CHAPTER-3

Motion in a Plane

Topics To Be Covered

- Scalar and vector quantities.
- Position and displacement vector.
- General vector and their notation.
- Equality of vectors
- Multiplication of vectors by a real number.
- Addition and subtraction of vectors.
- Unit vectors.
- Resolution of vector in a plane.
- Rectangular component
- Scalar & vector product of vectors.
- Case of uniform velocity, uniform acceleration, projectile motion.
- Uniform Circular motion.

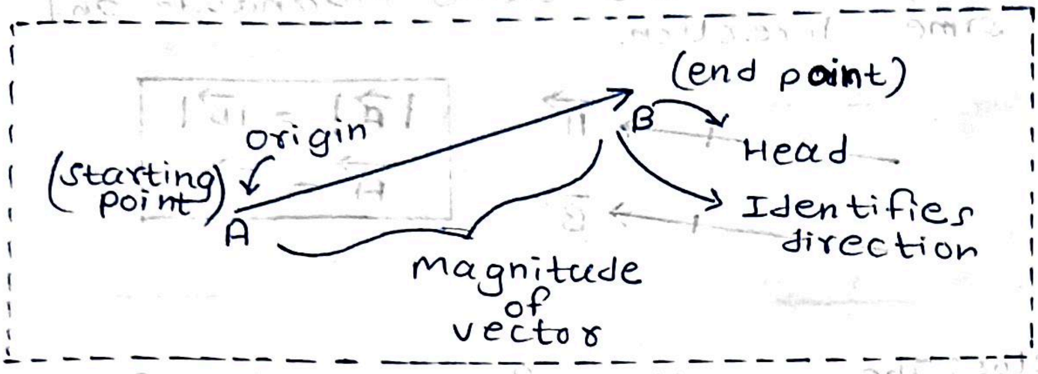
Scalar & Vector Quantities

Physical Quantities

Scalar	Vector
→ Magnitude	→ Magnitude
→ No direction	→ Have direction
→ Always +ve	→ Can be +ve, -ve or 0.
→ Follow simple (addition, subtraction, multiplication, division, algebraic rules).	→ Follow special algebraic rules.

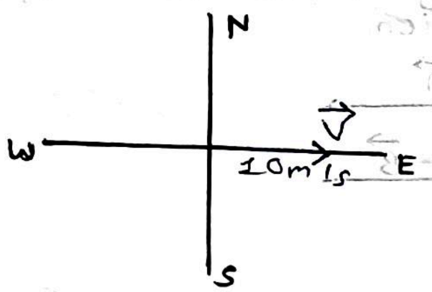
Representation of vectors

→ A vector quantity is represented by a straight line with an arrowhead over it.



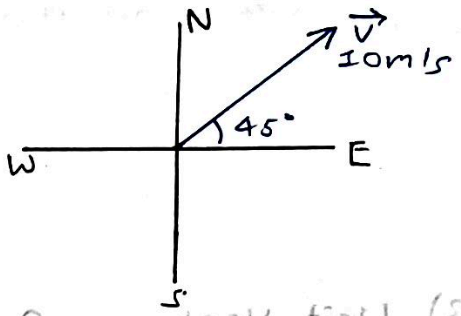
Eg: (i) 10 m/s east = $\vec{v}$

=>



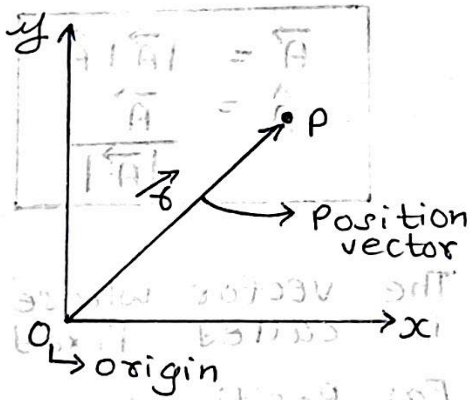
(ii) $\vec{v}$ = 10m/s (nort east)

=>

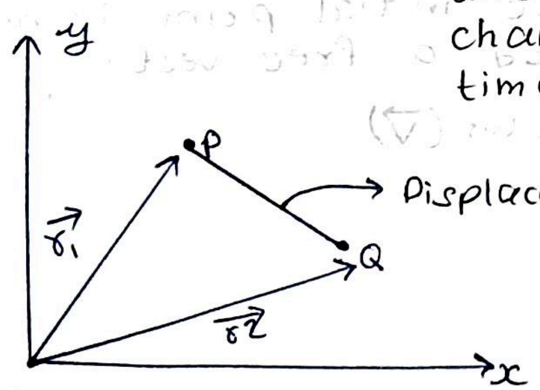


Position Vector : A vector which gives position of an object with reference to the origin of a co-ordinate system is called position vector.

Eg:

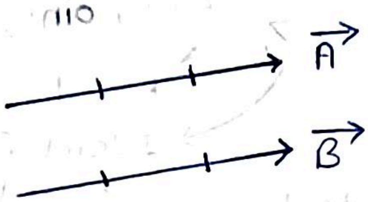


Displacement Vector : It is that vector which tell how much and in which direction an object has changed its position in a given time interval.



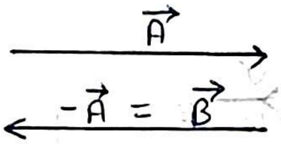
General Vectors and Their Notations

1) **Equal Vector:** Two vectors are said to be equal if they have the same magnitude and same direction.



$$\boxed{\begin{aligned} |\vec{A}| &= |\vec{B}| \\ \vec{A} &= \vec{B} \end{aligned}}$$

2) **Negative of a Vector:** The negative of a vector is defined as another vector having the same magnitude and opposite in direction.



3) **Unit Vector:** A unit vector is a vector of unit magnitude draw in the direction of a given vector.

$$\boxed{\begin{aligned} \vec{A} &= |\vec{A}|\,\hat{A} \\ \hat{A} &= \frac{\vec{A}}{|\vec{A}|} \end{aligned}}$$

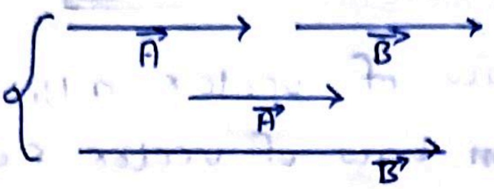
4) **Fixed Vector:** The vector whose initial point is fixed is called fixed vector.

Eg: Position vector

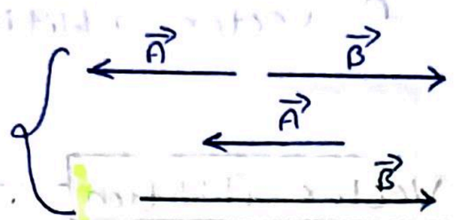
5) **Free Vector:** A vector whose initial point is not fixed is called a free vector.

Eg: Velocity vector ($\vec{V}$)

6) **Collinear Vector:** The vectors which either act along the same line or along parallel line are called collinear.

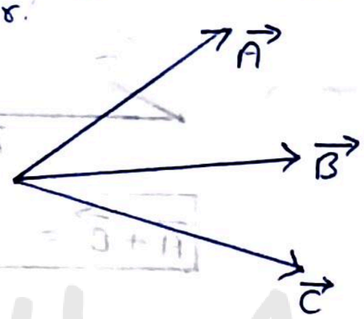


Like vector



Unlike vector

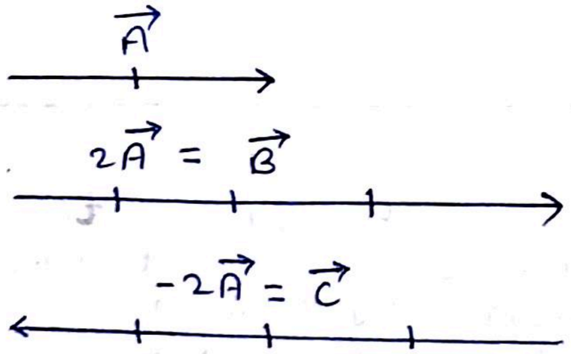
7) **Co-initial Vector:** The vectors which have the same initial point are called co-initial vector.



8) **Zero Vector:** A zero or null vector is a vector that has zero magnitude and an arbitrary direction. It is represented by '$\vec{0}$'.

Eg: The velocity vector of a stationary object is a zero vector.

