III. Ising Random Cluster

Parallel Multigrid Percolation Cluster Detection

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.62.1087

and lot of cool graph theoretic problem from Brower et al !

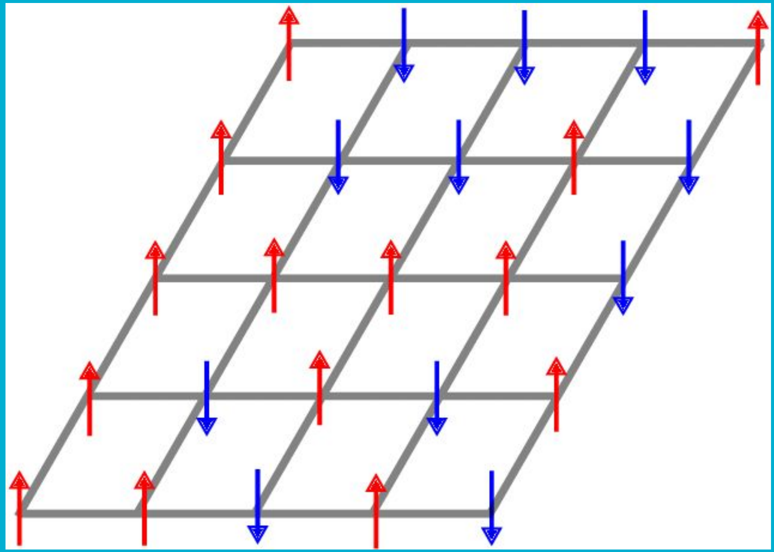
Ising Percolation Cluster Problem

Magnetic polarities tend to align.

We can count the energy of a plane of magnetic poles where we sum for each bonded/neighboring pair of poles -1 if they are the same and +1 if they are different by creating this as an undirected graph

We can relax this grid with the Swendsen-Wang algorithm by ‘percolating’ random bonds then randomly flipping where these bonded poles match

From a point we can serially find a cluster of like poles by BFS or DFS



Multigrid Algorithm

We are looking rather at relaxing based on a fixed point. We can derive the max distance of points in its cluster.

With a single V cycle of multigrid fine-coarse-fine we can distinguish where prior separate clusters can be combined by expanding a connectivity matrix by powers of two each iteration.

$$\Delta_{ij}^{(l)} = \begin{cases} M_{ij} & \text{if } i-j \text{ is a distance } \pm 2^l \text{ in a coordinate direction} \\ 0 & \text{otherwise} \end{cases} \qquad (13)$$

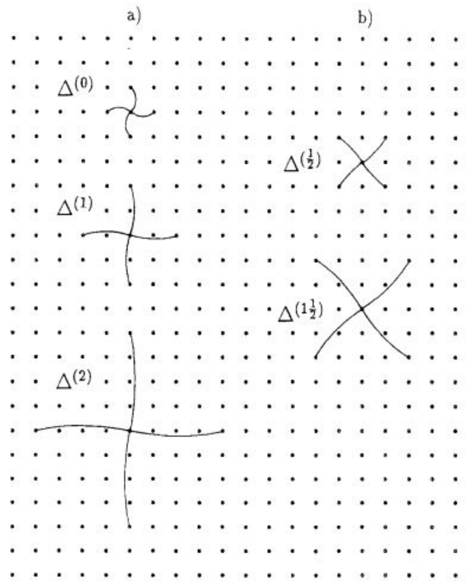


Fig. 1. (a) Multigrid connectivity matrices $\Delta^{(l)}$ connect neighbors at powers-of-two distances along coordinate axes. (b) The connectivity matrices for half-integer levels improve convergence by connecting neighbors along diagonals.

IV. HEAT EQUATION

(See Lect 11 in Class Lectures)