Finance 5350 Class Notes

Topic: The Bernoulli and Binomial Distributions

Introduction

This note is a brief review of concepts from discrete probability. We will cover the definition of a *dicrete random variable* and then turn our attention to a discrete probability distribution called the *Bernoulli Distribution* as well as its generalization in the *Binomial Distribution*.

These notes are based on the presentation in Bain and Engelhardt (1992).

Discrete Random Variables

Definition: Discrete Random Variable

If the set of all possible values of a random variable, X, is a countable set, $x_1, x_2, \ldots, x_n$, our $x_1, x_2, \ldots$, then X is called a **discrete random variable**. The function

$$f(x) = P[X = x] \qquad x = x_1, x_2, \ldots$$

that assigns the probability to each possible value x will be called the **discrete probability density function** (discrete pdf).

Definition: The Bernoulli Distribution

A random variable , X, that assumes only the values 0 and 1 is known as a **Bernoulli variable**, and a performance of an experiment with only two types of outcomes is called a **Bernoulli trial**. In particular, if an experiment can result only in "success" (E) or "failure" (E'), then the corresponding Bernoulli variable is

$$X(e) = \begin{cases} 1 & \text{if} \quad e \in E, \\ 0 & \text{if} \quad e \in E' \end{cases}$$

The pdf of X is given by $f(0) = 1 - \theta$ and $f(1) = \theta$. The corresponding distribution is known as a **Bernoulli distribution**, and its pdf can be expressed as

$$f(x) = \theta^x (1-\theta)^{1-x} \qquad x = 0, 1$$

Example: Drawing Marbles at Random

Consider drawing marbles at random from a collection of 10 black and 20 white marbles. In such a problem, we might regard "black" as success and "white" as failure, or vice versa, in a single draw. If obtaining a black marble is regarded as success, then $\theta = 10/30 = 1/3$ and $1 - \theta = 20/30 = 2/3$.

The Single-Period Binomial Model

Recall that in the single-period binomial option pricing model we could express it in its risk-neutral form as follows

$$f_0 = e^{-rh}[f_u p^* + f_d(1-p^*)]$$

where f_0 is value of the option (call or put) and f_u is the value of the option in the up state and f_d is the value of the option in the down state. Finally, $p^* = \frac{e^{(rh)}-u}{u-d}$ was called the ***risk-neutral probability***.

References

Bain, Lee J., and Max Engelhardt. 1992. *Introduction to Probability and Mathematical Statistics, 2nd Edition.* Duxbury Thomson Learning.