Kinematic Track Fit for Mu2e

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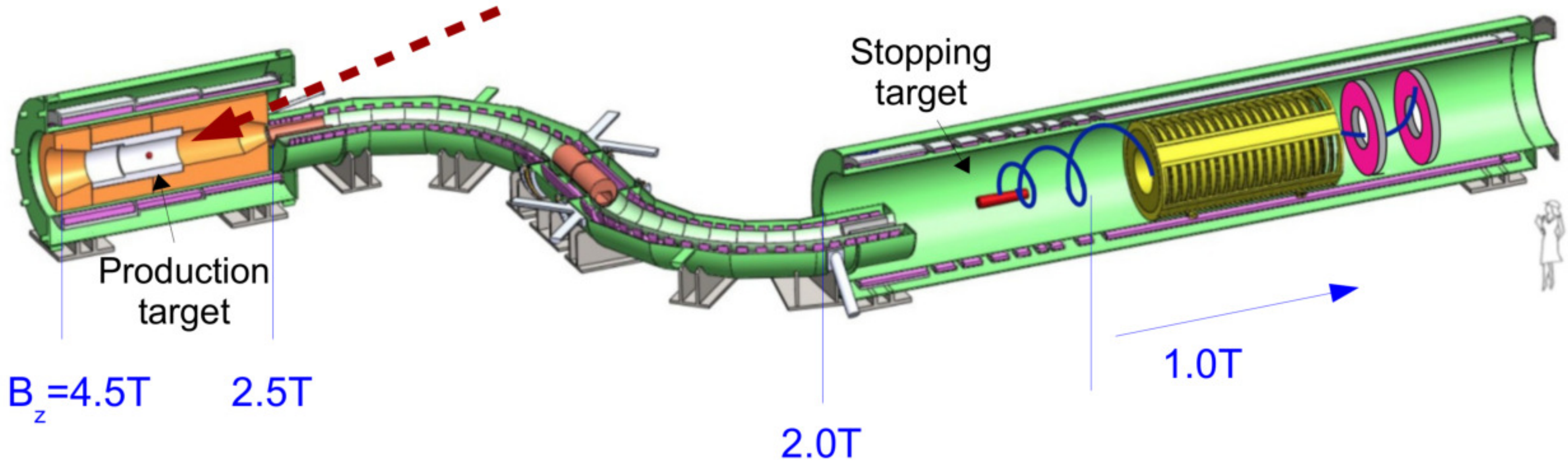
Kinematic Fit: What and Why?

- Kinematic track fit = time domain fit
  - time = parametric variable to describe trajectory
  - time = (primary) measurement dimension
  - t_0 = explicit fit parameter (+ 5 geometric params)
- Advantages
  - Integrates spatial and temporal measurements
  - Integrates t_0
  - Integrates timing-based PID with tracking
  - Directly constrains particle propagation direction
  - Natural relativistic kinematic interface (4-vectors)
  - Requires only D2T, not D2T and T2D, for drift sensors

Kinematic Fit: Who?

- Si Sensors
  - Excellent spatial information (segmentation)
  - Modest timing information, useful for pat. rec. but not fitting
  - Kinematic fit not required or useful
- Temporal sensors (Scintillator bars, Calorimetry, …)
  - Good timing (< 1 nsec)
  - Modest spatial information, useful for pat. rec. but not fitting
  - Kinematic fit can be useful
- Drift Chambers
  - Sensor identity gives rough spatial measurement (few mm→~1 cm)
  - Drift timing can refine spatial information to ~100 μm
  - Coupled time + position measurement ⇒ good application for kinematic fit
- Experiments generally have a mix of sensors
  - ⇒ good application for kinematic fit

