Chapter 6 - Commodity Forwards and Futures

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Section 6.1 Introduction to Commodity Forwards

Introduction to Commodity Forwards

- Commodity forward prices can be described by the same formula as that for financial forward prices

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

Introduction to Commodity Forward (Cont'd)

- For financial assets, δ is the dividend yield
- For commodities, δ is the commodity lease rate
  - The lease rate is the return that makes an investor willing to buy and then lend a commodity
  - The lease rate for a commodity can typically be estimated only by observing the forward prices

Introduction to Commodity Forward (Cont'd)

- Differences between commodities and financial assets include
  - Storage costs
  - Carry markets
  - Lease rate
  - Convenience yield

Introduction to Commodity Forward (Cont'd)

- The set of prices for different expiration dates for a given commodity is called the **forward rate** (or the **forward strip**) for that date
- If on a given date the forward curve is upward sloping, then the market is in **contango**. If the forward curve is downward sloping, the market is in **backwardation**
  - Note that forward curves can have portions in backwardation and portions in contango

Section 6.2 Equilibrium Pricing of Commodity Forwards

Equilibrium Pricing of Commodity Forwards

- As with financial forwards, the commodity forward price is a biased estimate of the expected spot price, $E(S_T)$, with the bias due to the risk premium on the commodity, $r - \alpha$. (***NB***: $r - \alpha = -(\alpha - r)$).

$$F_{0,T} = E_0(S_T)e^{-(\alpha - r)T}$$

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Equilibrium Pricing of Commodity Forwards (Cont'd)

- Different commodities have their distinct forward curves, reflecting different properties of
  - Storability
  - Storage costs
  - Production
  - Demand
  - Seasonality

Short-selling and the Lease Rate

- Suppose we engage in a reverse cash-and-carry for copper. The price of copper today is \$3 and the price of copper in one year is $F_{0,1}$. The risk-free rate is 10%.

TABLE 6.5 Reverse cash-and-carry for copper for 1 year, assuming that the commodity lender requires a lease payment of L.

	Cash Flows	
Transaction	Time 0	Time 1
Short-sell copper	S_0	$-S_1$
Lease payment	0	$-L$
Long forward	0	$S_1 - F_{0,1}$
Invest @ R	$-S_0$	$(1+R)S_0$
Total	0	$[(1+R)S_0 - F_{0,1}] - L$

- A copper borrower must make an extra payment, a lease payment, due to the difference in the current and forward prices.

Short-selling and the Lease Rate (Cont'd)

- The lease rate is the difference between the commodity discount rate, α, and the expected growth rate of the commodity price

$$\delta_1 = \alpha - \frac{1}{T} \ln \left[E_0(S_T)/S_0 \right]$$

- For a commodity owner who lends the commodity, the lease rate is like a dividend
  - With the stock, the dividend yield, δ, is an observable characteristic of the stock
  - With a commodity, the lease rate, δ_1, is income earned only if the commodity is loaned. It is not directly observable, except if there is a lease market

Short-selling and the Lease Rate (Cont'd)

- The lease rate has to be consistent with the forward price
- Therefore, when we observe the forward price, we can infer what the lease rate would have to be if a lease market existed
- The annualized lease rate

$$\delta_1 = r - \frac{1}{T} \ln F_{0,T}/S$$

Section 6.3 Pricing Commodity Forwards by Arbitrage

No-Arbitrage Pricing Incorporating Storage Costs

- A commodity for which the forward price compensates a commodity owner for costs of storage is called a **carry market**
- The cost of storing a physical item such as corn or copper can be large relative to its value
- Moreover, some commodities deteriorate over time, which is also a storage cost

No-Arbitrage Pricing Incorporating Storage Costs (Cont'd)

- Cash-and-carry arbitrage when the storage costs from time 0 to T are $\lambda(0, T)$

TABLE 6.4 Cash-and-carry for copper for 1 year, assuming that there is a 1-year storage cost of $\lambda(0, 1)$ payable at time 1, and an effective interest rate of R.

