Exercise 8.1. What vector in $\mathbb{R}^3$ has homogeneous coordinates $(10, 20, 30, 5)$?

Answer. We have $[10 : 20 : 30 : 5] = [2 : 4 : 6 : 1]$ which represents the point $(2, 4, 6) \in \mathbb{R}^3$.

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Exercise 8.2. Show that a rotation through γ can be represented as a composition of two shear-and-sclae transformations:

$$T_1 = \begin{pmatrix} 1 & 0 \\ \sin\gamma & \cos\gamma \end{pmatrix} \quad ; \quad \begin{pmatrix} \sec\gamma & -\tan\gamma \\ 0 & 1 \end{pmatrix}.$$

In what order the transformations should be taken?

Answer. Since we want $\cos\gamma$ in the $(1,1)$ position the correct order is as below. Multiplying out we have

$$T_2T_1 = \begin{pmatrix} \sec\gamma & -\tan\gamma \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \sin\gamma & \cos\gamma \end{pmatrix} = \begin{pmatrix} \frac{1-\sin^2\gamma}{\cos\gamma} & -\frac{\sin\gamma\cos\gamma}{\cos\gamma} \\ \\ \sin\gamma & \cos\gamma \end{pmatrix} = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}.$$

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Exercise 8.3. Multiplication of a 2-vector by an arbitrary 2×2 matrix usually requires four multiplications. Suppose a 2×1000 matrix D contains the coordinates of 1000 points in $\mathbb{R}^2$. How many multiplications are required to transform these points using two arbitrary 2×2 matrices A and B. Compare two possibilities: $A(BD)$ and $(AB)D$.

Answer. The question refers to the fact that in carrying out the calculation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by & cx + dy \end{pmatrix}$$

we have performed the four multiplications ax, by, cx, dy (we have also performed two addition operations). The number of such operations are a fundamental measure of algorithmic efficiency, which is why we care.
Now, when we multiply a 2×2 matrix $M_{2\times2}$ by a 2×1000 matrix $M_{2\times1000}$ we would need to perform four multiplications for each column of $M_{2\times1000}$ for a total of 4000 multiplication. Since the result is again an 2×1000 matrix, we would need 8000 multiplications for $A(BD)$.
In contrast, we require 8 multiplications for (AB) and since the result is again a 2×2 matrix, an additional 4000 multiplications for $(AB)D$, for a total of 4008 multiplications.
In summary, while the resulting matrix is the same $A(BD) = (AB)D$, the efficiency of computation is significantly different.

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Exercise 8.4. Write 4×4 matrix performing perspective projection to xy plane with center $(d_1, d_2, d_3)^T$.

Answer. As the texts says, we first translate by the fixed vector $(-d_1, -d_2, d_3)^T$. This is represented by a linear transformation in $\mathbb{R}^4$, as in the text. Then we apply the perspective projection onto the xy plane, also represented by a linear transformation in $\mathbb{R}^4$, as in the text. Finally, we translate by the fixed vector $(d_1, d_2, d_3)^T$.

$$\begin{pmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d_3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -d_1 \\ 0 & 1 & 0 & -d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d_1/d_3 & 0 \\ 0 & 1 & -d_2/d_3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d_3 & 1 \end{pmatrix}.$$

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Exercise 8.5. A transformation T in $\mathbb{R}^3$ is a rotation about the line $y = x + 3$ in the xy-plane through and angle γ. Write a 4×4 matrix corresponding to this transformation.

Answer. We first translate the line so it goes through the origin (this is the reason we need to work in $\mathbb{R}^4$, we want this translation to be a linear transformation). We then rotation the line in the xy-plane so it lies on the y-axis. We perform our rotation by an angle γ, and finally we reverse our transformations.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\gamma & 0 & -\sin\gamma & 0 \\ 0 & 1 & 0 & 0 \\ \sin\gamma & 0 & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$