Chapter-1

Units and Measurements

Physical Quantities

Those quantities which follow the laws of physics are called physical quantities.

Eg: Length, mass, time, force, energy, work, power, etc.

Types of Physical Quantities

1. **Fundamental Physical Quantities :-**

Those quanties which do not depends on another physical quantities known as fundamental physical quantities.

There are only seven fundamental physical quantities:-

(i) Length (distance between 2 points)
(ii) Mass (amount of matter contained in a body)
(iii) Time (interval between 2 events)
(iv) Temperature (measure of degree of hotness and coldness)
(v) Electric current (rate of flow of charge in definite direction)
(vi) Intensity of light (light energy emited per unit time)
(vii) Amount of substance (avogadro number)

2. **Derived ~~of~~ Physical Quantities :-**

Those quantities which depends on fundamental physical quantities known as derived physical quantities.

Eg: Speed, velocity, force, acceleration, energy, etc.

3. **Supplementry Physical Quantities :-**

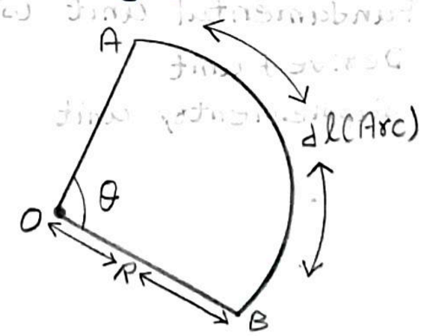
Those quantities which are neither fundamental nor derived known as supplementry physical quantities.

There are only 2 supplementry physical quantities:-

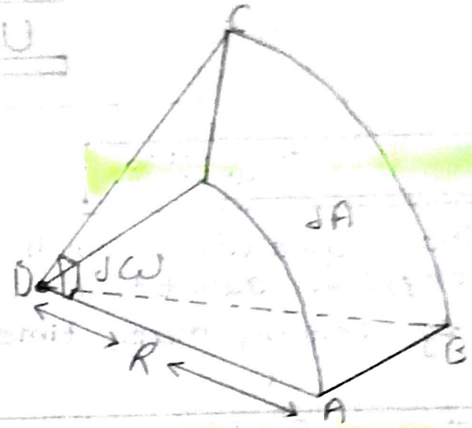
(i) Plane angle : Angle subtended by a line (arc) on any point is known as plane angle.

$$\boxed{\text{Plane angle} = \frac{\text{Arc}}{\text{Radius}}}$$

$$\boxed{d\theta = \frac{dl}{r}}$$



(ii) **Solid angle :** The angle subtended by any surface at any point is known as solid angle. It is represented by "ω" (omega).



$$\boxed{\text{Solid angle} = \frac{\text{Area}}{\text{Radius}^2}}$$

$$\boxed{d\omega = \frac{dA}{R^2}} \quad \text{or} \quad \boxed{d\Delta = \frac{\Delta A}{R^2}}$$

Note : $0° \leq \theta \leq 2\pi \quad [\pi = 180°]$

$0° \leq \omega \leq 4\pi \quad [\pi = 180°]$

Note: Physical Quantity = Numerical value X unit

$$Q = nu$$

Eg: Time = 2 hr → unit

2 ↓ numerical value (magnitude)

=> Numerical value define quantity while unit define nature of physical quantity.

Types of Physical Quantities on the basis of direction

1. Scaler
2. Vector
3. Tenser

Measurement

It is the process of comparission between two physical quantities of same nature.

Unit

It is the quantity of a constant magnitute which is used to measure the magnitude of other physical quantity of same nature.

Types of Unit :-

1. Fundamental unit (SI unit)
2. Derived unit
3. Supplementry unit

System of Units

1. **FPS System :-**
   - FPS = Foot, Pound, Second
   - Also known as British system.
   - In this system –

     Length → Foot

     Mass → Pound

     Time → second

2. **CGS System :-**
   - CGS = Centimetre, Gram, Second
   - In this system –

     Length → centimetre

     Mass → Gram

     Time → Second

3. **MKS System :-**
   - MKS = Metre, Kilogram, Second
   - In this system –

     Length → Metre

     Mass → Kilogram

     Time → Second

Standard International (SI) System of Units

→ It is the extended form of MKS system.

	Physical Quantity	SI Unit
Fundamental	Length (l)	metre (m)
	Mass (m)	kilogram (kg)
	Time (t)	second (s)
	Temperature (T)	kelvin (K)
	Electric Current (I)	Ampere (A)
	Intensity of Light (I_v)	candela (cd)
	Amount of substance (n)	mole (mol)
Supplementary	Plane angle (θ)	radian (rd)
	Solid angle (ω)	ste-radian (st-rd)

SI Unit of Derived Physical Quantity

S.No.	Physical Quantity	Formula	SI Unit	CGS Unit
1	Area	$L \times B$	m^2	cm^2
2	Volume	$L \times B \times H$	m^3	cm^3
3	Distance & Displacement	—	m	cm
4	Speed	$\frac{\text{Distance}}{\text{Time}}$	m/s	cm/s
5	Velocity	$\frac{\Delta d}{\Delta t}$	m/s	cm/s
6	Acceleration	$\frac{\Delta \text{velocity}}{\Delta \text{time}}$	m/s^2	cm/s^2
7	Linear momentum	mass X velocity	Kgm/s	~~gt~~ gcm/s
8	Force	mass X acceleration	Kgm/s^2 (Newton)	gcm/s^2 (dyne)
9	Pressure	$\frac{\perp r \text{ force}}{\text{Area}}$	N/m^2 (Pascal)	$dyne/cm^2$
10	Work	Force X displacement	Nm (Joule)	dyne-cm
11	Time period	—	second (s)	second (s)
12	Frequency	1/time period	$sec.^{-1}$ (Hz)	$sec.^{-1}$ (Hz)
13	Wavelength (λ)	—	m	cm
14	Kinetic energy	$\frac{1}{2}mv^2$	Kgm^2/s^2 (Joule)	gcm^2/s^2
15	Gravitational Potential energy	mgh	Kgm^2/s^2 (Joule)	gcm^2/s^2
16	Universal Gravitational Constant	$F = \frac{Gm_1m_2}{r^2}$ $G = \frac{F \cdot r^2}{m_1m_2}$	Nm^2/kg^2	$dynecm^2/g^2$