	Cash Flows	
Transaction	Time 0	Time 1
Buy copper	$-S_0$	S_1
Pay storage cost	0	$-\lambda(0, 1)$
Short forward	0	$F_{0,1} - S_1$
Borrow @ R	$+S_0$	$-(1+R)S_0$
Total	0	$F_{0,1} - [(1+R)S_0 + \lambda(0, 1)]$

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Borrow @ R	$+S_0$	$-(1+R)S_0$
Total	0	$F_{0,1} - [(1+R)S_0 + \lambda(0, 1)]$

- $F_{0,1}$ should not exceed $(1 + R)S_0 + \lambda(0, 1)$. If the forward price were greater, you could undertake a simple cash-and-carry after paying storage costs and interest

No-Arbitrage Pricing Incorporating Storage Costs (Cont'd)

- If $F_{0,T}$ is greater than or equal to $(1+R)S_0 + \lambda(0,1)$ then storage will occur because the forward premium is great enough that sale proceeds in the future compensate for the financial costs of storage (RS_0) and the physical costs of storage ($\lambda(0,1)$)
- When costly storage occurs, the forward rate can rise faster than the interest rate
- We can view storage costs as a negative dividend
- Storage costs can include depreciation of the commodity, which is less a problem for metals such as copper than it is for commodities such as strawberries or electricity

No-Arbitrage Pricing Incorporating Storage Costs (Cont'd)

- If interest rates and storage costs are paid continuously and are proportional to the value of the commodity, and there is no arbitrage

$$F_{0,T} = S_0 e^{(r+\lambda)T}$$

- If the forward price were greater, you could undertake a simple cash-and-carry and earn a profit after paying both storage costs and interest on the position

No-Arbitrage Pricing Incorporating Storage Costs (Cont'd)

- Some holders of a commodity receive benefits from physical ownership (e.g., a commercial user). This benefit is called the commodity's **convenience yield**
- If there is a continuously compounded convenience yield, c, then

$$F_{0,T} \geq S_0 e^{(r+\lambda-c)T}$$

- A user who buys and stores the commodity will be compensated for interest and physical storage costs less a convenience yield
- The commodity lease rate will be $\delta_1 = c - \lambda$

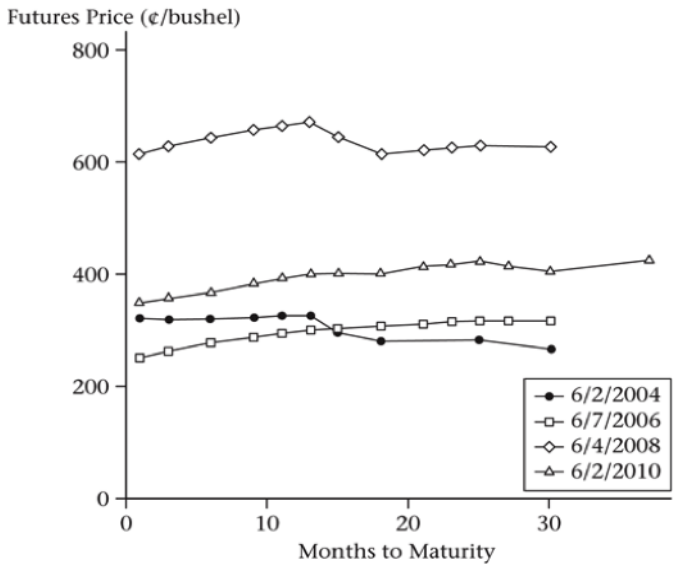
Section 6.5 Corn

Corn

- Corn is harvested primarily in the fall. In order to be consumed when it is not produced, it must be stored.

FIGURE 6.6

Forward curves for corn for four years.



Data from Datastream.

- In a typical year, once the harvest begins, storage is no longer necessary. Those storing corn will plan to deplete inventory as harvest approaches and to replenish inventory from the new harvest. The corn price will fall at harvest, only to begin rising again after the harvest

Section 6.6 Energy Markets

Energy Markets: Electricity

- Electricity has the following characteristics
  - It cannot be easily stored. Therefore, it is not possible to engage in arbitrage
  - At any point in time, the maximum supply of electricity is fixed
  - Demand for electricity varies substantially by season, by day of week, and by time of day

Energy Markets: Electricity (Cont'd)

- Given these characteristics, electricity forwards have large price swings over the day. Price swings reflect changes in the expected spot price, which in turn reflects changes in demand over the day

TABLE 6.7 Day-ahead price, by hour, for 1 megawatt-hour of electricity in New York City, March 21, 2011.