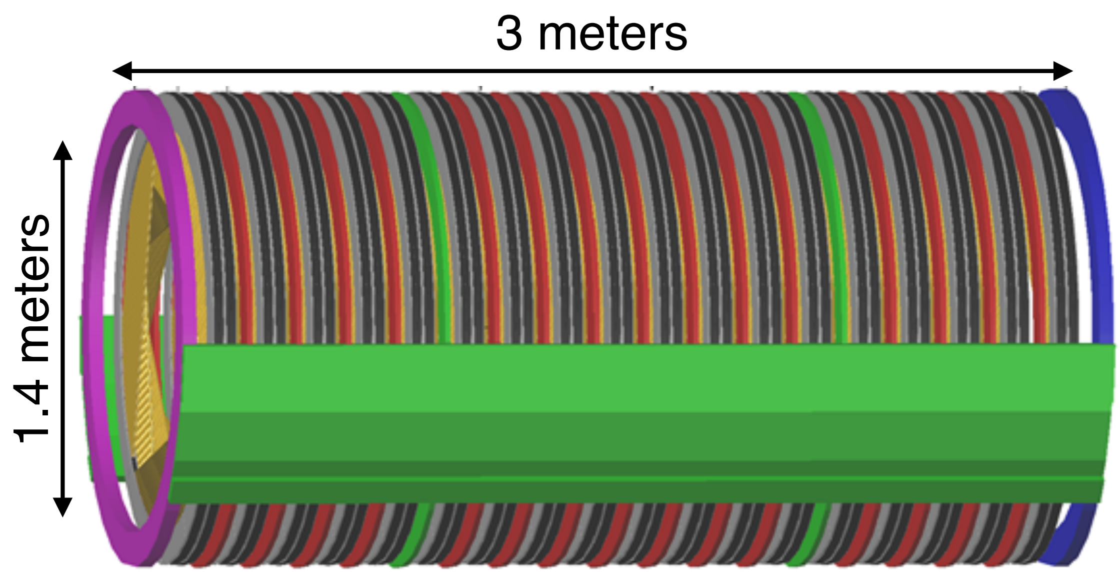
The Mu2e Experiment

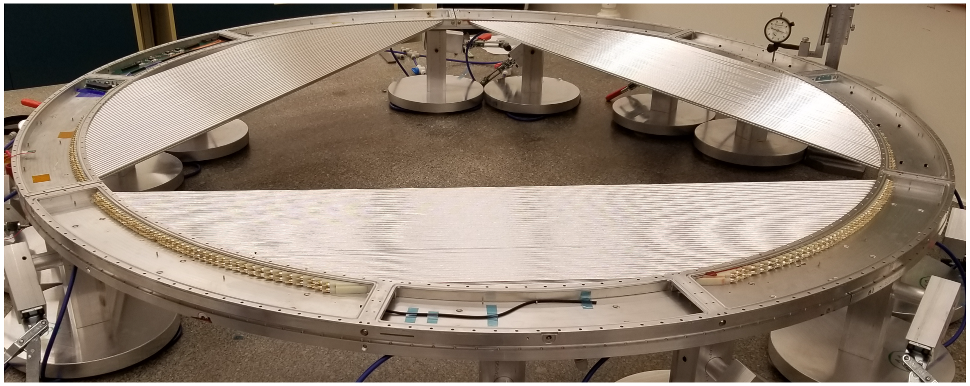


- Mu2e = Search for CLFV
  - $\mu^- \rightarrow e^\pm$ conversion on (Al) nuclei
- Target Sensitivity: $\Gamma_{\mu\rightarrow e}/\Gamma_{\mu\ \mathrm{capture}} \sim 10^{-16}$
- Straw Tracker
- CsI crystal calorimeter

The Mu2e Tracker

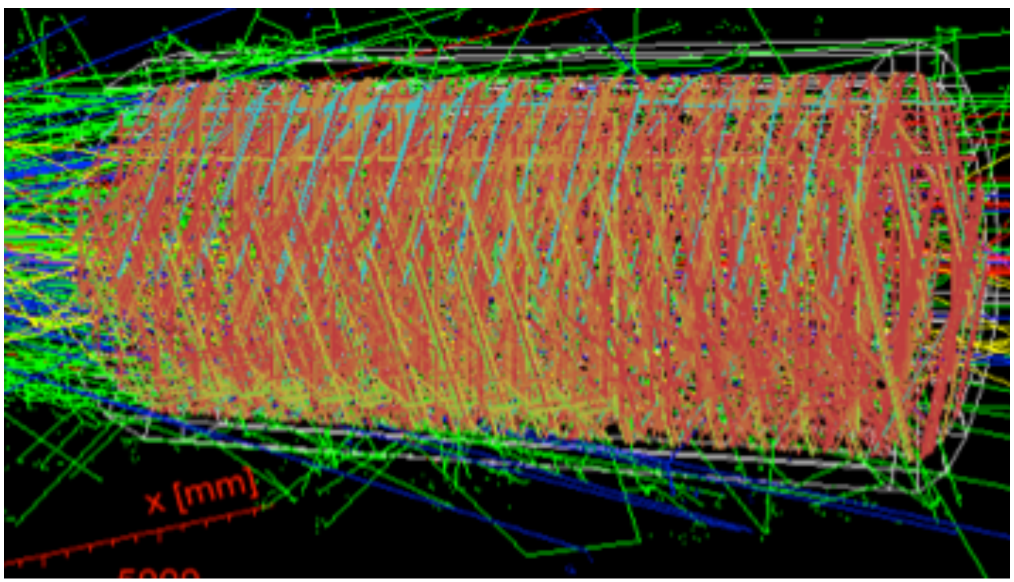
- 72 planes of 'MWPC' straws
- 5 mm diameter straws ⊥ to Z axis, 350 mm < L < 1.4 m
- 120° panels of 96 straws
- Central hole
- Large angle stereo
- ~1% X_0 total mass
- Operates in vacuum
- ~20,000 straws total
- Science goal: < 1% momentum resolution at 105 MeV/c



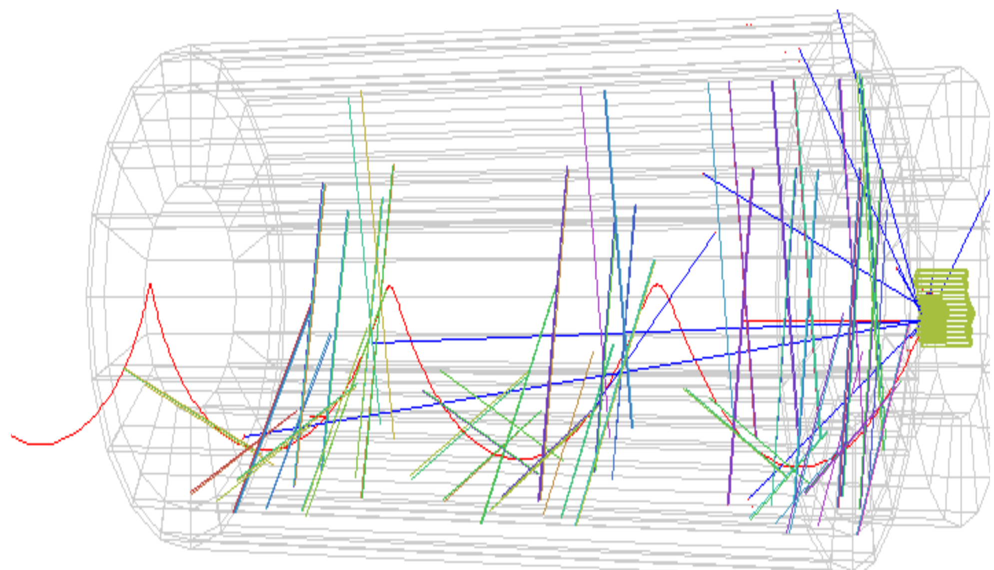


Mu2e Tracking Environment

- Single track signal, 105 MeV/c, 1.5 → 3 turns
- Random time origin (muon decay = signal source)
- Extended spatial origin (~1m x 7.5 cm diameter)
- High hit background rate (~100 KHz/straw)
  - Most hits are from Compton electrons