Multiplication of a Vector by a Real Number



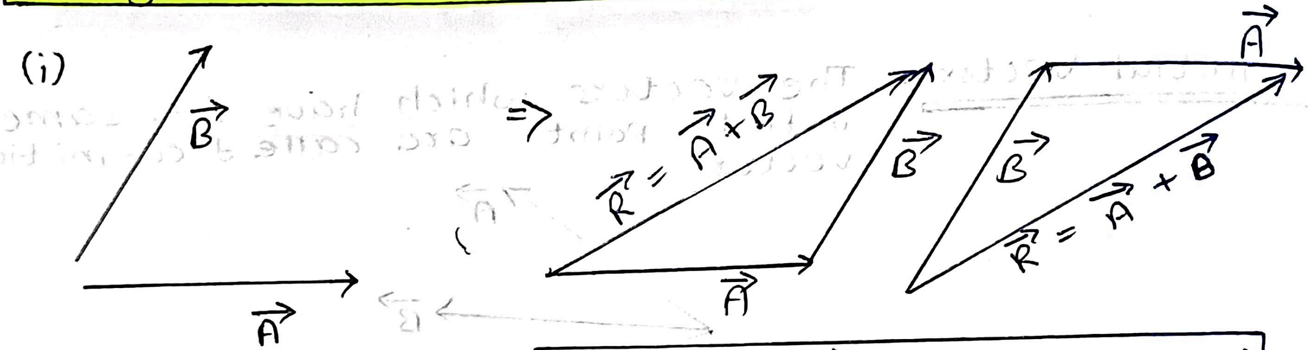
Addition and Subtraction of vector

Addition or composition of vector :-

1) Triangle Law of vector addition (For 2 vectors)
2) Parallelogram Law of vector addition (For 2 vectors)
3) Polygon Law of vector addition (For two or more than two vectors)

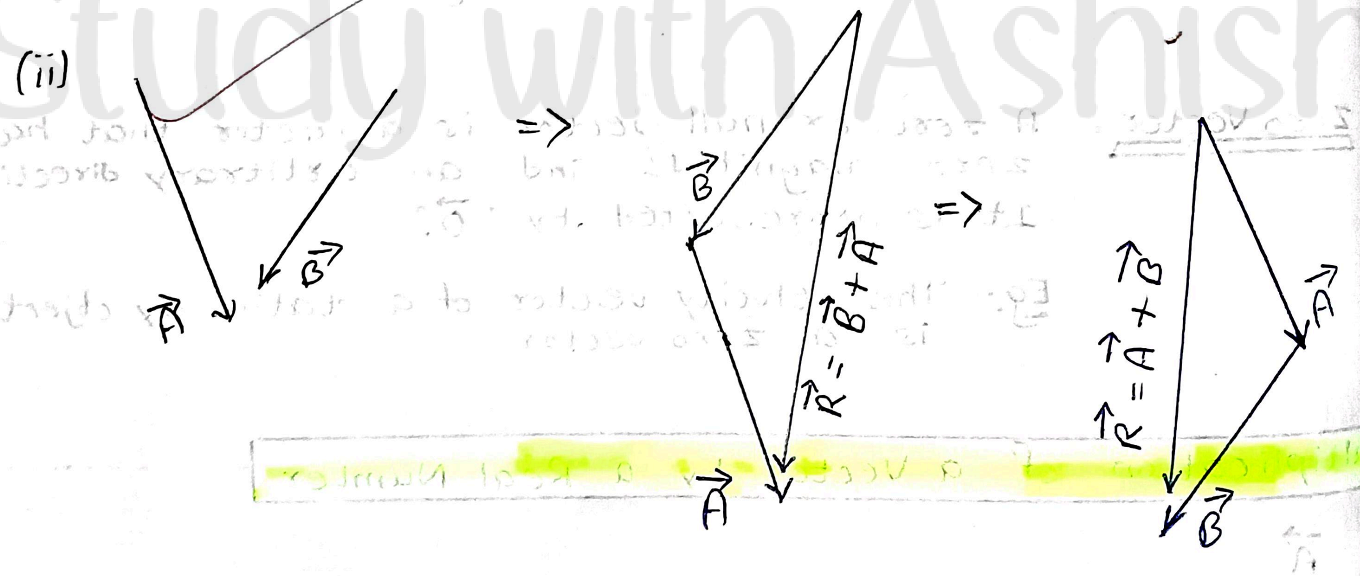
Triangle Law of Vector Addition

(i)

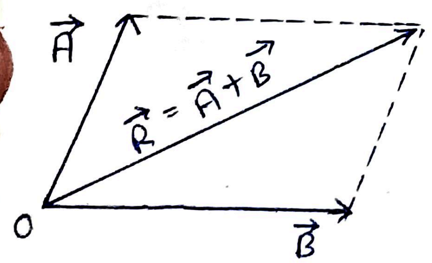


$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A} \text{ (Commutative Law)}}$

(ii)

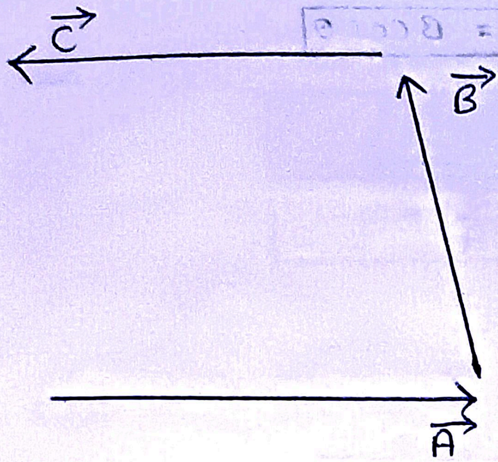


||gm Law of Vector Addition



If two vectors can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a common point, then their resultant is represented by the diagonal of the parallelogram passing through that point.

Polygon Law of Vector Addition

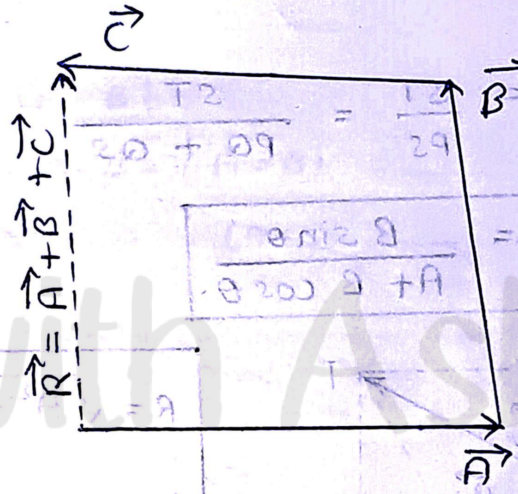


Join initial point of first vector to the final point of last vector.

Vector-1: $\vec{A}$

Vector-2: $\vec{B}$

Vector-3: $\vec{C}$



Magnitude of Resultant Vector & its direction

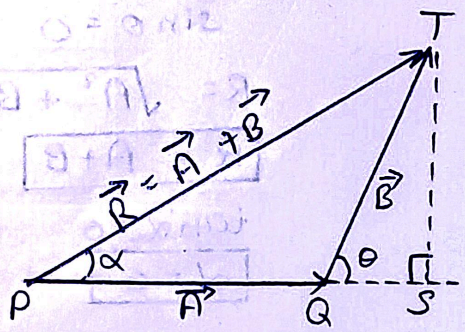
In ΔPST :-

$PT^2 = PS^2 + ST^2$

$PT^2 = (PQ + QS)^2 + ST^2$

$PT^2 = PQ^2 + QS^2 + 2.PQ.QS + ST^2$

$PT^2 = PQ^2 + (QS^2 + ST^2) + 2.PQ.QS$



Now, in ΔQST :-

$QT^2 = QS^2 + ST^2$

Then above becomes -

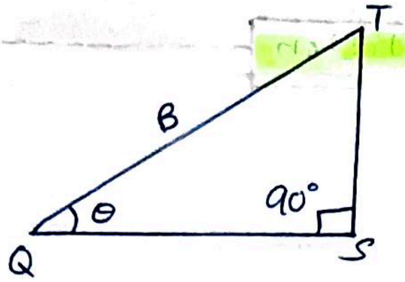
$PT^2 = PQ^2 + QT^2 + 2.PQ.QS$ ... (1)

From figure -

$PT = R$

$PQ = A$

$QT = B$



$\therefore \cos\theta = QS/QT$

$QS = QT \cdot \cos\theta$

$\boxed{QS = B\cos\theta}$

From eqⁿ ① we get -

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

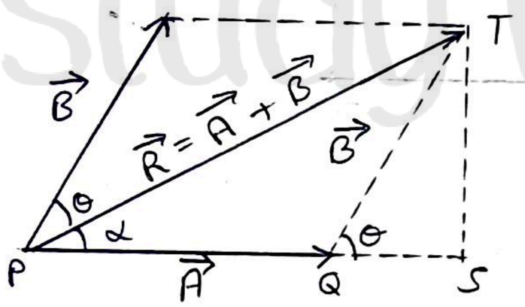
$$\therefore \boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Direction of $\vec{R}$:-

ΔPST

$$\tan\alpha = \frac{ST}{PS} = \frac{ST}{PQ + QS}$$

$$\therefore \boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$



$$R = \sqrt{A^2 + B^2 + AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Case - I : $\boxed{\text{If } \theta = 0°}$

$\vec{A} \parallel \vec{B}$

$\cos\theta = 1$

$\sin\theta = 0$

$$R = \sqrt{A^2 + B^2 + 2AB} = \sqrt{(A+B)^2}$$

$\boxed{R = A + B}$

$\tan\alpha = 0$

$\boxed{\alpha = 0}$

Case - II : $\boxed{\text{If } \theta = 90^\circ}$

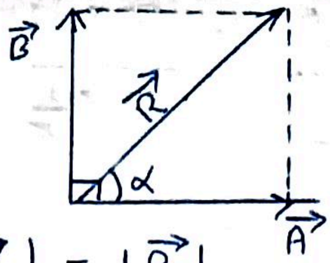
$\vec{A} \perp \vec{B}$

$\cos 90^\circ = 0$

$\sin 90^\circ = 1$

$$\boxed{R = \sqrt{A^2 + B^2}}$$

$$\boxed{\tan\alpha = \frac{B}{A}}$$



$|\vec{A}| = |\vec{B}|$

$R = \sqrt{A^2 + A^2} = \boxed{A\sqrt{2}}$

$\tan\alpha = 1$

$\boxed{\alpha = 45^\circ}$

Case - III : $\boxed{\text{If } \theta = 180^\circ}$

$\cos 180^\circ = 1$

$\sin 180^\circ = 0$

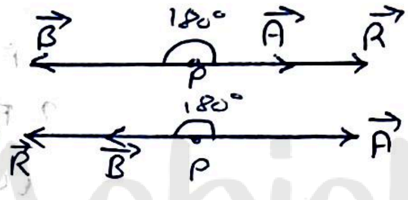
$R = \sqrt{(A-B)^2} = \boxed{A - B}$

$\tan\alpha = 0^\circ$

$\boxed{\alpha = 0}$

$\Rightarrow A - B = (A > B)$

$\Rightarrow B - A = (A < B)$



Q.1 A body is simultaneously given two velocities are 30 m/s due east and other 40 m/s due north. Find the resultant velocity.

