Lecture 1B – Syntactic Structure

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Programming Language

A large number of strings (typical infinite), is described by

- Syntax
- Semantics

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  - Specifies how programs in the language are built up
    E.g., context-free grammar, regular grammar, ...
- Semantics
  - Specifies what programs mean
    E.g., Operationals semantics, denotational semantics, ...

Grammar

A *finite* representation of a language, consists of

- **T**erminals
- **N**onterminals
- A **S**tart symbol
- A set of production **R**ules that defines a nonterminal in terms of a sequence of terminals or non-terminals

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$$\langle\ \mathbf{N}(\text{onterminals}),\ \mathbf{T}(\text{erminals}),\ \mathbf{S} \in \mathrm{N},\ \mathbf{R}(\text{ules})\ \rangle\ , \quad \text{where } \mathbf{N} \cap \mathbf{T} = \emptyset$$

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Context-Free Grammar (CFG): $R \subset N \times (N + T)^+$

e.g.: $\langle\{E, Int, Digit\},\ \{0, 1, ..., 9, +, *, (,)\},\ E,\ R\rangle$

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BNF (Backus-Naur Form) :

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\begin{array}{rrcl}
R = & \{\ E & \to & E + E \\
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EBNF (Extended BNF) :

What is more: { } , [ ] , | , ( )

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Real number?

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Real number?

$$\dots\dots\dots Int.Int \mid .Int$$

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$\ldots\ldots\ldots Int.Int \mid .Int \ldots\ldots\ldots:\quad [Int].Int$

Alternative Representation of Grammars

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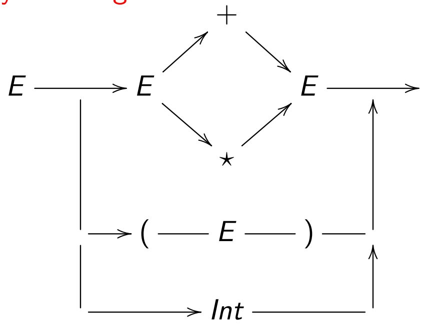
Syntax diagrams



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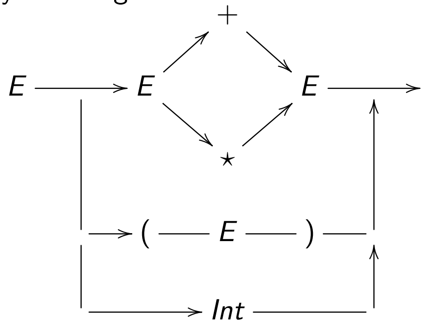

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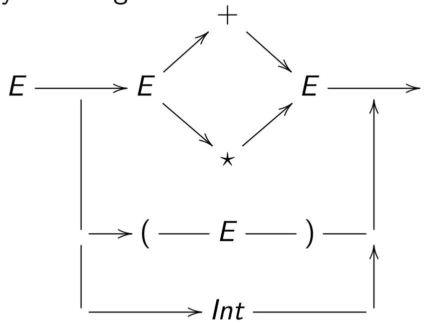

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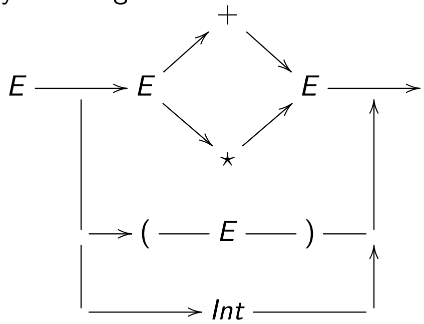

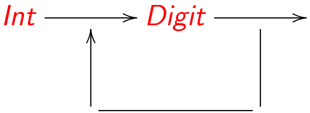

Expression Notations

Expression Notations

- Prefix
- Postfix
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Programming languages use a mix of them

Pre- and Postfix Notations (Polish and Reverse Polish)

Prefix

- **op** E_1 E_2

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Pre- and Postfix Notations (Polish and Reverse Polish)

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  $* + 2\ 3\ 4$

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Parenthesis-free

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- Ambigious to decode, e.g.,

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- Need rules for associativity and precedence

An operator can be either left or right associative

Left associative

Right associative

Associativity

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Left associative

E.g., -,+,*,/ : $1 - 2 - 3 = (1 - 2) - 3 \neq 1 - (2 - 3)$

$$E \to E - Int \mid Int$$

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E.g., expression $2^{3^2} = 2^{(3^2)} \neq (2^3)^2$

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What about . . .