S.No.	Physical Quantity	Formula	SI Unit	CGS Unit
17	Focal Length	—	m	cm
18	Radius of curvature	$F = \frac{R}{2}$ $R = 2F$	m	cm
19	Mirror formula	$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$	—	—
20	Power of ~~lens~~ mirror	$P_m = \frac{1}{f(m)}$	m^{-1}	cm^{-1}
21	Lens formula	$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$	—	—
22	Power of lens	$P_L = \frac{1}{f(m)}$	m^{-1}	cm^{-1}
23	Magnification of mirror	$m = \frac{h_i}{h_o} = \frac{-v}{u}$	—	—
24	Magnification of lens	$m = \frac{h_i}{h_o} = \frac{v}{u}$	—	—
25	Refractive index (μ)	$\mu = \frac{c}{v}$	—	—
26	Lens maker's formula	$\frac{1}{f} = (\mu - 1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	—	—
27	Electric charge	$Q = ne$	A-sec (coulumb)	stc-coulumb
28	Electric current	$I = \frac{Q}{t}$	Ampere (A) or c/s	—
29	Coulumb's constant	$F_e = \frac{k q_1 q_2}{r^2}$ $K = \frac{F_e \cdot r^2}{q_1 q_2}$	Nm^2/c^2	—
30	Electric potential	$V = \frac{W}{q}$	J/c (volt)	—
31	Resistance	$V = IR$ $R = \frac{V}{I}$	volt/ampere (ohm) (Ω)	—

S.No.	Physical Quantity	Formula	SI Unit	CGS Unit
32	Resistivity	$R = \rho \frac{l}{A}$ $\rho = \frac{RA}{l}$	$\frac{\Omega m^2}{\cancel{m}} = \Omega m$	—
33	Conductance (G)	$\frac{1}{\text{Resistance}}$ or $G = \frac{1}{R}$	Ω^{-} / ℧ / simon	—
34	Conductivity (σ)	$\frac{1}{\text{resistivity}}$ or $\sigma = \frac{l}{RA}$	$\frac{\cancel{m}}{\Omega m^2} = \frac{1}{\Omega m}$ Ωm^{-1} / Ω^{1}/m / ℧m^{-1} / $\frac{mho}{m}$	—

Important SI Prefix

Power of 10	Prefix	Symbol
18	exa	e
15	peta	p
12	tera	t
9	giga	G
6	mega	M
3	kilo	K
2	hecto	h
1	deca	da
-1	deci	d
-2	centi	c
-3	mili	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	femto	f
-18	alto	a

Eg: Kilogram = 10^3m
micro second = 10^{-6}m
mili watt = 10^{-3}watt

Dimensions & Dimensional Formula

- **Dimensions:** Dimensions of a physical quantity are the power to which the base quantity are raised to represent that quantity.

- **Dimensional Formula:** The dimensional formula of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

- **Dimensional Equation:** The equation obtained by equating a physical quantity with its dimensional formula is called a dimensional equation.

Eg: $[F] = [MLT^{-2}]$

$[V] = [M^0LT^{-1}]$

Note: 1. Dimensional formula is always written inside the [·] for convension.

2. Dimensional formula of any derived quantity is written by the help of dimensional formula of base quantity.

Base Quantity	Dimensional Formula
Length	[L]
Mass	[M]
Time	[T]
Temperature	[K] or [θ]
Electric Current	[A]
Intensity of Light	—
Amount of substance	[mol]

Method to Write Dimensional Formula

Step-1: Jiska dimensional formula likhna hai uska formula likho.

Step-2: Us formule me aane wale derived quantity ko base quantity me badlo.

Step-3: Ab fundamental ka dimensional formula likho.

Step-4: Solve karo aur dimensional formula pao.

Dimensional Formula of Derived Quantity

S.No.	Quantity	Formula	Calculation	Dimensional Formula
1	Area	$L \times B$	$[L^1]$ $[L]^1$	$[M^0L^2T^0]$
2	Volume	LBH	$[L^1][L^1][L^1]$	$[M^0L^3T^0]$
3	Distance / Displacement	—	$[L^1]$	$[M^0LT^0]$
4	Speed	$\frac{D}{T}$	$\frac{[L^1]}{[T^1]}$	$[M^0LT^{-1}]$
5	Velocity	$\frac{\text{Displacement}}{\text{Time}}$	$\frac{[L^1]}{[T^1]}$	$[M^0LT^{-1}]$
6	Acceleration	$\frac{\Delta v}{\Delta t}$	$\frac{[LT^{-1}]}{[T]}$	$[M^0L^1T^{-2}]$
7	Linear Momentum	mv	$[M][LT^{-1}]$	$[MLT^{-1}]$
8	Force	ma	$[M][LT^{-2}]$	$[MLT^{-2}]$
9	Work	Fd	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
10	Kinetic Energy	$\frac{1}{2}mv^2$	$[M][LT^{-1}]^2$	$[ML^2T^{-2}]$
11	Gravitational Potential Energy	mgh	$[M][LT^{-2}][L]$	$[ML^2T^{-2}]$
12	Pressure	$\frac{F}{A}$	$\frac{[MLT^{-2}]}{[L^2]}$	$[ML^{-1}T^{-2}]$
13	Weight	mg	$[M][LT^{-2}]$	$[MLT^{-2}]$
14	Power	$\frac{W}{T}$	$\frac{[ML^2T^{-2}]}{[T]}$	$[ML^2T^{-3}]$
15	Gravitational Constant (G)	$G = \frac{Fr^2}{m_1m_2}$	$\frac{[MLT^{-2}][L^2]}{[M^2]}$	$[M^{-1}L^3T^{-2}]$
16	Gravitational Potential (V)	$V = \frac{Gm}{r}$	$\frac{[M^{-1}L^3T^{-2}][M]}{[L]}$	$[M^0L^2T^{-2}]$
17	Plane angle (θ)	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{[L]}{[L]}$	$[M^0L^0T^0]$

