Polynomial Interpolants of Power Sums

LJ

1 Power Sum Definition

let,

$$\sum_{k=1}^{n} k^d = 1^d + 2^d + ... + n^d$$

define a "dth" order power sum of the first n integers.

2 Polynomial Interpolants of Power Sum

Bellow is the derivation for a method for finding an interpolating polynomial, $p_d \in \mathbb{P}_{d+1}$, of the dth order power sum, for $\forall d \in \mathbb{N}$, such that,

$$p_d(n) = \sum_{k=1}^{n} k^d, \quad \forall\, n \,\in\, \mathbb{N}$$

3 Sufficient Conditions for a Polynomial Interpolant

The following property holds for any dth order power sum,

$$\sum_{k=1}^{n+1} k^d = \sum_{k=1}^{n} k^d + (n+1)^d$$

By induction the following two properties are sufficient conditions for a polynomial, p, to interpolate a dth order power sum,

$$p_d(1) = \sum_{k=1}^{1} k^d, \tag{c1}$$

$$p_d(n+1) = p_d(n) + (n+1)^d,\ for\, all\, n\ \in\ \mathbb{N} \tag{c2}$$