5.1.

a) True

b) False. Vector space of all continuous functions over interval $[0, 1]$.

c) False. If $\mathbf{v}_1, \mathbf{v}_2 \ldots, \mathbf{v}_n$ is some basis then $\mathbf{v}_1 - \mathbf{v}_n, \mathbf{v}_2, \ldots, \mathbf{v}_1 + \mathbf{v}_n$ is also a basis.

d) True. Two bases have same length.

e) False. It's $n + 1$, $1, t, \ldots, t^n$.

f) True

g) False. A spanning set isn't necessarily linearly independent or a basis.

h) True. Theorem 5.5.

i) True

5.2. If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is linearly independent but not generating we can extend it to a basis in V, but that necessitates length of the basis strictly greater than n which is not possible.

If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ span V but are not linearly independent, then it should be possible to delete a vector from the list and not change its span (for the vector deleted would be a linear combination of other vectors in the list). Deleting a vector reduces length by one, while a generating set needs to have length of at least n.

5.3. If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a basis in V, by definition $\dim V = n$. However, if $\dim V = n$ and $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent then it's a basis following the same reasoning as in first half of 5.2.

5.4. Span of $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$ is same as span of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, less than 3.

5.5. Linear span of $\mathbf{u} + \mathbf{v} + \mathbf{w}, \mathbf{v} + \mathbf{w}, \mathbf{w}$ is same as linear span of $\mathbf{u}, \mathbf{v}, \mathbf{w}$, and the length of the list of vectors equals the dimension of vector space spanned by the basis.

5.6. Linear combination of the vectors,

$$\begin{bmatrix} 2\alpha_1 + 3\alpha_2 + \alpha_3 \\ -\alpha_1 - 2\alpha_2 + \alpha_3 \\ \alpha_1 + 50\alpha_3 \\ 5\alpha_1 - 921\alpha_3 \\ -3\alpha_1 \end{bmatrix}$$

α_1 is free in the last row, assume α_3, α_2 to be free in 4th and 2nd row. Then, adding vectors $(1, 0, 0, 0, 0)^T$ and $(0, 0, 1, 0, 0)^T$ completes the basis.