EC 504 – Spring 2023 – Homework 1

Due Wednesday Feb 8, 2023 at 8 PM Boston Time

NOTE: This HW is to be turned in to Gradescope as a PDF.

Start reading Chapters 1, 2, 3 and 4 in CLRS . They give a very readable introduction to Algorithms.. Also glance at Appendix A for math tricks which we will use from time to time.

1. (40 pts) In CRLS do Exercise 3.1-4 on page 53, Problem 3.2-7 on page 60, Problem 3-2 on page 61 , Problem 4.3-9 on page 88.

2. (30 pts) Place the following functions in order from asymptotically smallest to largest. As a convenience you may use $f(n) < O(g(n))$ to mean $f(n) \in O(g(n))$ and $f(n) = g(n)$ to mean $f(n) \in \Theta(g(n)$. Please use $=$ when you are sure that it is $\Theta(g(n))$.)

   $n^2 + 3n\log(n) + 5$, $n^2 + n^{-2}$, $n^{n^2} + n!$, $n^{\frac{1}{n}}$, n^{n^2-1} , $\ln n$, $\ln(\ln n)$, $3^{\ln n}$, 2^n ,

   $(1+n)^n$, $n^{1+\cos n}$, $\displaystyle\sum_{k=1}^{\log n} \frac{n^2}{2^k}$, 1 , $n^2 + 3n + 5$, $log(n!)$, $\displaystyle\sum_{k=1}^{n} \frac{1}{k}$, $\displaystyle\prod_{k=1}^{n}(1 - \frac{1}{k^2})$, $(1-1/n)^n$

   Giving the the algebra and explanation for the tricky cases can get some extra credit (even if you get it wrong!). Don't have to be perfect to get a good score.

1. (40 pts)

   (a) Given the equation, $T(n) = 2T(n/2) + n$, guess a solution of the form:

   $$T(n) = c_1 n + c_2\, nlog_2(n)\ .$$

   Find the coefficients c_1, c_2 to determine the exact solution assuming a value $T(1)$ at the bottom the recursion.

   (b) Generalize this to the case to the equation $T(n) = aT(n/b) + n^k$ and guess the solution of the form:

   $$T(n) = c_1 n^\gamma + c_2 n^k$$

   using $b^\gamma = a$ and assuming $\gamma \neq k$. First show if you drop the n^k using the **homogeneous** equation $T(n) = aT(n/b)$ the form $c_1 n^\gamma$ is a solution! (What is c_1?) Second drop $aT(n/b)$ and show $c_2 n^k$ is a solution (What is c_2?) With both terms (and $\gamma \neq k$) the full solution is just the sum of the to terms but only one or the other dominates!

   (c) What happens when the two solution collide (i.e have the same power, i.e $\gamma = \log(a)/\log(b) = k$.) Now show that the leading solution is as n goes to infinity is $T(n) = \Theta(n^k logn)$ [1]

[1] If you are ambitious you find exact solutions for part b and c above are of the form $T(n) = c_1 n^\gamma + givec_2 n^k$ and $T(n) = c_1 n^k + c_2 nlog(n) n^k$ explicitly determine the $c's$ for each case respectively. We did that in part for example. BUT note we already know the leading terms for the Master Equation without this extra effort! Neat trick!