Affine Gap Alignment

CS1810 Fall 2020

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Definitions: Inputs and Outputs

Inputs: $\langle X, Y, \alpha, \mu, \gamma, \tau \rangle$

- X, Y = strings of length m, n with characters indexed by i, j, respectively
- α = match score
- μ = mismatch penalty
- γ = gap opening penalty
- τ = gap extension penalty (single-letter gap penalty)

Output: An alignment which maximizes the following score:

 α (# matches) - μ (# mismatches) - γ (# gap clusters) - τ (# single-letter gaps)

Definitions: Auxiliary Data Structures

Matrices: V, G, E, F

- *V* = the best-score matrix
- G = the match-mismatch matrix
- E =the X-gap matrix
- F =the Y -gap matrix

The Algorithm:

- 1) Initialize the matrices
- 2) Apply the recurrence relations to fill each matrix
- 3) Traceback through *V* (not shown)

```
V(0,0) \leftarrow 0
for i \leftarrow 1 to m do
V(i,0) = E(i,0) \leftarrow -\gamma - i\tau
    end
        |V(0,j) = F(0,j) \leftarrow -\gamma - j\tau
     for i \leftarrow 1 to m, j \leftarrow 1 to n do
             G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}
E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \mu & \text{if } x_{i \neq j} \\ E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases} \\ F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases} \\ V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}
```

An Example:

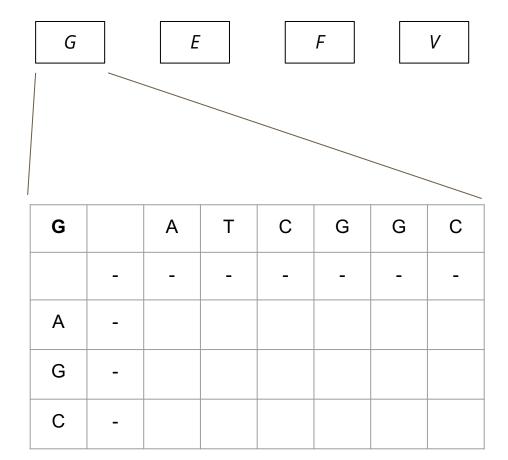
- X = ATCGGC
- Y = AGC
- $\alpha = 2$ (score = +2)
- $\mu = 1$ (penalty = -1)
- $\gamma = 2 \text{ (penalty = -2)}$
- $\tau = 1 \text{ (penalty = -1)}$

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end for $j \leftarrow 1$ to n do
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $i \leftarrow 1$ to m , $j \leftarrow 1$ to m do
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

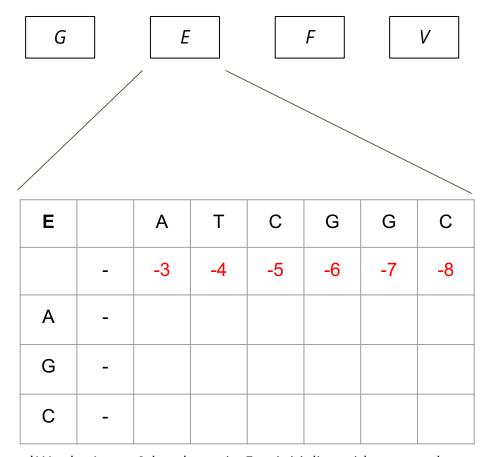
$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$



^{*}We don't use 0th row, 0th column in $G \rightarrow$ initialize with error values

 $V(0,0) \leftarrow 0$ for $i \leftarrow 1$ to m do $V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$ end for $j \leftarrow 1$ to n do $V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$ end for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do $G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$ $E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \mu & \text{if } x_i \neq y_j \\ E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases} \\ F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases} \\ V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases} \quad \boxed{\alpha = 2 \quad \gamma = 2 \\ \mu = 1 \quad \tau = 1}$ end



^{*}We don't use 0th column in $E \rightarrow$ initialize with error values

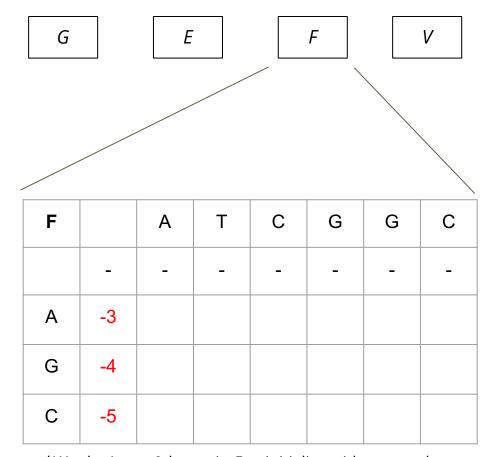
$$V(0,0) \leftarrow 0$$

$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$
end
$$| C(i,j) \leftarrow \sum_{i=1}^{n} (i,j) = \sum_{i=1}$$



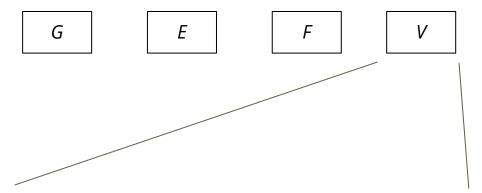
^{*}We don't use 0th row in $F \rightarrow$ initialize with error values

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) - \gamma - \tau | C(i,j) - \gamma | C(i,j) - \gamma - \tau | C(i,j) - \gamma | C(i,j) - \gamma | C(i,j) - \gamma | C(i,j) - \gamma | C(i$$



| V | | Α | Т | С | G | G | С |
|---|----|----|----|----|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | | | | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

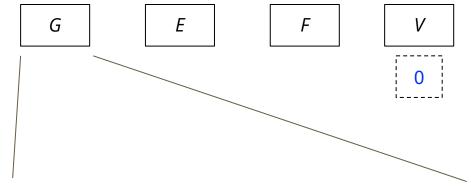
$$\begin{array}{l} V(0,0) \leftarrow 0 \\ \textbf{for } i \leftarrow 1 \textbf{ to } m \textbf{ do} \\ \mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ \textbf{end} \\ \textbf{for } j \leftarrow 1 \textbf{ to } n \textbf{ do} \\ \mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ \textbf{end} \end{array}$$

