Lecture 2 – Programming in Haskell

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What is it?

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Generally speaking

- Supports and encourages programming in a functional style

Functional Language

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- Functional programming

  – the basic method of computation is

  the application of functions to arguments

Functional Programming

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- Result depends only on the result of expressions
  no side-effects, not assignment to variables
- Implicit memory-management (including ‘garbage collection’)
- Functions are first-class objects

Background

1930s:

Alonzo Church

Developed the *lambda calculus*, a simple but powerful theory of **functions**

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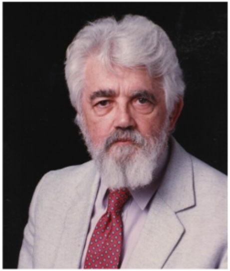
1950s:



John McCarthy

Developed *Lisp*

- The first functional language
- Some influences from the lambda calculus
- Variable assignments

FP language families

- LISP (1958), untyped, dialects:
    - Common Lisp
    - Scheme
- ML (1978), typed, dialects:
    - CAML, OCAML,
    - SMLNJ
    - Haskell

Useful resources

- Haskell homepage
  - http://www.haskell.org
- Learn in 10 minutes
  - http://www.haskell.org/haskellwiki/Learn_Haskell_in_10_minutes
- Real World Haskell
  - http://www.realwordlhaskell.org
- Programming in Haskell, slides
  - http://www.cs.nott.ac.uk/~gmh/book.html

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int total = 0;
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Haskell

```
sum [1..10]
```

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  - Type checking, type derivation
  - 'garbage collection'

Useful things (ghci/Hugs)

Programs are written in single file (text editor), and
the file is loaded (from ghci/Hugs) with
`:load filename.hs`, or
`:l filename` (can use `:cd` *dir* to change to source file directory)

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snd x y = x
plus x y = x + y
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One can use let
to define functions and variables "local" in
this session of ghci

- let `inc n = n + 1` . . . . – try "inc 13"
- let `two = 2` . . . . – try "inc two"
- let `fst (x,y) = x` . . . . – Obs: what do the parentheses mean?
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  where y = -1 .... error

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- **let** `y = x*x + x + 1` **where** `x = 2` .... ghci!
- **let** `x = 2 ; x = -1` **in** `x` .... error
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- `f 2` . . . . error

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- `let y =x*x ; x = 2 in y*y`

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- **let** `x = 2 ; y =x*x` **in** `y*y` 16
- **let** `y =x*x ; x = 2` **in** `y*y` . . . . 16 – lazy (!)

Evaluation

- `two = 2`
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Evaluation – lazy: "call by need", "outside-in"

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Parentheses!!! f a b $\neq$ f(a,b)

assume a :: A and b :: B

```
f :: A -> B -> ?
```

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a = b + c
      where
        b = 2
        c = 3
d = a * 2
	a = b + c
	      where
	        { b = 2 ; c = 3 }
	
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implicit	explicit

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implicit	explicit with {_}, not (_)

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No loops...

$$\text{fac n} = \prod_{i=1}^{i=n} i$$

```
fac n =
  res := 1
  for (i:=1, i++, n)
     res := res * n
  return res
```

