damped systems correspond to  $\zeta > 1$ . The transient response of critical damped and over-damped systems do not oscillate.

#### **Time Response of DC Motor Unit Feedback System:**

First, we consider time response of DC motor system to a unit-step input in three different cases:

#### (1) Under-damped case (0 $< \zeta < 1$ ):

$$\theta(t) = 1 - \frac{\omega_n}{\omega_d} e^{-\zeta \omega_n t} \sin(\omega_d t + \beta) \qquad \text{for } t \ge 0$$
 (6-2)

where  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ , the frequency  $\omega_d$  is called the damped natural frequency.  $\beta = \cos^{-1} \zeta$ . If the damping ration  $\zeta = 0$ , the response becomes undamped and oscillations continue indefinitely.

The response of zero damping is:

$$\theta(t) = 1 - \cos \omega_n t \tag{6-3}$$

Thus from Equation (6-3), we see that  $\omega_n$  represents the undamped natural frequency of the system. That is,  $\omega_n$  is that frequency at which the system would oscillate if the damping were decreased to zero. If the liner system has any amount of damping, the undamped natural frequency cannot be observed experimentally. The frequency that may be observed is the damped natural frequency  $\omega_d$ , which is equal to  $\omega_n\sqrt{1-\zeta^2}$ . This frequency is always lower than the undamped natural frequency. An increase in  $\zeta$  would reduce the damped natural frequency  $\omega_d$ . If  $\zeta$  is increased beyond unity, the response become overdamped and will not oscillate.

#### (2) Critically damped case ( $\zeta = 1$ ):

$$\theta(t) = 1 - e^{-\omega_n t} (1 + \omega_n t) \qquad \text{for } t \ge 0$$
 (6-4)

### (3) Over- damped case $(\zeta > 1)$ :

$$\theta(t) = 1 - e^{-(\zeta - \sqrt{\zeta^2 - 1})\omega_n t} \qquad \text{for } t \ge 0$$
 (6-5)

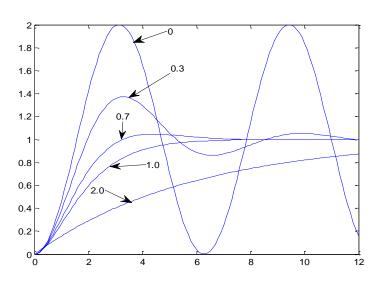


Figure 6-2

Transit time response specification are defined same as experiment 3. Referring to Figure 6-3:

 $T = \frac{2\pi}{\omega_d}$ **Period Time:** 

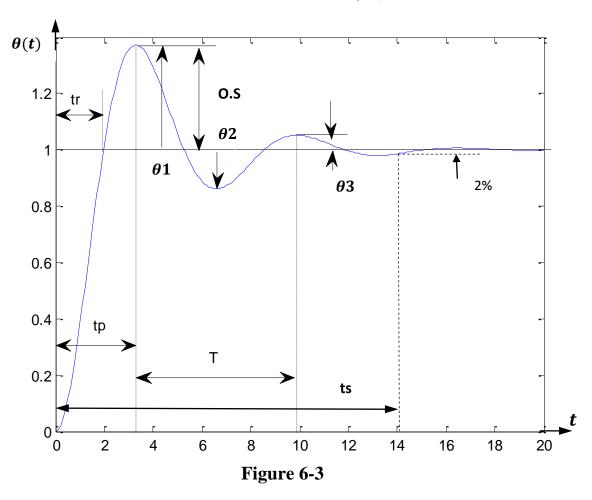
**Peak Time:** 

 $t_p$  = time taken to reach the first maximum,  $t_p \approx \frac{\pi}{\omega_d}$   $P.0 = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$ **Overshoot (P.O):** (6-6)

 $t_s \approx 4/\zeta \omega_n$ (2% settling time). **Settling Time:** 

**Logarithmic Decrement: Δ** 

$$\Delta = \ln \frac{\theta_1}{\theta_3} = \zeta \omega_n T = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$
 (6-7)



#### Frequency Response of DC Motor Unit Feedback System

The frequency response means the steady state response of a system to a sinusoidal input. The resulting output for a DC motor unit feedback system is sinusoidal in the steady state; it differs from the input waveform only in amplitude and phase angle.

Consider the DC motor described by Equation (6-1),  $\frac{\theta o(s)}{\theta i(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = G(s)$ The input  $\theta i(t)$  is sinusoidal and is given by

$$\theta i(t) = \sin \omega t$$

If the system is stable, then the output  $\theta(t)$  can be given by

$$\theta(t) = \theta \sin(\omega t + \phi)$$

Where

$$\theta = |G(j\omega)|$$

$$\phi = \angle G(j\omega) = \tan^{-1} \frac{imaginary\ part\ of\ G(j\omega)}{real\ part\ of\ G(j\omega)}$$

An example of input and output sinusoidal waveform is shown in Figure 6-4.