S.No.	Quantity	Formula	Calculation	Dimensional Formula
18	Solid angle (ω)	$\frac{Area}{Radius^2}$	$\frac{[L^2]}{[L^2]}$	$[M^0L^0T^0]$
19	Angular displacement (θ)	—	—	$[M^0L^0T^0]$
20	Angular velocity (ω)	$\frac{\Delta\theta}{\Delta t}$	$\frac{[M^0L^0T^0]}{[T]}$	$[M^0L^0T^{-1}]$
21	Angular Acceleration (α)	$\frac{\Delta\omega}{\Delta t}$	$\frac{[M^0L^0T^{-1}]}{[T]}$	$[M^0L^0T^{-2}]$
22	Moment of Inertia (M_i)	$M \times r^2$	$[M][L^2]$	$[ML^2T^0]$
23	Torque (τ)	$M_i \times \alpha$	$[ML^2T^0][M^0L^0T^{-2}]$	$[ML^2T^{-2}]$
24	Surface Tension	$\frac{Force}{Length}$	$\frac{[MLT^{-2}]}{[L]}$	$[ML^0T^{-2}]$
25	Frequency	$\frac{1}{Time}$	$\frac{1}{[T]}$	$[M^0L^0T^{-1}]$
26	Angular work	$(\tau.\theta)$	$[ML^2T^{-2}][M^0L^0T^0]$	$[ML^2T^{-2}]$
27	Charge	$Q = IT$	$[A][T]$	$[AT]$
28	Coulumb's Constant	$K = \frac{Fe-r}{q_1q_2}$	$\frac{[MLT^{-2}][L^2]}{[AT]^2}$	$[ML^3A^{-2}T^{-4}]$
29	Electric Potential	$\frac{W}{Q}$	$\frac{[ML^2T^{-2}]}{[A]}$	$[ML^2T^{-3}A^{-1}]$

S.No.	Quantity	Formula	Calculation	Dimensional Formula
30	~~Electric~~ Resistance	$\frac{V}{I}$	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}$	$[ML^2T^{-3}A^{-2}]$
31	Conductance	$\frac{1}{R}$	$\frac{1}{[ML^2T^{-3}A^{-2}]}$	$[M^{-1}L^{-2}T^3A^2]$
32	Resistivity	$\rho = \frac{RA}{l}$	$\frac{[ML^2T^{-3}A^{-2}][L^2]}{[L]}$	$[ML^3T^{-3}A^{-2}]$
33	Conductivity	$\frac{1}{\text{Resistivity}}$	$\frac{1}{[ML^3T^{-3}A^{-2}]}$	$[M^{-1}L^{-3}T^3A^2]$
34	Electric Field	$\frac{\text{Electric Force}}{\text{Charge}}$	$[MLT^{-2}]/[AT]$	$[MLT^{-3}A^{-1}]$
35	Electric Flux	Electric Field × Area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$
36	Heat	Work (= Force × distance)	$[MLT^{-2}][L]$	$[ML^2T^{-2}]$
37	Magnetic Field	$\frac{\text{Force}}{\text{Current} \times \text{Length}}$	$\frac{[MLT^{-2}]}{[A][L]}$	$[ML^0T^{-2}A^{-1}]$
38	Magnetic Flux	Magnetic Field × Area	$[ML^0T^{-2}A^{-1}][L^2]$	$[ML^2T^{-3}A^{-1}]$
39	Refractive index	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$	$\frac{[LT^{-1}]}{[LT^{-1}]}$	$[M^0L^0T^0]$
40	Power of lens & mirror	$\frac{1}{f(m)}$	$\frac{1}{[L]}$	$[M^0L^{-1}T^0]$

Some Important Dimensionless Physical Quantities

1. Trigonometric Ratio (sin, cos, tan, sec, cosec, cot)
2. Exponential : $e^{(-)}$
3. Logarithm
4. Pure number

Principle of Homogenity

→ According to this principle the physical quantities of same dimensions can be added and subtracted.

Questions Based on Principle of Homogenety

1. If $P = \frac{F}{A} + a$

   P = Pressure

   F = Force

   A = Area

   Then find 'a'.

