

Lecture 5 – Parsing

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What is parsing?

Parsing

- ▶ A process of **analysing** a piece of text to determine its *syntactic structure*.

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Parser

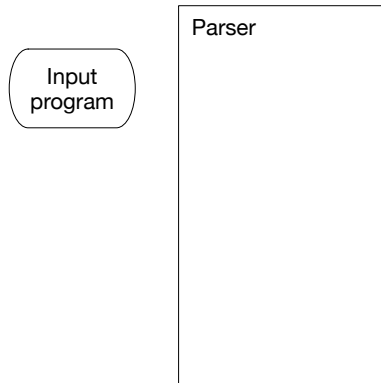
- ▶ A program that makes such syntactic analyses
 - takes some input data, usually text (e.g., program)
 - builds a data structure, like parse tree, abstract syntax tree

Parsing a program



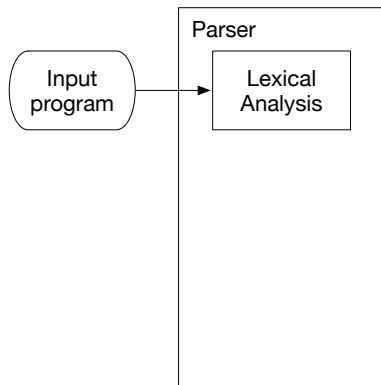
Input
program

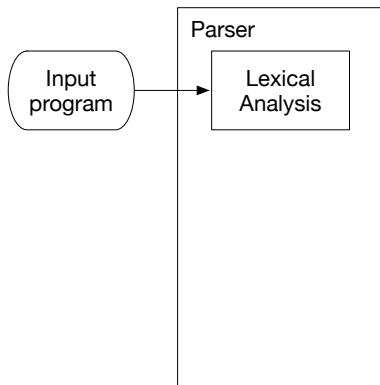
Parsing a program



Two steps

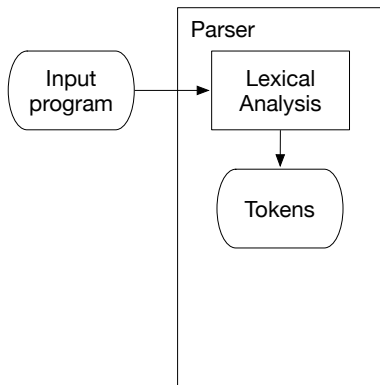
Lexical Analysis (lexer)





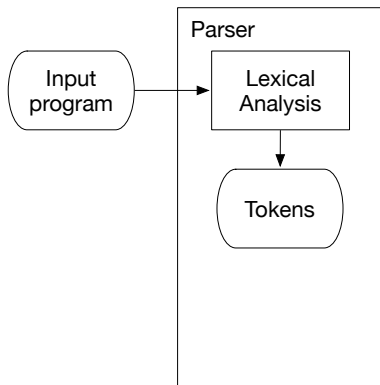
Lexical Analysis (lexer)

- Split into meaningful **terminals**



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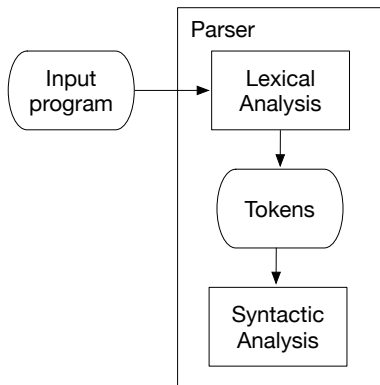


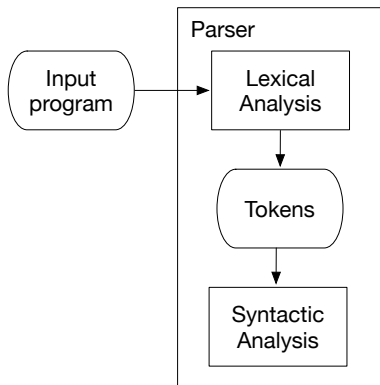
Lexical Analysis (lexer)