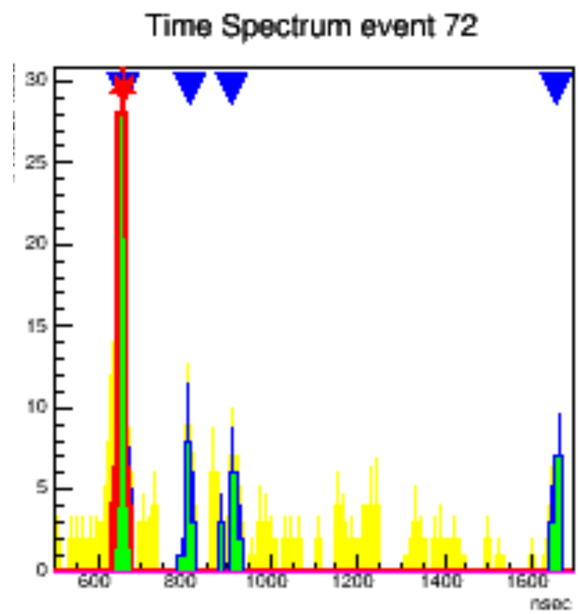
~2K background hits in 1 μsec

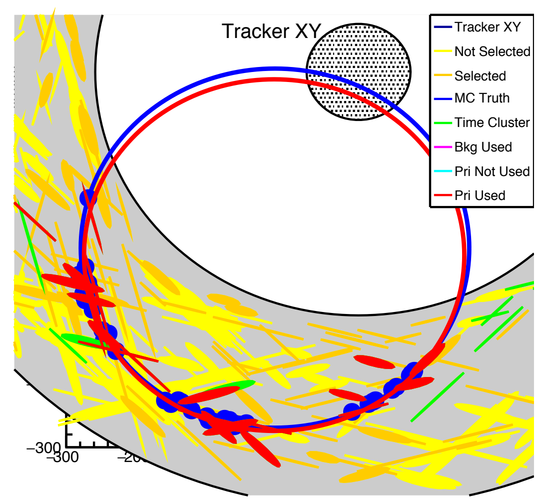


~40 signal hits
multiple turns

Mu2e Pat. Rec.

- ANN hit filter
  - Removes Compton e-, δ-rays, proton hits, …
- Time clustering of hits
  - Hit time resolution ~ 6 n-sec
  - 50 n-sec cluster in 1 μ-sec frame
- Helix fit using space ‘points’
  - Straw constrains 2 dimensions
    - σ ~ 3 mm
  - Hit time difference between straw ends constrains 3rd dimension
    - σ ~ 10-40 mm
- Fake rate < 10^{-4}, efficiency ~ 90%





Mu2e Fit Implementations

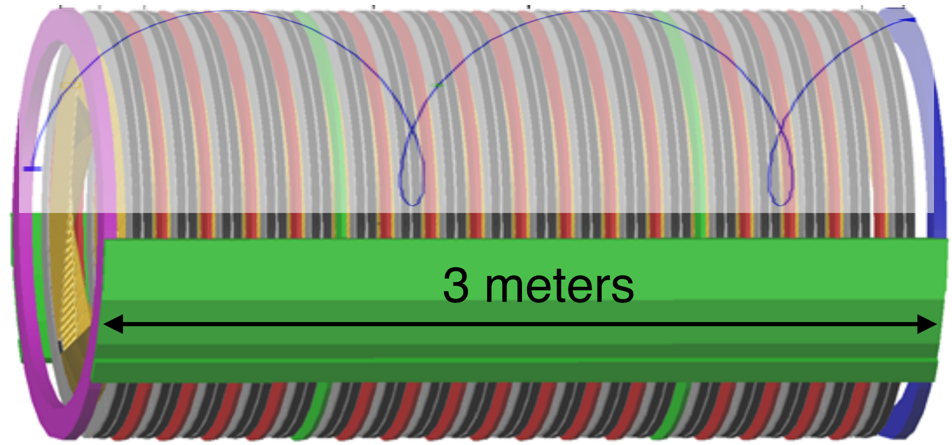
- Framework fit
  - 5-parameter geometric KF fit
  - Code inherited from BaBar (CLHEP, difAlgebra, …)
  - Annealing used for outlier removal
  - t_0, LR hit ambiguity determined through external iteration
- Kinematic fit
  - Being developed as a standalone package
  - Not yet integrated with Framework, Pat. Rec, …
- Rest of talk is about Kinematic fit

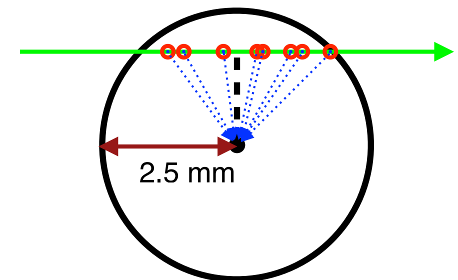
DOCA and TOCA

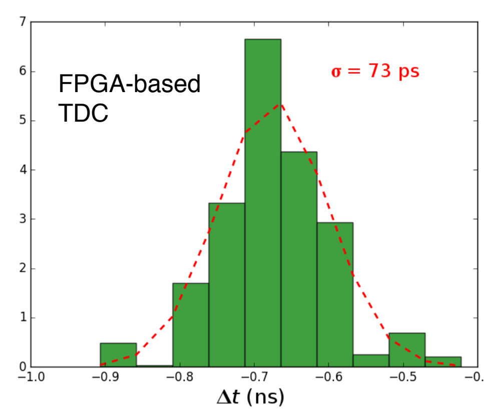
- Geometric tracking
  - Measurement = hit position
  - Fit used to predicted position near to sensor
    - DOCA
  - Residual = measurement - prediction = DOCA
- Kinematic tracking
  - Measurement = hit time
  - Fit used to predict hit time
    - TOCA (multiple contributions, see next slide)
  - Residual = measurement - prediction = t_{hit} - TOCA