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Precedence

Each precedence level requires a new non-terminal symbol, plus an extra symbol for the lowest elements:

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E &\rightarrow Int \mid (E)
\end{aligned}
$$

Precedence

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\begin{array}{lll}
E \rightarrow E + E & \qquad E \rightarrow G + E \mid G & \text{weakest} \\
E \rightarrow \ldots & \qquad .. \rightarrow \ldots & \\
E \rightarrow E * E & \qquad G \rightarrow L * G \mid L & \text{strongest} \\
E \rightarrow Int \mid (E) & \qquad L \rightarrow Int \mid (E) &
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Derive: 2 * 3 + 4 and 2 + 3 * 4

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Associativity

left: $E \rightarrow E - Int$

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Associativity

left: $E \rightarrow E - Int$ vs. right: $E \rightarrow Int^{E} \mid Int$

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Derive: 2 * 3 + 4 and 2 + 3 * 4

Associativity

left: $E \to E - Int$ vs. right: $E \to Int^E \mid Int$

On each level, all the symbols must associate equally

$E \to P * E \mid E + P$, with $P, E \to ...Int$, gives ambiguity:

2 * 3 + 4 can be (2 * 3) +4 or 2 * (3 + 4)

Mixfix Notation

Programming languages use a mix of prefix, infix and postfix notations

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```
f(x:int) {
  if x == 1
    then return x++;
    else return ++x;
}
```

Regular Language

$B \rightarrow b \mid B\ B$

Regular Language

$B \to b \mid B\ B$ vs. $B \to b \mid b\ B$

Regular Language

B $\rightarrow$ b | B B   vs.   B $\rightarrow$ b | b B

Regular language = can be defined with a **regular grammar**,
i.e., $\langle N, T, S \in N, R \rangle$, where each rule in R is of one of these forms:

Regular Language

B $\rightarrow$ b | B B   vs.   B $\rightarrow$ b | b B

Regular language = can be defined with a **regular grammar**,
i.e., $\langle N, T, S \in N, R \rangle$, where each rule in R is of one of these forms:
$N \rightarrow a$ for some $a \in T$

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$\mathrm{B} \to \mathrm{b} \mid \mathrm{B}\ \mathrm{B}$ vs. $\mathrm{B} \to \mathrm{b} \mid \mathrm{b}\ \mathrm{B}$

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- Regular grammar is always finite

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- Regular grammar is always finite

- Finite-state machines (FSM/FSA)

Ensure Unambiguity

z := 8 – 2 * 3 ;

Ensure Unambiguity

```
z := 8 – 2 * 3 ;
x := z + 7 ;
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Ensure Unambiguity

z := 8 – 2 * 3 ;
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$$\text{Pr} \to \text{Var} := \text{Pr} \mid \text{Pr} ; \text{Pr} \mid \text{E}$$

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$$Pr \rightarrow Var := Pr \mid Pr \; ; \; Pr \mid E$$

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Ensure Unambiguity

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P1 ; P2 ; P3 :	(P1 ; P2) ; P3	or	P1 ; (P2 ; P3)	?

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P1 ; P2 ; P3 :	(P1 ; P2) ; P3	or	P1 ; (P2 ; P3)	?
8 * 2 * 2 :	(8 * 2) * 2	or	8 * (2 * 2)	?

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Precedence:

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x := y := 5 :	x := (y := 5)	or	(x := y) := 5	?
x := 5 ; z := 3 :	(x := 5) ; (z := 3)	or	x := (5 ; z := 3)	?
8 − 2 * 3 :	(8 − 2) * 3	or	8 − (2 * 3)	?

Associativity:

P1 ; P2 ; P3 :	(P1 ; P2) ; P3	or	P1 ; (P2 ; P3)	?
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8 − 2 − 2 :	(8 − 2) − 2	or	8 − (2 − 2)	?