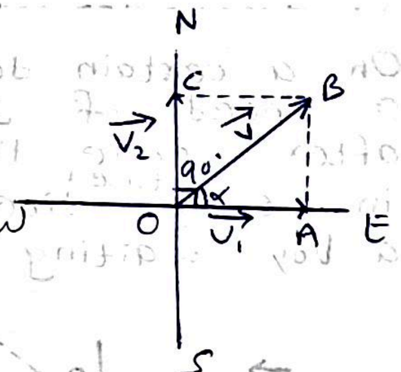
$\Rightarrow$ $V = \sqrt{v_1^2 + v_2^2}$

$= \sqrt{30^2 + 40^2}$

$= \sqrt{900 + 1600}$

$= \sqrt{2500}$

$V = \boxed{50 \text{ m/s}}$ Ans



$\Rightarrow \tan\alpha = \frac{V_2}{V_1} = \frac{40}{30} = \frac{4}{3}$

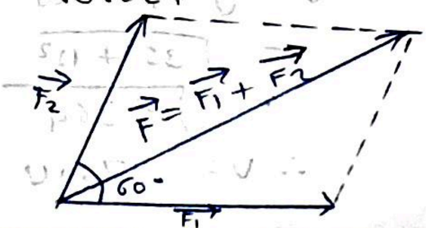
$\alpha = \boxed{\tan^{-1}\frac{4}{3}}$ Ans

Q.2 Two forces of 5N and 7N act on a particle with an angle of 60° between them. Find the resultant force.

$\Rightarrow$ $F_1 = 5\text{ N}$

$F_2 = 7\text{ N}$

$\theta = 60^\circ$



$F = \sqrt{25 + 49 + 70\cos 60^\circ}$

$= \sqrt{25 + 49 + 70 \times 1/2}$

$= \sqrt{25 + 49 + 35}$

$= \sqrt{109} = \boxed{10.44 \text{ N}}$ Ans

$\Rightarrow \tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta} = \dfrac{7\times\frac{\sqrt{3}}{\cancel{2}}}{12\times\frac{1}{\cancel{2}}} = \dfrac{7\sqrt{3}}{12}$

$\tan\alpha = \dfrac{7\sqrt{3}}{12}$

$\boxed{\alpha = \tan^{-1}\left(\dfrac{7\sqrt{3}}{12}\right)}$ Ans

Q.3 Two equal forces have their resultant equal to either. What is the inclination between them?

$\Rightarrow$ $F_1 = F$, $F_2 = F$, $F_R = F$

$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$F = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$

$F = \sqrt{2F^2 + 2F^2\cos\theta}$

$F^2 = 2F^2 + 2F^2\cos\theta$

$2F^2 = F^2 - 2F^2\cos\theta$

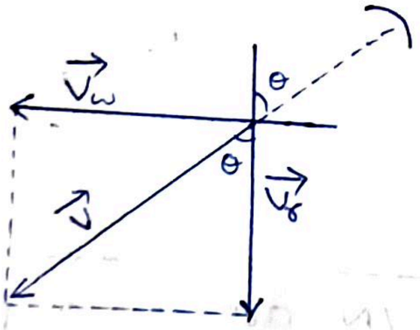
$2F^2\cos\theta = -F^2 \cancel{\cos\theta}$

$\cos\theta = \cancel{\dfrac{2F^2}{F^2}} \quad \dfrac{-\cancel{F^2}}{2\cancel{F^2}}$

$\cos\theta = -\dfrac{1}{2} = \boxed{120^\circ}$ Ans

Q.4 On a certain day rain was falling vertically with a speed of 35 m/s. A wind started blowing after some time with a speed of 12 m/s in east towards direction. In which direction should a boy waiting at a bus stop hold his umbrella?

$\Rightarrow$



$V_r = 35\ m/s$

$V_w = 12\ m/s$

$\tan\theta = \dfrac{V_w}{V_r} = \dfrac{12}{35}$

$\therefore \theta = \tan^{-1}\left(\dfrac{12}{35}\right)$

$V = \sqrt{V_r^2 + V_w^2}$

$= \sqrt{35^2 + 12^2}$

$= \sqrt{1369}$

$\therefore V = 37\ m/s$

Q.5 A river 800 m wide flows at rate of 5 km/h. A swimmer who can swim at 10 km/h in still water, wishes to cross the river straight.

(i) Along what direction must he strike?
(ii) What should be his resultant velocity?
(iii) How much time he would take?

=> (i) $\sin\theta = \frac{V_r}{V_s} = \frac{5}{10} = \frac{1}{2}$

$\sin\theta = \frac{1}{2} = \sin 30°$

Ans => $\boxed{\theta = 30°}$

=> (ii) Resultant velocity $= \sqrt{10^2 + 5^2 + 2\times 10.5 \times \frac{1}{2}}$

$= \sqrt{125 + (-50)}$

$= \cancel{120}\sqrt{125-50}$

$= \sqrt{75} = \boxed{5\sqrt{3}\ m/s}$ Ans

=> (iii) $T = D/S$

$S = 5\sqrt{3}$ km/h => $5\sqrt{3}\times\frac{5}{18}$ m/s

=> $\frac{800}{\frac{25\sqrt{3}}{18}} = \frac{800\times 18}{25\sqrt{3}} = \frac{800\times 18\times 25\sqrt{3}}{625\times 3} = 32\times 6\times\sqrt{3}$

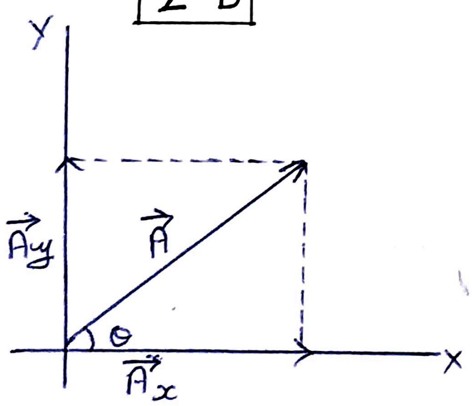
$= 192\sqrt{3}$

$= 192\times 1.732$

Ans => $\boxed{333.3\ sec}$

Resolution of a vector

2-D



$\vec{A_x} = A_x\hat{i} = A\cos\theta\,\hat{i}$

$\vec{A_y} = A_y\hat{j} = A\sin\theta\,\hat{j}$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

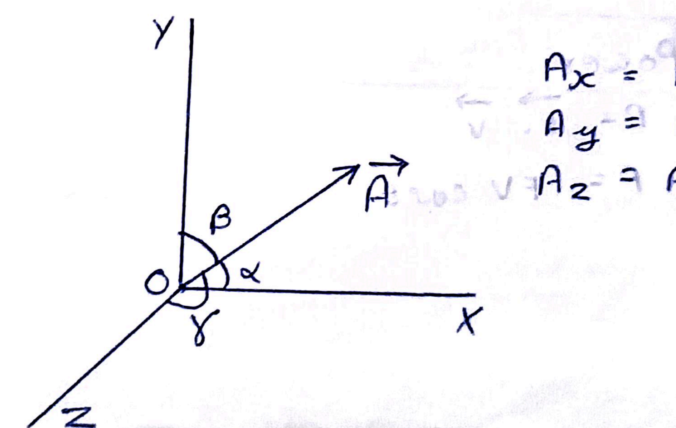
$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$A = \sqrt{A_x^2 + A_y^2}$

$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



$A_x = A\cos\alpha$

$A_y = A\cos\beta$

$A_z = A\cos\gamma$

3-D

$\boxed{Q.1}$ Find unit vector parallel to the resultant of the vectors –

$$\vec{A} = \hat{i} + 4\hat{j} - 2\hat{k}$$
$$\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$$

$$\Rightarrow \quad \vec{R} = \vec{A} + \vec{B} = 4\hat{i} - \hat{j} - \hat{k}$$

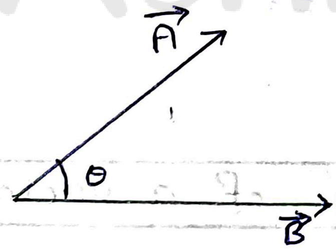
$$|\vec{R}| = \sqrt{4^2 - 1^2 - 1^2}$$
$$= \sqrt{16 + 1 + 1}$$
$$= \sqrt{18}$$
$$\Rightarrow 3\sqrt{2}$$

$$\Rightarrow \quad \vec{R} = R\hat{R}$$
$$\hat{R} = \frac{\vec{R}}{R} = \boxed{\frac{4\hat{i} - \hat{j} - \hat{k}}{3\sqrt{2}}} \quad \text{Ans}$$

Scalar Product of Two Vectors (Dot Product)

The scalar product or dot product ~~are~~ of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitude of $\vec{A}$ and $\vec{B}$ and cosine of the angle θ between them.

