HYD298 - Homework #2

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7.3 In flood protection planning, the 100year flood, which is an estimate of the quantile $x_{0.99}$, is often used as the design flow. Assuming that the floods in different years are independently distributed:

(a) Show that the probability of at least one 100year flood in a 5year period is 0.049.

```
#7.3a
#Prob. of 1 or more 100yr floods in 5yr is 1 minus the probability
# of not observing a 100year flood in 5 years
1-dbinom(0,5,1/100)
```

```
## [1] 0.04900995
```

(b) What is the probability of at least one 100 year flood in a 100 year period?

```
#7.3b
#Probability of 1 or more 100 year floods in 100 years is 1 minus
# probability of not observing flood in 100 years
1-dbinom(0,100,1/100)
```

```
## [1] 0.6339677
```

(c) If floods at 1000 different sites occur independently, what is the probability of at least one 100year flood at some site in any single year?

```
#7.3c
#Because sites are independent and we are only considering 1 year,
# we have 1 year * 1000 sites opportunities to observe a 100 year flood event
# and are virtually certain to observe at least one.
1-dbinom(0,1*1000,1/100)
```

```
## [1] 0.9999568
```

7.4 The price to be charged for water by an irrigation district has yet to be determined.

Currently it appears as if there is as 60% probability that the price will be $10 per unit of water and a 40% probability that the price will be $5 per unit. The demand for water is uncertain. The estimated probabilities of different demands given alternative prices are as follows:

```r
p=c(5,10) #prices
pp=c(0.4,0.6) #probability of prices

q=c(30,55,80,100,120) #discrete quantities; q

pqbar5=c(0,0.15,0.3,0.35,0.2) #conditional probabilities of levels of q given price = 5
pqbar10=c(0.2,0.3,0.4,0.1,0) # " " " price = 10
```

(a) What is the most likely value of future revenue from water sales?

The most likely value of future revenue is calculated from the levels of price and quantity with highest joint probability. We can calculate joint probabilities of price and quantity from $P_{XY} = P_{X|Y} * P_Y$ where X = quantity and Y = price and $P_{X|Y}$ are the probabilities for discrete quantities conditioned on price.

```r
#7.4a
jointpq=rbind(data.frame(joint=pqbar5*pp[1],price=p[1],qty=q),
          data.frame(joint=pqbar10*pp[2],price=p[2],qty=q))
most_likely=which(jointpq$joint==max(jointpq$joint))
mlrow=jointpq[most_likely,]
```

Most likely revenue (price*quantity) from water sales (in dollars):

```r
mlrow$price*mlrow$qty
```

```
## [1] 800
```

(b) What are the mean and variance of future water sales? (calculated for quantity, not revenue)

```r
#7.4b
mean_qty=sum(jointpq$joint*jointpq$qty)
var_qty=sum(((jointpq$qty-mean_qty)^2)*jointpq$joint)
```

Mean and variance of future water sales:

```r
mean_qty
```

```
## [1] 75.2
```

```r
var_qty
```

```
## [1] 634.96
```

(c) What is the median value and interquartile range of future water sales? (calculated for quantity, not revenue)

```r
#7.4c
combined=jointpq$joint[which(jointpq$price==5)]+jointpq$joint[which(jointpq$price==10)]
#probabilities assoc with each level of q, ignoring p
cfor=cumsum(combined) #will also hold for rev(combined)
median_qty=q[which(cfor >= 0.5)[1]]
firstq_qty=q[which(cfor >= 0.25)[1]]
thirdq_qty=q[which(cfor >= 0.75)[1]]
```

```r
median_qty #median
```

```
## [1] 80
```

```r
firstq_qty #first quartile
```

```
## [1] 55
```

```r
thirdq_qty #third quartile
```

```
## [1] 100
```

(d) What price will maximize the revenue from the sale of water?

```r
#7.4d
for(i in p) {
who=which(jointpq$price==i)
print(paste("Price $",i,"/unit has expected revenue: $",
        sum(jointpq$joint[who]*jointpq$price[who]*jointpq$qty[who]),sep=""))
}
```

```
## [1] "Price $5/unit has expected revenue: $182.5"
## [1] "Price $10/unit has expected revenue: $387"
```

Therefore price level of $10 will maximize revenue from water sales.