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Corrigendum

Corrigendum to "Delaunay Hodge star" [Comput. Aided Des. 45 (2013) 540–544]

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A B S T R A C T

A discrete diagonal Hodge star operator for pairwise Delaunay triangulations was described in Hirani et al. (2013). This note fixes errors in two figures and some indexing mistakes in the original paper. One consequence of this fix is that discrete exterior calculus may be applicable to a wider class of meshes.

In [1] discrete diagonal Hodge star operators of discrete exterior calculus (DEC) were studied for triangulations that are pairwise Delaunay. It was shown that for such triangulations that are non-degenerate the codimension 1 Hodge star assembled from elementary duals using the sign convention introduced in [1] has positive entries. It was further shown that for such two-dimensional triangulations embedded in $\mathbb{R}^2$ or $\mathbb{R}^3$ and for three-dimensional triangulations in $\mathbb{R}^3$, all other Hodge star operators also have positive entries when assembled from elementary duals using the sign convention of [1].

The main correction presented here is replacement of Fig. 1 of [1] by Fig. 1 of this note. This note also fixes a formula error and error with an illustration in Fig. 6 of the original paper. The main correction is described in Section 1 and other corrections in Section 2.

1. Main correction

The main aim of [1] was the statement and proof of positivity of entries of diagonal Hodge star matrix assembled from signed elementary duals introduced in [1]. The mathematical statements were made for meshes that are pairwise Delaunay, non-degenerate, and with a one-sidedness property for boundary simplices. There are no errors in the original paper in these mathematical statements (lemmas and theorems). In [1] no mathematical statements were made for meshes that violate the pairwise Delaunay condition and/or meshes that violate the one-sidedness condition. A numerical computation of solution of scalar Poisson's equation was shown for such meshes. These were shown in columns 3 and 4 of Fig. 1 in [1].

The authors recently discovered errors in some computer programs that were used for numerical experiments corresponding to columns 3 and 4 of Fig. 1 in [1]. Those figures in the original paper seemed to suggest that meshes which violate the pairwise Delaunay condition or one-sidedness condition may need to be avoided. After correcting the errors in the programs the authors noticed that DEC appears to produce the correct solution in the cases shown even for such meshes. In light of this, the following sentence from the Introduction in the original paper should be deleted: "That figure also shows the importance of the Delaunay property and of our boundary assumptions and the success of the signed dual volumes for such meshes that we describe in this paper.". Fig. 1 here replaces the incorrect Fig. 1 of [1]. The figures in columns 1 and 2 of Fig. 1 of [1] are unchanged by this correction — those figures in the original paper are correct. They are included here for completeness.

The programming error is now described. The error has been traced to generation and processing of the mesh used for Fig. 1, column 3 of [1]. The vertex and edge numberings for this mesh are shown in Fig. 2. (For clarity, only numbering of vertices and edges on the boundary are shown.) Notice that vertices numbered 0 and 19 overlap (bottom left corner) and vertices 14 and 15 overlap (top left corner). This led to an incorrect assignment of boundary conditions. (The boundary condition for the problem is inflow through left boundary, outflow through right, and no flow across the top and bottom boundaries.) As a result, the edge numbered 22 on the right was assigned an outflow velocity of 0 and edge 27 was assigned an outflow velocity of 1 pointing to the right. Due to normal to edge 27 pointing upwards this resulted in 0 flux through edge 27 (which is correct). However, edge 22 was assigned

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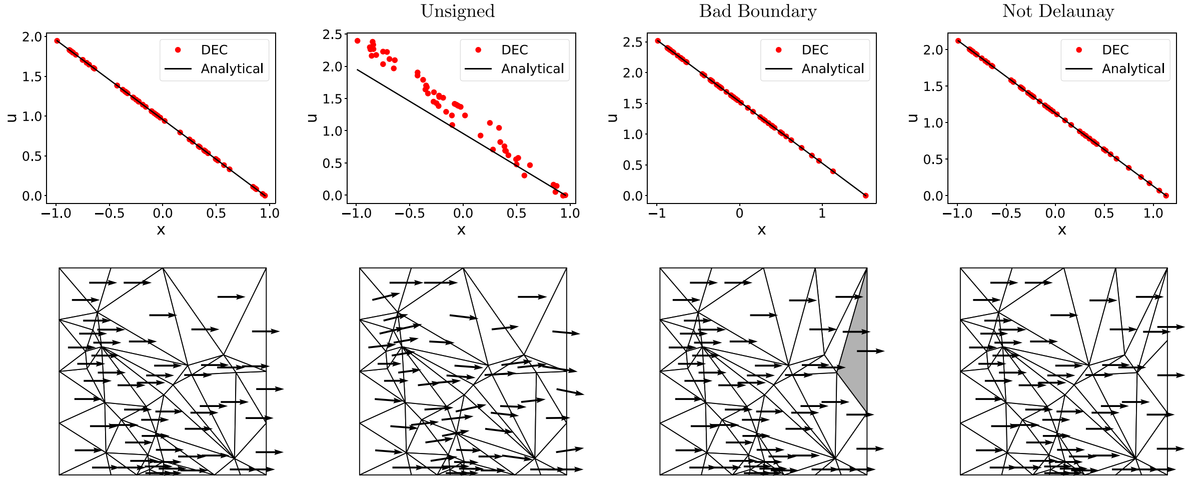