– Split into meaningful **terminals**

E.g.: "3 + 2" → "3", "+", "2"

Syntactic Analysis

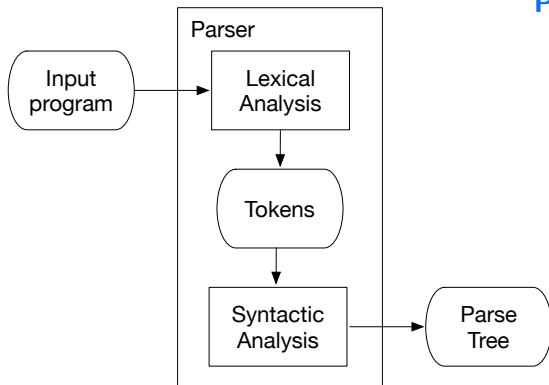




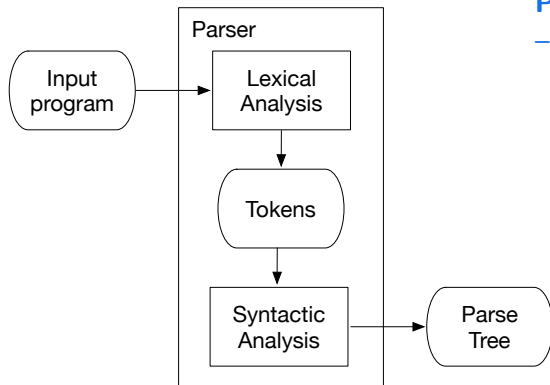
Syntactic Analysis

- Check whether the tokens form an illegal expression

Parse tree



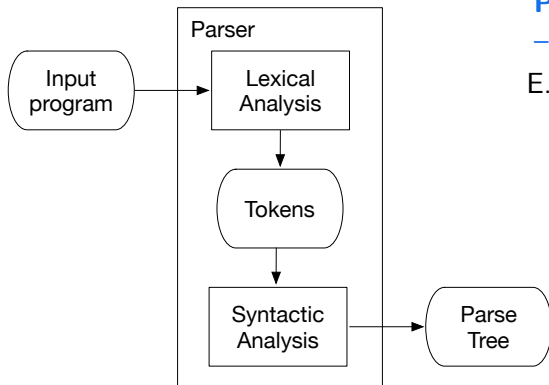
Parsing a program



Parse tree

– A data structure

Parsing a program

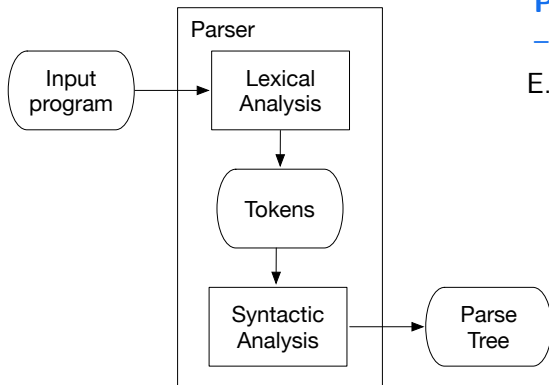


Parse tree

– A data structure

E.g.: “3”, “+”, “2”

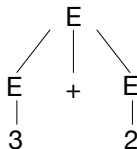
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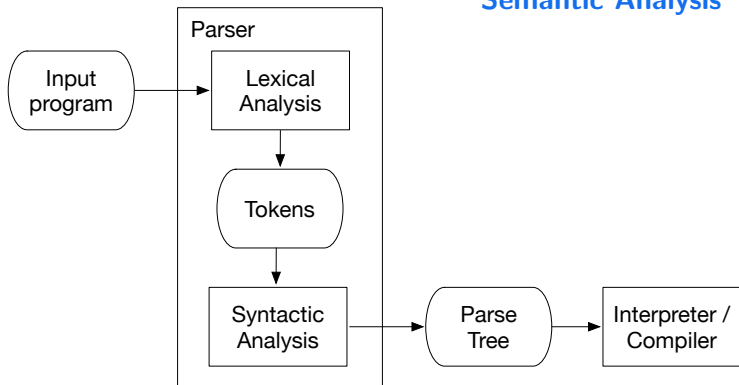
Parse tree

