
Problems
3-1 Asymptotic behavior of polynomials

Let

$$p(n) = \sum_{i=0}^d a_i n^i,$$

where $a_d > 0$, be a degree- d polynomial in n , and let k be a constant. Use the definitions of the asymptotic notations to prove the following properties.

- a. If $k \geq d$, then $p(n) = O(n^k)$.
- b. If $k \leq d$, then $p(n) = \Omega(n^k)$.
- c. If $k = d$, then $p(n) = \Theta(n^k)$.
- d. If $k > d$, then $p(n) = o(n^k)$.
- e. If $k < d$, then $p(n) = \omega(n^k)$.

3-2 Relative asymptotic growths

Indicate, for each pair of expressions (A, B) in the table below, whether A is O , o , Ω , ω , or Θ of B . Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with “yes” or “no” written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	\sqrt{n}	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

3-3 Ordering by asymptotic growth rates

- a. Rank the following functions by order of growth; that is, find an arrangement g_1, g_2, \dots, g_{30} of the functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, \dots , $g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.