Ensure Unambiguity

$$\mathsf{Pr} \rightarrow \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \; ; \; \mathsf{Pr} \mid \mathsf{E}$$
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	prec.
_;_	0
:=	1
+,–	2
*, /	3

Ensure Unambiguity

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	prec.	assos.
_;_	0	
:=	1	
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*, /	3	

Ensure Unambiguity

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	prec.	assos.
_;_	0	right
:=	1	right
+,–	2	
*, /	3	

Ensure Unambiguity

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_;_	0	right
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	prec.	assos.
_;_	0	right
:=	1	right
+,–	2	left
*, /	3	left

$$\text{Pr} \quad \rightarrow \quad \text{St} \mid \text{St} \; ; \; \text{Pr}$$

Ensure Unambiguity

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	prec.	assos.			
_;_	0	right	Pr	$\rightarrow$	St | St ; Pr
:=	1	right	St	$\rightarrow$	V := St | E
+,–	2	left			
*, /	3	left			

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	prec.	assos.			
_;_	0	right	Pr	$\rightarrow$	St | St ; Pr
:=	1	right	St	$\rightarrow$	V := St | E
+,–	2	left	E	$\rightarrow$	E + M |E – M | M
*, /	3	left			

Ensure Unambiguity

$$\mathsf{Pr} \to \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \ ; \ \mathsf{Pr} \mid \mathsf{E}$$
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	prec.	assos.			
_;_	0	right	Pr	$\to$	St | St ; Pr
:=	1	right	St	$\to$	V := St | E
+,–	2	left	E	$\to$	E + M |E – M | M
*, /	3	left	M	$\to$	M * F | M / F | F
			F	$\to$	int | V | (E)

Ensure Unambiguity

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_;_	0	right	Pr	$\rightarrow$	St | St ; Pr
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_;_	0	right	Pr	$\rightarrow$	St | St ; Pr
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			F	$\rightarrow$	int | V | (E)

Parse Tree (Concrete Syntax Tree)

- A representation of grammars in a tree-like form
- Pictorially shows how the start symbol of a grammar derives a string in the language
- Provide syntactic information

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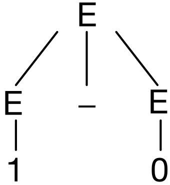
E ::= E − E | 0 | 1

Expression: 1 − 0



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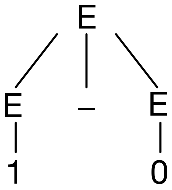
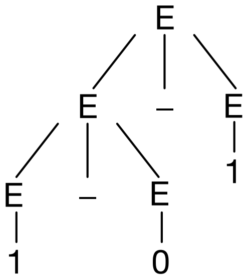


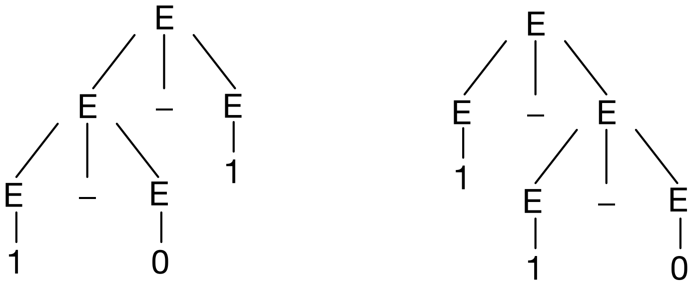
What about for 1 − 0 − 1

What about for $1 - 0 - 1$



Syntactical Ambiguity

What about for $1 - 0 - 1$



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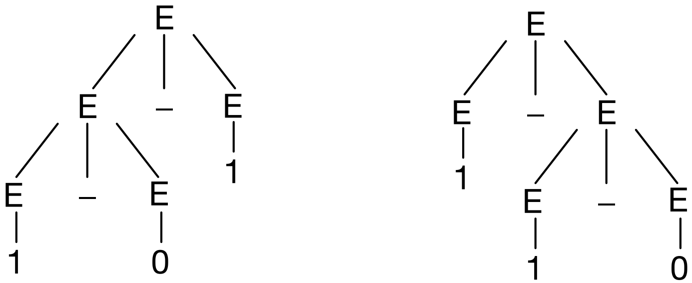