– A data structure

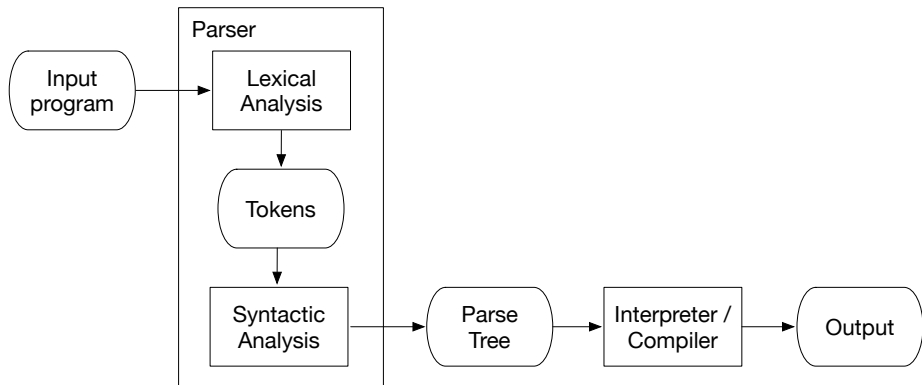
E.g.: "3", "+", "2"



Semantic Analysis



Parsing a program



Task

- ▶ To decide whether an input *can be* derived and *how* to derive from a starting symbol

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- ▶ Bottom-up

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- ▶ Top-down
 - start from the starting non-terminal
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 - attempt to find the leftmost derivation
 - may require *backtracking*
 - predictive grammar \leftarrow no backtracking
- ▶ Bottom-up

Task

- ▶ To decide whether an input *can be* derived and *how* to derive from a starting symbol

Mainly two ways:

- ▶ Top-down
 - start from the starting non-terminal
 - pick a production rule and try to match the input string
 - attempt to find the leftmost derivation
 - may require *backtracking*
 - predictive grammar \leftarrow no backtracking
- ▶ Bottom-up
 - start from the input string
 - locate the most basic elements, then
 - find the elements containing those identified ones, until
 - the starting non-terminal is reached

Top-down parsing

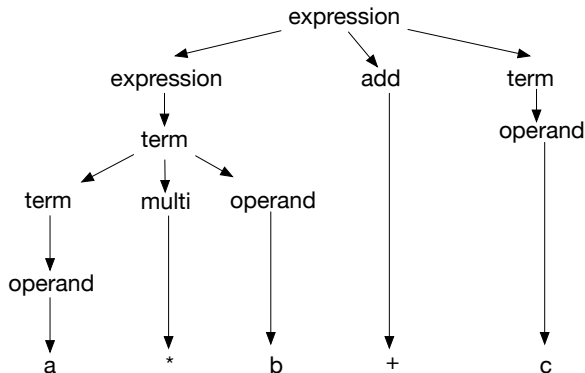
To parse “a * b + c”

expression ::= term | expression add term
term ::= operand | term multi operand
operand ::= a | b | c

add ::= +
multit ::= *

Top-down parsing

To parse “a * b + c”



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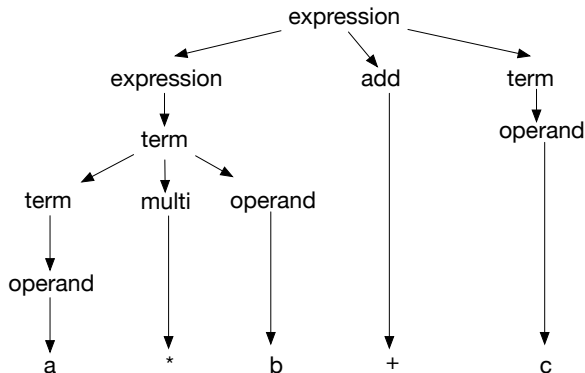
operand ::= a | b | c