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$
$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$



This can be written as :

$$\boxed{\begin{aligned} \vec{A} \cdot \vec{B} &= (A\cos\theta)\,B \\ \vec{A} \cdot \vec{B} &= (B\cos\theta)\,A \end{aligned}}$$

Eg: (i) $W = Fd\cos\theta$

$W = \vec{F} \cdot \vec{d}$

(ii) Power

$P = \vec{F} \cdot \vec{v}$

$P = FV\cos\theta$

Properties of Scalar Product:-

1) The scalar product is commutative

$$\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$$

2) It is distributive over addition.

$$\boxed{\begin{aligned} \vec{A}\cdot(\vec{B}+\vec{C}) &= \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C} \\ (\vec{B}+\vec{C})\cdot\vec{A} &= \vec{B}\cdot\vec{A} + \vec{C}\cdot\vec{A} \end{aligned}}$$

3) When two vectors are parallel to each other –

$$\vec{A} \parallel \vec{B} \Rightarrow \boxed{\vec{A}\cdot\vec{B} = AB\cos\theta = AB}$$

4) If $\vec{A}$ and $\vec{B}$ are parallel vectors having opposite direction.

$$\boxed{\vec{A} \downarrow\uparrow \vec{B} \Rightarrow \theta = 180^\circ,\ \cos 180^\circ = -1}$$

$$\boxed{\therefore \vec{A}\cdot\vec{B} = -AB}$$

5) Self Product –

$\vec{A}\cdot\vec{A} = A^2$	$\hat{i}\cdot\hat{i} = 1$
$\vec{B}\cdot\vec{B} = B^2$	$\hat{j}\cdot\hat{j} = 1$
	$\hat{k}\cdot\hat{k} = 1$

6) When two vectors are $\perp r$ to each other.

$$\boxed{\vec{A} \perp \vec{B},\ \theta = 90^\circ,\ \cos 90^\circ = 0}$$

$\vec{A}\cdot\vec{B} = 0$	$\hat{i}\cdot\hat{j} = 0$
$\vec{B}\cdot\vec{A} = 0$	$\hat{j}\cdot\hat{k} = 0$
	$\hat{k}\cdot\hat{i} = 0$

7) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

Numericals

1) If the magnitude of two vectors are 3 and 4 and their scalar product is 6 then find the angle between the two vectors.

$\Rightarrow$ $\vec{A}\cdot\vec{B} = AB\cos\theta$

$6 = 3\times4\cos\theta$

$6 = 12\cos\theta$

$\cos\theta = \frac{1}{2}$

Ans $\Rightarrow$ $\boxed{\theta = 60^\circ}$

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2). A force of $7\hat{i} + 6\hat{k}$ N makes a body moves on a rough plane with a velocity of $3\hat{j}+4\hat{k}$ m/s. Calculate the power.

$\Rightarrow$ $P = \hat{F}\cdot\hat{V}$

$P = (7\hat{i} + 6\hat{k})(3\hat{j} + 4\hat{k})$

$P = 21\times0 + 24\times0 + 18\times0 + 24\times1$

Ans $\Rightarrow$ $\boxed{P = 24 \text{ watt} \mid Nm/s}$

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3) Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ such that $\vec{A} = \vec{B} + \vec{C}$ and their magnitudes are 5, 4 and 3 respectively. Find the angle b/w $\vec{A}$ and $\vec{C}$

$\Rightarrow$ $\vec{A} = \vec{B} + \vec{C}$

$\vec{B} = \vec{A} - \vec{C}$

$\vec{B}\cdot\vec{B} = (\vec{A}-\vec{C})\cdot(\vec{A}-\vec{C})$

$B^2 = A^2 - \vec{A}\cdot\vec{C} - \vec{C}\cdot\vec{A} + C^2$

$B^2 = A^2 + C^2 - 2AC$

$4^2 = 5^2 - 2AC + 3^2$

$16 = 25 - 2\vec{A}\vec{C} + 9$

$2\vec{A}\vec{C} = 18$

$A\cdot C\cos\theta = 9$

$\cos\theta = \frac{3}{5}$

Ans $\Rightarrow$ $\boxed{\theta = 53^\circ}$

4) If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. Find the angle between $\vec{A}$ and $\vec{B}$.

$\Rightarrow$ $(\vec{A}+\vec{B}).(\vec{A}+\vec{B}) = (\vec{A}-\vec{B}).(\vec{A}-\vec{B})$

$\vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} \cancel{+ \vec{B}\cdot\vec{A}} + \vec{B}\cdot\vec{B}$

$= \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B}$

$\Rightarrow \cancel{A^2} + 2\vec{A}\vec{B} + \cancel{B^2} = \cancel{A^2} - 2\vec{A}\vec{B} + \cancel{B^2}$

$2\vec{A}\vec{B} = -2\vec{A}\vec{B}$

$4\vec{A}\vec{B} = 0$

$4AB\cos\theta = 0 \quad [\because A, B \neq 0$

$\cos\theta = 0$

Ans $\Rightarrow \boxed{\theta = 90°}$

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5) If $(\vec{A}+\vec{B} = \vec{C})$ and $(A^2+B^2 = C^2)$ then prove that $\vec{A}$ and $\vec{B}$ are $\perp r$ to each other.

$\Rightarrow$ $(\vec{A}+\vec{B}).(\vec{A}+\vec{B}) = \vec{C}.\vec{C}$

$\vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B} = \vec{C}\cdot\vec{C}$

$A^2 + 2\vec{A}\vec{B} + B^2 = C^2$

$(A^2+B^2) + 2\vec{A}\vec{B} = C^2$

$C^2 + 2\vec{A}\vec{B} = C^2$

$2\vec{A}\vec{B} = 0$

$AB\cos\theta = 0$

$\cos\theta = 0$

$\therefore \boxed{\theta = 90°}$

---

6) Under a force of $(10\hat{i} - 3\hat{j} + 6\hat{k})N$, a body of mass 5 kg is displaced from the position $6\hat{i} + 5\hat{j} - 3\hat{k}$ to the position $10\hat{i} - 2\hat{j} + 7\hat{k}$. Calculate the work done.

$\Rightarrow$ $\vec{r_1} = 6\hat{i} + 5\hat{j} - 3\hat{k}$

$\vec{r_2} = 10\hat{i} - 2\hat{j} + 7\hat{k}$

$\vec{d} = \vec{r_2} - \vec{r_1}$

$\vec{d} = (10\hat{i} - 2\hat{j} + 7\hat{k}) - (6\hat{i} + 5\hat{j} - 3\hat{k})$

$\vec{d} = 4\hat{i} - 7\hat{j} + 10\hat{k}$

$\vec{F} = 10\hat{i} - 3\hat{j} + 6\hat{k}$

$W = \vec{F}\cdot\vec{d} = 40 + 21 + 60 = \boxed{121 \text{ joule}}$ Ans

7) A force $\vec{F} = (5\hat{i} + 4\hat{j})$ N displaces a body through $\vec{d} = (3\hat{i} + 4\hat{k})$ m in 3 sec. Find the power.

=> $\vec{F} = (5\hat{i} + 4\hat{j})$ N

$\vec{d} = (3\hat{i} + 4\hat{k})$ m

$t = 3$ sec.

$P = ?$

=> $W = \vec{F} \cdot \vec{d}$

$W = (5\hat{i} + 4\hat{j})(3\hat{i} + 4\hat{k})$

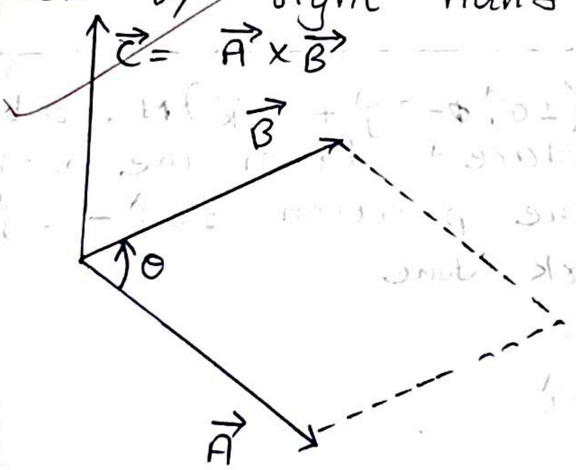
$W = 15 + 0 + 0 = 15$ Nm

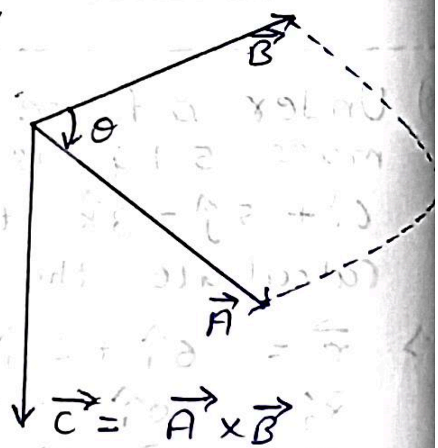
=> $P = W/t$

$= \frac{15}{3} = \boxed{5 \text{ watt / Nm/s}}$ Ans

Vector Product of two Vectors (Cross Product)

→ The vector or cross product of two vectors is defined as the vector whose magnitude is equal to the product of the magnitude of the two vectors and sine of the angle between them and whose direction is ⊥r to the plane of the two vectors and is given by right hand rule.





$$\boxed{|\vec{C}| = |\vec{A}||\vec{B}|}$$

$$\boxed{\vec{A} \times \vec{B} = (AB \sin\theta)\hat{n}}$$

Eg: (i) Torque (τ)
$\tau = \vec{r} \times \vec{F}$

(ii) Angular momentum
$\vec{L} = \vec{r} \times \vec{p}$

(iii) Instantaneous Velocity
$\vec{v} = \vec{\omega} \times \vec{r}$

Properties of Vector Product :-

1) Vector product is anti commutative.