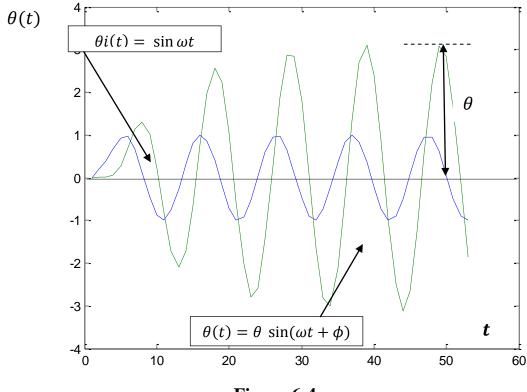


Figure 6-4

We can present frequency response characteristics in graphical forms, Bode Diagrams or Logarithmic Plots. A *Bode Diagram* consists of two graphs: one is a plot of the logarithm of the magnitude of a sinusoidal transfer function ( $20 \log |G(j\omega)|$ ), or called dB; the other is a plot of the phase angle (deg); both are plotted against the frequency in logarithmic scale. Figure 6-5 shows the Bode Diagram of DC motor unit feedback system.

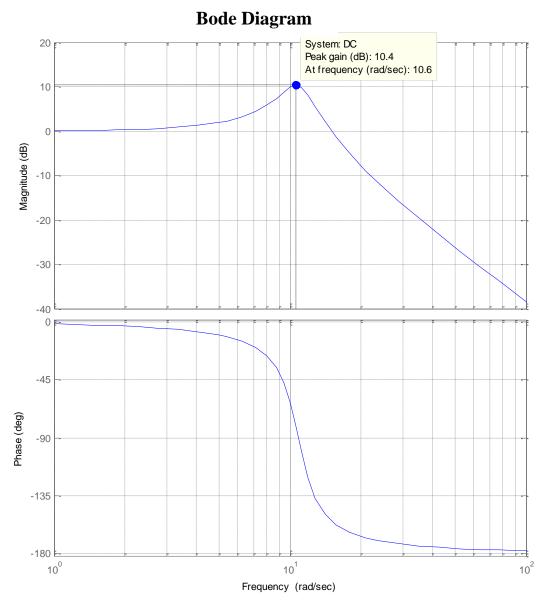


Figure 6-5

# **Experiment Outline**

In this experiment, first we will be examining time response of DC motors with unit feedback to step input, and then to sinusoidal input. As shown in Figure 6-6, we use function generator to get step and sinusoidal signal  $\theta$ i. Motor shaft position  $\theta$ o be measured by output potentiometer, The error signal  $\theta$ e =  $\theta$ i -  $\theta$ o will be adjusted through attenuator and amplified by servo amplifier. We can use Attenuator Module (AU150B) to adjust proportional gain Kp, thus change the damp ration of DC motor system, and get desired response.

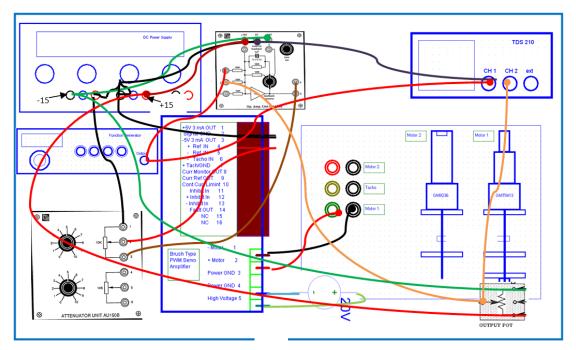


Figure 6-6

We notice that: 
$$\omega_n = \sqrt{\frac{Km}{N\tau_m}}$$
,  $= \sqrt{\frac{N}{4Km\tau_m}}$ , N=5.9,  $\tau_m = \frac{RJ}{BR + K_T K_b}$   
Km=Kp\*KM=Kp\*Kserv\*Kmotor, Where  $K_{motor} = \frac{K_T}{BR + K_T K_b}$ 

Kserv=5.

In the DC motor system, only the proportional gain Kp can be adjustable. Other parameters like motor constant,  $\tau_m$ , Kmotor (both can use value of Lab#2), and Kserv are all fixed.

