Journal of Economic Behavior & Organization 77 (2011) 97–106

Contents lists available at ScienceDirect

Journal of Economic Behavior & Organization

journal homepage: www.elsevier.com/locate/jebo

Option traders use (very) sophisticated heuristics, never the Black–Scholes–Merton formula

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ARTICLE INFO

Article history:
Received 30 November 2009
Received in revised form
14 September 2010
Accepted 15 September 2010
Available online 20 October 2010

JEL classification:
G12
G13

Keywords:
Options
Option hedging
Delta hedging
Put-call parity
Derivatives

ABSTRACT

Option traders use a heuristically derived pricing formula which they adapt by fudging and changing the tails and skewness by varying one parameter, the standard deviation of a Gaussian. Such formula is popularly called "Black–Scholes–Merton" owing to an attributed eponymous discovery (though changing the standard deviation parameter is in contradiction with it). However, we have historical evidence that: (1) the said Black, Scholes and Merton did not invent any formula, just found an argument to make a well known (and used) formula compatible with the economics establishment, by removing the "risk" parameter through "dynamic hedging", (2) option traders use (and evidently have used since 1902) sophisticated heuristics and tricks more compatible with the previous versions of the formula of Louis Bachelier and Edward O. Thorp (that allow a broad choice of probability distributions) and removed the risk parameter using put-call parity, (3) option traders did not use the Black–Scholes–Merton formula or similar formulas after 1973 but continued their bottom-up heuristics more robust to the high impact rare event. The paper draws on historical trading methods and 19th and early 20th century references ignored by the finance literature. It is time to stop using the wrong designation for option pricing.

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1. Breaking the chain of transmission

For us practitioners, theories about practice should arise from practice[1] or at least avoid conflict with it. This explains our concern with the "scientific" notion that practice should fit theory. Option hedging, pricing, and trading are neither philosophy nor mathematics, but an extremely rich craft rich with heuristics with traders learning from traders (or traders copying other traders) and tricks developing under evolution pressures, in a bottom-up manner. It is *technë*, not *ëpistemë*. Had it been a science it would not have survived—for the empirical and scientific fitness of the pricing and hedging theories offered are, we will see, at best, defective and unscientific (and, at the worst, the hedging methods create more risks than they reduce). Our approach in this paper is to ferret out historical evidence of *technë* showing how option traders went about their business in the past.

Options, we will show, have been extremely active in the pre-modern finance world. Complicated, tacitly transmitted tricks and heuristically derived methodologies in option trading and risk management of derivatives books have been developed over the past century, and used quite effectively by operators. In parallel, many derivations were produced

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[1] For us, in this discussion, a practitioner is deemed to be someone involved in repeated decisions about option hedging, not a support quant who writes pricing software or an academic who provides "consulting" advice.

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doi:10.1016/j.jebo.2010.09.013

by mathematical researchers.[2] The economics literature, however, did not recognize these contributions, substituting the rediscoveries or subsequent reformulations done by (some) economists. There is evidence of an attribution problem with Black–Scholes–Merton option "formula", which was developed, used, and adapted in a robust way by a long tradition of researchers and used heuristically by option market makers and "book runners". Furthermore, in a case of scientific puzzle, the exact formula called "Black–Sholes–Merton" was written down (and used) by Edward Thorp which, paradoxically, while being robust and realistic, has been considered unrigorous. This raises the following: (1) The Black–Scholes–Merton was, according to modern finance, just a neoclassical finance argument, no more than a thought experiment,[3] (2) we are not aware of traders using their argument or their version of the formula.

2. The Black–Scholes–Merton "formula" was an argument

Option traders call the formula they use the "Black–Scholes–Merton" formula without being aware that by some irony, of all the possible options formulas that have been produced in the past century, what is called the Black–Scholes–Merton "formula" (after Black and Scholes, 1973; Merton, 1973) is the one the furthest away from what they are using. In fact of the formulas written down in a long history it is the only formula that is fragile to jumps and tail events.

First, something seems to have been lost in translation: Black and Scholes (1973) and Merton (1973) actually never came up with a *new* option formula, but only an theoretical economic *argument* built on a new way of "deriving", rather re-deriving, an already existing – and well known – formula. The argument, we will see, is extremely fragile to assumptions. The foundations of option hedging and pricing were already far more firmly laid down before them. The Black–Scholes–Merton argument, simply, is that an option can be hedged using a certain methodology called "dynamic hedging" and then turned into a risk-free instrument, as the portfolio would no longer be stochastic. Indeed what Black, Scholes and Merton did was "marketing", finding a way to make a well-known formula palatable to the economics establishment of the time, little else, and in fact distorting its essence.