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

2) Vector product is distributive over addition.

$$\boxed{\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}}$$

3) Vector product of two parallel or anti-parallel vectors is a null vector (zero vector).

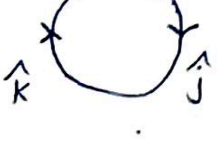
$\theta = 0^\circ$ or 180°
$\sin\theta = 0$
$\vec{A} \times \vec{B} = (0\hat{n}) = \vec{0}$

4) Vector product of a vector with itself is a null vector (self product).

$\theta = 0^\circ$
$\vec{A} \times \vec{A} = AA \sin 0\, \hat{n} = 0\hat{n} = \vec{0}$
$\hat{i} \times \hat{i} = 0$
$\hat{j} \times \hat{j} = 0$
$\hat{k} \times \hat{k} = 0$

5) The magnitude of the vector product of two ~~at~~ mutually $\perp$r is equal to the product their magnitudes.

$\theta = 90^\circ$
$\vec{A} \times \vec{B} = AB \sin 90^\circ \hat{n} = AB\hat{n}$
$|\vec{A} \times \vec{B}| = AB$



$\hat{i} \times \hat{j} = \hat{k}$	$\hat{i} \times \hat{k} = -\hat{j}$
$\hat{j} \times \hat{k} = \hat{i}$	$\hat{k} \times \hat{j} = -\hat{i}$
$\hat{k} \times \hat{i} = \hat{j}$	$\hat{j} \times \hat{i} = -\hat{k}$

$\rightarrow \vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \boxed{(A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} - (A_x B_y - A_y B_x)\hat{k}}$$

Q.1 Prove that the vectors $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel.

$\Rightarrow$ $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$

$\vec{A} \times \vec{B} = (2\hat{i} - 3\hat{j} - \hat{k}) . (-6\hat{i} + 9\hat{j} + 3\hat{k})$

$= (9 - 9)\hat{i} - (6 - 6)\hat{j} + (18 - 18)\hat{k}$

$= 0\hat{i} - 0\hat{j} + 0\hat{k}$

$\Rightarrow 0$

$\Rightarrow$ Hence the vectors, $\vec{A}$ and $\vec{B}$ are parallel.

Q.2 Calculate the area of a llgm whose two adjacent sides are formed by the vectors, $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = -3\hat{i} + 7\hat{j}$.

$\Rightarrow$ $\vec{A} = 3\hat{i} + 4\hat{j} + 0\hat{k}$

$\vec{B} = -3\hat{i} + 7\hat{j} + 0\hat{k}$

$\vec{A} \times \vec{B} = (3\hat{i} + 4\hat{j} + 0\hat{k})(-3\hat{i} + 7\hat{j} + 0\hat{k})$

$= (0 - 0)\hat{i} - (0 + 0)\hat{j} + (21 + 12)\hat{k}$

$= 33\hat{k}$

Area $= |\vec{A} \times \vec{B}| = \boxed{33 \text{ unit}^2}$ Ans

---

Q.3 If $\vec{A}$ and $\vec{B}$ denotes a side of allgm and its area is $\frac{AB}{2}$, find the angle between $\vec{A}$ and $\vec{B}$.

$\Rightarrow$ Area of llgm $= AB/2$

$\vec{A} \times \vec{B} = AB \sin\theta$

$\frac{\cancel{AB}}{2} = \cancel{AB} \sin\theta$

$\frac{1}{2} \sin\theta$

$\therefore \boxed{\theta = 30^\circ}$ Ans

Q.4 Determine a unit vector ⊥r to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

$\Rightarrow$ $\vec{A} = (2\hat{i} + \hat{j} + \hat{k})$

$\vec{B} = (\hat{i} - \hat{j} + 2\hat{k})$

$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{AB\sin\theta}$$

$\Rightarrow$ $\vec{A} \times \vec{B} = (2\hat{i} + \hat{j} + \hat{k})(\hat{i} - \hat{j} + 2\hat{k})$

$= (2+1)\hat{i} - (4-1)\hat{j} + (-2-1)\hat{k}$

$= 3\hat{i} - 3\hat{j} - 3\hat{k}$

$\Rightarrow$ $AB = |\vec{A} \times \vec{B}| = \sqrt{3^2 + (-3)^2 + (-3)^2}$

$= \sqrt{9+9+9}$

$= \sqrt{27}$

$\Rightarrow 3\sqrt{3}$

$\Rightarrow$ $$\hat{n} = \frac{\vec{A} \times \vec{B}}{AB\sin\theta}$$

$$= \frac{3\hat{i} - 3\hat{j} - 3\hat{k}}{3\sqrt{3}}$$

Ans. $\Rightarrow$ $$\boxed{\frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}}$$

Q.5 Find the area of triangle formed by the tip of the vectors: $\vec{a} = \hat{i} - \hat{j} - 3\hat{k}$

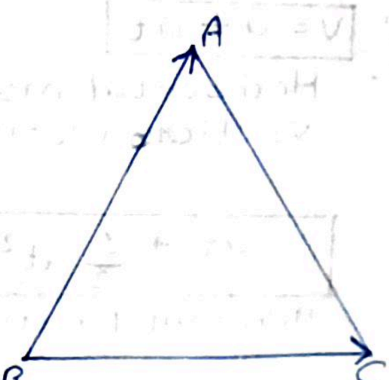
$\vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

$\Rightarrow$ $\vec{BA} = \vec{a} - \vec{b}$

$= (\hat{i} - \hat{j} - 3\hat{k}) - (4\hat{i} - 3\hat{j} + \hat{k})$

$= -3\hat{i} + 2\hat{j} - 4\hat{k}$



$\vec{BC} = \vec{c} - \vec{b}$

$= (3\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - 3\hat{j} + \hat{k})$

$= -\hat{i} + 2\hat{j} + \hat{k}$

$\Rightarrow$ Area of $\triangle ABC$:-

~~Mag.~~ $\vec{BA} \times \vec{BC}$

$= (2 \times 1 - (-4) \times 2)\hat{i} - [(-3 \times 1 - (-4)(-1)]\hat{j} - [(-3 \times 2 - 2 \times (-1)]\hat{k}$

$= 10\hat{i} + 7\hat{j} + 4\hat{k}$

$= \sqrt{10^2 + 7^2 + 4^2} = \sqrt{165}$

~~Ans~~ $\Rightarrow \boxed{12.8 \text{ unit}^2}$

= Area of ΔABC

$= \frac{1}{2} \times 12.8 \Rightarrow \boxed{6.4 \text{ unit}^2}$ Ans

---

Q.6 Find the moment about the point $(1, -1, -1)$ of the force $(3\hat{i} + 4\hat{j} - 5\hat{k})$ acting on the point $(1, 0, -2)$

$\Rightarrow$

$\vec{r_1} = \hat{i} - \hat{j} - \hat{k}$

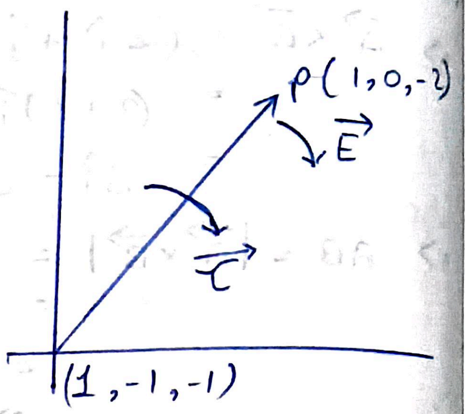
$\vec{r_2} = \hat{i} + 0\hat{j} - 2\hat{k}$

$\vec{r_1} + \vec{r} = \vec{r_2}$

$\vec{r} = \vec{r_2} - \vec{r_1} = (\hat{j} - \hat{k})$

$\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

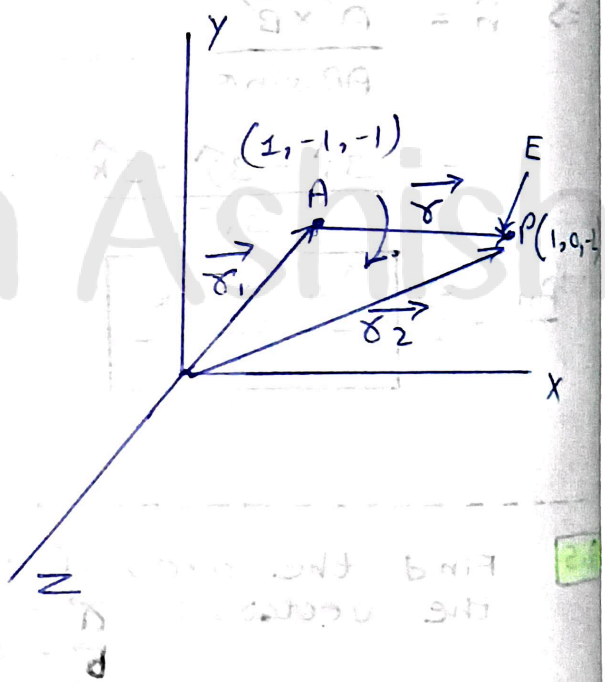
$\vec{\tau} = \vec{r} \times \vec{F}$



$\Rightarrow (\hat{j} - \hat{k})(3\hat{i} + 4\hat{j} - 5\hat{k})$

$= (-5+4)\hat{i} - 3\hat{j} + (-3)\hat{k}$

Ans $= \boxed{-\hat{i} - 3\hat{j} - 3\hat{k}}$



Motion in a Plane

1) $\boxed{v = u + at}$

$\Rightarrow$ Horizontal motion $\Rightarrow v_x = u_x + a_x t$

Vertical motion $\Rightarrow v_y = u_y + a_y t$

2) $\boxed{s = ut + \frac{1}{2}at^2}$

$\Rightarrow$ Horizontal motion $\Rightarrow x = u_x t + \frac{1}{2}a_x t^2$

Vertical motion $\Rightarrow y = u_y t + \frac{1}{2}a_y t^2$

3) $\boxed{v^2 = u^2 + 2as}$

$\Rightarrow$ Horizontal motion $\Rightarrow v_x^2 = u_x^2 + 2a_x$

Vertical motion $\Rightarrow v_y^2 = u_y^2 + 2a_y$

Ques. The position of a particle is given by $3t\hat{i} + 2t^2\hat{j} + 5\hat{k}$.