# **Experiment Procedure:**

#### **Step Response:**

- 1. Connect the function generator, Attenuator module, output pot, power supply and DSO as shown in Figure 6-6. The main power supply offer +15v, 0, -15v fix voltage to Op-Amp and Output Pot. The second power offer 0, 20v fix voltage exclusively to Serve-Amplifier. DO NOT TURN ON POWER SUPPLY YET!
- 2. Connect all ground to Power Supply Ground. It includes Function generator, OP150A 0v, AU150B #1, Serv-Amplifier #2 Ref in, Output Pot ground, Scope GND.
- 3. Signal flow: Function generator 50 Ω out, (Vi signal) add –Vo (from output pot #3) by Op-Amp 150A to get Ve, error signal. Vi→#1, Vo→#2, Ve out from #6 of 150A. Ve→#3 of AU150B, the adjusted Ve (out from #2 of 150B) go to #4 of Serv-Amplifier. The amplified control signal Vc (+ motor, motor) drive motor 1.
- 4. Set function generator to squire wave, amplitude to 4V, frequency to 0.5 Hz, Scope set to: CH 1 (2v/Div), CH 2 (2V/Div). M (50ms/Div), Ch2: Invert: on. **Don't turn on power supply yet**.

- 5. Set Kp =0.6 ~ 0.8, (reading 6 at AU150B). This setting depends on your step response. Make sure 3 peaks can get from this setting. **Ask lab instructor to check your connection before turn on both power supplies.** Use Run and Stop the oscilloscope to get the wave similar to Figurer 6-3.
- 6. From oscilloscope, press "Cursor" button $\rightarrow$ H bar, Choose source CH1, Adjust cursor to measure the CH 1 voltage (i.e. Delta=5.36, this is the input voltage Vi for motor), and choose source CH 2 to measure voltage (i.e. Delta=3.36, output Pot voltage, which can be converted into angular position). Referring to Figure 6-3, find the cursor of  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$  (if it exists) and switch to V Bar, then T, tr, tp, ts can be obtained. Draw a copy of screen display.
- 7. Change Kp=0.8~1.0, repeat from step 5.

#### **Frequency Response:**

- 1. Same connection as above step #1, only change function generator to sine wave. Kp=0.4~0.6, the step response should be 10% of overshoot and has one peak.
- 2. Use scope cursor to get ch1, ch2 peak to peak, and time delay of ch1 $\rightarrow$  ch2.
- 3. Change frequency of function generator as following table, get reading as step 2.
- 4. Repeat step 2, step 3 until fill up the following table. Note that between 7 Hz and 10 Hz, find peak frequency and make more measurement.

	T				T		
Freq.	T	t	Ch 1	Ch 2	dB	Phase (deg)	$\omega=2\pi f$
Generator	period	delay	Pk-Pk	Pk-Pk	Vo Vo	$_{o}$ 360t	
(real)			Vi	Vo	$=20\log \frac{Vo}{Vi} $	$\beta = \frac{360t}{T}$	(1/sec)
1 ( )							
2 ( )							
3							
4							
5							
6							
7							
7.5							
7.75							
8							
8.25							
8.5							
8.75							
9							
10							
11							
12							

## **Experiment Results:**

the two.

#### Time response:

- 1. Calculate the damping ratio  $\zeta$  using the PO equation (Equation 6-6) as well as the logarithmic decrement (Equation 6-7) for the two under damped plots which you obtained. In each case, compare the results obtained from the two methods and discuss any differences.
- 2. Calculate  $\omega_d$  using the value of Peak Time  $T_p$ , for the two plots and hence determine  $\omega_n$ , Compare this value with the theoretical value  $\sqrt{\frac{k_m}{N\tau_m}}$  and determine the error between
- 3. Simulate system using block diagram of Figure 6-1.

N=5.9,  $\tau_m=25$  ms (experiment result of Lab#2). Km=Kp\*KM=Kp\*Kserv\*Kmotor, Where  $K_{motor}=36$ Kserv=5 Kp =0.6 and Kp=0.8. Compare the result with experimental results.

#### Frequency response:

- 1. Calculate above table of dB, Phase delay, and  $\omega$ . Draw Bode diagram.
- 2. Refer to block diagram as in Figure 6-1, write a Matlab m script to get the unit feedback system transfer function (Hint: first, find transfer function of open loop first order motor system, second, using 'feedback' command to get closed loop unit feedback system transfer function, then, using 'bode' command to draw a Bode Diagram using system parameter as in experiment result), time response #3 of  $\tau_m = 25 \, ms \, and \, K_{motor} = 36$ , let Kp=0.4.
- 3. Compare Question#2's Bode diagram with Question #1 experimental Bode Diagram, comment on it.