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases} \qquad \alpha = 2 \quad \gamma = 2 \\ \mu = 1 \quad \tau = 1 \end{cases}$$
end



| G | | Α | Т | С | G | G | С |
|---|---|---|---|---|---|---|---|
| | - | - | - | - | - | - | - |
| Α | - | | | | | | |
| G | - | | | | | | |
| С | - | | | | | | |

$$\begin{array}{l} V(0,0) \leftarrow 0 \\ \textbf{for } i \leftarrow 1 \ \textbf{to} \ m \ \textbf{do} \\ \mid \ V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ \textbf{end} \\ \textbf{for } j \leftarrow 1 \ \textbf{to} \ n \ \textbf{do} \\ \mid \ V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ \textbf{end} \\ \textbf{for } i \leftarrow 1 \ \textbf{to} \ m, \ j \leftarrow 1 \ \textbf{to} \ n \ \textbf{do} \end{array}$$

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

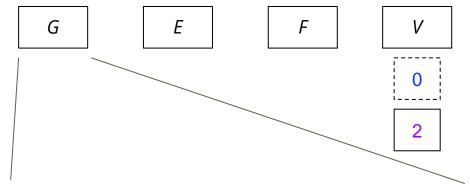
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2$$

$$\mu = 1 \quad \tau = 1$$
end



| G | | Α | Т | С | G | G | С |
|---|---|---|---|---|---|---|---|
| | - | - | - | - | - | - | - |
| Α | - | 2 | | | | | |
| G | - | | | | | | |
| С | - | | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end
for $i \leftarrow 1$ to m , $i \leftarrow 1$ to n do

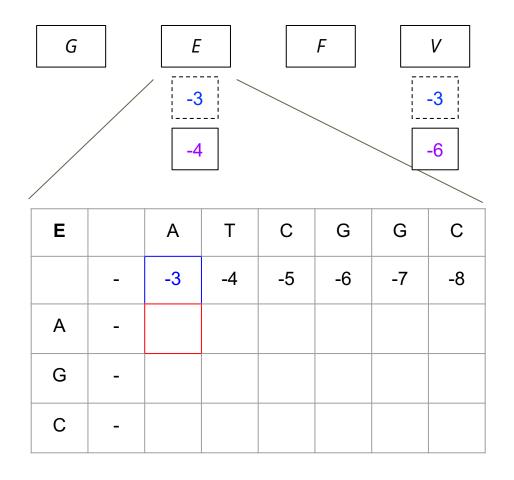
$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases} \qquad \boxed{\alpha = 2 \quad \gamma = 2}$$

$$\mu = 1 \quad \tau = 1$$

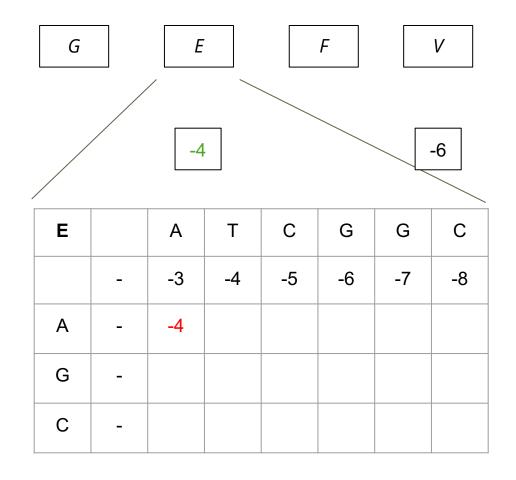


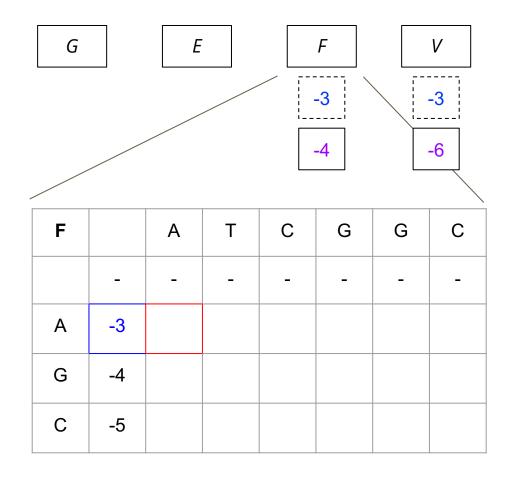
$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $j \leftarrow 1$ to m do
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) - \gamma - \tau | C(i,j) - \gamma | C(i,j) - \gamma | C(i,j) - \gamma | C(i,j) -$$





$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $j \leftarrow 1$ to m do
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

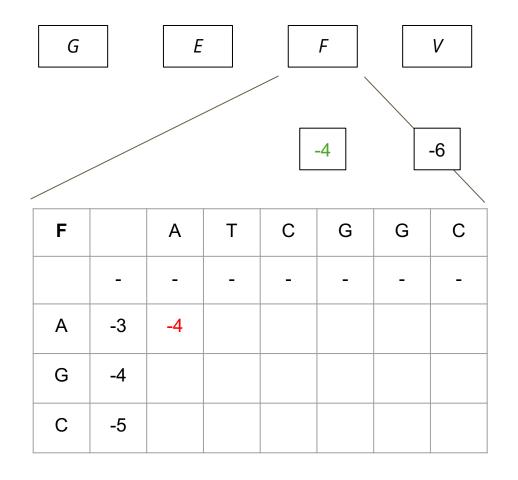
$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} F(i,j) - \tau \\ F(i,j) \leftarrow \tau \end{cases}$$