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Top-down parsing

To parse “a * b + c”



- Recursive descent parser
- LL parser

$\text{expression} ::= \text{term} \mid \text{expression add term}$

$\text{term} ::= \text{operand} \mid \text{term multi operand}$

$\text{operand} ::= a \mid b \mid c$

$\text{add} ::= +$

$\text{multi} ::= *$

Bottom-up parsing

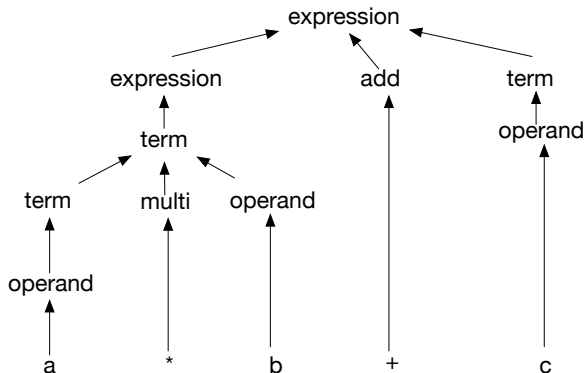
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Bottom-up parsing

To parse “a * b + c”



$\text{expression} ::= \text{term} \mid \text{expression add term}$

$\text{term} ::= \text{operand} \mid \text{term multi operand}$

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Some useful functions

```
takeWhile ::
```

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`takeWhile ::`

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Some useful functions

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takeWhile :: (a -> Bool) -> [a] -> [a]
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takeWhile (< 4) [1,2,1,4,1,2,3,4] ..... [1,2,1]
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dropWhile ::
```

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dropWhile ::
dropWhile (< 3) [1,2,3,4,5,1,2,3]
```

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Some useful functions

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`dropWhile (< 3) [1,2,3,4,5,1,2,3] [3,4,5,1,2,3]`

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`takeWhile p (x:xs)`

`| p x = x : takeWhile p xs`

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`dropWhile :: (a -> Bool) -> [a] -> [a]`

`dropWhile (< 3) [1,2,3,4,5,1,2,3] [3,4,5,1,2,3]`

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Some useful functions

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dropWhile :: (a -> Bool) -> [a] -> [a]
dropWhile (< 3) [1,2,3,4,5,1,2,3] ..... [3,4,5,1,2,3]
dropWhile (< 0) [1,2,3] ..... [1,2,3]

dropWhile _ [] = []
dropWhile p xs@(x:xs')
  | p x      = dropWhile p xs'
  | otherwise = xs
```

Some useful functions

`id ::`

Some useful functions

```
id ::  
id 3
```

Some useful functions

```
id ::
```

```
id 3.....3
```

Some useful functions

```
id ::
```

```
id 3.....3
```

```
id (1,2)
```

Some useful functions

```
id ::
```

```
id 3.....3
```

```
id (1,2).....(1,2)
```


Some useful functions

```
id :: a -> a                                identity function
id 3.....3
id (1,2).....(1,2)
```

Some useful functions

```
id :: a -> a                                identity function
id 3.....3
id (1,2).....(1,2)

