NB: IID = Independent, Identically Distributed

Bootstrap for Regression

I. Parametric bootstrap

- We can employ the IID-bootstrap to the residuals of a fitted OLS regression to generate pseudo-data sets, then re-fit the regression with those new data.

Algorithm (IID Parametric Bootstrap)

1. Fit the regression model by OLS, and obtain

$$\hat{y} = \hat{\alpha} + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

} $k=2$ w/o loss of generality

2. Form the estimated residuals as

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$= Y_i - \hat{\alpha} - \hat{\beta}_1 X_{1,i} - \hat{\beta}_2 X_{2,i}$$

3. Draw $u_1^{(1)}, u_2^{(1)}, u_3^{(1)}, \ldots, u_n^{(1)}$ with replacement from $\{\hat{u}_i\}_{i=1}^{n}$

4. Generate $Y_1^{(1)}, Y_2^{(1)}, \ldots, Y_n^{(1)}$ as

$$Y_i^{(1)} = \hat{\alpha} + \hat{\beta}_1 X_{1,i} + \hat{\beta}_2 X_{2,i} + \underbrace{u_i^{(1)}}_{\text{the bootstrapped residuals}}$$

5. Re-estimate the regression and save

$$\{\hat{\alpha}^{(1)}, \hat{\beta}_1^{(1)}, \hat{\beta}_2^{(1)}\}$$

} other items of interest as well

6. Repeat steps 3 – 5 for $b = 2, \ldots, B$

$B = 10{,}000$ or 50

7. Form histograms, kde's, hypothesis tests, confidence intervals

from $\left\{ \hat{\alpha}^{(b)}, \hat{\beta}_1^{(b)}, \hat{\beta}_2^{(b)} \right\}_{b=1}^{B}$

NB: use the empirical CDF as needed

II. Nonparametric Bootstrap for regression

Algorithm (IID Nonparametric Bootstrap)

1. Draw $\{(y_i^{(1)}, x_{1,i}^{(1)}, x_{2,i}^{(1)}), i=1,\dots,n\}$ from the original data (in row-wise pairs)

   $\{(y_i, x_{1,i}, x_{2,i}), i=1,\dots,n\}$ with replacement.

2. Estimate the regression

$$y_i^{(1)} = \alpha + \beta_1 x_{1,i}^{(1)} + \beta_2 x_{2,i}^{(1)} + \varepsilon_i$$

   and store $\{\hat{\alpha}^{(1)}, \hat{\beta}_1^{(1)}, \hat{\beta}^{(2)}\}$ } other items of interest

3. Repeat step 2 for $b=2,\dots,B$ and

   Save

$$\{\hat{\alpha}^{(b)}, \hat{\beta}_1^{(b)}, \hat{\beta}_2^{(b)}\}_{b=2}^{B}$$

4. Form histograms, Kde's, hypothesis tests, confidence intervals, empirical CDFs from

$$\left\{ \hat{\alpha}^{(b)}, \hat{\beta}_1^{(b)}, \hat{\beta}_2^{(b)} \right\}_{b=1}^{B}$$