Time	Price	Time	Price	Time	Price	Time	Price
0000	$36.77	0600	$44.89	1200	$53.84	1800	$56.18
0100	$34.43	0700	$58.05	1300	$51.36	1900	$63.51
0200	$32.22	0800	$52.90	1400	$50.01	2000	$54.99
0300	$32.23	0900	$54.06	1500	$49.55	2100	$47.01
0400	$32.82	1000	$55.06	1600	$49.71	2200	$40.26
0500	$35.84	1100	$55.30	1700	$51.66	2300	$37.29

Data from Bloomberg

- The forward prices in Table 6.7 provide price discovery, revealing otherwise unobtainable information about the future price of the commodity. The prices are best interpreted using equation (6.4)

Section 6.8 Synthetic Commodities

Synthetic Commodities

- A synthetic commodity can be created by combining a forward contract with a zero-coupon bond

Investment strategy	Cost at time 0	Payoff at time T
A long commodity forward contract at the price $F_{0,T}$	0	$S_T - F_{0,T}$
A zero-coupon bond that pays $F_{0,T}$ at time T	$F_{0,T}/(1+R)$	$F_{0,T}$
Total	$F_{0,T}/(1+R)$	S_T = the value unit of the commodity at time T

TABLE 6.1 Futures prices for various commodities, March 17, 2011.

Expiration Month	Corn (cents/ bushel)	Soybeans (cents/ bushel)	Gasoline (cents/ gallon)	Oil (Brent) (dollars/ barrel)	Gold (dollars/ ounce)
April	—	—	2.9506	—	1404.20
May	646.50	1335.25	2.9563	114.90	1404.90
June	—	—	2.9491	114.65	1405.60
July	653.75	1343.50	2.9361	114.38	—
August	—	—	2.8172	114.11	1406.90
September	613.00	1321.00	2.8958	113.79	—
October	—	—	2.7775	113.49	1408.20
November	—	1302.25	2.7522	113.17	—
December	579.25	—	2.6444	112.85	1409.70

Data from CME Group.

FIGURE 6.1

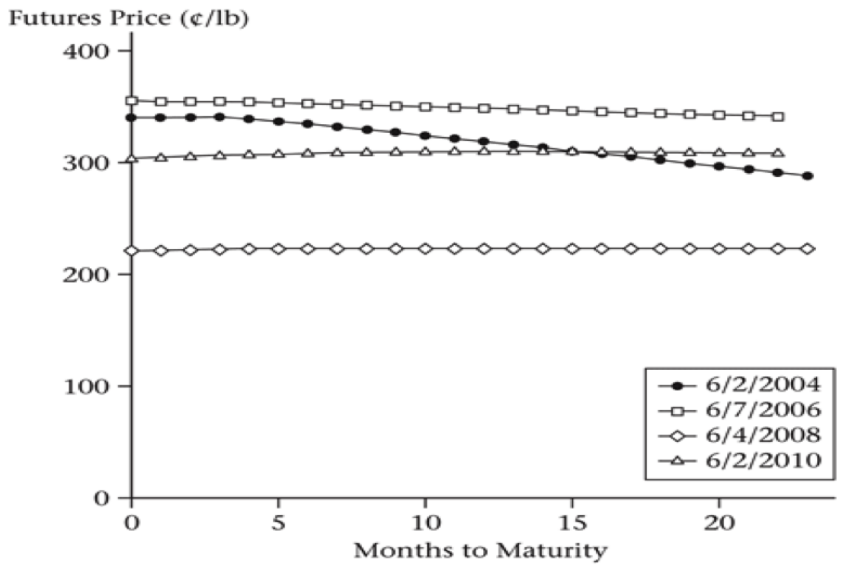
Specifications for the CME Group/COMEX high-grade copper contract.

Underlying	High-grade (Grade 1) copper
Where traded	CME Group/COMEX
Size	25,000 pounds
Months	24 consecutive months
Trading ends	Third-to-last business day of the maturing month
Delivery	Exchange-designated warehouse within the United States

Data from Datastream.

FIGURE 6.2

Forward curves for four dates for the CME Group high-grade copper futures contract.



Data from Datastream.

TABLE 6.2

Apparent reverse cash-and-carry arbitrage for copper if the copper forward price is $F_{0,1} < \$3.30$. These calculations *appear* to demonstrate that there is an arbitrage opportunity if the copper forward price is below \$3.30. S_1 is the spot price of copper in 1 year, and $F_{0,1}$ is the copper forward price. There is a logical error in the table.