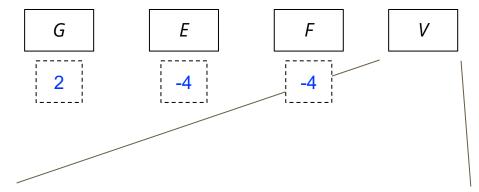
$$| C(i,j) \leftarrow \max \begin{cases} F(i,j) - \tau \\ F(i,j) \leftarrow \tau \end{cases}$$
 end
$$| C(i,j) \leftarrow \tau \rangle$$

$$| C(i,j) \leftarrow \tau \rangle$$

$$| C(i,j) \leftarrow \tau \rangle$$
 end
$$| C(i,j) \leftarrow \tau \rangle$$

$$| C(i,j)$$





| V | | Α | Т | С | G | G | С |
|---|----|----|----|----|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| А | -3 | | | | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

$$V(0,0) \leftarrow 0$$

$$\text{for } i \leftarrow 1 \text{ to } m \text{ do}$$

$$\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$

$$\text{end}$$

$$\text{for } j \leftarrow 1 \text{ to } n \text{ do}$$

$$\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$

$$\text{end}$$

$$\text{for } i \leftarrow 1 \text{ to } m, j \leftarrow 1 \text{ to } n \text{ do}$$

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

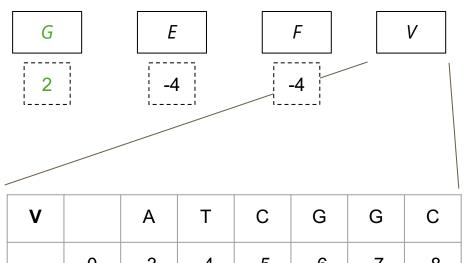
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$

$$C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$



| V | | Α | Т | С | G | G | С |
|---|----|----------------|----|----|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | | | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

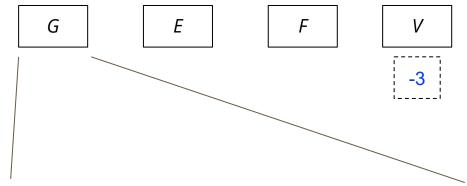
*Keep track of which matrix gave you the max score!

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \end{cases}$$
 end
$$| C(i,j) \leftarrow \max$$
 end
$$| C(i,j) \leftarrow$$



| G | | Α | Т | С | G | G | С |
|---|---|---|---|---|---|---|---|
| | - | - | - | - | - | - | - |
| Α | - | 2 | | | | | |
| G | - | | | | | | |
| С | - | | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

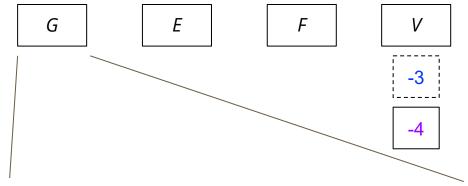
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2$$

$$\mu = 1 \quad \tau = 1$$
end



| G | | Α | Т | С | G | G | С |
|---|---|---|----|---|---|---|---|
| | - | - | - | - | - | - | - |
| А | - | 2 | -4 | | | | |
| G | - | | | | | | |
| С | - | | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end
for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do

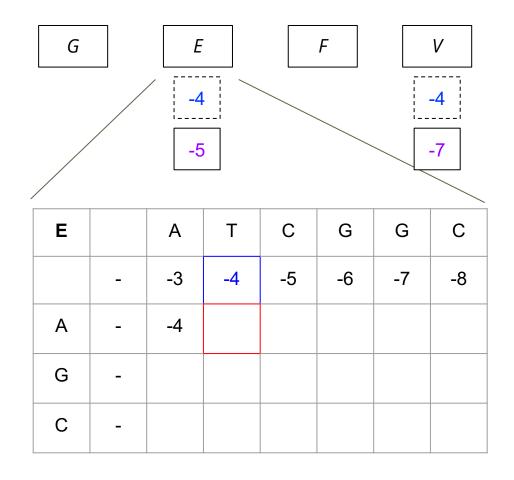
For
$$i \leftarrow 1$$
 to $m, j \leftarrow 1$ to n do
$$\begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

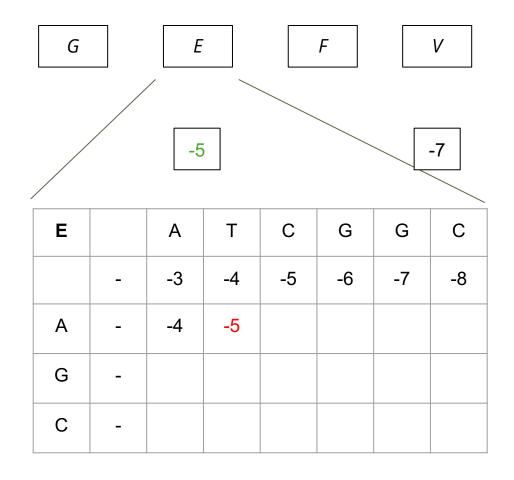
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

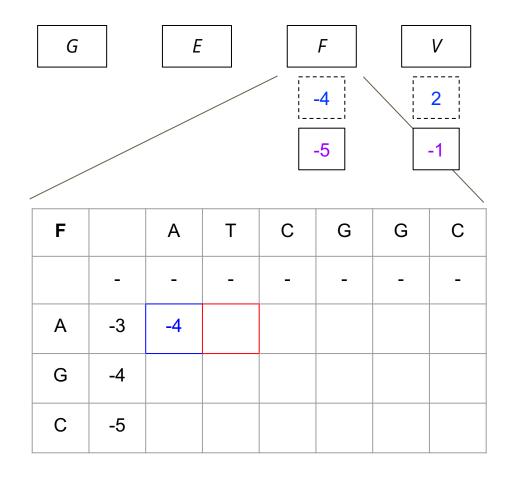
$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2 \quad \gamma = 1 \quad \gamma =$$







$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $j \leftarrow 1$ to m do
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$| F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} F(i,j) - \tau \\ F(i,j) \leftarrow \tau \end{cases}$$