=> By principle of homogeneity D.F. of 'a' $= \frac{F}{A}$

$$= \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

=> Here, D.F. of 'a' $= [ML^{-1}T^{-2}]$

Ans=> Hence, a = Pressure

2. If displacement, $x = a + bt + ct^2$, then find a, b and c.

=> D.F of 'a' = D.F. of bt = D.F. of ~~at~~ ct^2 = D.F. of displacement. [By principle of homogeneity]

=> $[a] = [L]$

=> $bt = [L]$

$[b][T] = [L]$

$[b] = \frac{[L]}{[T]}$

$\therefore [b] = [LT^{-1}]$

=> $ct^2 = [L]$

$[c] = \frac{[L]}{[T^2]}$

$\therefore [c] = [LT^{-2}]$

Ans=> $[a] = [L]$

$[b] = [LT^{-1}]$

$[c] = [LT^{-2}]$

3. $\boxed{\begin{array}{l} v = at + bt + ct^2 \\ \text{Find D.F. of } a, b, c. \end{array}}$

$\Rightarrow$ $v = [LT^{-1}]$

$\Rightarrow$ D.F. of 'a' = D.F. of bt = D.F. of ct^2 = D.F. of v
[By principle of homogeneity]

$\Rightarrow$ $\therefore [a] = [LT^{-1}]$

$\Rightarrow$ $bt = [LT^{-1}]$

$[b][T] = [LT^{-1}]$

$[b] = \dfrac{[LT^{-1}]}{[T]}$

$\therefore [b] = [LT^{-2}]$

$\Rightarrow$ $ct^2 = [LT^{-1}]$

$[c] = \dfrac{[LT^{-1}]}{[T^2]}$

$\therefore [c] = [LT^{-3}]$

Ans $\Rightarrow$ $[a] = [LT^{-1}]$
$[b] = [LT^{-2}]$
$[c] = [LT^{-3}]$

4. $\boxed{v = a + \dfrac{b}{c+t}}$

$\Rightarrow$ $v = [LT^{-1}]$

$\Rightarrow$ D.F. of v = D.F. of 'a' = D.F. of $\dfrac{b}{c+t}$.
[By principle of homogeneity]

$\Rightarrow$ $\therefore [a] = [LT^{-1}]$

$\Rightarrow$ $\dfrac{b}{c+t} = [LT^{-1}]$

$\dfrac{[b]}{[c]+[T]} = [LT^{-1}]$

$\dfrac{[b]}{[c]} = [LT^{-2}]$

$\Rightarrow$ By principle of homogeneity $[c] = [T]$
$\therefore [b] = [LT^{-1}]$

Ans $\Rightarrow$ $[a] = [LT^{-1}]$
$[b] = [LT^{-1}]$
$[c] = [T]$

5. $y = A\sin(a+bt)$ [y = displacement]
Find D.F. of A, a & b.

=> $\sin(a+bt)$ is dimensionless.

=> $\therefore [a] = [M^0L^0T^0]$

=> $bt = [M^0L^0T^0]$

$$[b] = \frac{[M^0L^0T^0]}{[T]}$$

$\therefore [b] = [M^0L^0T^{-1}]$

=> By principle of homogeneity D.F. of A = D.F. of y = [L]

Ans=> $[A] = [L]$

$[a] = [M^0L^0T^0]$

$[b] = [M^0L^0T^{-1}]$

6. $y = A\sin(\omega t + \phi)$
Find D.F. of A, ω, ϕ.

=> $\sin(\omega t + \phi)$ is dimensionless.

=> $\therefore \cancel{[\omega]} = [\phi] = [M^0L^0T^0]$

=> $[\omega t] = [M^0L^0T^0]$

$$[\omega] = \frac{[M^0L^0T^0]}{[T]}$$

$\therefore [\omega] = [M^0L^0T^{-1}]$

=> By principle of homogeneity ~~of~~ D.F. of A = D.F. of y = [L]

Ans=> $[A] = [L]$

$[\omega] = [M^0L^0T^{-1}]$

$[\phi] = [M^0L^0T^0]$

7. A student of Class 11th writing equations of motion as follows :-

(i) $v = A + Bt$

(ii) $S = a't + \frac{1}{2}bt^2$

(iii) $v^2 = \alpha^2 + 2as$

He detects some unknown quantity : A, B, a', α, b
Find A, B, a'b, α.

$\Rightarrow [v] = [LT^{-1}]$

$\therefore [A] = [LT^{-1}]$ [By principle of homogeneity]

$\Rightarrow [BT] = [LT^{-1}]$

$[B] = \frac{[LT^{-1}]}{[T]}$

$\therefore [B] = [LT^{-2}]$

$\Rightarrow s = [L]$

$[a't] = [L]$

$[a'] = \frac{[L]}{[T]}$

$\Rightarrow [a'] = [LT^{-1}]$

$\Rightarrow [bt^2] = [L]$

$[b] = \frac{[L]}{[T^2]}$

$\Rightarrow [b] = [LT^{-2}]$

$\Rightarrow [LT^{-1}][LT^{-1}] = \alpha^2$

$[LT^{-1}]^{\cancel{2}} = \alpha^{\cancel{2}}$

$\therefore \alpha = [LT^{-1}]$

Ans=> $[A] = [LT^{-1}]$ = Velocity

$[B] = [LT^{-2}]$ = Acceleration

$[a'] = [LT^{-1}]$ = velocity

$[b] = [LT^{-2}]$ = Acceleration

$[\alpha] = [LT^{-1}]$ = velocity

8. If $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ (F → Force, $q_1 q_2$ → Charge, r^2 → Distance)

Find D.F. of ϵ_0.