	Cash Flows	
Transaction	**Time 0**	**Time 1**
Long forward @ $F_{0,1}$	0	$S_1 - F_{0,1}$
Short-sell copper	+\$3.00	$-S_1$
Lend short-sale proceeds @ 10%	−\$3.00	\$3.30
Total	0	$\$3.30 - F_{0,1}$

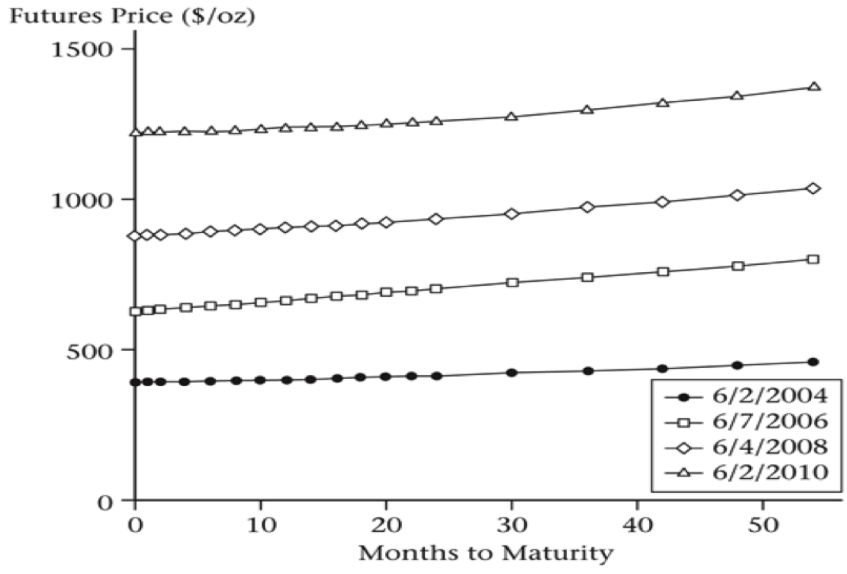
FIGURE 6.3

Specifications for the CME Group gold futures contract.

Underlying	Refined gold bearing approved refiner stamp
Where traded	CME Group/NYMEX
Size	100 troy ounces
Months	February, April, August, October, out 2 years. June, December, out 5 years
Trading ends	Third-to-last business day of maturity month
Delivery	Any business day of the delivery month

FIGURE 6.4

The forward curve for gold on four dates, from NYMEX gold futures prices.



Data from Datastream.

TABLE 6.6

Gold forward and prepaid forward prices on 1 day for gold delivered at 1-year intervals, out to 6 years. The continuously compounded interest rate is 6% and the lease rate is assumed to be a constant 1.5%.

Expiration Year	Forward Price ($)	Prepaid Forward Price ($)
1	313.81	295.53
2	328.25	291.13
3	343.36	286.80
4	359.17	282.53
5	375.70	278.32
6	392.99	274.18

FIGURE 6.5

Specifications for the CME Group/CBOT corn futures contract.

Underlying	#2 Yellow, with #1 Yellow deliverable at a $0.015 premium and #3 Yellow at a $0.015 discount.
Where traded	CME Group/CBOT
Size	5000 bushels (~127 metric tons)
Months	March, May, July, September, and December, out 2 years
Trading ends	Business day prior to the 15th day of the month.
Delivery	Second business day following the last trading day of the delivery month

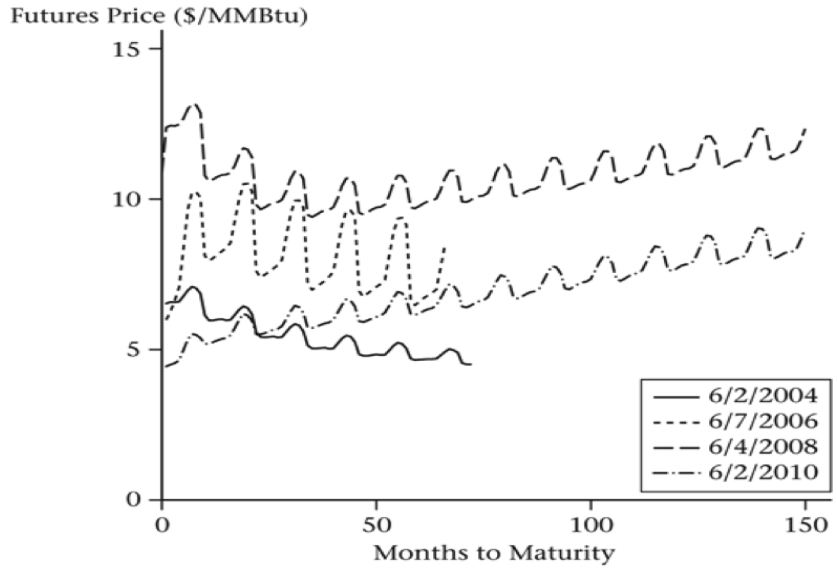
FIGURE 6.7

Specifications for the NYMEX Henry Hub natural gas contract.

