6.1. We'll prove this with contradiction. Assume the list of vectors $A\mathrm{v}_1, \ldots, A\mathrm{v}_n$ is not a basis in W. If they're linearly independent in W but not generating, then there exists a vector w in W that admits no representation as linear combination of $A\mathrm{v}_1, \ldots, A\mathrm{v}_n$. An inverse map $A^{-1} : W \to V$ can be defined such that $AA^{-1} = I_W$ and $A^{-1}A = I_V$, given A is an isomorphism. Then, $A^{-1}\mathrm{w} = \mathrm{v}$ for some v in V, but v admits a unique representation in $\alpha_1\mathrm{v}_1 + \ldots + \alpha_n\mathrm{v}_n$. It follows, $AA^{-1}\mathrm{w} = \mathrm{w} = A\mathrm{v} = \alpha_1 A\mathrm{v}_1 + \ldots + \alpha_n A\mathrm{v}_n$.

Assume $A\mathrm{v}_1, \ldots, A\mathrm{v}_n$ are not linearly independent, then for some scalars $\alpha_1, \ldots, \alpha_n$ not all zero,

$$\alpha_1 A\mathrm{v}_1 + \ldots + \alpha_n A\mathrm{v}_n = 0_W$$

Multiplying by A^{-1}, we get $\alpha_1\mathrm{v}_1 + \ldots + \alpha_n\mathrm{v}_n = 0_V$ which is only true when all scalars are zero.

6.2. Suppose $\begin{bmatrix} a & b \end{bmatrix}^T$ is the right inverse, then product $a + b$ must be 1. It's apparent no right inverse is left inverse for the product can not be defined.

6.6.

$$AB(AB)^{-1} = A(B(AB)^{-1}) = I$$
$$(AB)^{-1}AB = ((AB)^{-1}A)B = I$$

6.7. If A and AB are invertible, then B can be written as $A^{-1}(AB)$ which is a product two invertible matrices and must be invertible itself.

6.8. Suppose not, let A be invertible. Then, $A^{-1}A = AA^{-1} = I$ implies $A^{-1}A^2A^{-1} = I$, but $A^2 = 0$, and we get a contradiction.

6.9. Suppose $A : \mathbb{F}^n \to \mathbb{F}^m$, then $AB = (Ab_1, \ldots, Ab_k)$ where b_i is the ith column of B for some postive integer k. A column of AB can be written as linear combination of columns of A, $Ab_1 = a_1b_{1,1} + a_2b_{1,2} + \ldots + a_nb_{1,n}$. Given $AB = 0$ for some non-zero matrix B, A is non-invertible for its linear combination of columns admits non-trivial representation of zero, $Ab_1 = \cdots = Ab_k = 0$.

6.10. In case of T_1 notice that $T_1^2 = I$. For inverse of T_2, define $T_2^{-1} : \mathbb{F}^5 \to \mathbb{F}^5$ such that $T_2^{-1}((x_1, x_2, x_3, x_4, x_5)^T) = (x_1, x_2 - ax_4, x_3, x_4, x_5)^T$.

$$T_2T_2^{-1}\mathrm{x} = T_2\left(\begin{bmatrix} x_1 \\ x_2 - ax_4 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

6.13. $(AA^{-1})^T = (A^{-1})^TA = A^{-1}A$, or $(A^{-1})^T = A^{-1}$.