Such argument requires strange far-fetched assumptions: some liquidity at the level of transactions, knowledge of the probabilities of future events (in a neoclassical Arrow–Debreu style),[4] and, more critically, a certain mathematical structure that requires "thin-tails", or mild randomness, on which, later. The entire argument is indeed, quite strange and rather inapplicable for someone clinically and observation-driven standing outside conventional neoclassical economics. Simply, the dynamic hedging argument is dangerous in practice as it subjects you to blowups; it makes no sense unless you are concerned with neoclassical economic theory. The Black–Scholes–Merton argument and equation flow a top-down general equilibrium theory, built upon the assumptions of operators working *in full knowledge* of the probability distribution of future outcomes—in addition to a collection of assumptions that, we will see, are highly invalid mathematically, the main one being the ability to cut the risks using continuous trading which only works in the very narrowly special case of thin-tailed distributions (or, possibly, jumps of a well-known structure). But it is not just these flaws that make it inapplicable: option traders do not "buy theories", particularly speculative general equilibrium ones, which they find too risky for them and extremely lacking in standards of reliability. A normative theory is, simply, not good for decision-making under uncertainty (particularly if it is in chronic disagreement with empirical evidence). Operators may take decisions based on heuristics under the impression of using speculative theories, but avoid the fragility of theories in running their risks.

Yet professional traders, including, initially, the authors (and, alas, the Swedish Academy of Science) have operated under the illusion that it was the Black–Scholes–Merton "formula" they actually used—we were told so. This myth has been progressively reinforced in the literature and in business schools, as the original sources have been lost or frowned upon as "anecdotal" (Merton, 1992). As Fig. 1 shows, these simple random jumps represent too large a share of the variability of returns to make the Black-Scholes-Merton argument scientifically acceptable – the Swedish Academy does not grant the Nobel in Medicine to works that are grounded in the assumption that men were mice.

This discussion will present our real-world, ecological understanding of option pricing and hedging based on what option traders actually do and did for more than a hundred years.

This is a very general problem. As we said, option traders develop a chain of transmission of *technë*, like many professions. But the problem is that the "chain" is often broken as universities do not store the acquired skills by operators. Effectively plenty of robust heuristically derived implementations have been developed over the years, but the economics establishment has refused to quote them or acknowledge them. This makes traders need to relearn matters periodically. Failure of dynamic hedging in 1987, by such firm as Leland O'Brien Rubinstein, for instance, does not seem to appear in the academic literature

[2] Heuristics as tacit knowledge: Gigerenzer and Todd (2000).

[3] Here we question the notion of confusing thought experiments in a hypothetical world, of no predictive power, with either science or practice. The fact that the Black–Scholes–Merton argument works in a Platonic world and appears to be "elegant" does not mean anything since one can always produce a Platonic world in which a certain equation works, or in which a "rigorous" proof can be provided, thanks to a process called reverse-engineering.

[4] Of all the misplaced assumptions of Black Scholes that cause it to be a mere thought experiment, though an extremely elegant one, a flaw shared with modern portfolio theory, is the certain knowledge of future delivered variance for the random variable (or, equivalently, all the future probabilities). This is what makes it clash with practice—the rectification by the market fattening the tails is a negation of the Black–Scholes thought experiment.

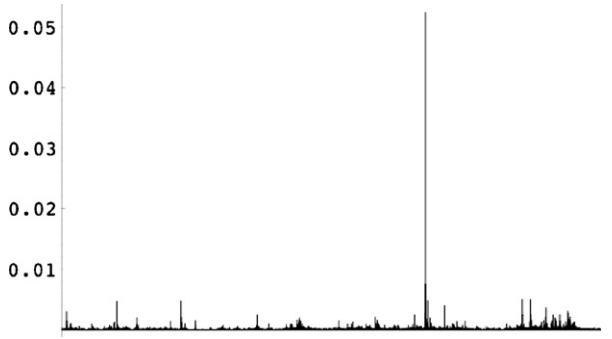

Fig. 1. The typical "risk reduction" performed by the Black–Scholes–Merton argument. These are the variations of a dynamically hedged portfolio. BSM indeed "smoothes" out risks but exposes the operator to massive tail events—reminiscent of such blowups as LTCM. Other option formulas are robust to the rare event and make no such claims.

published after the event[5] (Merton, 1992; Rubinstein, 1998; Ross, 2005); to the contrary dynamic hedging is held to be a standard operation.

There are central elements of the real world that can escape them—academic research without feedback from practice (in a *practical* and applied field) can cause the diversions we witness between laboratory and ecological frameworks. This explains why some many finance academics have had the tendency to make smooth returns, then "blow up" (that, is experience a terminal or near-terminal sharp loss) using their own theories.[6] We started the other way around, first spending years trading options and performing millions of hedges and option trades. We did this in combination with the investigating the forgotten and ignored ancient knowledge in option pricing and trading we will explain some common myths about option pricing and hedging.

There are indeed two myths:

- That we had to wait for the Black–Scholes–Merton options formula to trade the product, price options, and manage option books. In fact the introduction of the Black, Scholes and Merton argument increased our risks and set us back in risk management. More generally, it is a myth that traders rely on theories, even less a general equilibrium theory, to price options.
- That we "use" the Black–Scholes–Merton options "pricing formula". We, simply don't.