Underlying	Natural gas delivered at Sabine Pipe Lines Co.'s Henry Hub, Louisiana
Where traded	New York Mercantile Exchange
Size	10,000 million British thermal units (MMBtu)
Months	72 consecutive months
Trading ends	Third-to-last business day of month prior to maturity month
Delivery	As uniformly as possible over the delivery month

FIGURE 6.8

Forward curves for natural gas for four years. Prices are dollars per MMBtu, from CME Group/NYMEX.



Data from Datastream.

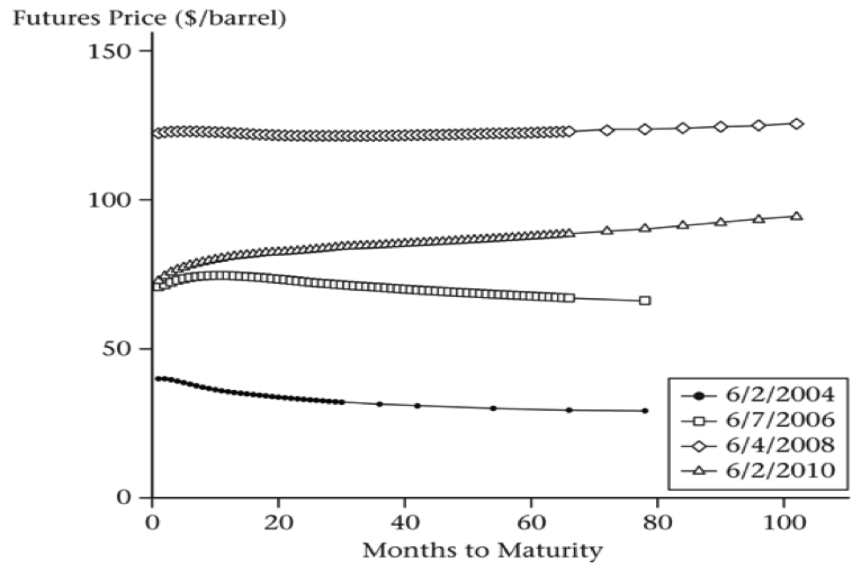
FIGURE 6.9

Specifications for the NYMEX light sweet crude oil contract.

Underlying	Specific domestic crudes delivered at Cushing, Oklahoma
Where traded	New York Mercantile Exchange
Size	1000 U.S. barrels (42,000 gallons)
Months	30 consecutive months plus long-dated futures out 7 years
Trading ends	Third-to-last business day preceding the 25th calendar day of month prior to maturity month
Delivery	As uniformly as possible over the delivery month

FIGURE 6.10

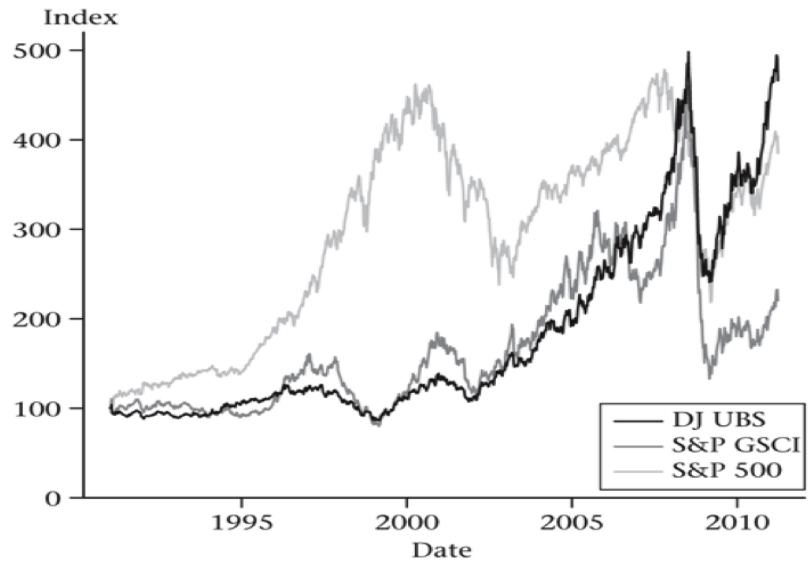
Multi-year strips of NYMEX crude oil futures prices, $/barrel, for four different dates.



Data from Datastream.

FIGURE 6.11

Value of S&P GSCI and DJ UBS indexes from 1991 to 2011, plotted against the S&P 500 index.



Source: Datastream