Fig. 1. Solution of Poisson's equation $-\Delta u = f$ in mixed form. In mixed form this equation is the system $\sigma = -\text{grad } u$ and $\text{div } \sigma = f$. The boundary condition is constant influx on left and outflux on right. The correct solution is linear u which varies only along x-direction and a constant horizontal σ. The top row shows u and bottom row shows σ. The first column shows the correct solution using the results of this paper on a Delaunay mesh with correct boundary simplices. The second column is for unsigned duals using the same mesh as first column. This fails to produce the correct solution. The next two columns are instances in which the 1-Hodge star mass matrix is not positive definite. The third column has a single bad (i.e., not one-sided) boundary triangle shown shaded in a Delaunay mesh. The fourth column is a non Delaunay mesh. It appears that discrete exterior calculus produces correct solution even for these cases for meshes used in columns 3 and 4.

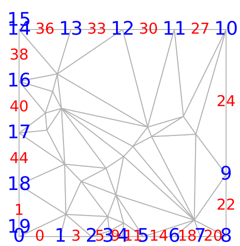

Fig. 2. The cause of the programming error was an incorrect mesh. Notice the overlapping vertices on top left and bottom. This led to an incorrect assignment of boundary conditions for columns 3 and 4 of Fig. 1 in the original paper as explained in the text.

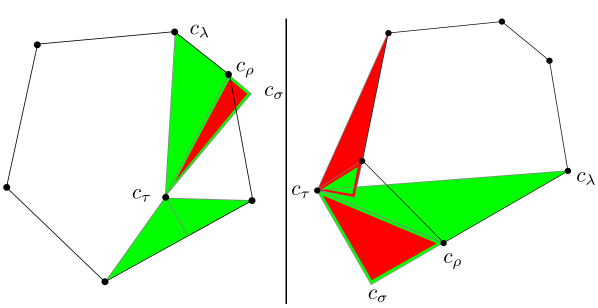

Fig. 3. Representative elementary dual simplices of $\star\tau$ when it intersects τ (left side) and does not intersect τ (right side) corresponding to the two cases shown in Fig. 5 of [1]. An angle in a triangle and a color of another triangle have been corrected.

an incorrect flux of 0. All outgoing flux was thus assigned to edge 24 on the right.

For column 4 in Fig. 1 of [1], the non-Delaunay mesh was obtained by starting from the erroneous mesh of column 3. In particular, one of the triangles of the mesh from column 3 was subdivided to yield a non-Delaunay pair. Consequently, the non-Delaunay mesh inherited the edge indexing problems of the mesh from column 3. In addition, due to modification of edge markers during subdivision, some internal edges were misidentified as being boundary edges. This again led to an incorrect assignment of boundary conditions.

2. Other corrections

The other corrections in [1] are not related to the main correction described above. In Section 2 of [1], the formula for sign s that appears in the last line of first paragraph on second column of page 541 should be changed to $s = s_p\, s_{p+1} \ldots, s_{n-1}$ (and not end at s_n as given incorrectly in the original paper). The subscripts in the subsequent examples in the next two paragraphs of the original paper that use this formula should also be changed accordingly.

The illustration in Fig. 6 of [1] should be replaced by one in Fig. 3 given here. The changes are as follows. In the left figure of Fig. 6 of [1], the gray line from c_τ to the base of the green triangle at the bottom should have been perpendicular to the base. This is correctly shown now in Fig. 3. In the right figure of Fig. 6 in [1], triangle $c_\tau c_\sigma c_\rho$ should be red as shown here and not green as in the original.

References

[1] Hirani Anil N, Kalyanaraman Kaushik, VanderZee Evan B. Delaunay Hodge star. Comput Aided Des 2013;45(2):540–4.