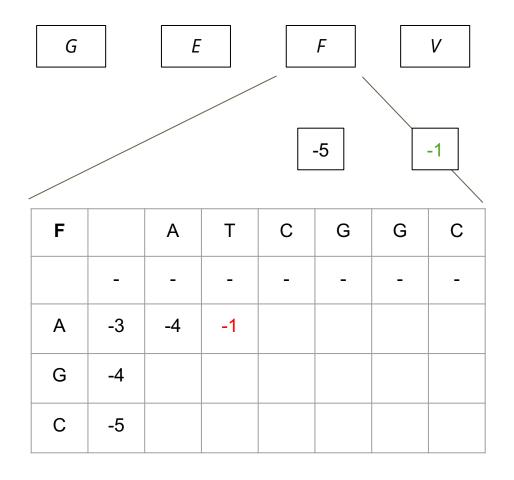
$$| C(i,j) \leftarrow \max \begin{cases} F(i,j) - \tau \\ F(i,j) \leftarrow \tau \end{cases}$$
 end
$$| C(i,j) \leftarrow \tau \rangle$$

$$| C(i,j) \leftarrow \tau \rangle$$

$$| C(i,j) \leftarrow \tau \rangle$$
 end
$$| C(i,j) \leftarrow \tau \rangle$$

$$| C(i,j) \leftarrow \tau \rangle$$
 end
$$| C(i,j) \leftarrow \tau \rangle$$

$$|$$

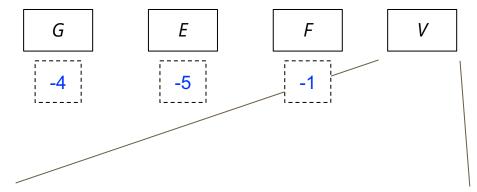


$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $j \leftarrow 1$ to m do
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

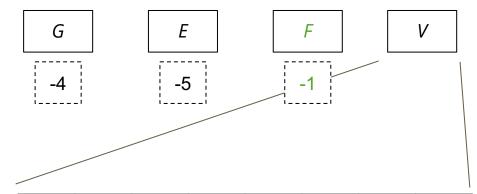
$$| F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ F(i,j) \leftarrow T \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} F(i,j) - \tau \\ F(i,j) \leftarrow T \end{cases}$$

$$| C(i,j) \leftarrow \max \begin{cases} F(i,j) - \tau \\ F(i,j) \leftarrow T \end{cases}$$
 end
$$| C(i,j) \leftarrow T \rangle$$
 end
$$| C(i,$$



| V | | Α | Т | С | G | G | С |
|---|----|----------------|----|----|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | | | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

$$\begin{split} V(0,0) &\leftarrow 0 \\ \text{for } i \leftarrow 1 \text{ to } m \text{ do} \\ &\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ \text{end} \\ \text{for } j \leftarrow 1 \text{ to } n \text{ do} \\ &\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ \text{end} \\ \text{for } i \leftarrow 1 \text{ to } m, j \leftarrow 1 \text{ to } n \text{ do} \\ &\mid G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases} \\ E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases} \\ F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases} \\ V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases} \\ \text{end} \end{cases}$$



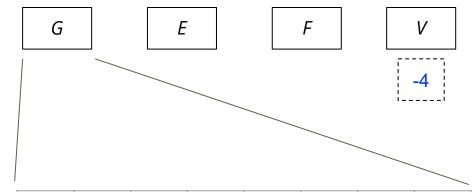
| V | | Α | Т | С | G | G | С |
|---|----|----------------|-----------------|----|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| А | -3 | 2 _G | -1 _F | | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ G(i,j) \\ G(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ G(i,j) \\ G(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ G(i,j) \\ G(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ G(i,j) \\ G(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ G(i,j) \\ G(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ G(i,j) \\ G(i,j) \end{cases}$$



| G | | Α | Т | С | G | G | С |
|---|---|---|----|---|---|---|---|
| | - | - | - | - | - | - | - |
| Α | - | 2 | -4 | | | | |
| G | - | | | | | | |
| С | - | | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

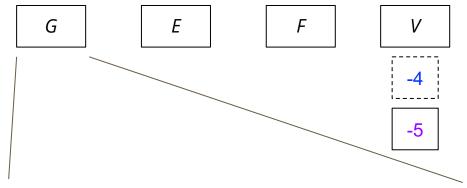
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2$$

$$\mu = 1 \quad \tau = 1$$
end



| G | | Α | Т | С | G | G | С |
|---|---|---|----|----|---|---|---|
| | - | - | - | - | - | - | - |
| А | - | 2 | -4 | -5 | | | |
| G | - | | | | | | |
| С | - | | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end
for $i \leftarrow 1$ to m , $i \leftarrow 1$ to n do

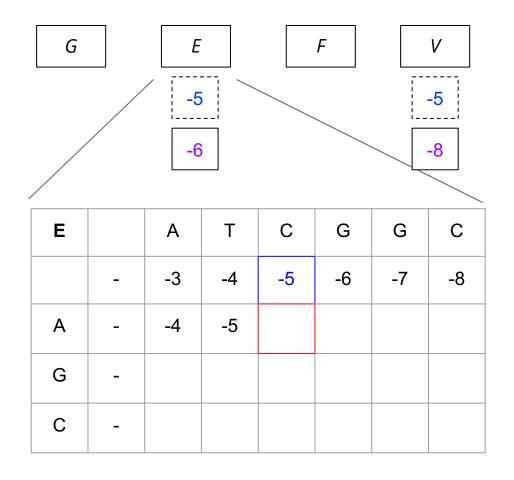
For
$$i \leftarrow 1$$
 to $m, j \leftarrow 1$ to n do
$$\begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2 \quad \gamma = 1 \quad \gamma =$$

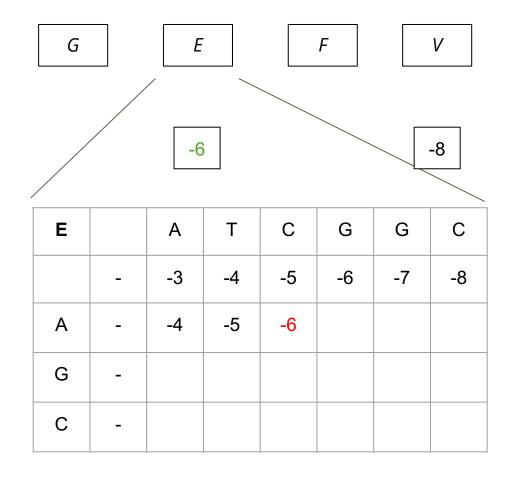


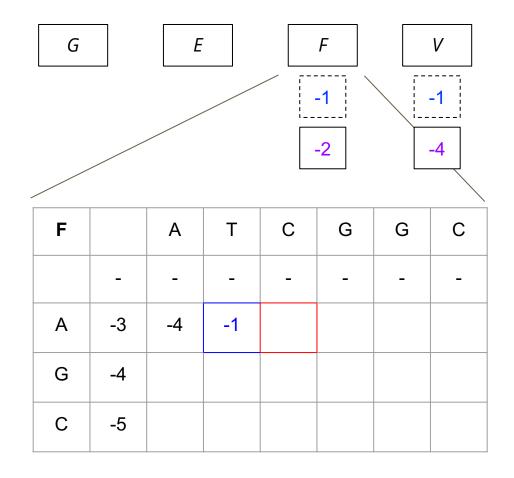
$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