In our discussion of these myths we will focus on the bottom-up literature on option theory that has been hidden in the dark recesses of libraries. And that addresses only recorded matters—not the actual practice of option trading that has been lost.

3. Myth 1: people did not properly "price" options before the Black–Scholes–Merton theory

It is assumed that the Black–Scholes–Merton theory is what made it possible for option traders to calculate their delta hedge (against the underlying) and to price options. This argument is highly debatable, both historically and analytically.

Options had been actively trading at least in 1600 as described by De La Vega (1688)—implying some form of *technë*, a heuristic method to price them and deal with their exposure. De La Vega describes option trading in the Netherlands, indicating that operators had some expertise in option pricing and hedging. He diffusely points to put-call parity, and his book was not even meant to teach people about the technicalities in option trading. De Pinto (1771) even more explicitly points out how to convert call options into put options.[7] Our insistence on the use of put-call parity is critical for the following reason: the Black–Scholes–Merton's claim to fame is removing the necessity of a risk-based drift from the underlying security—to make the trade "risk-neutral". But one does not need dynamic hedging for that: simple put-call parity can suffice (Derman and Taleb, 2005), as we will discuss later. And yet it is this central removal of the "risk-premium" that apparently was behind the decision by the Nobel committee to grant Merton and Scholes the (then called) Bank of Sweden Prize in Honor of Alfred Nobel: "Black, Merton and Scholes made a vital contribution by *showing* that it is in fact not necessary to use any risk premium when valuing an option. This does not mean that the risk premium disappears; instead it is already included

[5] For instance how mistakes never resurface into the consciousness, Mark Rubinstein was awarded in 1995 the Financial Engineer of the Year award by the International Association of Financial Engineers. There was no mention of portfolio insurance and the failure of dynamic hedging.

[6] For a standard reaction to a rare event, see the following: "Wednesday is the type of day people will remember in quant-land for a very long time," said Mr. Rothman, a University of Chicago Ph.D. who ran a quantitative fund before joining Lehman Brothers. "Events that models only predicted would happen once in 10,000 years happened every day for three days." One 'Quant' Sees Shakeout For the Ages – '10,000 Years' By Kaja Whitehouse, August 11, 2007; Page B3.

[7] See Poitras (2009).

in the stock price."[8] It is for having removed the effect of the drift on the value of the option, *using a thought experiment*, that their work was originally cited, something that was mechanically present by any form of trading and converting using far simpler techniques.

Options have a much richer history than shown in the conventional literature. Forward contracts seems to date all the way back to Mesopotamian clay tablets dating all the way back to 1750 B.C. Gelderblom and Jonker (2005) show that Amsterdam grain dealers had already used options and forward in 1550 (but Amsterdam is not the earliest, as even more sources document even earlier uses in Europe[9]).

In the late 1800 and the early 1900 there were active option markets in London and New York as well as in Paris and several other European exchanges. Markets it seems, were active and extremely sophisticated option markets in 1870. Kairys and Valerio (1997) discuss the market for equity options in USA in the 1870s, indirectly showing that traders were sophisticated enough to price for tail events.[10] In a recent paper Mixon (2009a,b) looks at option pricing in the past versus the present and concludes that:

> "Traders in the nineteenth century appear to have priced options the same way that twenty-first century traders price options. Empirical regularities relating implied volatility to realized volatility, stock prices, and other implied volatilities (including the volatility skew), are qualitatively the same in both eras."

There was even active option arbitrage trading taking place between some of these markets. There is a long list of missing treatises on option trading: we traced at least 10 German treatises on options written between the late 1800s and the hyperinflation episode.[11]

Mixon (2009a,b) describes a relatively active Foreign Exchange Option Market from 1917 to 1921. The currency option market at that time evolved from one involving relatively large sums of money per transaction to one focused on tiny retail transactions.

A study by Moore and Juh (2006) looks at warrants traded on the Johannesburg Stock Exchange as well as call options written on 112 stocks in the early 20th century. The authors find that the warrant prices were surprisingly accurately priced in the pre Black-Scholes era.

When Cyrus Field finally succeeded in joining Europe and America by cable in 1866, intercontinental arbitrage was made possible. Although American securities had been purchased in considerable volume abroad after 1800, the lack of quick communication placed a definite limit on the amount of active trading in securities between London and New York markets, (see Weinstein, 1931). Furthermore, one extant source, Nelson (1904), speaks volumes: an option trader and arbitrageur, S.A. Nelson published a book "The A B C of Options and Arbitrage" based on his observations around the turn of the twentieth century. The author states that up to 500 messages per hour and typically 2000–3000 messages per day were sent between the London and the New York market through the cable companies. Each message was transmitted over the wire system in less than a minute. In a heuristic method that was repeated in *Dynamic Hedging* by one of the authors, (Taleb, 1997), Nelson, describe in a theory-free way many rigorously clinical aspects of his arbitrage business: the cost of shipping shares, the cost of insuring shares, interest expenses, the possibilities to switch shares directly between someone being long securities in New York and short in London and in this way saving shipping and insurance costs, as well as many more similar tricks.