Contributions to TOCA Estimate

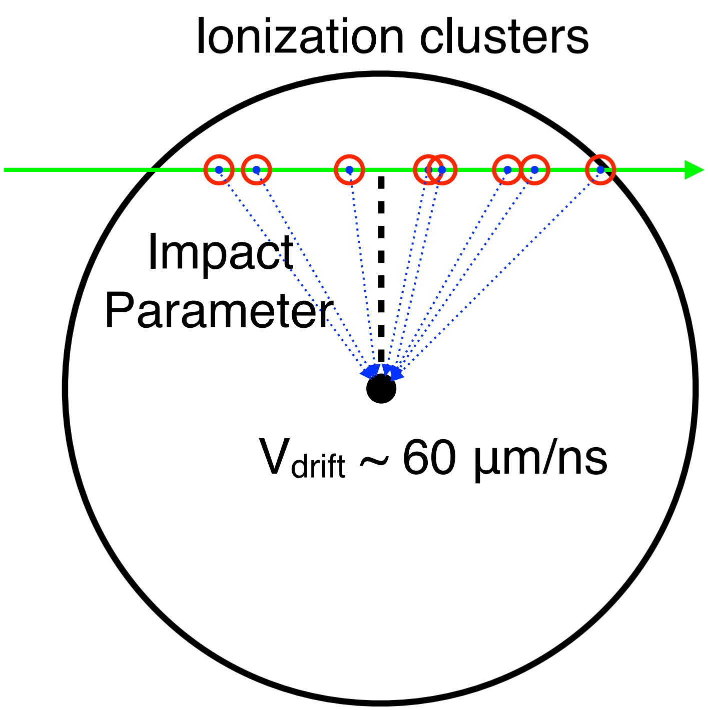
- Particle propagation t_{pprop}
  - $10 < \tau < 20$ ns
  - $\sigma < 100$ ps
- Ionization drift t_{drift}
  - $0 < \tau < 40$ ns
  - $\sigma \sim 3$ ns (with long tails, see next slide)
- Straw signal propagation t_{sprop}
  - 350 mm $< L_{straw} <$ 1400 mm
  - $0.5\,c < V_{eff} < 0.9\,c$ (dispersion + slewing)
  - $\sigma \sim 500$ ps without slewing correction
  - $\sigma \sim 200$ ps with slewing correction
- Electronics signal propagation t_{eprop}
  - TDC $\sigma < 100$ ps
  - Clock distribution $\sigma \sim 150$ ps

