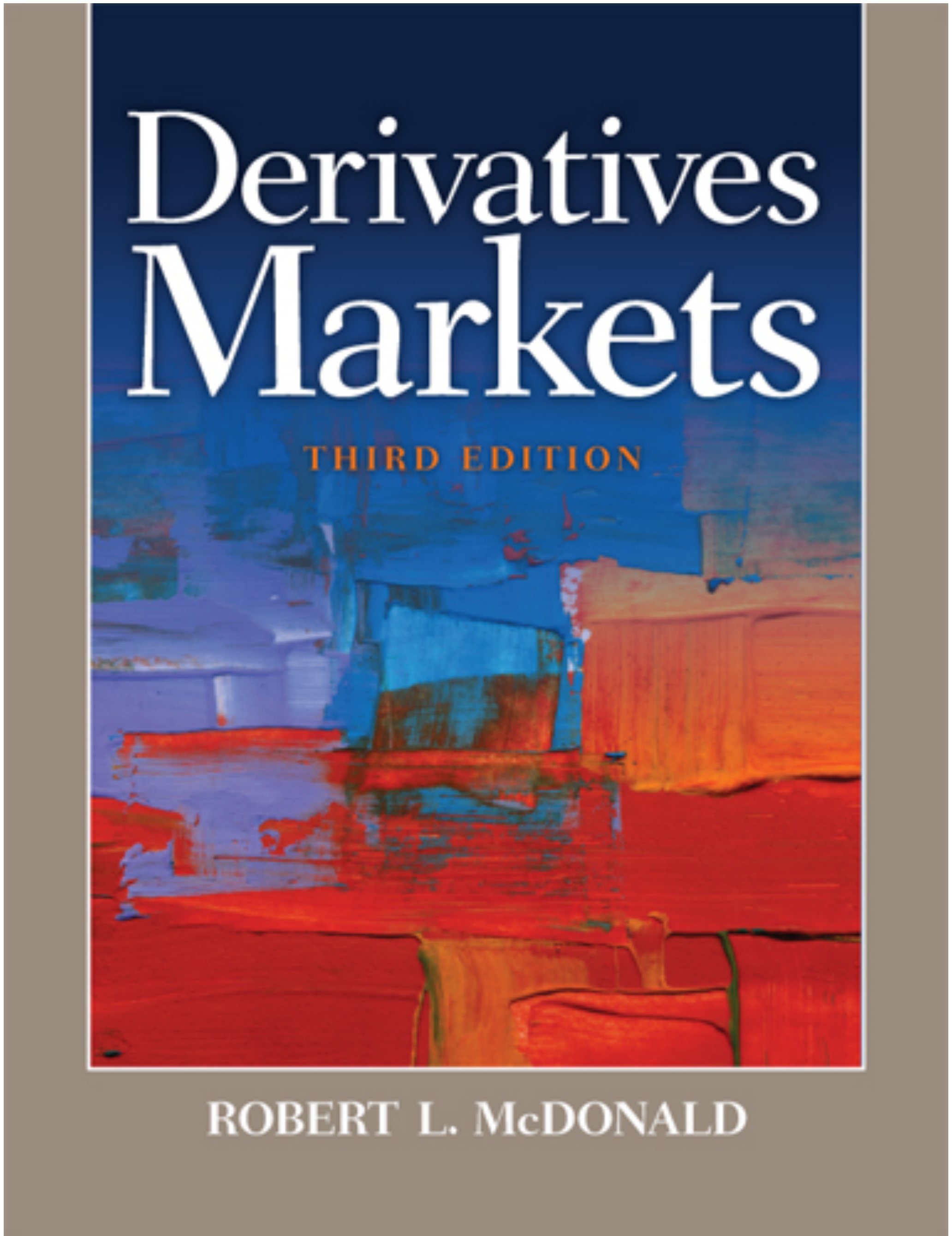


Chapter 9

Parity and Other Option Relationships

IBM Option Quotes

TABLE 9.1 IBM option prices, dollars per share, May 6, 2011. The closing price of IBM on that day was $168.89.

		Calls		Puts	
Strike	**Expiration**	**Bid ($)**	**Ask ($)**	**Bid ($)**	**Ask ($)**
160	June	10.05	10.15	1.16	1.20
165	June	6.15	6.25	2.26	2.31
170	June	3.20	3.30	4.25	4.35
175	June	1.38	1.43	7.40	7.55
160	October	14.10	14.20	5.70	5.80
165	October	10.85	11.00	7.45	7.60
170	October	8.10	8.20	9.70	9.85
175	October	5.80	5.90	12.40	12.55

Source: Chicago Board Options Exchange.