The formal financial economics canon does not include historical sources from outside economics, a mechanism discussed in Taleb (2007). The put-call parity was according to the formal option literature first fully described by Stoll (1969), but neither he nor others in the field even mention Nelson. Not only was the put-call parity argument fully understood and described in detail by Nelson (1904), but he, in turn, makes frequent reference to Higgins (1902). Just as an example Nelson (1904) referring to Higgins (1902) writes:

> "It may be worthy of remark that 'calls' are more often dealt than 'puts' the reason probably being that the majority of 'punters' in stocks and shares are more inclined to look at the bright side of things, and therefore more often 'see' a rise than a fall in prices.

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[8] See http://www.Nobel.se.

[9] See Bell et al. (2007)—we thank Barkley Rosser for ferreting out earlier uses.

[10] The historical description of the market is informative until Kairys and Valerio try to gauge whether options in the 1870s were underpriced or overpriced (using Black–Scholes–Merton style methods). There was one tail-event in this period, the great panic of September 1873. Kairys and Valerio find that holding puts was profitable, but deem that the market panic was just a one-time event: "However, the put contracts benefit from the "financial panic" that hit the market in September, 1873. Viewing this as a "one-time" event, we repeat the analysis for puts excluding any unexpired contracts written before the stock market panic." Using references to the economic literature that also conclude that options in general were overpriced in the 1950s 1960s and 1970s they conclude: "Our analysis shows that option contracts were generally overpriced and were unattractive for retail investors to purchase". They add: "Empirically we find that both put and call options were regularly overpriced relative to a theoretical valuation model." These results are contradicted by the practitioner Nelson (1904): ". . .the majority of the great option dealers who have found by experience that it is the givers, and not the takers, of option money who have gained the advantage in the long run".

[11] Here is a partial list: Bielschowsky, R (1892): Ueber die rechtliche Natur der Prämiengeschäfte, Bresl. Genoss.-Buchdr Granichstaedten-Czerva, R (1917): Die Prämiengeschäfte an der Wiener Börse, Frankfurt am Main Holz, L. (1905) Die Prämiengeschäfte, Thesis (doctoral)–Universität Rostock Kitzing, C. (1925): Prämiengeschäfte : Vorprämien-, Rückprämien-, Stellagen- u. Nochgeschäfte ; Die solidesten Spekulationsgeschäfte mit Versicherg auf Kursverlust, Berlin Leser, E, (1875): Zur Geschichte der Prämiengeschäfte Szkolny, I. (1883): Theorie und praxis der prämiengeschäfte nach einer originalen methode dargestellt., Frankfurt am Main Author Unknown (1925): Das Wesen der Prämiengeschäfte, Berlin : Eugen Bab & Co., Bankgeschäft.

> This special inclination to buy 'calls' and to leave the 'puts' severely alone does not, however, tend to make 'calls' dear and 'puts' cheap, for it can be shown that the adroit dealer in options can convert a 'put' into a 'call,' a 'call' into a 'put', a 'call o' more' into a 'put- and-call,' in fact any option into another, by dealing against it in the stock. We may therefore assume, with tolerable accuracy, that the 'call' of a stock at any moment costs the same as the 'put' of that stock, and half as much as the Put-and-Call."

The Put-and-Call was simply a put plus a call with the same strike and maturity, what we today would call a straddle. Nelson describes the put-call parity over many pages in full detail. Static market neutral delta hedging was also known at that time, in his book Nelson for example writes:

> "Sellers of options in London as a result of long experience, if they sell a Call, straightway buy half the stock against which the Call is sold; or if a Put is sold; they sell half the stock immediately."

We must interpret the value of this statement in the light that standard options in London at that time were issued at-the-money (as explicitly pointed out by Nelson); furthermore, all standard options in London were European style. In London in- or out-of-the-money options were only traded occasionally and were known as "fancy options". It is quite clear from this and the rest of Nelson's book that the option dealers were well aware that the delta for at-the-money options was approximately 50%. As a matter of fact, at-the-money options trading in London at that time were adjusted to be struck to be at-the-money forward, in order to make puts and calls of the same price. We know today that options that are at-the-money forward and do not have very long time to maturity have a delta very close to 50% (naturally minus 50% for puts). The options in London at that time typically had one month to maturity when issued.

Nelson also diffusely points to dynamic delta hedging, and that it worked better in theory than practice (see Haug, 2007). It is clear from all the details described by Nelson that options in the early 1900 traded actively and that option traders at that time in no way felt helpless in either pricing or in hedging them.

