Simulation Lab(MC503)

Assignment-6

Try to solve all the problems

Maximum Likelihood Estimation (MLE):

Suppose we have a random sample $X_1, X_2, \ldots, X_n$ whose assumed probability distribution depends on some unknown parameters θ. MLE is a technique used for estimating the parameters θ of a given distribution using some observed data.
For example: If a population is known to follow a normal distribution but the mean and variance are unknown. MLE can be used to estimate them using a limited sample of the population, by finding particular values of the mean and variance so that the observation is the most likely result to have occurred.

Definition: Let $x_1, x_2, \ldots, x_n$ be observations from n independent and identically distributed random variables drawn from a Probability distribution that depends on some parameters θ. The MLE maximizes the likelihood function:

$$L = f(x_1, x_2, \ldots, x_n; \theta) = \prod_{i=1}^{n} f(x_i; \theta)$$

For maximization, we have

$$\frac{dL}{d\theta} = 0; \; \frac{d^2L}{d\theta^2} < 0.$$

Since logarithm is a non-decreasing function, so for maximizing L, it is equivalently correct to maximize $\log L$, i.e.

$$\frac{1}{L}\frac{dL}{d\theta} = 0 \implies \frac{d \log L}{dL} = 0.$$

In other words, the log-likelihood function is easier to work.

Problem: Find the maximum likelihood estimation(MLE) of given distribution function for the 1000 samples.

1. Distributions with the given CDF as:

   i) Lindley Distribution

   $$F_x(x;\theta) = 1 - \frac{\theta + 1 + \theta x}{\theta + 1} e^{-\theta x}, \; x > 0, \; \theta > 0.$$

   ii) Modified Weibull Distribution

   $$F_X(x, \gamma, \beta, \lambda) = 1 - e^{-\beta x^{\gamma} e^{\lambda x}}$$

2. Distributions with the given PDF as:

i) Normal Distribution

$$f_X(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x-\mu)^2}{\sigma^2}\right\},\ x\in\mathcal{R},\ \mu\in\mathcal{R},\ \sigma>0.$$

ii) Generalized exponential distribution

$$f_X(x;\alpha,\beta) = \alpha\beta e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha-1},\ x>0,\ \alpha,\beta>0.$$

iii) Kumaraswamy Distribution

$$f_X(x;\alpha,\beta) = \alpha\beta x^{\alpha-1}(1-x^{\alpha})^{\beta-1},\ x\in(0,1),\ \alpha,\beta>0.$$

* Here, first find the CDF of distribution, then apply probability integral transformation to generate the samples. To verify whether the generated sample is correct if the mean of generated samples is approximately equal to the mean of the distribution.

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