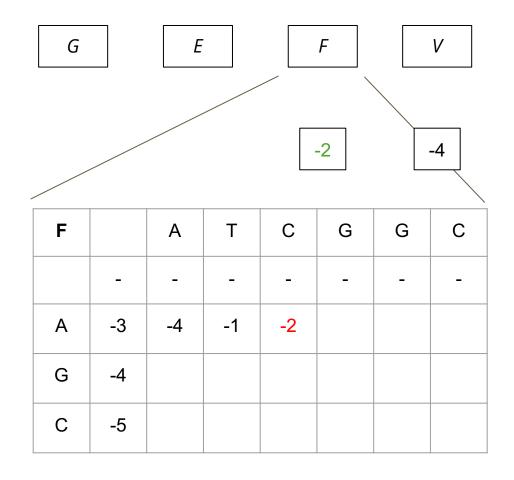
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

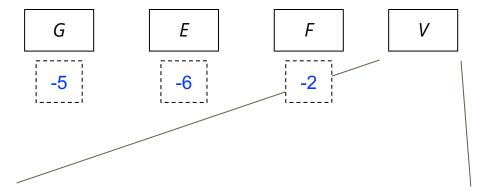
$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$
 end
$$e = 2 \quad \gamma = 2$$
 end
$$\gamma = 1 \quad \gamma = 1$$
 end
$$\gamma = 1 \quad \gamma = 1$$
 end
$$\gamma = 1 \quad \gamma = 1$$
 end
$$\gamma = 1 \quad \gamma = 1$$









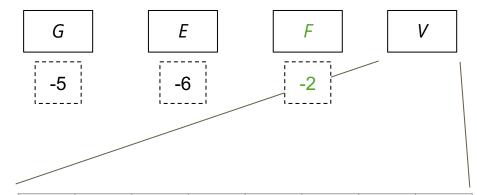
| V | | Α | Т | С | G | G | С |
|---|----|----------------|-----------------|----|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| А | -3 | 2 _G | -1 _F | | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \sum_{j=1}^{n} (i,j) \leftarrow \sum_{j=1}^{n$$



| V | | Α | Т | С | G | G | С |
|---|----|----------------|-----------------|-----------------|----|----|----|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| А | -3 | 2 _G | -1 _F | -2 _F | | | |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

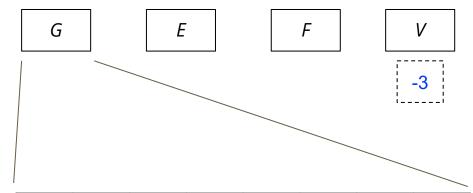
$$\begin{split} V(0,0) &\leftarrow 0 \\ &\text{for } i \leftarrow 1 \text{ to } m \text{ do} \\ &\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ &\text{end} \\ &\text{for } j \leftarrow 1 \text{ to } n \text{ do} \\ &\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ &\text{end} \\ &\text{for } i \leftarrow 1 \text{ to } m, \ j \leftarrow 1 \text{ to } n \text{ do} \\ &\mid G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases} \\ E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases} \\ F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases} \\ V(i,j) \leftarrow \begin{cases} G(i,j) \\ E(i,j) \end{cases} \\ \text{end} \end{cases} \end{split}$$

And so on...

The algorithm continues like so until all the matrices are filled.

We'll skip ahead, stopping at some interesting intermediate states which cover new branches in the algorithm.

As an exercise, try filling out these matrices on your own and checking the values against our final solution!



| G | | Α | Т | С | G | G | С |
|---|---|---|----|----|----|----|----|
| | - | - | - | - | - | - | - |
| Α | - | 2 | -4 | -5 | -6 | -7 | -8 |
| G | - | | | | | | |
| С | - | | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

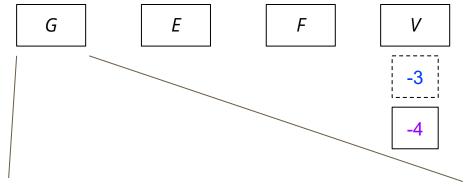
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2$$

$$\mu = 1 \quad \tau = 1$$
end



| G | | А | Т | С | G | G | С |
|---|---|----|----|----|----|----|----|
| | - | - | - | - | - | - | - |
| Α | - | 2 | -4 | -5 | -6 | -7 | -8 |
| G | - | -4 | | | | | |
| С | - | | | | | | |

$$\begin{array}{l} V(0,0) \leftarrow 0 \\ \textbf{for } i \leftarrow 1 \textbf{ to } m \textbf{ do} \\ \mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ \textbf{end} \\ \textbf{for } j \leftarrow 1 \textbf{ to } n \textbf{ do} \\ \mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ \textbf{end} \\ \textbf{for } i \leftarrow 1 \textbf{ to } m, j \leftarrow 1 \textbf{ to } n \textbf{ do} \end{array}$$

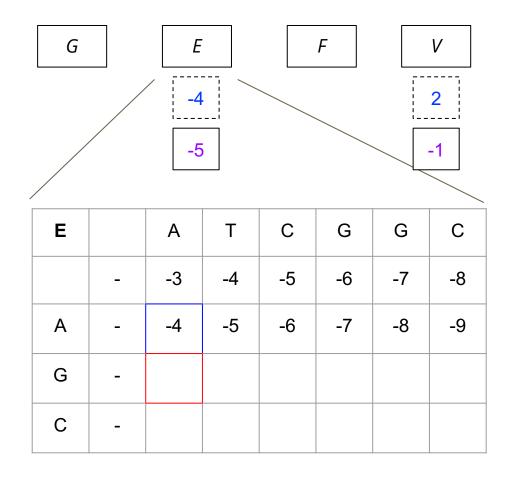
For
$$i \leftarrow 1$$
 to $m, j \leftarrow 1$ to n do
$$\begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