1) Find the velocity and acceleration of the particle.
2) Find the magnitude and direction of velocity at $t = 3$ sec.

$\Rightarrow$ Let $\vec{r} = 3t\hat{i} + 2t^2\hat{j} + 5\hat{k}$

$\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(3t\hat{i} + 2t^2\hat{j} + 5\hat{k})$

$= 3\hat{i} + 4t\hat{j}$ Ans

$\Rightarrow$ At $t = 3$ sec.

$|\vec{v}| = \sqrt{3^2 + (4\times3)^2}$

$= \sqrt{153}$ m/s Ans

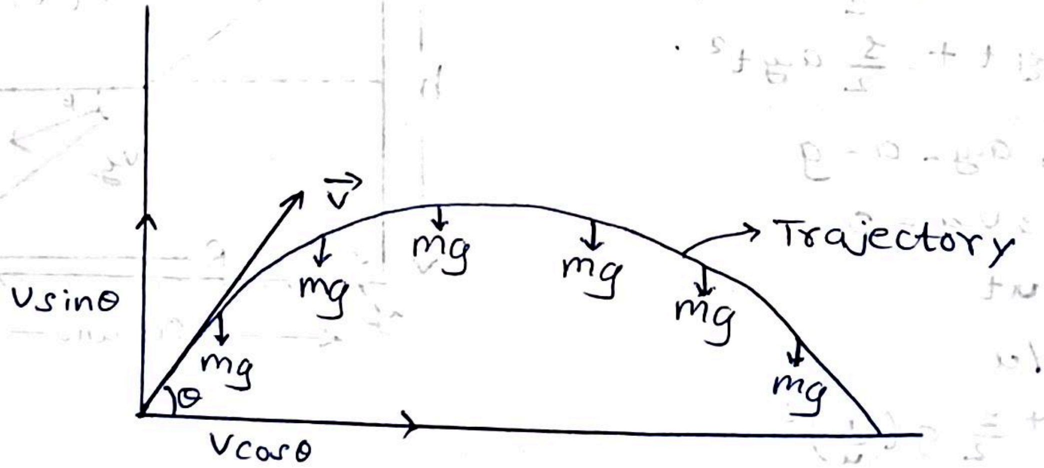
$\Rightarrow |\vec{a}| = 4 \text{ m/s}^2$

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{12}{3} = 4 \; ; \; \theta = \tan^{-1}(4)$

$\therefore \theta = 76°$

Projectile Motion

A projectile is the name given to any body which ones thrown into space with some initial velocity, moves thereafter under influence of gravity alone, without being propelled by the engine or fuel. The path followed by the projectile is called trajectory.



Eg: A bullet fired from a rifle.

Principle of Physical Independence of Motion

The two motion of a projectile along horizontal and vertical direction are independent of each other which is called principle of physical independence of motion.

→ $\boxed{\text{Vertical Component}} = \boxed{u \sin\theta}$

$\theta = 30° = u\sin\theta = u/2$

$\theta = 45° = u\sin\theta = u/\sqrt{2}$

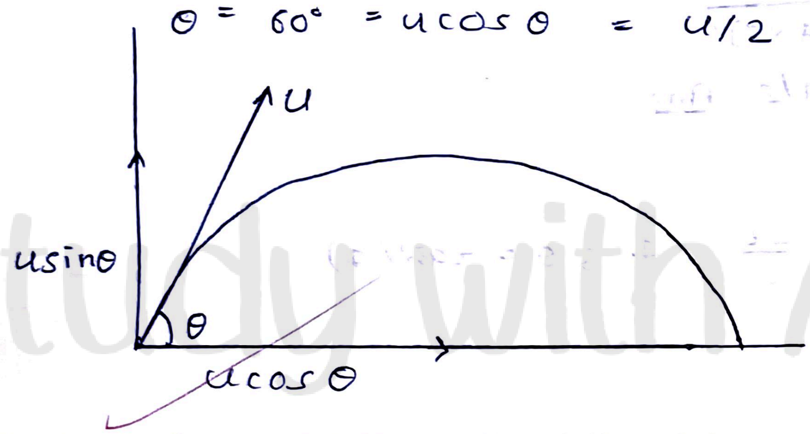
$\theta = 60° = u\sin\theta = \frac{\sqrt{3}u}{2}$

→ $\boxed{\text{Horizontal Component}} = \boxed{u \cos\theta}$

$\theta = 30° = u\cos\theta = \sqrt{3}u/2$

$\theta = 45° = u\cos\theta = u/\sqrt{2}$

$\theta = 60° = u\cos\theta = u/2$



Horizontal Projection given by Projectile

$\Rightarrow V_x = u_x + a_x t$

$v_y = u_y + a_y t$

$x = u_x t + \frac{1}{2} a_x t^2$

$y = u_y t + \frac{1}{2} a_y t^2$

$a_x = 0, \; a_y = a = g$

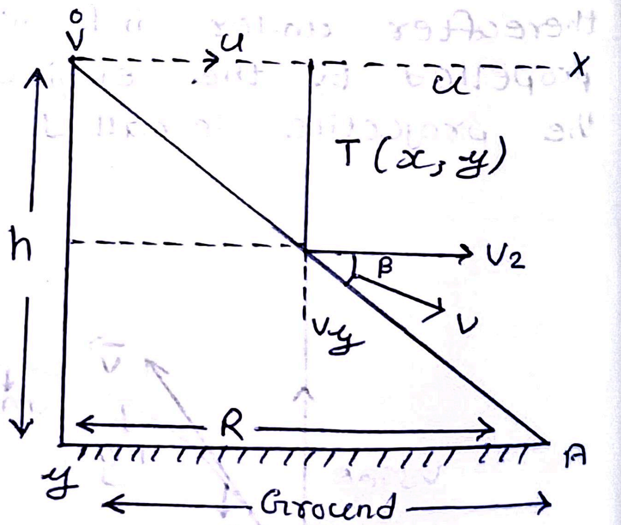
$v_x = v, \; v_y = 0$

$\therefore x = ut$

$\Rightarrow t = x/u$

$u = 0 + \frac{1}{2} g \left(\frac{x}{u}\right)^2$

$y = \left(\frac{g}{2v^2}\right) x^2$



$\Rightarrow y = kx^2$

$k = g/2v^2$ (constant)

→ This is the equation of parabolla. Therefore, the trajectory of projectile is parabolla.

→ **Time of Flight :-**

$a_y = g$

$t = T$

$u_y = 0$

$y = h$

$h = 0 + \frac{1}{2} g t^2$

$\Rightarrow \frac{2h}{g} = T^2$

$\therefore \boxed{T = \sqrt{\frac{2h}{g}}}$

→ **Horizontal range :-**

$\boxed{R = uT}$

→ **Velocity of projectile at any instant :-**

$v = u + at$

$v_x = u_x + a_x t$

$v_x = u$

$\vec{v_x} \perp \vec{v_y}$

$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{v^2 + gt^2}$

$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{v}$

$v_y = v_y + a_x t$

$v_y = gt$

$\boxed{\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u}}$

Q.1 A hiker stand on the edge of cliff 490m above the ground throws a stone horizontally with an initial speed 15 m/s neglecting air resistance, find the time taken by stone to reach the ground and the speed ~~with its~~ when it hits the ground.

$\Rightarrow$ Given :- $h = 490\text{ m}$

$s = 15\text{ m/s}$

$\Rightarrow T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4900}{9.8}} = \sqrt{100} = \boxed{10\text{ sec.}}$ Ans

$\Rightarrow v = \sqrt{u^2 + (gt)^2} = \sqrt{15^2 + (9.8 \times 10)^2}$

$= \sqrt{225 + 96.04 \times 100}$

$= \sqrt{9829} = \boxed{98.5\text{ m/s}}$ Ans

Q.2 A projectile fired horizontally with a velocity of 98 m/s from the top of hill 490 m high. Find :-

(i) The time taken to reach the ground.
(ii) The distance of the target from the hill.
(iii) velocity.