Put-Call Parity

- For European options with the same strike price and time to expiration the parity relationship is

Call – put = *PV* (forward price – strike price)

or

$$C(K,T) - P(K,T) = PV_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K)$$

- Intuition
  - Buying a call and selling a put with the strike equal to the forward price ($F_{0,T} = K$) creates a synthetic forward contract and hence must have a zero price

Parity for Options on Stocks

- If underlying asset is a stock and $PV_{0,T}(\text{Div})$ is the present value of the dividends payable over the life of the option, then $e^{-rT} F_{0,T} = S_0 - PV_{0,T}(\text{Div})$, therefore

$$C(K,T) = P(K,T) + [S_0 - \text{PV}_{0,T}(\text{Div})] - e^{-rT}(K)$$

- Rewriting above

$$S_0 = C(K,T) - P(K,T) + \text{PV}_{0,T}(\text{Div}) + e^{-rT}(K)$$

- For index options, $S_0 - \text{PV}_{0,T}(\text{Div}) = S_0 e^{-\delta T}$, therefore

$$C(K,T) = P(K,T) + S_0 e^{-\delta T} - \text{PV}_{0,T}(K)$$

Parity for Options on Stocks (cont'd)

- Examples 9.1 & 9.2
  - Price of a non-dividend-paying stock: \$40, r=8%, option strike price: \$40, time to expiration: 3 months, European call: \$2.78, European put: \$1.99. ➡ $\$2.78=\$1.99+\$40 - \$40e^{-0.08\times0.25}$
  - Additionally, if the stock pays \$5 just before expiration, call: \$0.74, and put: \$4.85. ➡ $\$0.74-\$4.85=(\$40 - \$5e^{-0.08\times0.25}) - \$40e^{-0.08\times0.25}$
- Synthetic security creation using parity
  - Synthetic stock: buy call, sell put, lend PV of strike and dividends
  - Synthetic T-bill: buy stock, sell call, buy put (conversion)
  - Synthetic call: buy stock, buy put, borrow PV of strike and dividends
  - Synthetic put: sell stock, buy call, lend PV of strike and dividends

Generalized Parity Relationship

$$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q)$$

TABLE 9.2 Payoff table demonstrating that there is an arbitrage opportunity unless $-C(S_t, Q_t, T-t) + P(S_t, Q_t, T-t) + F^P_{t,T}(S) - F^P_{t,T}(Q) = 0$.

		Expiration	
Transaction	**Time 0**	$S_T \leq Q_T$	$S_T > Q_T$
Buy call	$-C(S_t, Q_t, T-t)$	0	$S_T - Q_T$
Sell put	$P(S_t, Q_t, T-t)$	$S_T - Q_T$	0
Sell prepaid forward on A	$F^P_{t,T}(S)$	$-S_T$	$-S_T$
Buy prepaid forward on B	$-F^P_{t,T}(Q)$	Q_T	Q_T
Total	$-C(S_t, Q_t, T-t)$ $+P(S_t, Q_t, T-t)$ $+F^P_{t,T}(S) - F^P_{t,T}(Q)$	0	0

Properties of Option Prices

- European versus American Options
  - Since an American option can be exercised at anytime, whereas a European option can only be exercised at expiration, an American option must always be at least as valuable as an otherwise identical European option

$$C_{Amer}(S, K, T) \geq C_{Eur}(S, K, T)$$

$$P_{Amer}(S, K, T) \geq P_{Eur}(S, K, T)$$

Properties of Option Prices (cont'd)

- Maximum and Minimum Option Prices
  - Call price cannot
    - be negative
    - exceed stock price
    - be less than price implied by put-call parity using zero for put price:

$$S > C_{Amer}(S,K,T) \geq C_{Eur}(S,K,T) \geq \max[0, \mathrm{PV}_{0,T}(F_{0,T}) - \mathrm{PV}_{0,T}(K)]$$

  - Put price cannot
    - be more than the strike price
    - be less than price implied by put-call parity using zero for put price:

$$K > P_{Amer}(S,K,T) \geq P_{Eur}(S,K,T) \geq \max[0, \mathrm{PV}_{0,T}(K) - \mathrm{PV}_{0,T}(F_{0,T})]$$

Properties of Option Prices (cont’d)

- Early exercise for American options
  - An American call option on a non-dividend-paying stock should not be exercised early, because

$$C_{Amer} \geq C_{Eur} \geq S_T - K$$

  - That means, one would lose money be exercising early instead of selling the option
  - If there are dividends, it may be optimal to exercise early, just prior to a dividend
  - It may be optimal to exercise a non-dividend-paying put option early if the underlying stock price is sufficiently low

