7.1. X, Y are subspaces of a vector space V, that is they are closed under vector addition and scalar multiplication. It follows $X \cap Y$ is closed under vector addition and scalar for it only contains vectors in both X and Y.

7.2. Let $\mathrm{v}_{x,1}, \ldots, \mathrm{v}_{x,n}$ be a basis in X and $\mathrm{v}_{y,1}, \ldots, \mathrm{v}_{y,m}$ be a basis in Y. Then, linear combination of the two bases generates $X + Y$ by definition. Any vectors $\mathrm{v}_1, \mathrm{v}_2$ in $X + Y$ are linear combination of the two bases, and closed under vector addition. The same holds for scalar multiplication which merely scales the vector, multiplies all co-efficients by a scalar.

Note: it's not claimed linear combination of two bases will admit unique representation for a vector in $X + Y$.

7.3. Assume $\mathrm{v} + \mathrm{x}$ is in some subpsace V_0 for any x in V_0 and v being not. For $\mathrm{x} = 0$, v is in V_0, and we contradict our assumption.

7.4. Suppose v is a vector in subspace $X \cup Y$. Note X is a subset of $X \cup Y$, for x in X and v not in X, $\mathrm{v} + \mathrm{x}$ is in Y. With Y being a subpsace, it must contain v and its additive inverse, and $\mathrm{v} + \mathrm{x} - \mathrm{v} = \mathrm{x}$ is in Y by closure. Or, X is a subset of Y.

7.5. Define $e_{i,j}$ as matrix with all entries 0 but (i, j)th one being 1. The smallest subpsace for upper triangular matrices of 4×4 would be linear span of vectors $e_{i,j}$ with $0 < i \leq j \leq 4$ as it only contains upper-triangular matrices and zero.