Herbert Filer was another option trader that was active from 1919 to the 1960s. Filer (1959) describes what must be considered a reasonable active option market in New York and Europe in the early 1920s and 1930s. Due to World War II, there was no trading on European Exchanges; the London markets did not resume until 1958. Since in the early 1900s, option traders in London were considered to be the most sophisticated, according to Nelson, it could well be that World War II and the subsequent shutdown of option trading for many years was the reason known robust arbitrage principles about options were forgotten and almost lost, to be partly re-discovered by finance professors such as Stoll (1969).

The put-call parity in the older literature seems to serve two main purposes:

1. As a pure arbitrage constrain,
2. but also as a tool to create calls out of puts, puts out of calls and straddles out of calls or puts for the purpose of hedging options with options. In other words more than simply arbitrage constraint, but a very important tool to transfer risk between options.

The original descriptions and uses of the put-call parity concept, unlike later theories, consider that supply and demand for options will affect option prices. Even if the Black, Scholes, Merton model in strict theoretical sense is fully consistent with the arbitrage constrain of the put-call parity, the model is actually not consistent with the original invention and use of the put-call parity.

In 1908, Vinzenz Bronzin published a book deriving several option pricing formulas, and a formula very similar to what today is known as the Black–Scholes–Merton formula, see Hafner and Zimmermann (2007, 2009). Bronzin based his *risk-neutral* option valuation on robust arbitrage principles such as the put-call parity and the link between the forward price and call and put options—in a way that was rediscovered by Derman and Taleb (2005).[12] Indeed, the put-call parity restriction is sufficient to remove the need to incorporate a future return in the underlying security—it forces the lining up of options to the forward price.[13]

Again, in 1910 Henry Deutsch describes put-call parity but in less detail than Higgins and Nelson. In 1961 Reinach again described the put-call parity in quite some detail. Traders at New York stock exchange specializing in using the put-call parity to convert puts into calls or calls into puts was at that time known as Converters, Reinach (1961):

[12] The argument of Derman and Taleb (2005) was present in Taleb (1997) but remained unnoticed.

[13] Ruffino and Treussard (2006) accept that one could have solved the risk-premium by happenstance, not realizing that put-call parity was so extensively used in history. But they find it insufficient. Indeed the argument may not be *sufficient* for someone who subsequently complicated the representation of the world with some implements of modern finance such as "stochastic discount rates"—while simplifying it at the same time to make it limited to the Gaussian and allowing dynamic hedging. They write that "the use of a non-stochastic discount rate common to both the call and the put options is inconsistent with modern equilibrium capital asset pricing theory." Given that we have never seen a practitioner use "stochastic discount rate", we, like our option trading predecessors, feel that put-call parity is sufficient & does the job. The situation is akin to that of scientists lecturing birds on how to fly, and taking credit for their subsequent performance—except that here it would be lecturing them the wrong way.

> "Although I have no figures to substantiate my claim, I estimate that over 60 per cent of all Calls are made possible by the existence of Converters."

In other words the *Converters* (dealers) who basically operated as market makers were able to operate and hedge most of their risk by "statically" hedging options with options. Reinach wrote that he was an option trader (Converter) and gave examples on how he and his colleagues tended to hedge and arbitrage options against options by taking advantage of options embedded in convertible bonds:

> "Writers and traders have figured out other procedures for making profits writing Puts & Calls. Most are too specialized for all but the seasoned professional. One such procedure is the ownership of a convertible bond and then writing of Calls against the stock into which the bonds are convertible. If the stock is called converted and the stock is delivered."

Higgins, Nelson and Reinach all describe the importance of put-call parity and *hedging options with options*. Option traders were in no way helpless in hedging or pricing before the Black–Scholes–Merton formula. As already mentioned static market-neutral delta hedging was described by Higgins and Nelson in 1902 and 1904. Also, Gann (1937) discusses market neutral delta hedging for at-the-money options, but in much less details than Nelson (1904). Gann also indicates some forms of auxiliary dynamic hedging.

Mills (1927) illustrates how jumps and fat tails were present in the literature in the pre-Modern Portfolio Theory days. He writes: "A distribution may depart widely from the Gaussian type because the influence of one or two extreme price changes."

4. Option formulas and delta hedging

Which brings us to option pricing formulas. The first identifiable one was Bachelier (1900). Sprenkle (1961) extended Bacheliers work to assume lognormal rather than normally distributed asset price. It also avoids discounting (to no significant effect since many markets, particularly the U.S., option premia were paid at expiration).

Boness (1964) also assumed a lognormal asset price. He derives a formula for the price of a call option that is actually identical to the Black–Scholes–Merton, 1973 formula. This is among several others also pointed out by Rubinstein (2006):

> "The real significance of the formula to the financial theory of investment lies not in itself, but rather in how it was derived. Ten years earlier the same formula had been derived by Case M. Sprenkle (1961) and A. James Boness (1964)."

Samuelson (1965) and Thorp (1969) published somewhat similar option pricing formulas to Boness and Sprenkle. Thorp (2007) claims that he actually had an identical formula to the Black–Scholes–Merton formula programmed into his computer years before Black, Scholes and Merton published their theory.