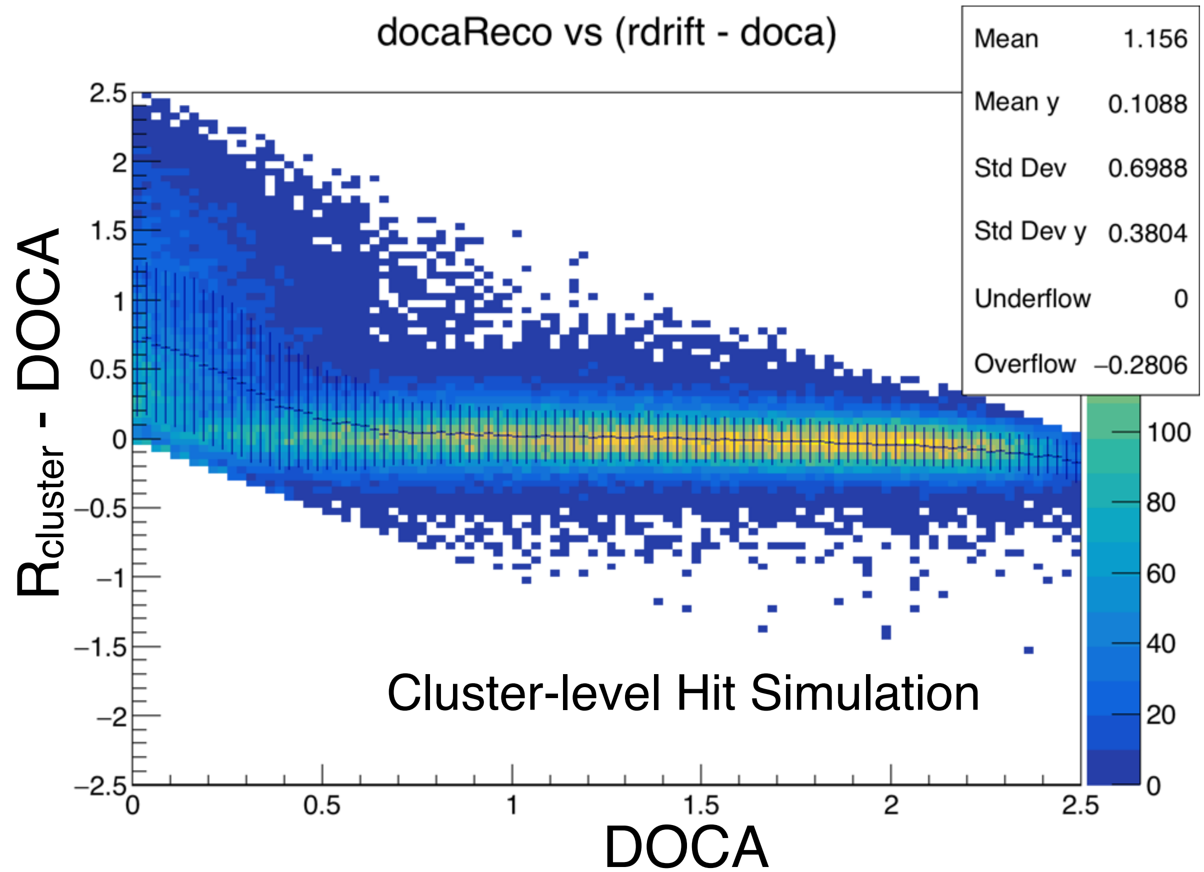






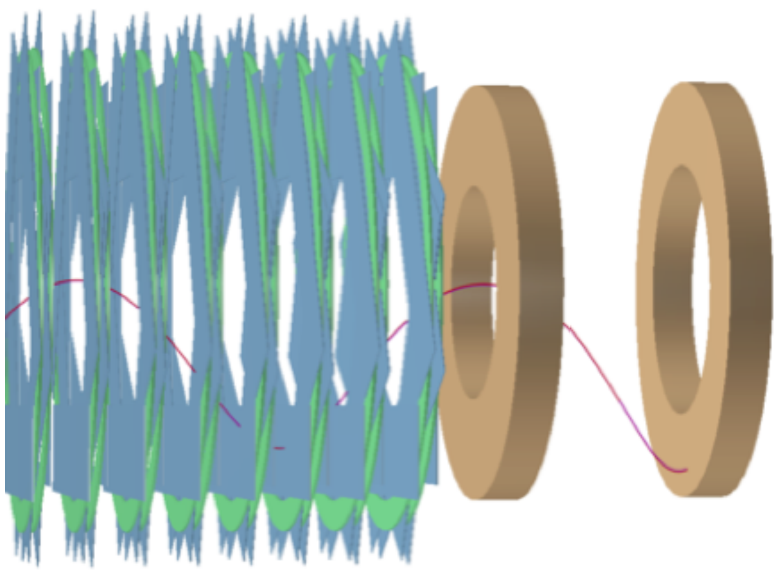
Drift Resolution

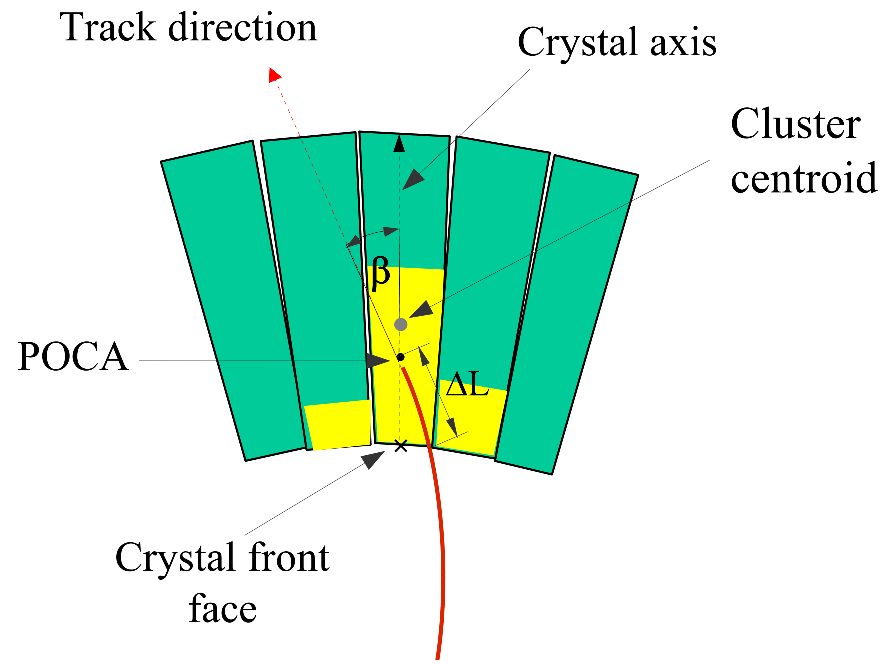




- Far from wire ⇒ hit time directly related to impact parameter
  - Use residual = t_{hit} - TOCA
- Near to wire ⇒ hit time poor approximation to impact parameter
  - Use residual = wire DOCA (no drift information)
- Fit DOCA used to separate regimes (iterative)

Calorimeter in Tracking

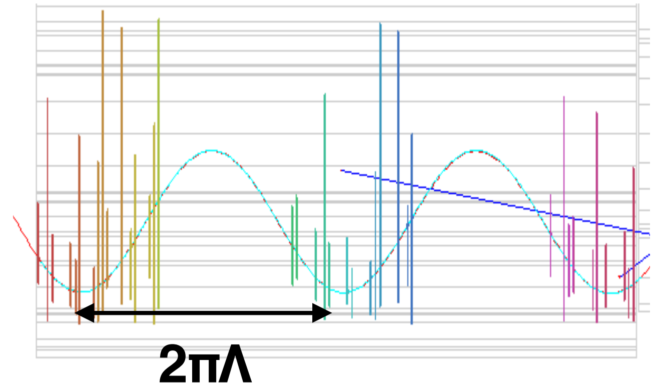


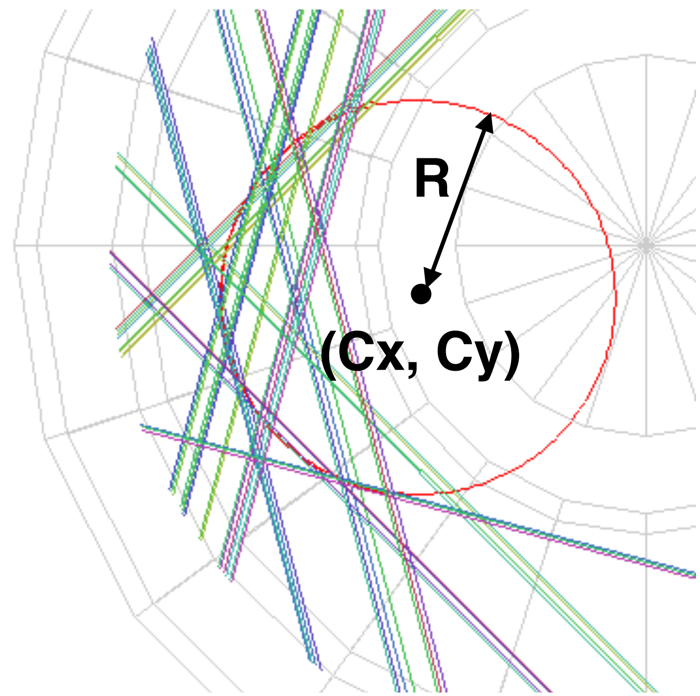


- CsI calorimeter cluster properties
  - $\sigma_E \sim$ few MeV (with long tails from leakage) used for electron PID (E/p)
  - $\sigma_t \sim$ 300 psec
  - $\sigma_{xy} \sim$ few cm, $\sigma_z \sim$ 10cm
  - ~90% acceptance
- Clusters associated with track using POCA in early Pat. Rec.
- Calorimeter time residual used in Kalman fit as a ‘TrkCaloHit’

Kinematic Helix Parameterization

- Λ =Longitudinal wavelength
  - sign(Λ) ≡ -sign(q · Bz · Pz)
  - sign(Λ) = helicity
- **R** = Signed transverse radius
  - sign(**R**) ≡ -sign(q · Bz)
  - sign(**R**) = helicity · direction
- C_x, C_y = center position
- ϕ_0 = atan2(Py,Px) at z=0
- t_0 = time at z=0