const ::
```

Some useful functions

`id :: a -> a` `identity function`

`id 3.....3`

`id (1,2).....(1,2)`

`const ::`

`const 1 2`

Some useful functions

`id :: a -> a` `identity function`

`id 3.....3`

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`const 1 2.....1`

Some useful functions

`id :: a -> a` `identity function`

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`const 1 2.....1`

`const 10 'a'`

Some useful functions

`id :: a -> a` `identity function`

`id 3.....3`

`id (1,2).....(1,2)`

`const ::`

`const 1 2.....1`

`const 10 'a'.....10`

Some useful functions

`id :: a -> a` `identity function`

`id 3.....3`

`id (1,2).....(1,2)`

`const :: a -> b -> a` `constant function`

`const 1 2.....1`

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Some useful functions

`id :: a -> a` `identity function`

`id 3.....3`

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`const 1 :: ???`

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`id (1,2).....(1,2)`

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`const 1 2.....1`

`const 10 'a'.....10`

`const 1 :: b -> a`

Some useful functions

`id :: a -> a` `identity function`

`id 3.....3`

`id (1,2).....(1,2)`

`const :: a -> b -> a` `constant function`

`const 1 2.....1`

`const 10 'a'.....10`

`const 1 :: Num a => b -> a`

Example – a dictionary

As a list of pairs

type Dict = [(Key, Value)] for some types **Key** and **Value**

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emptyDict =

Example – a dictionary

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type Dict = [(Key, Value)] for some types **Key** and **Value**

As a function

type Dict = Key -> Value

emptyDict :: Dict

emptyDict = _ -> **Nothing**

Example – a dictionary

As a list of pairs

type Dict = [(Key, Value)] for some types **Key** and **Value**

As a function

type Dict = Key -> Value

emptyDict :: Dict

emptyDict = **const Nothing**

Example – a dictionary

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type Dict = [(Key, Value)] for some types **Key** and **Value**

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type Dict = Key -> **Maybe** Value

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Example – a dictionary

As a list of pairs

type Dict = [(Key, Value)] for some types **Key** and **Value**

As a function

type Dict = Key -> **Maybe** Value

emptyDict :: Dict

emptyDict = **const Nothing**

lookup ::

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lookup d k =

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lookup d k = d k

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add d k v = \x -> **if** x == k

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Example – a dictionary

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e.g. [“012”, “*”, “(”, “3”, “+”, “11”, “)”]

Lexical analysis – tokenize :: String -> [String]

- ▶ “012 * (3 + 11)” must be divided into meaningful substrings (tokens)
e.g. [“012”, “*”, “(”, “3”, “+”, “11”, “)”]
- ▶ tokenize [] = []

Lexical analysis – tokenize :: String -> [String]

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- ▶ tokenize [] = []
tokenize (' ':xs) = tokenize xs

Lexical analysis – tokenize :: String -> [String]

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tokenize (' ':xs) = tokenize xs
tokenize ('(' :xs) = "(" : tokenize xs

Lexical analysis – tokenize :: String -> [String]

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tokenize '('('':xs) = "(" : tokenize xs

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- ▶ tokenize [] = []
 - tokenize (' ':xs) = tokenize xs
 - tokenize ('(':xs) = "(" : tokenize xs
 - tokenize (')':xs) = ")" : tokenize xs
 - tokenize (x:xs) = if isDigit x then –

Lexical analysis – tokenize :: String -> [String]

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tokenize (x:xs) = if isDigit x then –
collect all digits until something which is not a digit ...

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(takeWhile isDigit (x:xs)) : tokenize (dropWhile isDigit xs)

Lexical analysis – tokenize :: String -> [String]

▶ “012 * (3 + 11)” must be divided into meaningful substrings (tokens)
e.g. [“012”, “*”, “(”, “3”, “+”, “11”, “)”]

▶ tokenize [] = []

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etc. ...

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make a list with simple signs that **are** tokens:

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t = "*+()" == ['*', '+', '(', ')']
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  | elem x t = [x] : tokenize xs t s
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  | otherwise = (takeWhile (notin t++s) (x:xs)) :  
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Recursive descent parsing

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AST:

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`parse :: [String] -> (Ast, [String])`

`E ::= T + E | T` `parseE (s)`

`T ::= F * T | G` `parseT (s)`

`G ::= - G | F` `parseG (s)`

`F ::= int | (E)` `parseF (s)`

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`data Ast = V Int | N Ast | A Ast Ast | M Ast Ast`

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$T ::= F * T \mid G$	parseT (s)	M Ast Ast
$G ::= - G \mid F$	parseG (s)	N Ast
$F ::= \text{int} \mid (E)$	parseF (s)	V Ast

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– error "expect +"

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LL grammar – allows a parser to read input from Left and construct a Leftmost derivation

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- ▶ LL(k) parsers does not require backtracking