$\Rightarrow F = [MLT^{-2}]$

$[MLT^{-2}] = \frac{1}{4\pi\epsilon_0} \frac{[A^2T^2]}{[L^2]}$

$\Rightarrow \epsilon_0 = \frac{1}{[MLT^{-2}]} \frac{[A^2T^2]}{[L^2]}$

$\epsilon_0 = [M^{-1}L^{-1}T^2][L^{-2}T^2A^2]$

Ans=> Hence, $\epsilon_0 = [M^{-1}L^{-3}T^4A^2]$

Practise Questions

1. The SI unit of electrochemical equivalent is
⇒ $Kg\ C^{-1}$

2. The sum of numbers 436.32, 227.2 and 0.301 in appropriate significant figures is

⇒
$$\begin{array}{r} 436.320 \\ 227.200 \\ +\ 0.301 \\ \hline 663.821 \\ \hline \end{array}$$

Ans ⇒ 663.8

3. A sextant is used to measure
⇒ Height of an object.

4. A pressure of 10^6 dyne cm^{-2} is equivalent to
⇒ $10^5\ Nm^2$

5. Universal time is based on
⇒ Earth's orbital motion around the sun.

6. Which of the following cannot be regarded as an essential characteristics of a unit measurement?
⇒ Reproductibility

7. The mean length of an object is 5 cm. Which of the following is most accurate?
⇒ 4.9 cm

8. Energy per unit volume represent
⇒ Pressure

9. Which of the following pairs of physical quantities does not have same dimensional formula?
⇒ Tension and surface tension

10. ~~A pressure of 106~~ Which one of the following pairs of quantities and their unit is properly matched?
⇒ Magnetic flux - weber/m^2

11. One light year is defined as the distance travelled by light in one year. The speed of light is $3 \times 10^8\ ms^{-1}$. The same in metre is.
⇒ 9.461×10^{15} m

12. If the acceleration due to gravity is $10\,ms^{-2}$ and the units of length and time are changed in kilometer and hour respectively, the numerical value of acceleration?
=> 129600

13. What is the power of a 100 W bulb in CGS unit?
=> $10^9\ erg^{-1}$

14. The fundamental unit, which has the same power in the dimensional formula of surface tension and viscosity is
=> Mass

15. If the unit of M and L are increased three times, then the unit of energy will be increased by
=> 27 times

16. SI unit of intensity of wave is
=> Wm^{-2}

17. A suitable unit for gravitational constant
=> $Nm^2\,kg^{-2}$

18. One yard in SI unit is equal to
=> 0.9144 m

19. If L denotes the inductance of an inductor through which a current I is flowing, then the d.f. of LI^2 is
=> $\cancel{ML^2T}$ $[ML^2T^{-2}]$

20. Farad is not equal to
=> qV^2

21. The dimensional formula of magnetic permeability is
=> $[MLT^{-2}A^{-2}]$

~~41~~ 22. $[ML^2T^{-2}]$ Represents d.f. of which quantity?
=> Energy

23. The dimensions of emf in MKS is
=> $[ML^2T^{-2}Q^{-1}]$

24. The quantity which has the d.f. $[M^1T^{-3}]$ is
=> Surface tension

25. If I is the moment of ~~int~~ inertia and ω the angular velocity, what is the d.f. of rotational kinetic energy.

$\Rightarrow$ $[ML^2T^{-2}]$

26. The dimensions of power of lens are

$\Rightarrow$ $[M^0L^{-1}T^0]$

Applications of Dimensional Formula

1. To check the dimensional correctness of equation :-

→ Any equation will be correct if dimensional formula of LHS quantity is equal to the dimensional formula of RHS quantity.

Eg: (i) 2nd equation of motion derived by a student is given as $s = ut + at^2$, where s = displacement, u = velocity, t = time, a = acceleration. Then check the correctness of the equation.

$\Rightarrow$ $$s = ut + at^2$$

$$[L] = [L\cancel{T^{-1}}][\cancel{T}] + [L\cancel{T^{-2}}][\cancel{T^2}]$$

$\Rightarrow$ $[L] = [L]$

LHS = RHS

Hence, the given equation is correct.

(ii) 4 students A, B, C and D derived the equation of centripetal force, then check which student's equation is correct.

$\Rightarrow$ A $\Rightarrow$ $F_c = \frac{m}{v^2 - r}$, B $\Rightarrow$ $\frac{1}{2}\frac{mv^2}{r}$, C $\Rightarrow$ $\frac{mv^2}{r}$, D $\Rightarrow$ $F_c = \frac{r}{mv^2}$

$\Rightarrow$ A : $F_c = \frac{m}{v^2 - r}$

$\Rightarrow$ $[MLT^{-2}] = \frac{[M]}{[LT^{-1}] - [L]}$ $\Rightarrow$ Here, LHS $\neq$ RHS
Hence, This eqn is incorrect.

$\Rightarrow$ B : $F_c = \frac{1}{2}\frac{mv^2}{r}$

$\rightarrow$ $[MLT^{-2}] = \frac{[M][LT^{-1}]^2}{[L]} = \frac{[ML^{\cancel{2}}T^{-2}]}{[\cancel{L}]} = [MLT^{-2}]$

$\Rightarrow$ Here, LHS = RHS
Hence, this eqn is correct.

$\Rightarrow$ C : $F_c = \dfrac{mv^2}{r}$

$\Rightarrow [MLT^{-2}] = \dfrac{[M][LT^{-1}]^2}{[L]} = \dfrac{[ML^2T^{-2}]}{[L]} = [MLT^{-2}]$

$\Rightarrow$ Here, LHS = RHS
Hence, this eq^n is correct.

$\Rightarrow$ D : $F_c = \dfrac{r}{mv^2}$

$\Rightarrow [MLT^{-2}] = \dfrac{[L]}{[M]\,[LT^{-1}]^2} = \dfrac{[L]}{[ML^2T^{-2}]} = [M^{-1}L^{-1}T^{2}]$

$\Rightarrow$ Here, LHS $\neq$ RHS
Hence, this eq^n is incorrect.

2. Conversion of Units from one system to another :-

→ Physical Quantity = $n.u$; where, n = magnitude and u = unit.