Static Fit Parameters

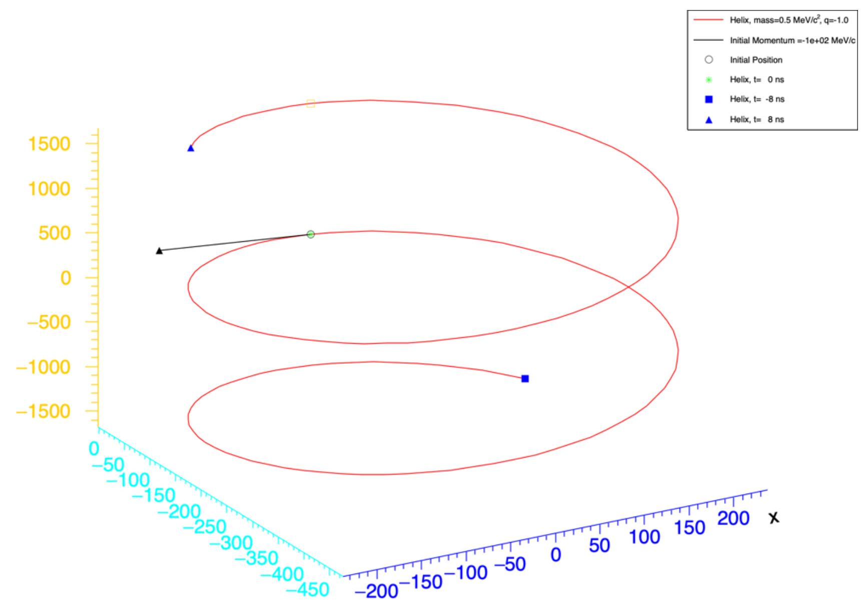
- External parameters
  - Bz = axial magnetic field (T)
  - c = speed of light ≅ 300 mm/nsec
  - Coordinate origin = center of tracker
- Discrete Parameters (a-priori to the fit)
  - particle direction (upstream or downstream)
  - m = particle mass $\in \{m_e, m_\mu, m_\pi, m_K, m_P, m_D, \ldots\}$
  - q = particle charge (Ne)
  - Carried with the dynamic fit parameters

Helix Equations

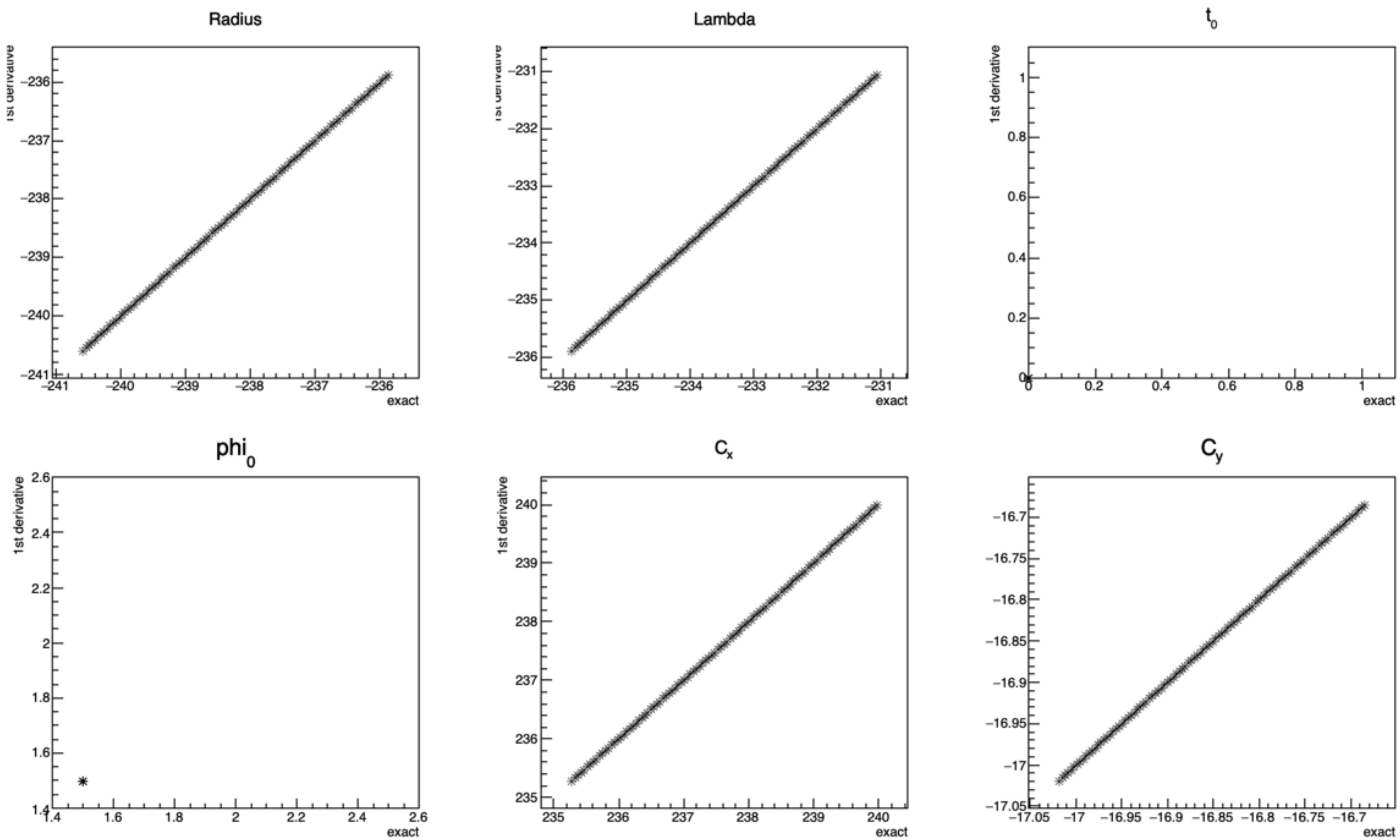
- $\mathbf{Q} \equiv -qcBz$
- $\bar{\mathbf{m}} \equiv$ reduced mass $= m/\mathbf{Q}$ (unit = length!)
- $\Omega \equiv d\Phi/dt = c \cdot \mathrm{sign}(\mathbf{Q})/\mathrm{sqrt}(\mathbf{\Lambda}^2 + \mathbf{R}^2 + \bar{\mathbf{m}}^2)$
- Position
- $x(t) = \mathbf{C_x} + \mathbf{R} \cdot \sin(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $y(t) = \mathbf{C_y} - \mathbf{R} \cdot \cos(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $z(t) = \mathbf{\Lambda}\Omega(t-\mathbf{t_0})$
- Momentum
- $P_x(t) = \mathbf{Q} \cdot \mathbf{R} \cdot \cos(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $P_y(t) = \mathbf{Q} \cdot \mathbf{R} \cdot \sin(\Omega(t-\mathbf{t_0}) + \boldsymbol{\phi_0})$
- $P_z(t) = \mathbf{Q} \cdot \mathbf{\Lambda}$
- $|P| = |\mathbf{Q}| \cdot \mathrm{sqrt}(\mathbf{R}^2 + \mathbf{\Lambda}^2)$
- $E = |\mathbf{Q}| \cdot \mathrm{sqrt}(\mathbf{R}^2 + \mathbf{\Lambda}^2 + \bar{\mathbf{m}}^2)$