Properties of Option Prices (cont’d)

- Time to Expiration
  - An American option (both put and call) with more time to expiration is at least as valuable as an American option with less time to expiration. This is because the longer option can easily be converted into the shorter option by exercising it early
  - A European call option on a non-dividend-paying stock will be more valuable than an otherwise identical option with less time to expiration.
  - European call options on dividend-paying stock and European puts may be less valuable than an otherwise identical option with less time to expiration
  - When the strike price grows at the rate of interest, European call and put prices on a non-dividend-paying stock increases with time to maturity

Properties of Option Prices (cont'd)

- Different strike prices ($K_1 < K_2 < K_3$), for both European and American options
  - A call with a low strike price is at least as valuable as an otherwise identical call with higher strike price

$$C(K_1) \geq C(K_2)$$

  - A put with a high strike price is at least as valuable as an otherwise identical call with low strike price

$$P(K_2) \geq P(K_1)$$

  - The premium difference between otherwise identical calls with different strike prices cannot be greater than the difference in strike prices

$$C(K_1) - C(K_2) \leq K_2 - K_1$$

Properties of Option Prices (cont'd)

- Different strike prices ($K_1 < K_2 < K_3$), for both European and American options
  - The premium difference between otherwise identical puts with different strike prices cannot be greater than the difference in strike prices

$$P(K_1) - P(K_2) \leq K_2 - K_1$$

  - Premiums decline at a decreasing rate for calls with progressively higher strike prices. (Convexity of option price with respect to strike price)

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} \leq \frac{C(K_2) - C(K_3)}{K_3 - K_2}$$

Properties of Option Prices (cont'd)

TABLE 9.6

Panel A shows call option premiums for which the change in the option premium ($6) exceeds the change in the strike price ($5). Panel B shows how a bear spread can be used to arbitrage these prices. By lending the bear spread proceeds, we have a zero cash flow at time 0; the cash outflow at time T is always greater than $1.

Panel A

Strike	50	55
Premium	18	12

Panel B

		Expiration or Exercise		
Transaction	**Time 0**	**$S_T < 50$**	**$50 \leq S_T \leq 55$**	**$S_T \geq 55$**
Buy 55-strike call	−12	0	0	$S_T - 55$
Sell 50-strike call	18	0	$50 - S_T$	$50 - S_T$
Total	6	0	$50 - S_T$	−5

Properties of Option Prices (cont'd)

TABLE 9.7 The example in Panel A violates the proposition that the rate of change of the option premium must decrease as the strike price rises. The rate of change from 50 to 59 is 5.1/9, while the rate of change from 59 to 65 is 3.9/6. We can arbitrage this convexity violation with an asymmetric butterfly spread. Panel B shows that we earn at least \$3 plus interest at time T.

Panel A

Strike	50	59	65
Call premium	14	8.9	5

Panel B

		Expiration or Exercise			
Transaction	**Time 0**	$S_T < 50$	$50 \le S_T \le 59$	$59 \le S_T \le 65$	$S_T > 65$
Buy four 50-strike calls	-56	0	$4(S_T - 50)$	$4(S_T - 50)$	$4(S_T - 50)$
Sell ten 59-strike calls	89	0	0	$10(59 - S_T)$	$10(59 - S_T)$
Buy six 65-strike calls	-30	0	0	0	$6(S_T - 65)$
Lend \$3	-3	$3e^{rT}$	$3e^{rT}$	$3e^{rT}$	$3e^{rT}$
Total	0	$3e^{rT}$	$3e^{rT} + 4(S_T - 50)$	$3e^{rT} + 6(65 - S_T)$	$3e^{rT}$

Properties of Option Prices (cont'd)

TABLE 9.8 Arbitrage of mispriced puts using asymmetric butterfly spread.