It is also worth to mention that McKean (1965) derives a formula for perpetual American put option, but without assuming continuous delta hedging. The formula was later modified by Merton (1973) to assume risk neutrality based on continuous dynamic hedging.

Now, delta hedging. As already mentioned static market-neutral delta hedging was clearly described by Higgins (1902) and Nelson (1904). Thorp and Kassouf (1967) presented market neutral static delta hedging in more details, not only for at-the-money options, but for options *with any delta*. In his 1969 paper Thorp is shortly describing market neutral static delta hedging, also briefly pointed in the direction of some dynamic delta hedging, not as a central pricing device, but a risk-management tool. Filer also points to dynamic hedging of options, but without showing much knowledge about how to calculate the delta. Another "ignored" and "forgotten" text is Bernhard (1970), a book/booklet published in 1970 by Arnold Bernhard & Co. The authors are clearly aware of market neutral static delta hedging or what they name "balanced hedge" for any level in the strike or asset price. This book has multiple examples of how to buy warrants or convertible bonds and construct a market neutral delta hedge by shorting the right amount of common shares. Arnold Bernhard & Co also published deltas for a large number of warrants and convertible bonds that they distributed to investors on Wall Street.

Referring to Thorp and Kassouf (1967), Black, Scholes and Merton took the idea of delta hedging one step further, Black and Scholes (1973):

> "If the hedge is maintained continuously, then the approximations mentioned above become exact, and the return on the hedged position is completely independent of the change in the value of the stock. In fact, the return on the hedged position becomes certain. This was pointed out to us by Robert Merton."

This may be a brilliant mathematical idea, but option pricing is not mathematical theory.

Just after Black, Scholes and Merton published their papers, Thorp (1973) showed how there could not be risk-neutrality once one moved away from continuous delta hedging. And given that continuous delta hedging was obviously impossible in practice, the paper showed how a similar option formula derived under discrete time delta hedging in the limit (of continuous hedging) was equivalent with the Black, Scholes, Merton model. However, his point was that the continuous time delta hedging of the formula was not correct since continuous hedging is impossible and such hedging is very non-robust – see Thorp (2002).

5. Myth 2: option traders today "use" the Black–Scholes–Merton formula

5.1. Traders do not do "valuation"

First, operationally, a price is not quite "valuation". Valuation requires a strong theoretical framework with its corresponding fragility to both assumptions and structure of a model. For traders, a "price" produced to buy an option when one has no knowledge of the probability distribution of the future is not "valuation", but an expedient. Such price could change. Their beliefs do not enter such price. It can also be determined by his inventory.

This distinction is critical: traders are engineers, whether boundedly rational (or even non-interested in any form of probabilistic rationality), they are not privy to informational transparency about the future states of the world and their probabilities. So they do not need a general theory to produce a price—merely the avoidance of Dutch-book style arbitrages against them, and the compatibility with some standard restriction: in addition to put-call parity, a call of a certain strike K cannot trade at a lower price than a call $K+\Delta K$ (avoidance of negative call and put spreads), a call struck at K and a call struck at $K+2\Delta K$ cannot be more expensive than twice the price of a call struck at $K+\Delta K$ (negative butterflies), horizontal calendar spreads cannot be negative (when interest rates are low), and so forth. The degrees of freedom for traders are thus reduced: they need to abide by put-call parity and compatibility with other options in the market.

In that sense, traders do not perform "valuation" with some "pricing kernel" until the expiration of the security, but, rather, produce a price of an option compatible with other instruments in the markets, with a holding time that is stochastic. They do not need top-down "science".

5.2. When do we value?

If you find traders operated solo, in a desert island, having for some to produce an option price and hold it to expiration, in a market in which the forward is absent, then some valuation would be necessary—but then their book would be minuscule. And this thought experiment is a distortion: people would not trade options unless they are in the business of trading options, in which case they would need to have a book with offsetting trades. For without offsetting trades, we doubt traders would be able to produce a position beyond a minimum (and negligible) size as dynamic hedging not possible. (Again we are not aware of many non-blownup option traders and institutions who have managed to operate in the vacuum of the Black–Scholes–Merton argument.) It is to the impossibility of such hedging that we turn next.

5.3. On the mathematical impossibility of dynamic hedging

Finally, we discuss the severe flaw in the dynamic hedging concept. It assumes, nay, requires all moments of the probability distribution to exist.[14]

Assume that the distribution of returns has a scale-free or fractal property that we can simplify as follows: for x large enough, (i.e. "in the tails"), $P[X>n\,x]/P[X>x]$ depends on n, not on x. In financial securities, say, where X is a daily return, there is no reason for $P[X>20\%]/P[X>10\%]$ to be different from $P[X>15\%]/P[X>7.5\%]$. This self-similarity at all scales generates power-law, or Paretian, tails, i.e., above a crossover point, $P[X>x]=Kx^{-\alpha}$. It happens, looking at millions of pieces of data, that such property holds in markets—all markets, baring sample error. For overwhelming empirical evidence, see Mandelbrot (1963), which predates Black–Scholes–Merton (1973) and the jump-diffusion of Merton (1976); see also Stanley et al. (2000), and Gabaix et al. (2003). The argument to assume the scale-free is as follows: the distribution might have thin tails at some point (say above some value of X). But we do not know where such point is—we are epistemologically in the dark as to where to put the boundary, which forces us to use infinity.