→ For conversion : $n_1u_1 = n_2u_2 = n_3u_3 \ldots\ldots$

$\Rightarrow n_1u_1 = n_2u_2$

$\Rightarrow n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$

$\Rightarrow n_2 = \dfrac{n_1[M_1^a L_1^b T_1^c]}{[M_2^c L_2^b T_2^c]}$

$$\therefore \boxed{n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

Eg: (i) Convert one newton to dyne.

$\Rightarrow$ Newton = Dyne

$\Rightarrow$ Force = $[MLT^{-2}]$

$\Rightarrow$ From formula, $a = 1, b = 1, c = -2$

$\Rightarrow n_2 = n_1\left[\dfrac{M_1}{M_2}\right]^1\left[\dfrac{L_1}{L_2}\right]^1\left[\dfrac{T_1}{T_2}\right]^{-2}$

$= 1\left[\dfrac{1kg}{1g}\right]^1\left[\dfrac{1m}{1cm}\right]^1\left[\dfrac{\cancel{1sec}}{\cancel{1sec}}\right]^{-2}$

$= \left[\dfrac{1000\cancel{g}}{1\cancel{g}}\right]\left[\dfrac{100\cancel{cm}}{1\cancel{cm}}\right] = 10^3\times10^2$

Ans:→ 10^5 dyne

(ii) $\boxed{\text{Convert 1 Joule to erg.}}$

$\Rightarrow$ Energy $= [ML^2T^{-2}]$

$\Rightarrow$ From formula, $a = 1, b = 2, c = -2$

$$\Rightarrow n_2 = n_1 \left[\frac{M_1}{M_2}\right] \left[\frac{L_1}{L_2}\right] \left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1 \left[\frac{1\,kg}{1\,g}\right]^1 \left[\frac{1\,m}{1\,cm}\right]^2 \left[\frac{1\,sec}{1\,sec}\right]^{-2}$$

$$= \left[\frac{1000\,\cancel{g}}{1\,\cancel{g}}\right] \left[\frac{100\,\cancel{cm} \times 100\,\cancel{cm}}{1\,\cancel{cm} \times 1\,\cancel{cm}}\right]$$

$$= [10^3] \times [10^4]$$

Ans $\Rightarrow$ 10^7 ergs.

3. <u>Derivation of formula of any physical quantity</u> :-

<u>Eg</u>: (i) $\boxed{\text{Force on any object depends on mass of object \& acceleration, then,}}$

$\Rightarrow F \propto m^x a^y$

$\Rightarrow F = K\, m^x a^y$

$\Rightarrow [MLT^{-2}] = [M]^x\, [LT^{-2}]^y$

$\Rightarrow$ <u>On comparing</u>,

$x = 1, y = 1$

$\Rightarrow [MLT^{-2}] = [M^x L^y T^{-2y}]$

$\Rightarrow F = K \cdot m^1 . a^1$

$\therefore \boxed{F = ma}$

(ii) A particle of mass (m) moving on a circular ~~part~~ path of radius (r) with speed (v), the centripetal force acting on the particle depends on mass, speed and radius, then,

$\Rightarrow F \propto m^a . v^b . r^c$

$\Rightarrow F = K . m^a v^b r^c$

$\Rightarrow [MLT^{-2}] = [M]^a [LT^{-1}]^b [L]^c$

$\Rightarrow [MLT^{-2}] = [M^a L^{b+c} T^{-b}]$

$\Rightarrow$ <u>On comparing</u> :

$a = 1, \; b+c = 1, \; \cancel{-}b = \cancel{-}2 \Rightarrow 2 + c = 1, \; c = -1$

$\Rightarrow \quad F = K m^1 v^2 c^{-1}$

$\Rightarrow \quad F = K \frac{mv^2}{r}$

$\therefore \quad \boxed{F = \frac{mv^2}{r}}$

Significant Figures

→ Significant figures are those digits in a measured quantity which are known with certainity including the one which is uncertain.

→ These are the numbers of digits in a value after a measurement which contribute to the degree of accuracy of the value.

Rules for counting significant figures :-

Case-1 : For a number greater than 1 :-

1. All non zero digits are significant. (Decimal doesn't matter).
   Eg: 22.52 (4 S.F.), 235 (3 S.F.)

2. All zeros between non zeros are significant. (Decimal doesn't matter).
   Eg: 2305 (4 S.F.), 22.003 (5 S.F.), 22001 (5 S.F.)

3. All zeros to the right of non zeros are significant (no. without decimal).
   Eg: 250000 (2 S.F.), 20100 (3 S.F.), 200 (1 S.F.)

4. All zeros to the right of non zeros are significant in decimal number.
   Eg: 2.000 (4 S.F.), 20.10 (4 S.F.), 2010 (3 S.F.), 2.300 (4 S.F.)

5. In measured value of any physical quantity, terminating zeros are always significant.
   Eg: 86400 (3 S.F.) but 86400 sec. (5 S.F.)

6. Power of ten multiplied in any value does not affect the significant figures.
   Eg: 1.5×10^2 (2 S.F.), 3.49×10^{-3} (3 S.F.)

7. Number of significant figures doesn't depend on system of units.
   Eg: 16.4 cm (3 S.F.), 0.164 m (3 S.F.), 0.000164 km (3 S.F.)

Note: If original measured value is converted from one unit to another the S.F. should be same.

Eg: $\boxed{52\ km = 52000\ m = 52 \times 10^3 m}$

Here, "52 km" has 2 significant figures.
"52000 m" has 5 significant figures.
"$52 \times 10^3 m$" has 2 significant figures.

$\therefore$ Because of different S.F.,
52 km = 52000 m ✗
And because of same S.F.,
52 km = $52 \times 10^3 m$ ✓

Eg: Measured height of Parvej Sir is 1.5m. Convert in cm, mm and km.