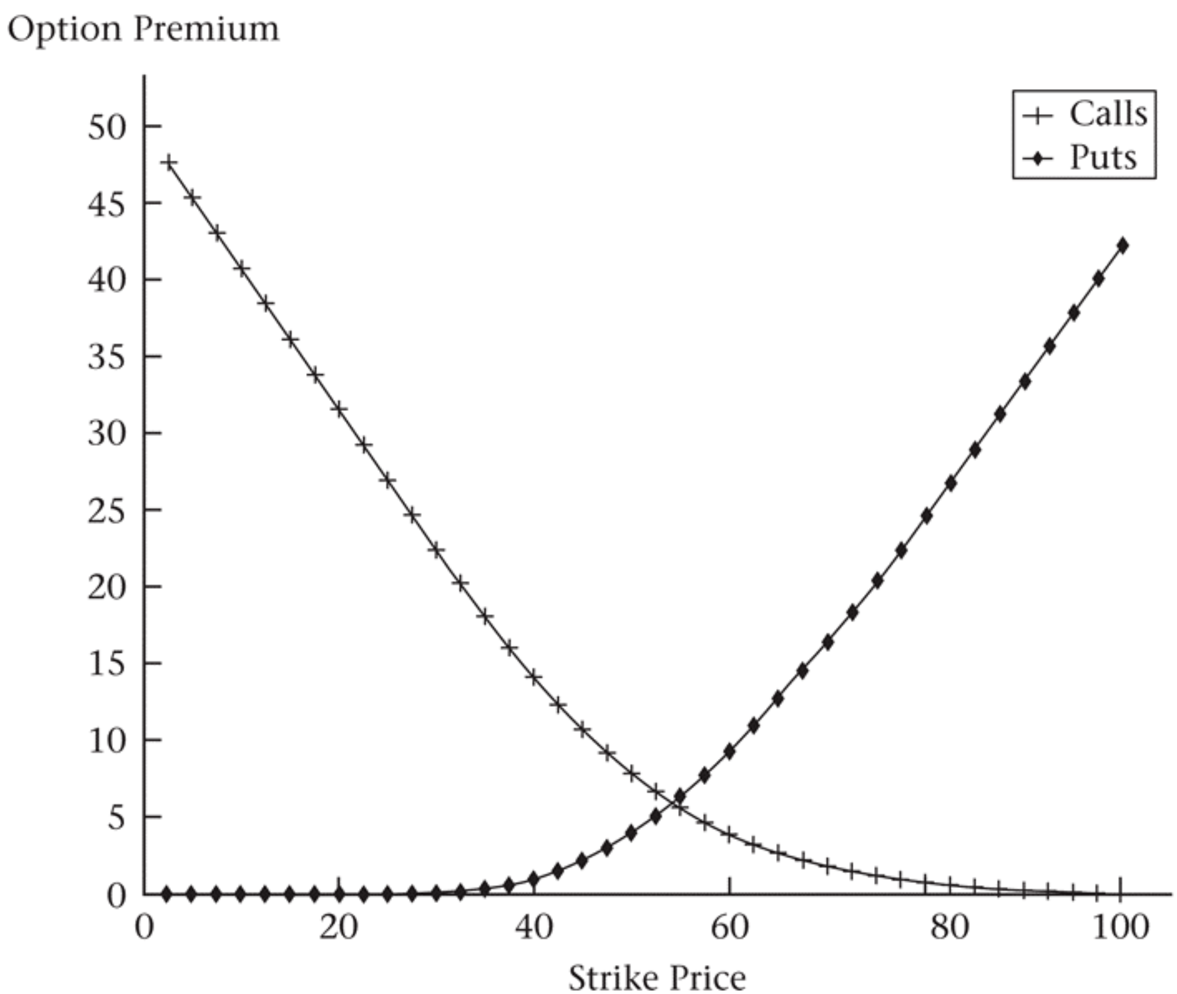
Panel A

Strike	50	55	70
Put premium	4	8	16

Panel B

		Expiration or Exercise			
Transaction	**Time 0**	$S_T < 50$	$50 \le S_T \le 55$	$55 \le S_T \le 70$	$S_T > 70$
Buy three 50-strike puts	-12	$3(50 - S_T)$	0	0	0
Sell four 55-strike puts	32	$4(S_T - 55)$	$4(S_T - 55)$	0	0
Buy one 70-strike put	-16	$70 - S_T$	$70 - S_T$	$(70 - S_T)$	0
Lend \$4	-4	$4e^{rT}$	$4e^{rT}$	$4e^{rT}$	$4e^{rT}$
Total	0	$4e^{rT}$	$4e^{rT} + 3(S_T - 50)$	$4e^{rT} + 70 - S_T$	$4e^{rT}$

Properties of Option Prices (cont'd)



Summary of Parity Relationships

TABLE 9.9 Versions of put-call parity. Notation in the table includes the spot currency exchange rate, x_0; the risk-free interest rate in the foreign currency, r_f; and the current bond price, B_0.

