FIN 5330 - Homework 2

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Problem 1

1. Simulate $T = 500$ observations from an $AR(1)$ process for $\phi = \{0.25, 0.5, 0.75, 0.8, 0.9\}$.

$$y_t = \phi y_{t-1} + \epsilon_t$$

2. Treat the artificial from the simulations above as observed data by an econometrician.
3. Estimate each model via OLS.
4. Test the standard null hypothesis of $\phi = 0$ with a standard t-test for significance levels $\{0.01, 0.05, 0.10\}$ for each one and report the results in a table. Provide test statistics, standard errors, critical values, p-values, etc.
5. Pick one of the parameter values for the models above and do the following:
   - Use the Central Limit Theorem to derivive a sampling distribution for $\hat{\phi}$. Present the parameter values of the sampling distribution. Produce a graph of the distribution.
   - Use parametric Monte Carlo to simulate the sampling distribution. Use $M = 10,000$ repititions. Use the sample mean and standard deviation to estimate the parameter values of the distribution. Produce a histogram.
   - Use the IID Bootstrap to simulate the sampling distribution. Use $B = 10,000$ repititions. Use the sample mean and standard deviation to estimate the parameter values of the distribution. Produce a histogram.
   - Compare all three methods.
   - Can you interpret the last two distributions as predictive densities?
6. Return to the problem in 5 above and redo the simulation from step one, but replace the error distribution with a Student-T distribution with $df = 5$ (degrees of freedom parameter). Even though we know at the generation stage that the errors come from the Student-T distribution, the econometrician assumes a normal distribution when using the CLT and parametric Monte Carlo. The bootstrap obviously does not need to make such assumptions. Compare to the results above.

Problem 2

Simulate an $AR(1)$ process with parameter $\phi = 0.8$ by using the $MA(\infty)$ representation. **Hint:** you will have to truncate the $MA(\infty)$ representation, yielding an approximation to the $AR(1)$. Recall that the $AR(1)$ can be represented by the following (i.e. the $MA(\infty)$ representation):

$$x_t = x_0 + \sum_{j=0}^{\infty} \phi^j \varepsilon_{t-j} \qquad \text{where } x_0 \text{ is the initial condition.}$$

- Plot the simulated time series.
- Estimate $\hat{\phi}$ via OLS. Report the usual suspects.

Problem 3

Take the $AR(1)$ model above:

$$y_t = \phi y_{t-1} + \varepsilon_t \qquad \text{with } \phi = 0.8$$

- Run the following simulation:
  - Set $\varepsilon_0 = 1.0$
  - Set $y_0 = 0.0$
  - Set all $\varepsilon_t = 0.0$ for $t > 0$
  - Plot the simulated process $\{y\}_{t=0}^{T}$ as a function of time. This is called the ***impluse response function***. Interpret it in terms of the MA (θ) coefficients for the $AR(1)$ representation.
  - Simulate $T = 50$ time steps in the process.

Problem 5

Simulate $T = 500$ observations from the following ARMA model:

$$y_t = \phi_1 yt - 1 + \phi_2 y_{t-2} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t$$

Choose appropriate values for the AR and MA coefficients, as well as for σ_ε.

- Plot the simulated time series.
- Calculate γ_0, γ_1 and γ_2.

Problem 6

- Simulate $T = 500$ time steps for the following two equations:

$$y_t = y_{t-1} + u_{1,t}$$
$$x_t = x_{t-1} + u_{2,t}$$

- where $u_{j,t} \quad j = 1, 2$ are independent standard white noise processes.
- Next regress y_t on x_t and estimate β (slope coefficient) via OLS in the following regression

$$y_t = \alpha + \beta x_t + \epsilon_t$$

- Test the null hypothesis $H_0 : \beta = 0$ against the alternative $H_a : \beta \neq 0$. Use the standard t-test with standard significance levels (0.01, 0.05, and 0.10). What should you find? What do you find?
- Repeat the process $M = 50,000$ times and store the β coefficients for each run of the simulation.
- Summarize the simulated sampling distribution for β.
- Make a histogram plot of the simulated coefficients.

Problem 7

- Repeat the exercise in Problem 1 above for $\phi = 1.0$.
- Comment on your findings.

Problem 8

- Simulate $T = 500$ time steps from the random walk model

$$x_t = x_{t-1} + u_{1,t}$$

- Next simulate $T = 500$ time steps from the model

$$y_t = \alpha + \beta x_t + \epsilon_t$$

- Where $\alpha = 0.22$ and $\beta = 2.50$.
- $\epsilon \sim N(0, 1)$ (white noise process)
- Use the Augmented Dickey-Fuller Test to check for the presence of a unit-root in both y_t and x_t. What do you find? What should you find?
- Implement the Engle-Granger two-step method by:
  - First, test for cointegration by submitting $\hat{\epsilon}_t$ to the ADF Test. What do you find?
  - Obtain $\hat{\beta}$ via OLS.
  - Estimate the error-correction model with $p = 1$ and include contemporaneous x_t.