...Only recursion

- `fac n = if n <= 0 then 1 else n * fac (n-1)`

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  fib n = if n <= 1 then 1
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- Important difference between the last two: running time
  ```

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- head :: [a] $\rightarrow$ a — head [1,2,3,4,5] $=$ 1
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- tail :: $[a] \to [a]$ — tail [1,2,3,4,5] $=$ [2,3,4,5]
- $++$ :: $[a] \to [a] \to [a]$ — [1,2,3]$++$[4,5] $=$ [1,2,3,4,5]
- length :: $[a] \to$ Int — length [1,2,3,4,5] $= 5$

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- reverse :: [a] $\rightarrow$ [a] — reverse [1,2,3,4,5] = [5,4,3,2,1]

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- ++ :: [a] $\rightarrow$ [a] $\rightarrow$ [a] — [1,2,3]++[4,5] = [1,2,3,4,5]
- length :: [a] $\rightarrow$ Int — length [1,2,3,4,5] = 5
- reverse :: [a] $\rightarrow$ [a] — reverse [1,2,3,4,5] = [5,4,3,2,1]
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- !! :: [a] $\rightarrow$ Int $\rightarrow$ a — [5,6,7,8] !! 2 = 7

Some list functions

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- $++$ :: [a] $\to$ [a] $\to$ [a] — [1,2,3]$++$[4,5] $=$ [1,2,3,4,5]
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  list !! length list — <u>*** Exception. Prelude(!!): Index too large</u>
- sum :: Num a => [a] $\rightarrow$ a — sum [1,2,3,4] = 10
- product :: Num a => [a] $\rightarrow$ a — product [1,2,3,4] = 24

Examples

- last $[x] = x$

Examples

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  last (x:y:ys) = last (y:ys)

Examples

- last [x] = x
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Examples: list division

- divide the list into 2 (almost) even parts

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  di [1,2,3,4,5,6] = ([1,3,5],[2,4,6])

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ms [] = []
ms [x] = [x]
ms xs = let  (aa,bb) = split xs              – split:: [a] -> ([a],[a])
           in merge (ms aa) (ms bb)
```

```
split [] = ( [], [] )
split xs = let h = (length xs) ‘div‘ 2 in ( take h xs, drop h xs )
```

Example - quicksort

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   grtr x (y:ys) = **if** (x<=y) **then** y:grtr x ys **else** grtr x ys

B. qs (x:xs) = qs (less x xs) ++ [x] ++ qs (grtr x xs)

C. less x ys = [y | y <– ys, y < x]

qs (x:z) = qs [y | y <– z, y < x] ++ [x] ++ qs [y | y <– z, y >=x]

Example - quicksort

qs [] = []

? find smaller/larger (than pivot) elements from the list:

A. ~~less x ls = for i in [0..length(ls)-1] ...~~

B. less x [] = []
less x (y:ys) = **if** (x > y) **then** (y:less x ys) **else** (less x ys)

B+. grtr x [] = []
grtr x (y:ys) = **if** (x<=y) **then** y:grtr x ys **else** grtr x ys

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QuickCheck – quicksort

- import Test.QuickCheck – at the top of the program

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Is the result sorted?

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  sorted [x] = True

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Is the result sorted?

- sorted [] = True

  sorted [x] = True

  sorted (x:y:xs) = x <= y && sorted (y:xs)

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> quickCheck (\li -> sorted (qs li))

- import Test.QuickCheck – at the top of the program

```
qs (x:z) = qs [ y | y <- z, y < x ] ++ [x] ++ qs [ y | y <- z, y >= x ]
```

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sorted [x] = True
sorted (x:y:xs) = x <= y && sorted (y:xs)
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+++ OK, passed 100 tests.
```

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- quickCheck generates random test data for most of the types (in particular, for basic types like Int, [Int], Char, String)

QuickCheck – mergesort

1. merge two sorted lists into a sorted list:

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merge xs [] = xs
merge (x:xs) (y:ys) =
        if x<y then x:(merge xs (y:ys)) else y:(merge (x:xs) ys)
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   split [] = ([], [])
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2. split a list into two parts:

```
split [] = ( [], [] )
split xs = let h = (length xs) `div` 2 in ( take h xs, drop h xs )
```

3. mergesort:

```
ms [] = []
ms [x] = [x]
ms xs = let (aa,bb) = split xs in merge (ms aa) (ms bb)
```

quickCheck – mergesort. . .

4. quickCheck (\li -> sorted (ms li))

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5. properties can be collected in the file – name should start with prop_

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```
prop_merge x y =
      if (sorted x && sorted y) then sorted (merge x y)
                                else True
         where types = x::[Int]
```

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prop_merge x y =
        if (sorted x && sorted y) then sorted (merge x y)
                  else True
           where types = x::[Int]
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```
> quickCheck prop_merge
```