Two or more parse trees $\Rightarrow$ ambigious

Parse trees can be very complicated:

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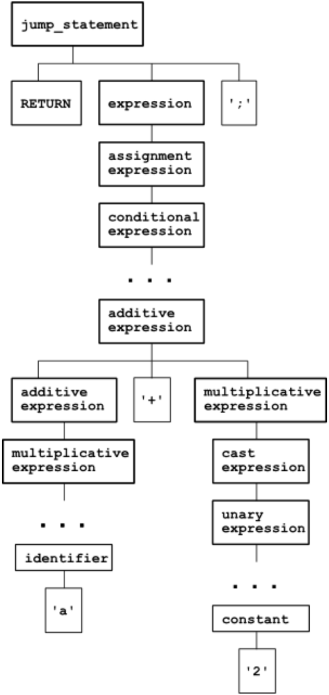
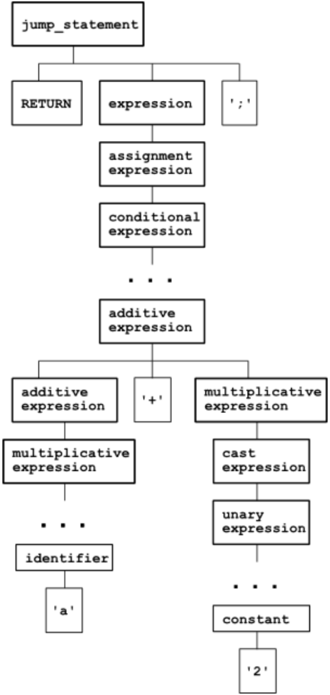


Parse trees can be very complicated:

E.g., the C statement, **return** a + 2;

- Hard to analyse, and processing



Abstract Syntax Tree (AST)

- A simplified syntactic representation of the source code
- Different from parse tree:
  - Does not show the whole syntactic mess
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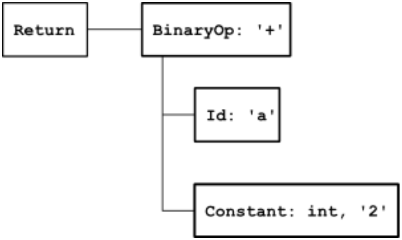
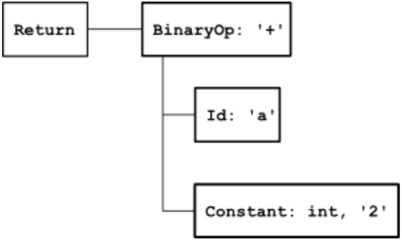
  - Pictorises the parsed string in a structured way



- A simplified syntactic representation of the source code
- Different from parse tree:
  - Does not show the whole syntactic mess
  - Pictorises the parsed string in a structured way



CST to AST

$$\mathsf{Pr} \rightarrow \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \ ; \ \mathsf{Pr} \mid \mathsf{E}$$
$$\mathsf{E} \rightarrow \mathsf{E} \ (+ \mid - \mid * \mid / \ ) \ \mathsf{E} \mid \mathsf{V} \mid \mathsf{int} \mid (\mathsf{E})$$

$$\mathsf{Pr} \to \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr}\ ;\ \mathsf{Pr} \mid \mathsf{E}$$
$$\mathsf{E} \to \mathsf{E}\ (\ + \mid - \mid * \mid /\ )\ \mathsf{E} \mid \mathsf{V} \mid \mathsf{int} \mid (\mathsf{E})$$

	prec.	assos.			
_;_	0	right	Pr	$\to$	St | St ; Pr
:=	1	right	St	$\to$	V := St | V := E
+,–	2	left	E	$\to$	E + M | E – M | M
*, /	3	left	M	$\to$	M * F | M / F | F
			F	$\to$	int | V | (E)

Remove left recursion (generally)

$$\mathsf{Pr} \to \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \ ; \ \mathsf{Pr} \mid \mathsf{E}$$
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*, /	3	left	M	$\to$	M * F | M / F | F
			F	$\to$	int | V | (E)

Remove left recursion (generally)