Some criticism of these "true fat-tails" accept that such property might apply for daily returns, but, owing to the Central Limit Theorem, the distribution is held to become Gaussian under aggregation for cases in which α is deemed higher than 2. Such argument does not hold owing to the preasymptotics of scalable distributions: Bouchaud and Potters (2003) and Mandelbrot and Taleb (2010) argue that the preasymptotics of fractal distributions are such that the effect of the Central Limit Theorem are exceedingly slow in the tails—in fact irrelevant. Furthermore, there is sampling error as we have less data for longer periods, hence fewer tail episodes, which give an in-sample illusion of thinner tails. In addition, the point that aggregation thins out the tails does not hold for dynamic hedging—in which the operator depends necessarily on high frequency data and their statistical properties. So long as it is scale-free at the time period of dynamic hedge, higher moments become explosive, "infinite" to disallow the formation of a dynamically hedge portfolio. Simply a Taylor expansion is impossible as moments of higher order that 2 matter critically—one of the moments is going to be infinite.

The mechanics of dynamic hedging are as follows: Assume the risk-free interest rate of 0 with no loss of generality. The canonical Black–Scholes–Merton package consists in selling a call and purchasing shares of stock that provide a hedge against instantaneous moves in the security. Thus the portfolio π locally "hedged" against exposure to the first moment of

[14] Merton (1992) seemed to accept the inapplicability of dynamic hedging but he perhaps thought that these ills would be cured thanks to his prediction of the financial world "spiraling towards dynamic completeness". Fifteen years later, we have, if anything, spiraled *away* from it.

Fig. 2. A 25% gap in ericsson, one of the most liquid stocks in the world. Such move can dominate hundreds of weeks of dynamic hedging.

the distribution is the following:

$$\pi = -C + \frac{\partial C}{\partial S} S$$

where C is the call price, and S the underlying security.

Take the discrete time change in the values of the portfolio

$$\Delta\pi = -\Delta C + \frac{\partial C}{\partial S}\Delta S$$

By expanding around the initial values of S, we have the changes in the portfolio in discrete time. Conventional option theory applies to the Gaussian in which all orders higher than ΔS^2 and disappears rapidly.

$$\Delta\pi = -\frac{\partial C}{\partial S}\Delta t - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\Delta S^2 + O(\Delta S^3)$$

Taking expectations on both sides, we can see here very strict requirements on moment finiteness: all moments need to converge. If we include another term, of order ΔS^3, such term may be of significance in a probability distribution with significant cubic or quartic terms. Indeed, although the nth derivative with respect to S can decline very sharply, for options that have a strike K away from the center of the distribution, it remains that the delivered higher orders of ΔS are rising disproportionately fast for that to carry a mitigating effect on the hedges.

So here we mean *all* moments—no approximation. The logic of the Black–Scholes–Merton so-called solution thanks to Ito's lemma was that the portfolio collapses into a deterministic payoff. But let us see how quickly or effectively this works in practice.

The actual replication process is as follows: the payoff of a call should be replicated with the following stream of dynamic hedges, the limit of which can be seen here, between t and T

$$\lim_{\Delta t \to 0} \left(\sum_{i=1}^{n=(T/\Delta t)} \frac{\partial C}{\partial S} \Big| S = S_{1+(t=1)\Delta t}, t = t + (i-1)\Delta t, \quad S_{t+i\Delta t} - S_{t+(i-1)\Delta t} \right)$$

Such policy does not match the call value: the difference remains stochastic (while according to Black Scholes it should shrink). Unless one lives in a fantasy world in which such risk reduction is possible.[15]

Further, there is an inconsistency in the works of Merton making us confused as to what theory finds acceptable: in Merton (1976) he agrees that we can use Bachelier-style option derivation in the presence of jumps and discontinuities – no dynamic hedging – but only when the underlying stock price is uncorrelated to the market. This seems to be an admission that dynamic hedging argument applies only to *some* securities: those that do not jump and are correlated to the market (Fig. 2).

5.4. The robustness of the Gaussian

The success of the "formula" last developed by Thorp, and called "Black–Scholes–Merton" was due to a simple attribute of the Gaussian: you can express *any* probability distribution in terms of Gaussian, even if it has fat tails, by varying the standard deviation σ at the level of the density of the random variable. It does not mean that you are using a Gaussian, nor does it mean that the Gaussian is particularly parsimonious (since you have to attach a σ for every level of the price). It simply mean that the Gaussian can express anything you want if you add a function for the parameter σ, making it function of strike price and time to expiration.

