Homework 0: The Single-Period Binomial Model

Introduction

The purpose of this "homework" problem set is to help you master the single-period binomial option pricing model. It is important that you have facility with the model numerically and theoretically.

Please submit your answers in a Quarto document with Julia code using `motoro.jl`.

Numerical Workout Problems

1. Please do the following from McDonald Chapter 10
    - 10.1 - 10.15
    - 10.20 - 10.23

Theoretical and Philosophical Problems

2. Draw the binomial tree for problem 10.20 above.
    - What is the probability law for this problem?
    - What is the experiment being conducted? By whom?
    - What is the sample space for this problem?
    - What is the probability mass function?
    - In what sense is the probability law a prediction model?
    - What is the empirical part of this problem?
    - Does Tom Sargent's "communism of models" apply to this problem? Why or why not? Explain in depth.
3. Again using problem 10.20 above.

- Outline the single-period option pricing model as a numerical algorithm. Carefully enumerate and label each of the steps.
- With your algorithm in place, please carefully explain the logic of no-arbitrage.
- What is a boundary condition? How and where does it apply in this model?
- What are the knowledge requirements for this no-arbitrage logic?
- Explain carefully the economic and probabilistic logic of the no-arbitrage pricing equation. How are they connected?

4. What are the two probability density (mass) functions of concern in the single-period binomial model?

   - Explain both models carefully.
   - How is the second concept of probability derived from the first? Why is it needed?
   - Explain carefully the economic and probabilistic logic of the risk-neutral pricing equation. What is its economic meaning? What are its probability foundations? Why is it useful?

5. Carefully explain the logic of American option pricing using the single-period binomial option pricing model.

   - Use both the no-arbitrage and risk-neutral pricing equations.
   - Explain the American option pricing problem as an "optimal stopping" problem.