E $\to$ **E Rs** | **Trm**

$$\mathrm{Pr} \to \mathrm{V} := \mathrm{Pr} \mid \mathrm{Pr}\ ;\ \mathrm{Pr} \mid \mathrm{E}$$
$$\mathrm{E} \to \mathrm{E}\ (\ +\ \mid\ -\ \mid\ *\ \mid\ /\ )\ \mathrm{E} \mid \mathrm{V} \mid \mathrm{int} \mid (\mathrm{E})$$

	prec.	assos.			
_;_	0	right	Pr	$\to$	St | St ; Pr
:=	1	right	St	$\to$	V := St | V := E
+,–	2	left	E	$\to$	E + M | E – M | M
*, /	3	left	M	$\to$	M * F | M / F | F
			F	$\to$	int | V | (E)

Remove left recursion (generally)

E $\to$ **E Rs** | **Trm** becomes **E** $\to$ **Trm New** and **New** $\to$ **Rs New** | ε

$$\mathsf{Pr} \to \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr}\ ;\ \mathsf{Pr} \mid \mathsf{E}$$
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			F	$\to$	int | V | (E)

Remove left recursion (generally)

E $\to$ **E Rs** | **Trm** becomes **E** $\to$ **Trm New** and **New** $\to$ **Rs New** | ε

e.g., E $\to$ M New

$$\mathsf{Pr} \rightarrow \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \; ; \; \mathsf{Pr} \mid \mathsf{E}$$
$$\mathsf{E} \rightarrow \mathsf{E} \; (+ \mid - \mid * \mid / \;) \; \mathsf{E} \mid \mathsf{V} \mid \mathsf{int} \mid (\mathsf{E})$$

	prec.	assos.			
_;_	0	right	Pr	$\rightarrow$	St | St ; Pr
:=	1	right	St	$\rightarrow$	V := St | V := E
+,–	2	left	E	$\rightarrow$	E + M | E – M | M
*, /	3	left	M	$\rightarrow$	M * F | M / F | F
			F	$\rightarrow$	int | V | (E)

Remove left recursion (generally)

E $\rightarrow$ **E Rs** | **Trm** becomes **E** $\rightarrow$ **Trm New** and **New** $\rightarrow$ **Rs New** | ε

e.g., E $\rightarrow$ M New and New $\rightarrow$ + M New | – M New | ε

$$\mathsf{Pr} \to \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \; ; \; \mathsf{Pr} \mid \mathsf{E}$$
$$\mathsf{E} \to \mathsf{E} \; (+ \mid - \mid * \mid / \;) \; \mathsf{E} \mid \mathsf{V} \mid \mathsf{int} \mid (\mathsf{E})$$

	prec.	assos.			
_;_	0	right	Pr	$\to$	St | St ; Pr
:=	1	right	St	$\to$	V := St | V := E
+,–	2	left	E	$\to$	E + M | E – M | M
*, /	3	left	M	$\to$	M * F | M / F | F
			F	$\to$	int | V | (E)

Remove left recursion (generally)

E $\to$ **E Rs** | **Trm** becomes **E** $\to$ **Trm New** and **New** $\to$ **Rs New** | ε

e.g., E $\to$ M New and New $\to$ + M New | – M New | ε

change associativity to right!

$$\mathsf{Pr} \to \mathsf{V} := \mathsf{Pr} \mid \mathsf{Pr} \ ; \ \mathsf{Pr} \mid \mathsf{E}$$
$$\mathsf{E} \to \mathsf{E} \ (+ \mid - \mid * \mid / \ ) \ \mathsf{E} \mid \mathsf{V} \mid \mathsf{int} \mid (\mathsf{E})$$

	prec.	assos.			
_;_	0	right	Pr	$\to$	St | St ; Pr
:=	1	right	St	$\to$	V := St | V := E
+,–	2	left	E	$\to$	E + M | E – M | M
*, /	3	left	M	$\to$	M * F | M / F | F
			F	$\to$	int | V | (E)

Remove left recursion (generally)

E $\to$ **E Rs** | **Trm** becomes **E** $\to$ **Trm New** and **New** $\to$ **Rs New** | ε

e.g., E $\to$ M New and New $\to$ + M New | – M New | ε

change associativity to right! – manipulation of left-recursion requires something more than 'recursive descent parsing'