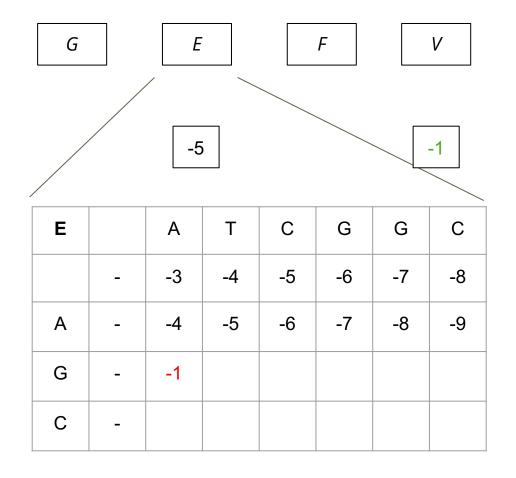
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

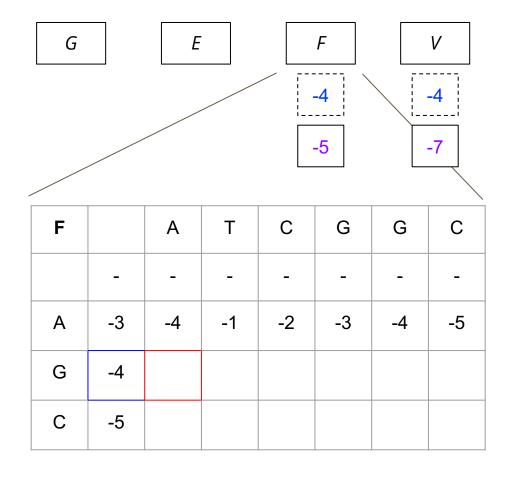
$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

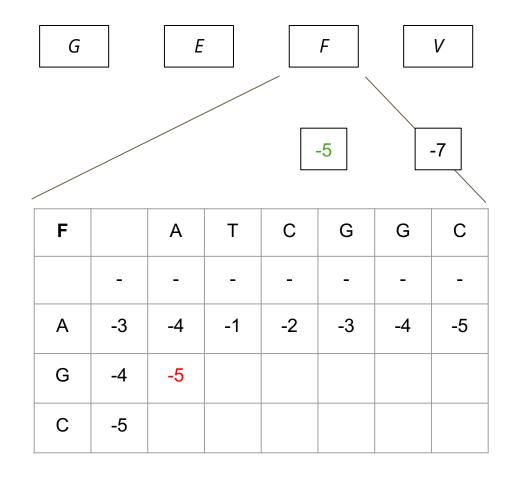
$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

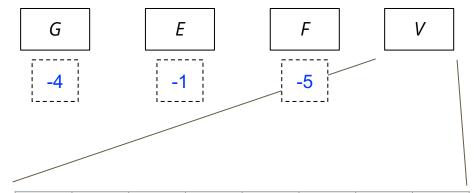
$$\alpha = 2 \quad \gamma = 2 \quad \gamma = 1 \quad \gamma =$$











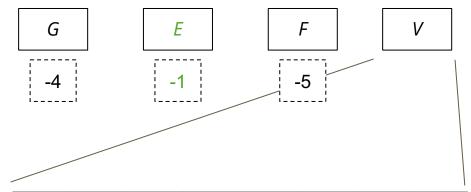
| V | | Α | Т | С | G | G | С |
|---|----|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | | | | | | |
| С | -5 | | | | | | |

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end for $i \leftarrow 1$ to m , $j \leftarrow 1$ to m do
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \sum_{j=1}^{m} | C(i,$$



| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| А | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | | | | | |
| С | -5 | | | | | | |

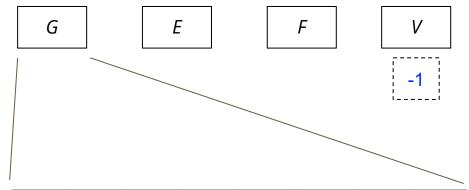
$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 and
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases}$$
 and
$$| C(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \leftarrow \max \end{cases}$$
 and
$$| C(i,j) \leftarrow \max \end{cases}$$
 end
$$| C(i,j) \leftarrow \max \end{cases}$$





| G | | Α | Т | С | G | G | С |
|---|---|----|----|----|----|----|----|
| | - | - | - | - | - | - | - |
| Α | - | 2 | -4 | -5 | -6 | -7 | -8 |
| G | - | -4 | 1 | -2 | 0 | -1 | -5 |
| С | - | -5 | | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end
for $i \leftarrow 1$ to m is a 1-to m do

for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do

$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

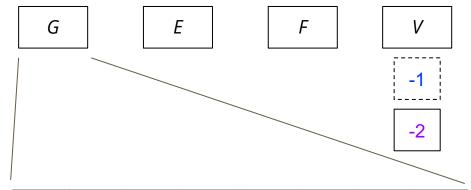
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2$$

$$\mu = 1 \quad \tau = 1$$
end



| G | | Α | Т | С | G | G | С |
|---|---|----|----|----|----|----|----|
| | - | - | - | - | - | - | - |
| Α | - | 2 | -4 | -5 | -6 | -7 | -8 |
| G | - | -4 | 1 | -2 | 0 | -1 | -5 |
| С | - | -5 | -2 | | | | |

$$V(0,0) \leftarrow 0$$

for $i \leftarrow 1$ to m do
 $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$
end
for $j \leftarrow 1$ to n do
 $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$
end
for $i \leftarrow 1$ to m , $i \leftarrow 1$ to n do

for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do

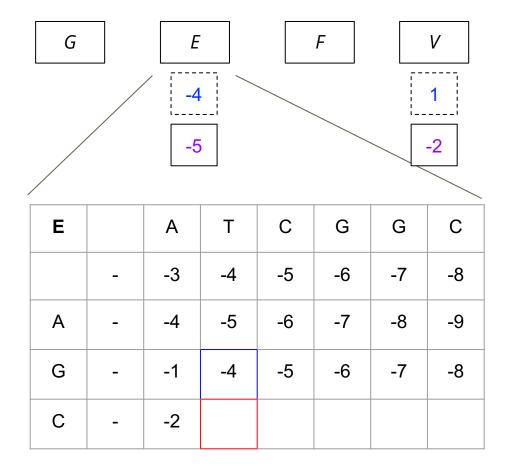
For
$$i \leftarrow 1$$
 to $m, j \leftarrow 1$ to n do
$$\begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