Underlying Asset	Parity Relationship
Futures contract	$e^{-rT}F_{0,T} = C(K,T) - P(K,T) + e^{-rT}K$
Stock, no-dividend	$S_0 = C(K,T) - P(K,T) + e^{-rT}K$
Stock, discrete dividend	$S_0 - \text{PV}_{0,T}(\text{Div}) = C(K,T) - P(K,T) + e^{-rT}K$
Stock, continuous dividend	$e^{-\delta T}S_0 = C(K,T) - P(K,T) + e^{-rT}K$
Currency	$e^{-r_f T}x_0 = C(K,T) - P(K,T) + e^{-rT}K$
Bond	$B_0 - \text{PV}_{0,T}(\text{Coupons}) = C(K,T) - P(K,T) + e^{-rT}K$

FIGURE 9.1

Cash flows for outright purchase of stock and for synthetic stock created by buying a 40-strike call and selling a 40-strike put.

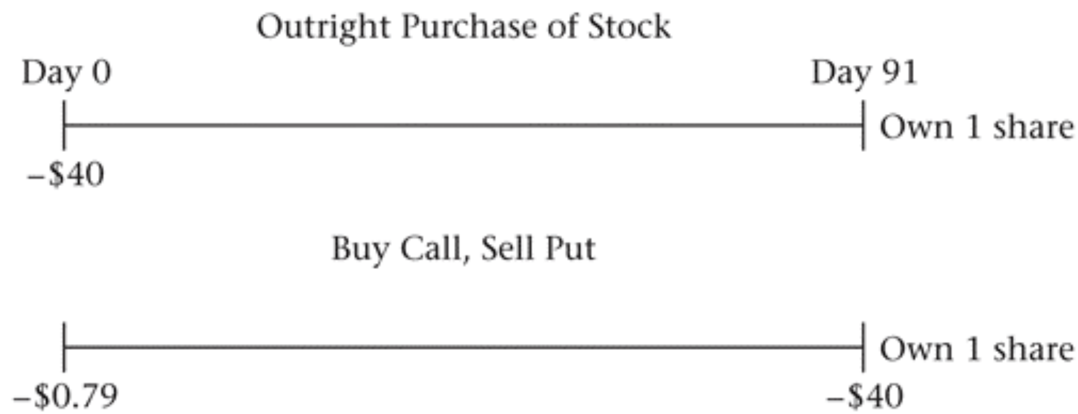


TABLE 9.3 The equivalence of buying a dollar-denominated euro call and a euro-denominated dollar put. In transaction I, we buy one dollar-denominated call option, permitting us to buy €1 for a strike price of \$1.20. In transaction II, we buy 1.20 euro-denominated puts, each with a premium of €0.04363, and permitting us to sell \$1 for a strike price of €0.833.

		Year 0		Year 1			
				$x_1 < 1.20$		$x_1 \geq 1.20$	
	Transaction	\$	€	\$	€	\$	€
I:	Buy 1 euro call	–0.06545	—	0	0	–1.20	1
II:	Convert dollars to euros,	–0.06545	0.05236				
	buy 1.20 dollar puts		–0.05236	0	0	–1.20	1

TABLE 9.4 Demonstration of arbitrage if a call option with price C sells for less than $S_t - Ke^{-r(T-t)}$ and the stock pays no dividends. Every entry in the row labeled "Total" is nonnegative.

		Expiration or Exercise, Time T	
Transaction	**Time t**	$\mathbf{S_T < K}$	$\mathbf{S_T > K}$
Buy call	$-C$	0	$S_T - K$
Short stock	S_t	$-S_T$	$-S_T$
Lend $Ke^{-r(T-t)}$	$-Ke^{-r(T-t)}$	K	K
Total	$S_t - Ke^{-r(T-t)} - C$	$K - S_T$	0

TABLE 9.5 Demonstration that there is an arbitrage if $P(T) \leq P(t)$ with $t < T$. The strike on the put with maturity t is $K_t = Ke^{rt}$, and the strike on the put with maturity T is $K_T = Ke^{rt}$. If the option expiring at time t is in-the-money, the payoff, $S_t - K_t$, is reinvested until time T. If $P(t) \geq P(T)$, all cash flows in the "total" line are nonnegative.

		Payoff at Time T			
		$S_T < K_T$		$S_T > K_T$	
		Payoff at Time t			
Transaction	**Time 0**	$S_t < K_t$	$S_t > K_t$	$S_t < K_t$	$S_t > K_t$
Sell $P(t)$	$P(t)$	$S_T - K_T$	0	$S_T - K_T$	0
Buy $P(T)$	$-P(T)$	$K_T - S_T$	$K_T - S_T$	0	0
Total	$P(t) - P(T)$	0	$K_T - S_T$	$S_T - K_T$	0