$\Rightarrow$ 1.5 m = 150 cm (✗) = 1.5×10^2 cm (✓)

$\Rightarrow$ 1.5m = 1500 mm (✗) = 1.5×10^3 mm (✓)

$\Rightarrow$ 1.5m = 0.0015 km (✗) = 1.5×10^{-3} km (✓)

Case - 2 : When number is less than 1 :-

→ Any zero right to non zero digit is significant.
→ Any zero ~~right~~ before non zero digit is not significant.
→ Any zero between non zero digits is significant.

Eg: 0.0750 (3 S.F.)
0.00010 (2 S.F.)
0.007040 (4 S.F.)

Rules for Rounding off

1. If digit is draft greater than 5 then preceeding digit is raised by 1.
   Eg: 3.577 = 3.58

2. If digit is draft by less than 5 then preceeding digit remains same.
   59.72 = 59.7

3. If digit is draft by 5 or 5 followed by 0, then preceeding digit is increased by 1 if it is odd and remains same if it is even.

Eg: 25.73(5) = 25.74

2.72(5) = 2.72

26.27(50) = 26.28

4. If 5 is followed by further non zero digit then preceeding digit is raised by 1.

Eg: 13.3(52) = 13.4

15.4(53) = 15.5

Significant Figures in Calculation

Case-1 : In addition and subtraction :-

→ In such operations result should have smallest decimal place according to numbers involve.

Eg: (i) 2.29 + 62.7 = 64.99

Ans ⇒ 64.9

(ii) 82.29 − 62.7 = 9.59

Ans ⇒ 9.6

Case-2 : In division and multiplication :-

→ In such operations result should have equal significant figure according to least no. of significant figures of the numbers involve in calculation.

Eg: (i) 2.30 × 0.4 = 0.920

Ans ⇒ 0.9

(ii) $\frac{3500}{7.52}$ = 465.42

Ans ⇒ 470

Limitation of Dimensional Analysis

1. We can't find out derived quantity is vector or scalar.
2. The relation derived from this method gives no information about the dimensionless constants.
3. With the help of this method we can't derive an equation which depends on more than 3 physical quantities.
4. We can't check complete correctness of any equation by this method.
5. We can't decide the actual physical quantity by this method.

Error Analysis

→ **Error** : Any value different from original value known as error value & difference between true value & measured value is known as error.

i.e., $\boxed{\text{error} = \text{True value} - \text{Measured value}}$

Eg: If measured mass of a motorcycle is 200.12 kg, then find error in measurement if original mass is 200 kg.

=> $200.12 - 200.00 = 0.12$ (error)
Ans

→ **Types of Error** :-

1. Systematic Error
2. Random Error
3. Gross Error

• **Systematic Error** : That error which causes are known are called systematic error.
This error can be minimised.

Types of Systematic Error :-

(i) Error due to external factors. Eg: Temperature, pressure, humidity, etc.
(ii) Error due to imperfection. Eg: Neglegence of air resistance.
(iii) Instrumental error
(iv) Personal error

- **Random Error** : When repeated measurements of the quantity yeild different results under the same conditions, this is referred to as random error. This error occurs for unknown reasons.

- **Gross Error**: It is a significant, unpredictable mistake caused by human error that often leads to large ~~de~~ discrepancies.

Absolute Error

→ Let us consider measured value of a quantity in small & observations are $x_1, x_2, x_3, \ldots\ldots x_n$.

→ Then its true value $$\boxed{(x_m) = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}}$$

→ And, absolute error in $x_1 = |x_m - x_1| \ (|\Delta x_1|)$
absolute error in $x_2 = |x_m - x_2| \ (|\Delta x_2|)$
absolute error in $x_3 = |x_m - x_3| \ (|\Delta x_3|)$
absolute error in $x_4 = |x_m - x_4| \ (|\Delta x_4|)$

→ Similarly, absolute error in $x_n = |x_m - x_n| \ (|\Delta x_n|)$

- **Mean Absolute ~~e~~ error** : It is the arithmetic mean of all absolute errors.

i.e., $$\boxed{\overline{\Delta x}_m = \frac{|\Delta x_1| + |\Delta x_2| + \ldots\ldots + |\Delta x_n|}{n}}$$

- **Relative error** : Relative error (R.E.) = $\frac{\text{Mean absolute error}}{\text{True value}}$

i.e., $$\boxed{R.E. = \frac{\overline{\Delta x}_m}{x_m}}$$

- **Percentage error** : Percentage error = Relative error × 100

i.e., $$\boxed{\%\ \text{error} = \frac{\overline{\Delta x}_m}{x_m} \times 100}$$

Q.1: Length of a white board measured by 5 students are :
2.1m, 2.22m, 1.98m, 2.0m, 2.2m
Find absolute error, mean absolute error and percentage error.

$\Rightarrow$
$x_1 = 2.1m$
$x_2 = 2.22m$
$x_3 = 1.98m$
$x_4 = 2.0m$
$x_5 = 2.2m$

$\Rightarrow$ True value (x_m) $= \dfrac{2.1 + 2.22 + 1.98 + 2.0 + 2.2}{5}$

$= \dfrac{10.5}{5} = \boxed{2.1}$ (True value)

$\Rightarrow$ Absolute error in,

$$\left.\begin{aligned} x_1 &= |2.1 - 2.1| = 0 \\ x_2 &= |2.1 - 2.22| = 0.12 \\ x_3 &= |2.1 - 1.98| = 0.12 \\ x_4 &= |2.1 - 2.0| = 0.1 \\ x_5 &= |2.1 - 2.2| = 0.1 \end{aligned}\right] \text{Absolute error}$$

$\Rightarrow$ Mean absolute error $(\Delta\bar{x}_m)$ $= \dfrac{0 + 0.12 + 0.12 + 0.1 + 0.1}{5}$

$= \dfrac{0.44}{5} = \boxed{0.088}$ $(\Delta\bar{x}_m)$

$\Rightarrow$ Percentage error $= \dfrac{\Delta\bar{x}_m}{x_m} \times 100$

$= \dfrac{0.088}{2.1} \times 100 = \boxed{4.19\,\%}$ (%age error)

Q.2: Repeated measurement of a certain quantity in an experiment gave the following values:
1.29, 1.34, 1.35, 1.36, 1.32, 1.30, 1.33, 1.33
Find absolute error, mean absolute error, relative error and percentage error.