Code

- https://github.com/brownd1978/KinematicHelixFit
- Helix Parameter class + kinematic interfaces
  - mom. + pos. 4-vectors ⟺ Helix Parameters
- Home-made 4-vector class
  - Switch to GenVector?
- Simple root visualization
- Unit tests
- Toy MC for validation
  - 40 straws + calorimeter (timing)
  - Gaussian scattering, hit smearing, …
- KF implemented using SVector, SMatrix

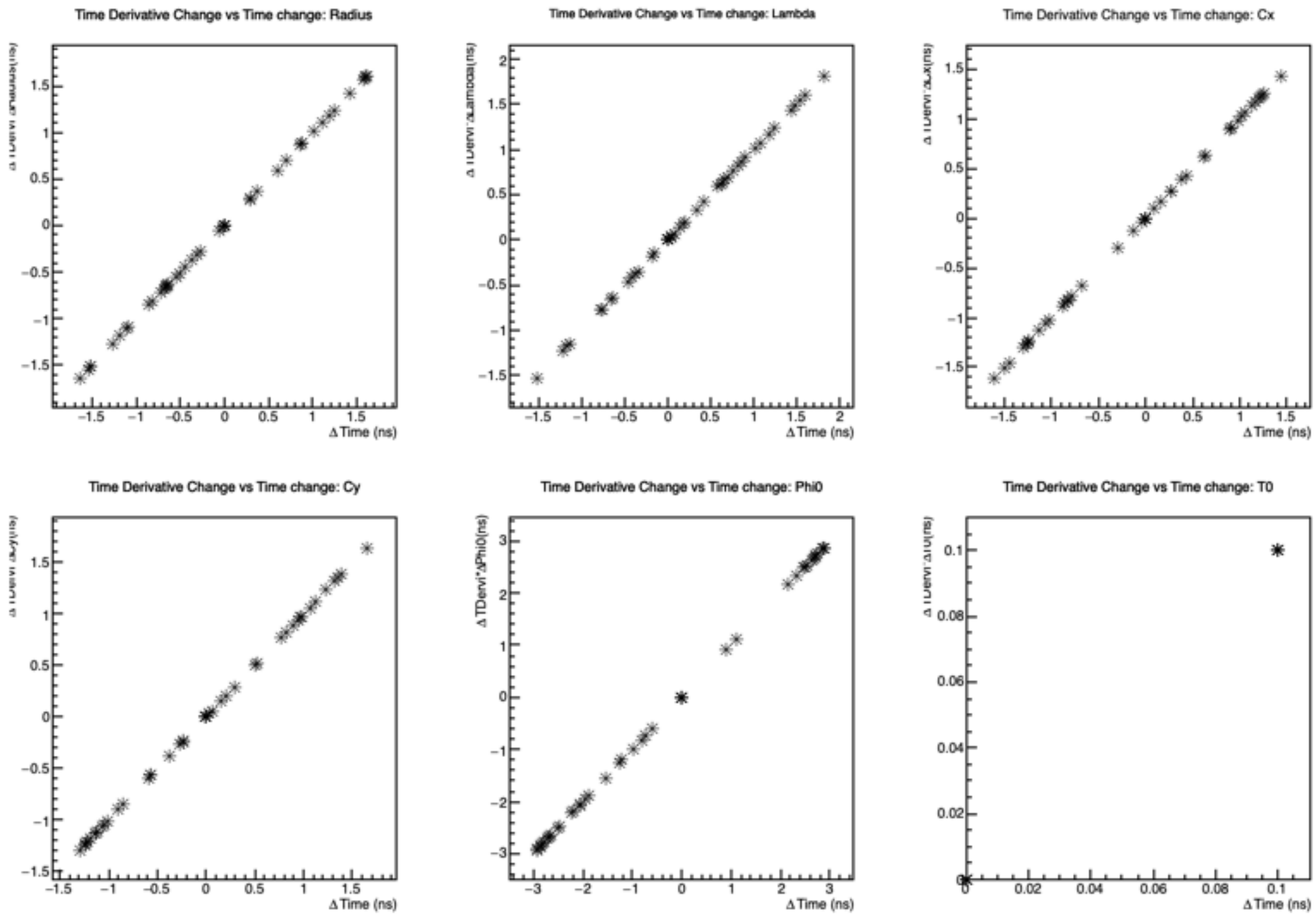


Scattering Unit Test (polar angle)



