EC 504 – Fall 2025 – Homework 1

Due Tuesday Sept 23, 2025 at 11:59 PM Boston Time. You should put the .pdf solution in directory `/projectnb/ec504rb/students/yourname/HW1`

Read Chapters 1, 2, 3 and 4 in CLRS . They give a very readable introduction to Algorithms.. Also glance at Appendix A for math tricks which we will use from time to time.

Start working on this right a way and then in class on Tuesday Sept 16 and Thursday Sept 18 ask any question you want to get help to finish it!

1. (40 pts) In CRLS do Exercise 3.1-4 on page 53, Problem 3.2-7 on page 60, Problem 3-2 on page 61 , Problem 4.3-9 on page 88. GitHUB has these pages from CRLS posted.

2. (30 pts) Place the following functions in order from asymptotically smallest to largest. As a convenience you may use $f(n) < O(g(n))$ to mean $f(n) \in O(g(n))$ and $f(n) = g(n)$ to mean $f(n) \in \Theta(g(n)$. Please use $=$ when you are sure that it is $\Theta(g(n))$.)

$$n^2 + 3n\log(n) + 5\ ,\ \ n^2 + n^{-2}\ ,\ n^{n^2} + n!\ ,\ n^{\frac{1}{n}}\ ,\ n^{n^2-1}\ ,\ \ln n\ ,\ \ln(\ln n)\ ,\ 3^{\ln n}\ ,\ 2^n\ ,$$

$$(1+n)^n\ ,\ n^{1+\cos n}\ ,\ \sum_{k=1}^{\log n} \frac{n^2}{2^k}\ ,\ 1\ ,\ n^2 + 3n + 5\ ,\ log(n!)\ ,\ \sum_{k=1}^{n} \frac{1}{k}\ ,\ \prod_{k=1}^{n}(1 - \frac{1}{k^2})\ ,\ (1 - 1/n)^n$$

Giving the the algebra and explanation for the tricky cases can get some extra credit (even if you get it wrong!). Don't have to be perfect to get a good score.

3. (40 pts)

   (a) Given the equation, $T(n) = 2T(n/2) + n$, guess a solution of the form:

   $$T(n) = c_1 n + c_2\ nlog_2(n)\ .$$

   Find the coefficients c_1, c_2 to determine the exact solution assuming a value $T(1)$ at the bottom the recursion.

   (b) Generalize this to the case to the equation $T(n) = aT(n/b) + n^k$ and guess the solution of the form:

   $$T(n) = c_1 n^\gamma + c_2 n^k$$

   using $b^\gamma = a$ and assuming $\gamma \neq k$. First show if you drop the n^k using the **homogeneous** equation $T(n) = aT(n/b)$ the form $c_1 n^\gamma$ is a solution! (What is c_1?) Second drop $aT(n/b)$ and show $c_2 n^k$ is a solution (What is c_2?) With both terms (and $\gamma \neq k$) the full solution is just the sum of the to terms but only one or the other dominates!

   (c) What happens when the two solution collide (i.e have the same power, i.e $\gamma = \log(a)/\log(b) = k$.) Now show that the leading solution is as n goes to infinity is $T(n) = \Theta(n^k logn)$ [1]

[1] If you are ambitious , find the exact solutions for part b and c by `guessing` the form $T(n) = c_1 n^\gamma + c_2 n^k$ and $T(n) = c_1 n^k + c_2 nlog(n) n^k$ to determine the $c's$ for each case respectively. You need to assume a starting value $T(1)$ is given.