11 Suggested Project Topic & References

One possible Project Topic:

The general topic is the use of 2D Linear solvers and paralleliztion. The engineering problem is to understand temperture distribution on a 2D chip. The basic heat equation is described in detail in:

https://en.wikipedia.org/wiki/Heat_equation

The simplest problem can be static (no time dependence) but the local Jacobi interation does give you the time dependence for free! If you are ambitious you many extend the input heat to random thermal fluctuatation. Start simple and see how far you can go.

Here is start for reference: https://en.wikipedia.org/wiki/Thermal_simulations_for_integrated_circuits We all can collaborate in googling for useful information.

The topic is flexible so you should do research on this topic and pick a realistic goal given what the course has taught and teams talents. **In the remaining meetings there will be a lot of hands on help in the Lab to suggest solutions.** Each project should include some use of MPI and/or openMP to speed up the problem and a discussion of performance and accuracy. Part of the report can be library research on more advance methods that you might pursue if you were to take on this professionally!

11.1 Some ideas and References

Part of the project is for each team to go off and find neat ideas to explore. First let us consider the basic time dependent equation described in https://en.wikipedia.org/wiki/Heat_equation

We are looking at this in 2D. So let think about it. Heat gets "pumped" in at rate of $\dot{q}(x, y, t)$ per unit area at location x, y. It has a flux of $k(x, y)\nabla T$ where is k the conductivity for heat to flow from high temperture to low and its divergence ($\nabla \cdot k\nabla T(x, y, t)$) tells how much total heat is conducted away from that point at x, y. Finally what heat remains raises the temperture $\rho c_p \dot{T}(x, y, t$ depending on the specific heat times the density of the material $\rho(x, y)c_p(x, y)$. All this result in this equation!

$$\dot{q} = \rho c_p \frac{\partial T(x, y, t)}{\partial t} - \nabla \cdot (k\nabla T(x, y, t)) \tag{126}$$

If we ignore the x, y dependence of k, ρ, c_p this is just the equation we are using to solver the `Ax = b` problem in descrete form!

$$\frac{T[x, y, t + \delta t] - T[x, y, t]}{\delta t} = k\frac{T[x + h, y, t] + T[x - h, y, t] + T[x, y + h, t] + T[x, y - h, t] - 4T[x, y, t]}{h^2} + \dot{q}[x, y, t] \tag{127}$$

where for simplicity we have redefined $k/\rho c_p \to k$ and $\dot{q}/\rho c_p \to \dot{q}$. Next multiplying through by δt,

$$T[x, y, t+\delta t] = (1-4k\delta t/h^2)T[x, y, t]+(k\delta t/h^2)(T[x+h, y, t]+T[x+h, y.t]+T[x+h, y, t]+T[x+h, y, t])+\delta t\dot{q} \tag{128}$$

and defining $\alpha = 4k\delta t/h^2$. and

$$T[x, y, t+1] = (1-\alpha)T[x, y, t]+\alpha(T[x+h, y, t]+T[x-h, y, t]+T[x, y+h, t]+T[x, y-h, t])/4+\alpha b(x, y) \tag{129}$$

This is just the Jacobi iteration with an overrelaxation perameter:$\alpha = 4k\delta t/h^2$. Small α is small time steps (very under-relaxed). The standard solver used $\alpha = 1$. Ok now we see that we can do all sorts of real heat conduction problems.