$\Rightarrow$
$x_1 = 1.29$
$x_2 = 1.34$
$x_3 = 1.35$
$x_4 = 1.36$
$x_5 = 1.32$
$x_6 = 1.30$
$x_7 = 1.33$
$x_8 = 1.33$

$\Rightarrow$ True value (x_m) $= \dfrac{1.29 + 1.34 + 1.35, 1.36 + 1.32 + 1.30 + 1.33 + 1.33}{8}$

$= \dfrac{10.62}{8} = \boxed{1.33}$ (True value)

$\Rightarrow$ Absolute error in,

$$\left.\begin{aligned} x_1 &= |1.33 - 1.29| = 0.04 \\ x_2 &= |1.33 - 1.34| = 0.01 \\ x_3 &= |1.33 - 1.35| = 0.02 \\ x_4 &= |1.33 - 1.36| = 0.03 \\ x_5 &= |1.33 - 1.32| = 0.01 \\ x_6 &= |1.33 - 1.30| = 0.03 \\ x_7 &= |1.33 - 1.33| = 0.00 \\ x_8 &= |1.33 - 1.33| = 0.00 \end{aligned}\right] \text{Absolute error}$$

$\Rightarrow$ Mean absolute error $(\overline{\Delta x_m})$ =

$$= \frac{0.04 + 0.01 + 0.02 + 0.03 + 0.01 + 0.03 + 0.00 + 0.00}{8}$$

$$= \frac{0.14}{8} = \boxed{0.018} \text{ (Mean absolute error)}$$

$\Rightarrow$ Relative error $= \frac{\overline{\Delta x_m}}{x_m} = \frac{0.018}{1.33} = \boxed{0.013}$ (Relative error)

$\Rightarrow$ Percentage error $= \frac{\overline{\Delta x_m}}{x_m} \times 100 = \frac{0.018}{1.33} \times 100 = \boxed{1.3\%}$ (%age error)

Error in Mathematical operation

1. Error in Summation :-

→ Let $x = a + b$ ... ①

→ Then maximum absolute error in x is given as:

$$\boxed{\Delta x = \pm(\Delta a + \Delta b)} \quad \text{...②}$$

→ Here, Δa = ~~error in~~ absolute error in a.
Δb = absolute error in b.

→ Relative error :-

$$\frac{\Delta x}{x} = \boxed{\frac{(\Delta a + \Delta b)}{(a+b)}}$$

→ And percentage error :

$$\frac{\Delta x}{x} \times 100 = \boxed{\frac{(\Delta a + \Delta b)}{(a+b)} \times 100}$$

2. Error in difference :-

→ Let $x = a - b$ ... ①

→ Then maximum absolute error in x is given as:

$$\boxed{\Delta x = \pm(\Delta a + \Delta b)}$$

→ Relative error => $\frac{\Delta x}{x} = \boxed{\frac{(\Delta a + \Delta b)}{(a-b)}}$

→ Percentage error => $\frac{\Delta x}{x} \times 100 = \boxed{\frac{(\Delta a + \Delta b)}{(a-b)} \times 100}$

3. Error in Multiplication :-

→ Let $x = a.b$

→ Then, relative error in x is given by:

$$\frac{\Delta x}{x} = \boxed{\frac{\Delta a}{a} + \frac{\Delta b}{b}}$$

→ Percentage error $= \frac{\Delta x}{\Delta x} \times 100 = \boxed{\frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100}$

→ Absolute error $= \Delta x = \boxed{\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) x}$

4. Error in Division :-

→ Let $x = \frac{a}{b}$

→ Then, relative error in x is given by :

$$\frac{\Delta x}{x} = \boxed{\frac{\Delta a}{a} + \frac{\Delta b}{b}}$$

→ Percentage error $= \frac{\Delta x}{x} \times 100 = \boxed{\frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100}$

→ Absolute error $= \Delta x = \boxed{\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) x}$

5. Error in Power :-

→ Let $x = \frac{a^{n_1} b^{n_2}}{c^{n_3}}$

→ The, relative error is given by:

$$\boxed{\frac{\Delta x}{x} = n_1\left(\frac{\Delta a}{a}\right) + n_2\left(\frac{\Delta b}{b}\right) + n_3\left(\frac{\Delta c}{c}\right)}$$

→ Percentage error:

$$\boxed{\frac{\Delta x}{x} \times 100 = n_1\left(\frac{\Delta a}{a} \times 100\right) + n_2\left(\frac{\Delta b}{b} \times 100\right) + n_3\left(\frac{\Delta c}{c} \times 100\right)}$$

Trick: In addition and subtraction, first write absolute error and in division and multiplication, first write relative error and find other errors with the help of them.

Note:

1. Errors should be always added.
2. No error is considered in fixed number and constants.
3. Significant figures in any calculated objects is always taken as infinity.

Eg: 10 Mangoes, 5 Pens, 20 Men — ∞ S.F.

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