$\Rightarrow T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = \sqrt{100} = 10\text{ sec.}$ Ans

$\Rightarrow R = uT = 98 \times 10 = 980\text{ m.}$ Ans

$\Rightarrow v = \sqrt{u^2 + g^2 t^2} = \sqrt{98^2 + (9.8)^2 \times 10^2} = 98\sqrt{1}\text{ m/s}$ Ans

Q.3 A body is thrown horizontally from the top of a tower and strike the ground after 3 sec. at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected. [Take $g = 9.8\ m/s^2$]

$\Rightarrow$ $T = \sqrt{\frac{2h}{g}} \Rightarrow 3 = \sqrt{\frac{2h}{9.8}} = 3^2 = \frac{2 \times h}{9.8}$

$\Rightarrow$ $9 \times 9.8 = 2h$

$h = \frac{88.2}{2}$

$h = 44.1\,m$ Ans

$\Rightarrow$ $\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u}$

$\tan 45° = \frac{gt}{u}$

$\tan 45° = 1$

$u = gt = 9.8 \times 3$

$\therefore u = 29.4\,m/s$ Ans

Q.4 A ball is projected horizontally from a tower with velocity of 4 m/s. Find the velocity of the ball after ~~the~~ 0.7 sec. [$g = 10\,m/s^2$]

$\Rightarrow$ $V_x = u$

$V_y = gt$

$V = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{u^2 + g^2t^2} = \sqrt{4^2 + 10^2 \times 0.7^2}$

$= \sqrt{16 + 100 \times 0.49}$

$= \sqrt{64} = 8.06\,m/s$ Ans

Projectile Fixed At an angle θ with the Horizontal

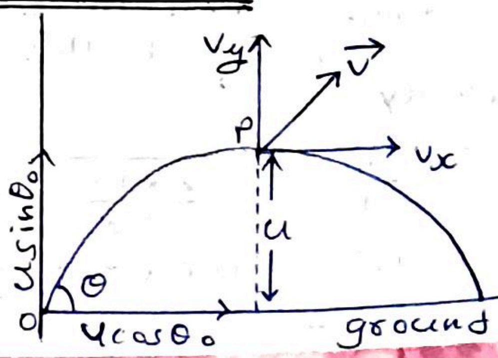
→ **Equation of Trajectory of a Projectile :-**

$x = u_x t + \frac{1}{2} a_x t^2$

$y = u_y t + \frac{1}{2} a_y t^2$

$a_x = 0, \quad a_y = g$

$u_x = u\cos\theta, \quad u_y = u\sin\theta$



$$\boxed{x = u\sin\theta_0 t}$$

$$\boxed{t = \frac{x}{u\cos\theta_0}}$$

$$y = u\sin\theta_0\left(\frac{x}{u\cos\theta_0}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta_0}\right)^2$$

$$y = (\tan\theta_0)x - \left(\frac{g}{2u^2\cos^2\theta_0}\right)x^2$$

Let, $A = \tan\theta$

$B = \frac{g}{2u^2\cos^2\theta}$ → constant

$$\therefore \boxed{y = Ax - Bx^2}$$

Note: This is the equation of a parabolla, therefore, the ~~part~~ path of projectile is parabolic.

→ **Time of maximum height :-**

It is the time taken by the projectile to reach the max. height.

$$v = u + at$$

$$v_y = u_y + a_y t$$

$$0 = u\sin\theta_0 - gt$$

$$g t_m = u\sin\theta_0$$

$$\boxed{t_m = \frac{u\sin\theta_0}{g}}$$

$$\begin{bmatrix} v_y = 0 \\ u_y = u\sin\theta_0 \\ a_y = -g \\ t = t_m \end{bmatrix}$$

→ **Time of Flight :-**

$$T_f = 2t_m$$

$$\boxed{T_f = \frac{2u\sin\theta}{g}}$$

Note: Time of accending is equal to the time of decending under gravity.

→ **Maximum Height :-**

$$v^2 = u^2 + 2as$$

$$v_y^2 = u_y^2 + 2a_y y$$

$$0 = (u\sin\theta_0)^2 - 2gH$$

$$2gH = u^2\sin^2\theta_0$$

$$\boxed{H = \frac{u^2\sin^2\theta_0}{2g}}$$

$$\begin{bmatrix} y = H \\ a_y = g \\ u_y = u\sin\theta_0 \\ v_y = 0 \end{bmatrix}$$

→ Horizontal Range :-

$$R = u_x \times T_f$$

$$R = (u\cos\theta_0) \cdot \left(\frac{2u\sin\theta_0}{g}\right)$$

$$R = u^2 \left(\frac{2\cos\theta_0 \cdot \sin\theta_0}{g}\right)$$

$[\because \sin 2\theta = 2\cos\theta \cdot \sin\theta]$

$$\therefore \boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Q.1 A cricket ball is thrown at a speed of 28 m/s in a direction 30° above the horizontal. Calculate –

(i) Maximum Height

$$\Rightarrow H = \frac{u^2\sin^2\theta}{2g} = \frac{28^2 \sin^2 30°}{2\times 9.8} = \frac{784\times\frac{1}{4}}{2\times 9.8} = \frac{980}{98} = \boxed{10m}$$ Ans

(ii) The total time taken by the ball to return to the same level.

$$\Rightarrow T_f = \frac{2u\sin\theta}{g} = \frac{2\times \cancel{28}^{14} \times\frac{1}{\cancel{2}}}{9.8} = \frac{28^2}{\cancel{9.8}_{0.7}} = \boxed{2.85}$$ sec. Ans

(iii) The horizontal distance from the thrower to the point where the ball returns to the same level.

$$\Rightarrow R = \frac{u^2\sin 2\theta}{g} = \frac{28^2 \times 2\sin\theta\cos\theta}{9.8}$$

$$= \frac{784 \times 2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}}{9.8} = \frac{3920}{98}\times\sqrt{3}$$

$$= \boxed{40\sqrt{3}m}$$ Ans

Q.2 A projectile has range of 50 m. Max. height = 10m. Calculate the angle at which the projectile is fixed.

$\Rightarrow$ $R = 50m$

$H = 10m$

$\theta = ?$

$$\Rightarrow R = \frac{u^2\sin 2\theta}{g}$$

$$= \frac{u^2(2\sin\theta\cos\theta)}{g}$$

$$\Rightarrow H = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \frac{R}{H} = \frac{\cancel{u^2}\cdot 2\cancel{\sin\theta}\cos\theta}{\cancel{g}} \times \frac{2g}{\cancel{u^2}\sin^{\cancel{2}}\theta}$$

$$= \frac{4\cos\theta}{\sin\theta}$$

$\Rightarrow \frac{H}{R} = \frac{\tan\theta}{4}$

$\tan\theta = 4\frac{H}{R} = 4 \times \frac{1\cancel{0}}{5\cancel{0}}$

$\Rightarrow \tan\theta = 0.8$

$\therefore \boxed{\theta = \tan^{-1}(0.8)}$ Ans

Q.3 A boy stand at 39.2 m from a building and thrown a ball which just passes throw a window 19.6 m above the ground. Calculate the velocity of projection of the ball.

$\Rightarrow R = 39.2\,m + 39.2\,m$

$H = 19.6\,m$

$\theta = ?$

$\Rightarrow \tan\theta = \frac{19.6 \times 4}{39.2 + 39.2}$

$\tan\theta = 1$

$\therefore \boxed{\theta = 45^\circ}$

$\Rightarrow H = \frac{u^2 \sin 2\theta}{g}$

$u^2 = \frac{2gH}{\sin^2\theta} = \frac{2 \times 9.8 \times 19.6}{\left(\frac{1}{\sqrt{2}}\right)^2}$

Ans $\Rightarrow \boxed{19.6\sqrt{2}\ m/s}$

Uniform Circular Motion

If a particle move along a circular path with a constant speed. Then, its motion is called uniform circular motion.

Angular Displacement :-

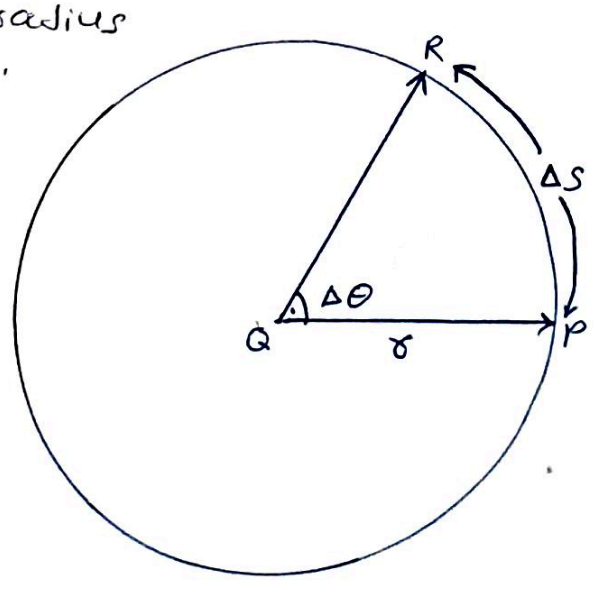
The angular displacement of a particle moving along a circular path is defined as the angle swept out by the radius vector in a given time interval.