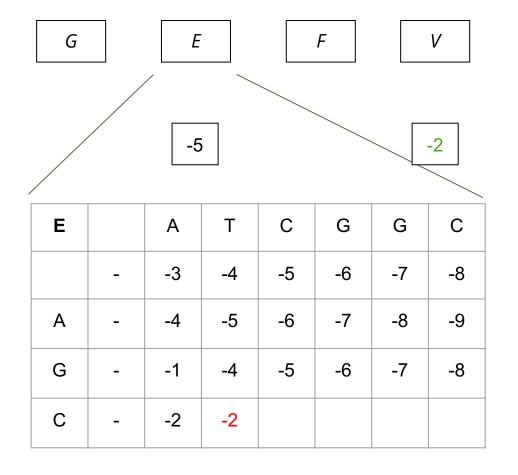
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

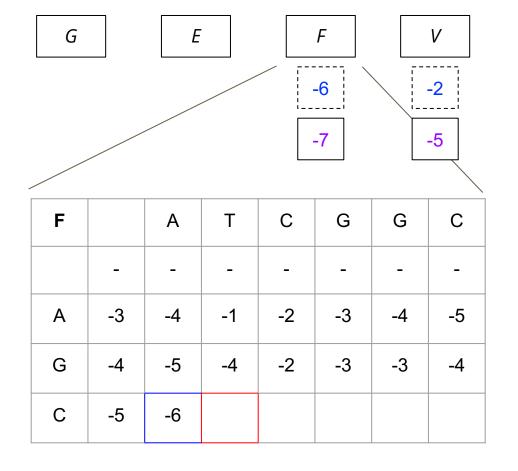
$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

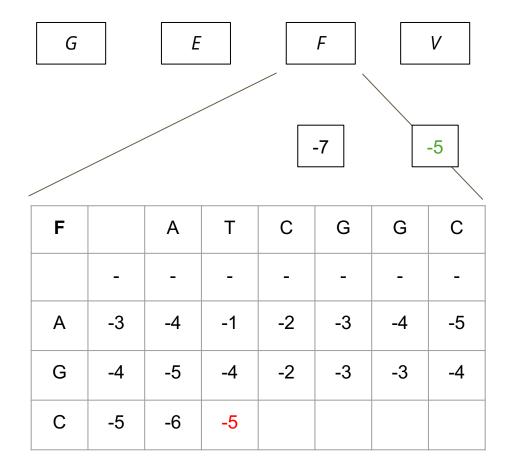
$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$

$$\alpha = 2 \quad \gamma = 2 \quad \gamma = 1 \quad \gamma =$$







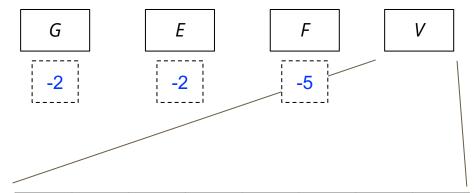


$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$| F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$| V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \sum_{j=1}^{n} (i,j) + \sum_{j=1$$



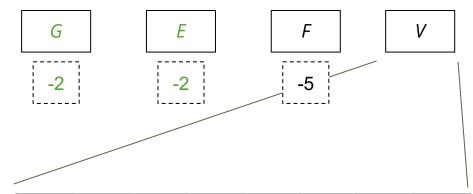
| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-----------------|-------------------|-----------------|-----------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | | | | | |

$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do
$$| V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$$
 end
$$| G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

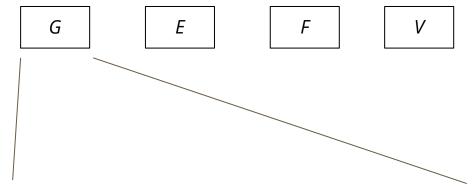
$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \\ F(i,j) \end{cases}$$
 end
$$| C(i,j) \leftarrow \sum_{j=1}^{n} (i,j) \leftarrow \sum_{j=1}^{n$$



| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-----------------|-----------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | | | | |

$$\begin{split} V(0,0) &\leftarrow 0 \\ \text{for } i \leftarrow 1 \text{ to } m \text{ do} \\ &\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ \text{end} \\ \text{for } j \leftarrow 1 \text{ to } n \text{ do} \\ &\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ \text{end} \\ \text{for } i \leftarrow 1 \text{ to } m, j \leftarrow 1 \text{ to } n \text{ do} \\ &\mid G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases} \\ E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases} \\ F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases} \\ V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases} \\ \text{end} \end{cases}$$





| G | | Α | Т | С | G | G | С |
|---|---|----|----|----|----|----|----|
| | - | - | - | - | - | - | - |
| Α | - | 2 | -4 | -5 | -6 | -7 | -8 |
| G | - | -4 | 1 | -2 | 0 | -1 | -5 |
| С | - | -5 | -2 | 3 | -3 | -1 | 1 |

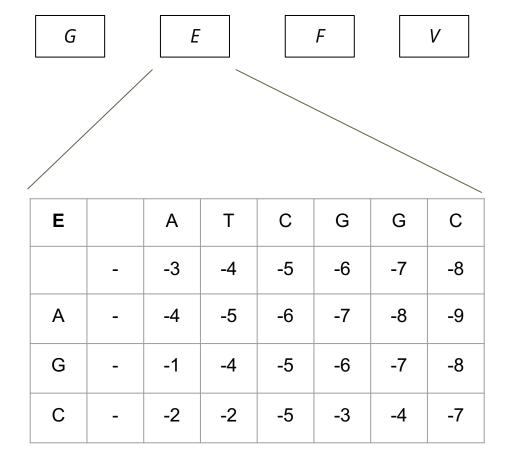
$$V(0,0) \leftarrow 0$$
 for $i \leftarrow 1$ to m do $\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau$ end for $j \leftarrow 1$ to n do $\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau$ end for $i \leftarrow 1$ to $m, j \leftarrow 1$ to n do $\mid G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$