[15] We often hear the misplaced comparison to Newtonian mechanics. It supposedly provided a good approximation until we had relativity. The problem with the comparison is that the thin-tailed distributions are *not approximations* for fat-tailed ones: there is a deep qualitative difference.

This "volatility smile", i.e., varying one parameter to produce $\sigma(K)$, or "volatility surface", varying two parameter, $\sigma(S,t)$ is effectively what was done in different ways by Dupire (1994, 2005) and Derman and Kani (1994, 1998), see Gatheral (2006). They assume a volatility process not because there is necessarily such a thing—only as a method of fitting option prices to a Gaussian. Furthermore, although the Gaussian has finite second moment (and finite all higher moments as well), you can express a scalable with infinite variance using Gaussian "volatility surface". One strong constrain on the σ parameter is that it must be the same for a put and call with same strike (if both are European-style), and the drift should be that of the forward.[16]

Indeed, ironically, the volatility smile is inconsistent with the Black–Scholes–Merton theory. This has lead to hundreds if not thousands of papers trying extend (what was perceived to be) the Black–Scholes–Merton model to incorporate stochastic volatility and jump-diffusion. Several of these researchers have been surprised that so few traders actually use stochastic volatility models. It is not a model that says how the volatility smile should look like, or evolves over time; it is a hedging method that is robust and consistent with an arbitrage free volatility surface that evolves over time.

In other words, you can use a volatility surface as a map, not a territory. However, it is foolish to justify Black–Scholes–Merton on grounds of its use: we repeat that the Gaussian bans the use of probability distributions that are not Gaussian–whereas non-dynamic hedging derivations (Bachelier, Thorp) are not grounded in the Gaussian.

5.5. Order flow and options

It is clear that option traders are not necessarily interested in probability distribution at expiration time—given that this is abstract, even metaphysical for them. In addition to the put-call parity constrains, we can hedge away inventory risk in options with other options. One very important implication of this method is that if you hedge options with options then option pricing will be largely demand and supply based.[17] This in strong contrast to the Black–Scholes–Merton (1973) theory in which demand and supply for options simply should not affect the price of options. If someone wants to buy more options the market makers can simply manufacture them by dynamic delta hedging that will be a perfect substitute for the option itself.

This raises a critical point: option traders do not "estimate" the odds of rare events by pricing out-of-the-money options. They just respond to supply and demand. The notion of "implied probability distribution" is merely a Dutch-book compatibility type of proposition.

5.6. Bachelier–Thorp

The argument often casually propounded attributing the success of option volume to the quality of the Black Scholes formula is rather weak. It is particularly weakened by the fact that options had been so successful at different time periods and places.

Furthermore, there is evidence that while both the Chicago Board Options Exchange and the Black–Scholes–Merton formula came about in 1973, the model was "rarely used by traders" before the 1980s (O'Connell, 2001). When one of the authors (Taleb) became a pit trader in 1992, almost two decades after Black–Scholes–Merton, he was surprised to find that many traders still priced options heuristically "sheets free", "pricing off the butterfly", and "off the conversion", without recourse to any formula.

Even a book written in 1975 by a finance academic appears to credit Thorp and Kassouf (1967)—rather than Black and Scholes (1973), although the latter was present in its bibliography, Auster (1975).

Sidney Fried wrote on warrant hedges before 1950, but it was not until 1967 that the book Beat the Market by Edward O. Thorp and Sheen T. Kassouf rigorously, but simply, explained the "short warrant/long common" hedge to a wide audience.

We conclude with the following remark. Sadly, all the equations, from the first (Bachelier), to the last pre-Black–Scholes–Merton (Thorp) accommodate a scale-free distribution. The notion of explicitly removing the expectation from the forward was present in Keynes (1924) and later by Blau (1944)—and long a Call short a put of the same strike equals a forward. These simple and effective arbitrage relationships appeared to be well known heuristics in 1904.

One could easily attribute the explosion in option volume to the computer age and the ease of processing transactions, added to the long stretch of peaceful economic growth and absence of hyperinflation. From the evidence (once one removes the propaganda), the development of scholastic finance appears to be an epiphenomenon rather than a cause of option trading. Once again, lecturing birds how to fly does not allow one to take subsequent credit.

This is why we call the equation Bachelier–Thorp. We have been using it all along and gave it the wrong name, after the wrong method and with attribution to the wrong persons. It does not mean that dynamic hedging is out of the question; it is just not a central part of the pricing paradigm. It led to the writing down of a certain stochastic process that may have its uses, some day, should markets "spiral towards dynamic completeness". But not in the present.

[16] See Breeden and Litzenberger (1978), Gatheral (2006). See also Bouchaud and Potters (2001) for hedging errors in the real world.

[17] See <fn0085>Gârleanu et al. (2009).

Acknowledgements

We thank Russ Arbuthnot, John (Barkley) Rosser, and others for useful comments.

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