Cointegration

Tyler J. Brough

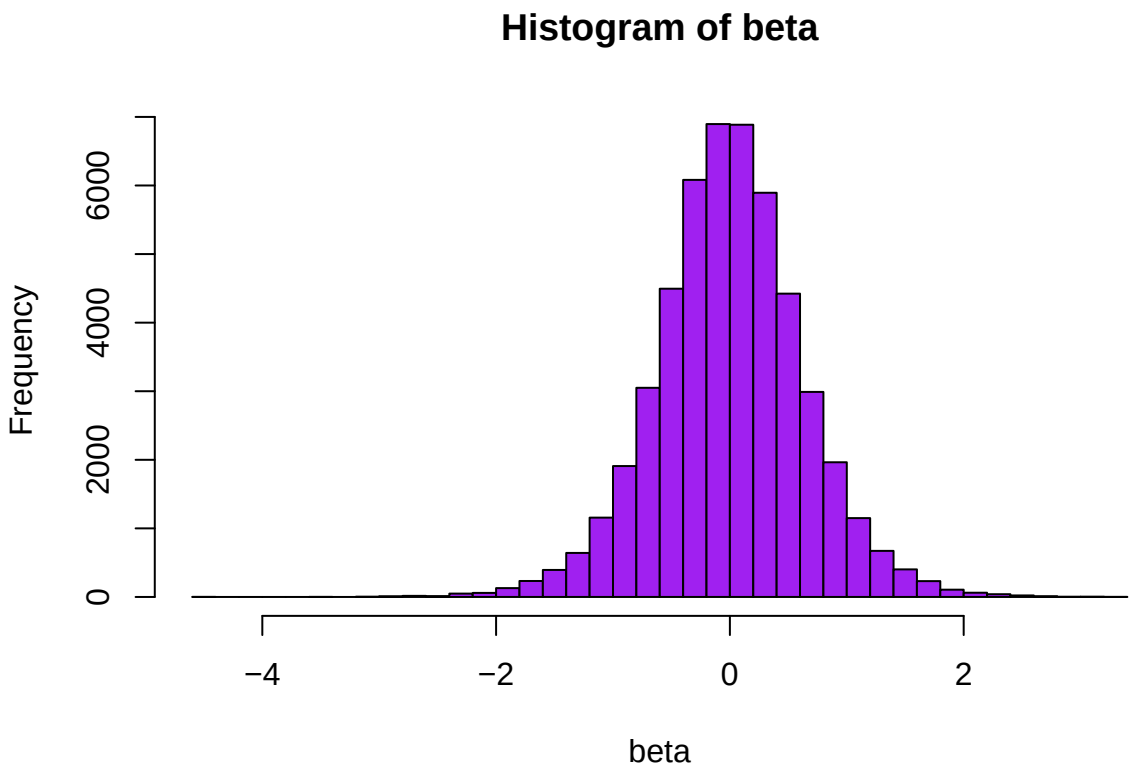
February 28, 2019

Spurious Regression

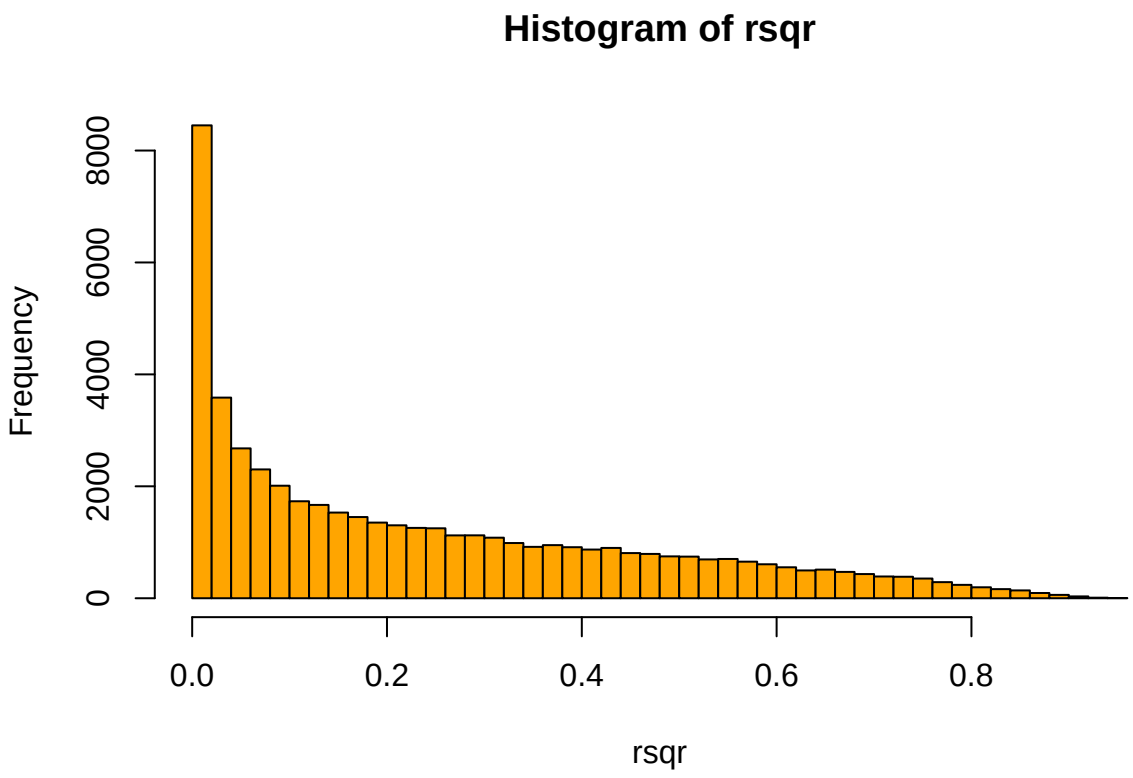
```
N <- 500
M <- 50000
beta <- rep(0, M)
rsqr <- rep(0, M)

for(i in 1:M) {
  y <- cumsum(rnorm(N))
  x <- cumsum(rnorm(N))
  fit <- lm(y ~ x)
  beta[i] <- fit$coefficients[2]
  rsqr[i] <- summary(fit)$r.squared
}
```

```
hist(beta, breaks=50, col="purple")
```




```
hist(rsqr, breaks=50, col="orange")
```



Cointegration and Error-Correction

```
N <- 500
a <- 0.22
b <- 2.5
u <- rnorm(N)
x <- cumsum(rnorm(N))
y <- a + b * x + u
fit <- lm(y ~ x)
z <- fit$residuals
```

```
Dy <- diff(y)
Dx <- diff(x)
fit <- lm(Dy[3:N] ~ z[3:N] + Dy[2:(N-1)] + Dx[2:(N-1)])
summary(fit)
```

```
##
## Call:
## lm(formula = Dy[3:N] ~ z[3:N] + Dy[2:(N - 1)] + Dx[2:(N - 1)])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.5445 -1.7215 -0.0189  1.8122  7.6361
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -0.04462    0.11810  -0.378    0.706
## z[3:N]        -1.04806    0.17237  -6.080 2.41e-09 ***
```

```
## Dy[2:(N - 1)]  0.13031    0.12156   1.072    0.284
## Dx[2:(N - 1)] -0.20628    0.32255  -0.640    0.523
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.632 on 493 degrees of freedom
##   (1 observation deleted due to missingness)
## Multiple R-squared:  0.1078, Adjusted R-squared:  0.1024
## F-statistic: 19.85 on 3 and 493 DF,  p-value: 3.655e-12
```