NB: Notes on Point Estimation
from Chp. 9 of
Introduction to Probability and mathematical statistics 2nd Ed by Bain & Engelhardt

## Bayes and Minimax Estimators

### Loss Function

If $T$ is an estimator of $\tau(\theta)$, then a loss function is any real-valued function, $L(t;\theta)$ such that

$$L(t;\theta) \geq 0 \quad \text{for every } t$$

and

$$L(t;\theta) = 0 \quad \text{when } t = \tau(\theta)$$

### Risk Function

The risk function is defined to be the expected loss

$$R_T(\theta) = E\left[L(T;\theta)\right]$$

## Admissable Estimator

An estimator $T_1$ is a better estimator than $T_2$ if and only if

$$R_{T_1}(\theta) \le R_{T_2}(\theta) \qquad \text{for all } \theta \in \Omega$$

and

$$R_{T_1}(\theta) < R_{T_2}(\theta) \qquad \text{for at least one } \theta$$

An estimator $T$ is admissable if and only if there is no better estimator

## Minimax Estimator

An estimator $T_1$ is a minimax estimator if

$$\max_{\theta} R_{T_1}(\theta) \leq \max_{\theta} R_T(\theta)$$

for every estimator $T$.

## Bayes Risk

For a random sample from $f(x;\theta)$, the Bayes Risk of an estimator $T$ relative to a risk function $R_T(\theta)$ and pdf $p(\theta)$ is the average risk with respect to $p(\theta)$,

$$A_T = E_\theta[R_T(\theta)] = \int_\Omega R_T(\theta)\, p(\theta)\, d\theta$$

## Bayes Estimator

For a random sample from $f(x;\theta)$, the Bayes estimator $T^*$ relative to the risk function $R_T(\theta)$ and pdf $p(\theta)$ is the estimator with minimum expected risk,

$$E_\theta[R_{T^*}(\theta)] \le E_\theta[R_T(\theta)]$$

for every estimator $T$.