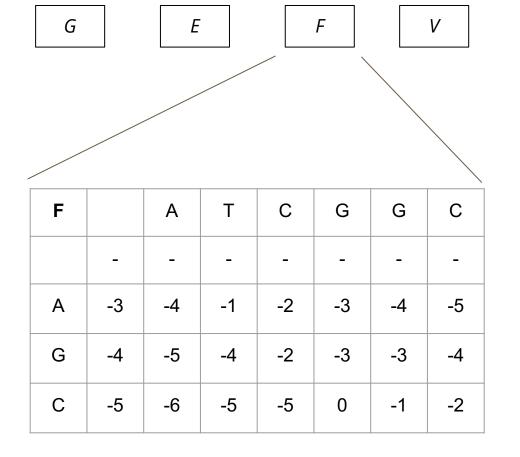
$$G(i,j) \leftarrow \begin{cases} V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \\ V(i,j-1) - \mu & \text{if } x_i \neq y_j \end{cases}$$

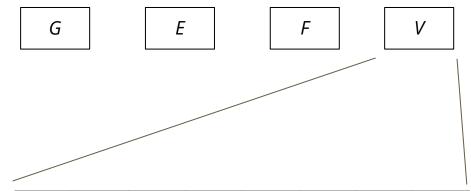
$$E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases}$$

$$F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases}$$

$$V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ E(i,j) \\ F(i,j) \end{cases} \qquad \alpha = 2 \quad \gamma = 2 \\ \mu = 1 \quad \tau = 1 \end{cases}$$
end







| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2_{G} | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

$$\begin{split} V(0,0) &\leftarrow 0 \\ &\textbf{for } i \leftarrow 1 \textbf{ to } m \textbf{ do} \\ &\mid V(i,0) = E(i,0) \leftarrow -\gamma - i\tau \\ &\textbf{end} \\ &\textbf{for } j \leftarrow 1 \textbf{ to } n \textbf{ do} \\ &\mid V(0,j) = F(0,j) \leftarrow -\gamma - j\tau \\ &\textbf{end} \\ &\textbf{for } i \leftarrow 1 \textbf{ to } m, j \leftarrow 1 \textbf{ to } n \textbf{ do} \\ &\mid G(i,j) \leftarrow \begin{cases} V(i-1,j-1) + \alpha & \text{if } x_i = y_j \\ V(i-1,j-1) - \mu & \text{if } x_i \neq y_j \end{cases} \\ E(i,j) \leftarrow \max \begin{cases} E(i,j-1) - \tau \\ V(i,j-1) - \gamma - \tau \end{cases} \\ F(i,j) \leftarrow \max \begin{cases} F(i-1,j) - \tau \\ V(i-1,j) - \gamma - \tau \end{cases} \\ V(i,j) \leftarrow \max \begin{cases} G(i,j) \\ F(i,j) \end{cases} \quad \alpha = 2 \quad \gamma = 2 \\ \mu = 1 \quad \tau = 1 \end{split}$$

Traceback

We use the backpointers in our *V* matrix to reconstruct our alignment.

At each position, we can recover the single-letter alignment of the prior two characters based on which matrix produced our maximum score.

Starting from V(m,n), at every V(i,j):

- If argmax = $G \rightarrow$ recover a match/mismatch; recurse on V(i-1, j-1)
- If argmax = $E \rightarrow$ recover a gap in X; recurse on V(i, j-1)
- If argmax = $F \rightarrow$ recover a gap in Y; recurse on V(i-1, j)

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

Score: +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3_{G} | 0 _F | -1 _{G/F} | 1 _G |

Score: +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|----------------|-------------------|---------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | | | | | | -4 _F | |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | | | | | | -1 _{G/F} | |

Score: +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-----------------|-------------------|-----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | | | | | | -1 _{G/F} | |

C Score:

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

G C Score: G C +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

G C Score: G C +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|-------------------|-------------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F | - 3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

G G C Score:
- G C +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-----------------|--------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F | -2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | | | | | | -1 _{G/F} | |

G G C Score: - G C +1

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|-------------------|----------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F 💠 | - -2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

| V | | Α | Т | С | G | G | С |
|---|----|-----------------|--------------------------|---------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G | -1 _F ◆ | - 2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

C G G C Score:
- G C +1

| V | | А | Т | С | G | G | С |
|---|----|------------------|---------------------------------|---------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G 🔙 | <mark></mark> -1 _F ◆ | - 2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

| V | | Α | Т | С | G | G | С |
|---|----|------------------|-------------------|-------------------|----------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | | 2 _G ◀ | | | | | |
| G | -4 | -1 _∈ | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |



| V | | Α | Т | С | G | G | С |
|---|----|------------------|---------------------------|----------------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G • | - 1 _F ◆ | - 2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

A T C G G C Score:
A - - - G C +1

| V | | Α | Т | С | G | G | С |
|---|----|--------------------------------|---------------------------|----------------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G ♦ | - 1 _F ◆ | - 2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |



| V | | Α | Т | С | G | G | С |
|---|----|--------------------------------|---------------------------|----------------------------------|-------------------|-------------------|-----------------|
| | 0 | -3 | -4 | -5 | -6 | -7 | -8 |
| Α | -3 | 2 _G ♦ | - 1 _F ◆ | - 2 _F ◆ | ■ -3 _F | -4 _F | -5 _F |
| G | -4 | -1 _E | 1 _G | -2 _{G/F} | 0_{G} | -1 _G | -4 _F |
| С | -5 | -2 _E | -2 _{G/E} | 3 _G | 0 _F | -1 _{G/F} | 1 _G |

Results:

A T C G C C A - - G C

...is our optimal alignment with score +1!