Distance = Arc = Δs

Radius vector = r

Ang. displacement = $\frac{\text{arc}}{\text{radius}}$

$\boxed{\Delta\theta = \frac{\Delta s}{r}}$ radian



Angular Velocity (ω) :-

The time rate of change of angular displacement of a particle is called its angular velocity.

$$\boxed{\omega = \frac{d\theta}{dt} \text{ rad/sec.}}$$

For Full Rotation :-

$\theta = 2\pi$

$t = T$

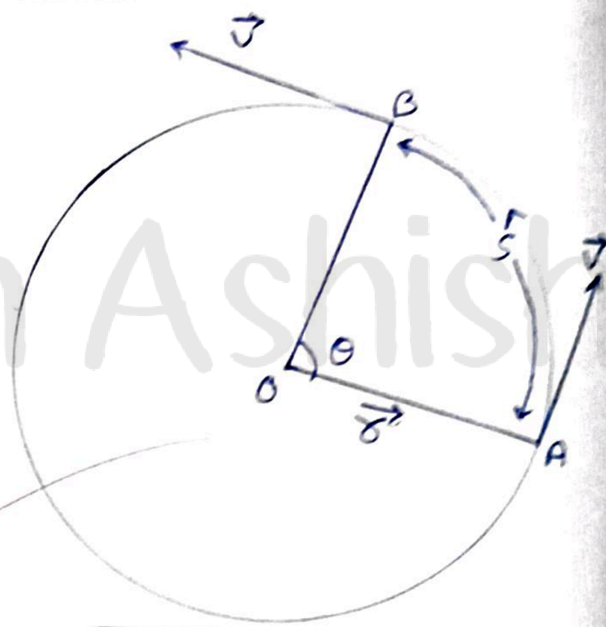
$$\boxed{\omega = \frac{2\pi}{T}}$$

Frequency :-

$n = 1/T$

$$\boxed{\omega = 2\pi n}$$

Relation b/w Linear Velocity & Angular velocity :-



S = Linear distance

θ = Angular displacement

$\theta = \frac{s}{r}$

$\left(\frac{d\theta}{dt}\right) = \frac{1}{r}\left(\frac{ds}{dt}\right)$

$\omega = \frac{1}{r} v$

$\therefore \boxed{v = r\omega}$

In vector form -
$\vec{v} = \vec{\omega} \times \vec{r}$

Angular Acceleration :-

The time rate of change of angular velocity of a particle is called its angular acceleration.

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

Relation b/w Angular acceleration & Linear acceleration :-

$v = r\omega$

$\left(\frac{dv}{dt}\right) = r\left(\frac{d\omega}{dt}\right)$

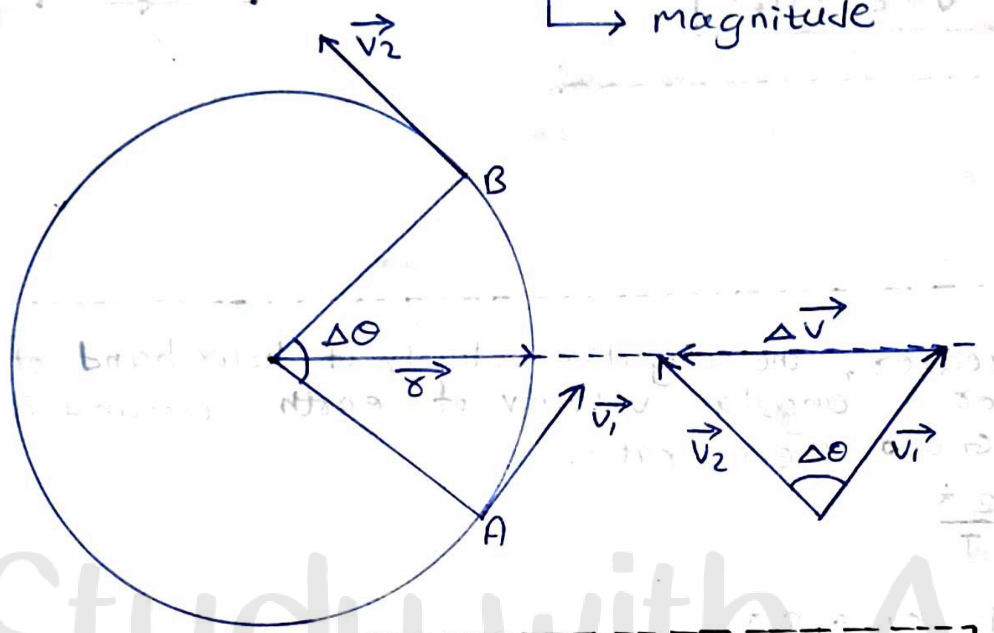
$$\boxed{a = r\alpha}$$

$$\boxed{\vec{a} = \vec{r} \times \vec{\alpha}}$$

Centripetal Acceleration :-

A body undergoing uniform circular motion is acted upon by an acceleration which is directed along the radius towards the centre of the circular path. This acceleration is called centripetal acceleration. It is also called centre seeking acceleration or radial acceleration.

→ Velocity is a vector quantity.
- → Direction
- → magnitude



$$\Delta\theta = \frac{\Delta v}{v}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{v}\cdot\frac{\Delta v}{\Delta t}$$

$$\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = \frac{1}{v}\lim_{t\to 0}\frac{\Delta v}{\Delta t}$$

$$\left(\frac{d\theta}{dt}\right) = \frac{1}{v}\left(\frac{dv}{dt}\right)$$

$$\omega = \frac{1}{v}.a_c$$

$$\therefore \boxed{a_c = v\omega}$$

$$a_c = v\omega$$

$$v = r\omega$$

$$\therefore \boxed{a_c = r\omega^2}$$

$$\omega = v/r$$

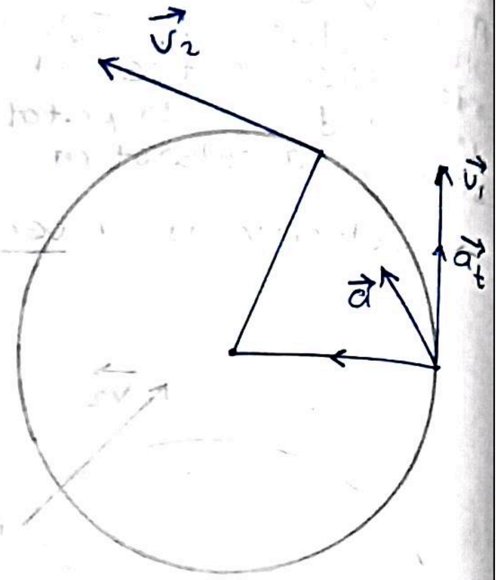
$$\therefore \boxed{a_c = \frac{v^2}{r}}$$

Circular Motion with Variable Speed

a_t = tangential acceleration
a_c = centripetal acceleration

$a_c \perp a_t$

$\therefore \boxed{a = \sqrt{a_c^2 + a_t^2}}$



Q.1 Which is greater, the angular velocity of hour hand of a watch or angular velocity of earth around its own axis? Give their ratio.

$\Rightarrow \quad \omega = \frac{\theta}{t} = \frac{2\pi}{T}$

For watch, $\omega_1 = \frac{2\pi}{12}$

For earth, $\omega_2 = \frac{2\pi}{24}$

$\therefore \boxed{\omega_1 > \omega_2}$ Ans

$\Rightarrow$ Ratio $= \frac{\omega_1}{\omega_2} = \frac{\frac{\cancel{2\pi}}{\cancel{12}}}{\frac{\cancel{2\pi}}{\cancel{24}\,2}} = \frac{2}{1}$ Ans $\Rightarrow \boxed{2:1}$

Q.2 Calculate the angular speed of fly wheel making 420 revolution per minute.

$\Rightarrow \quad \omega = 2\pi n$

$= 2 \times \frac{22}{7} \times \left(\frac{420}{60}\right)$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{7}$

$= 2 \times 22$

Ans $\Rightarrow \boxed{44 \text{ rad/s}}$

Q.3 A body of mass 10 kg revolve in a circle of diameter 0.4 m making 1000 revolution per minute. Calculate it linear velocity and centripetal acceleration.

=> Linear velocity $= \vec{\omega} \times \vec{r} = \left(\frac{1000}{60} \times 2\pi\right) \times 0.2$

$= \frac{100\pi}{3} \times 0.2 \Rightarrow \frac{20}{3}\pi$ Ans

=> $a_c = \frac{v^2}{r} = \frac{\left[\frac{20}{3}\pi\right]^2}{0.2} = \frac{\cancel{400}^{200} \times \pi^2}{9} \times \frac{1}{\cancel{0.2}\,0.1} = \frac{200\pi^2}{0.9}$ Ans

Q.4 Find the magnitude of centripetal acceleration of a particle on the tip of a fan blade, 0.3 m in diameter rotating at 1200 rpm

=> $a_c = r\omega^2$

$a_c = 0.15\,(2\pi 20)^2$

$= 0.15 \times \frac{22}{7} \times \frac{22}{7} \times 20 \times 20$

$\therefore \boxed{a_c = 2366.3040 \text{ rad/s}}$ Ans

Q.5 The radius of earth's orbit around the sun is 1.5×10^{11} m. Calculate the angular and linear velocity of earth. ~~How~~ Through how much angle does the earth revolve in 2 days ?

=> $r = 1.5 \times 10^{11}$ m

$T = 365$ days $= 365 \times 24 \times 60 \times 60$

$\omega = \frac{2\pi}{T} = \boxed{1.99 \times 10^{-7} \text{ rad/sec.}}$ Ans

$v = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7} = \boxed{2.99 \times 10^{4} \text{ m/s}}$ Ans

Angle made by earth' revolution in 2 days = $365 \rightarrow 2\pi$

$1d = \frac{2\pi}{365}$

$2d = \frac{4\pi}{365}$

Ans => $\boxed{0.0344 \text{ rad}}$