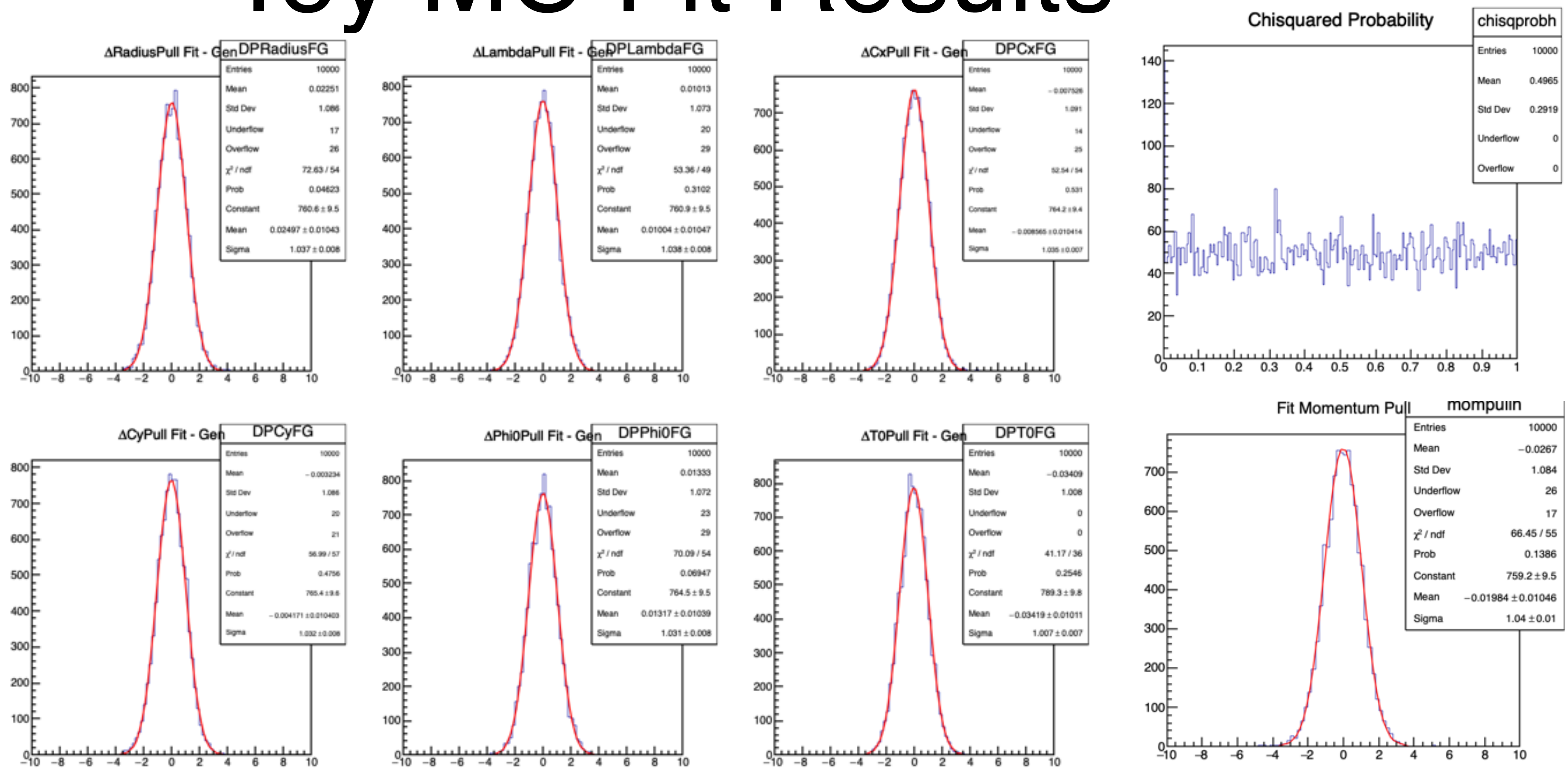
- Tweak momentum direction, everything else fixed
- Compare exact parameter change with 1st derivative prediction

Time Derivatives Unit Test



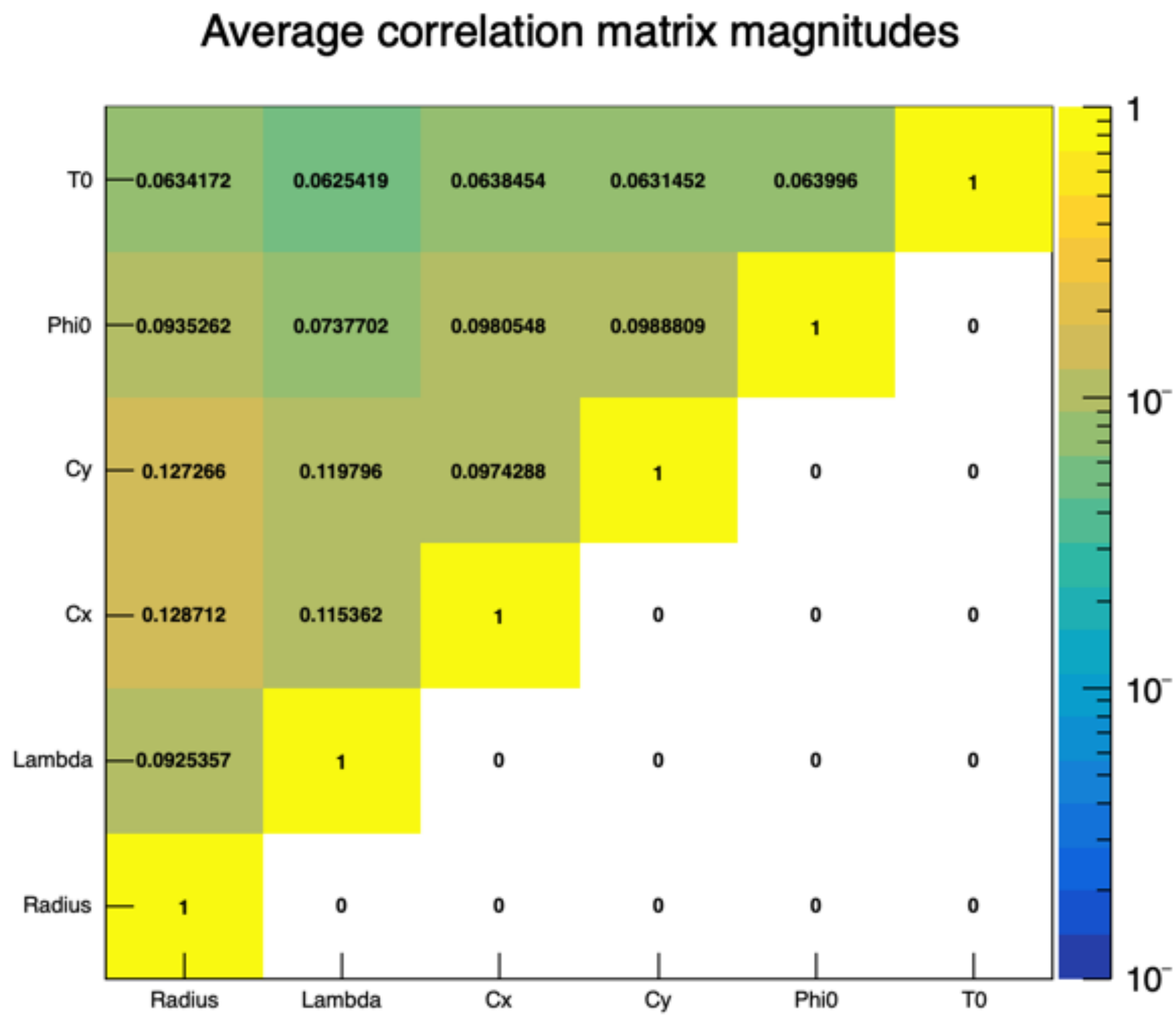
- Tweak individual parameters (R, Λ, …)
- Compare exact change in hit time prediction with 1st derivative prediction

Toy MC Fit Results



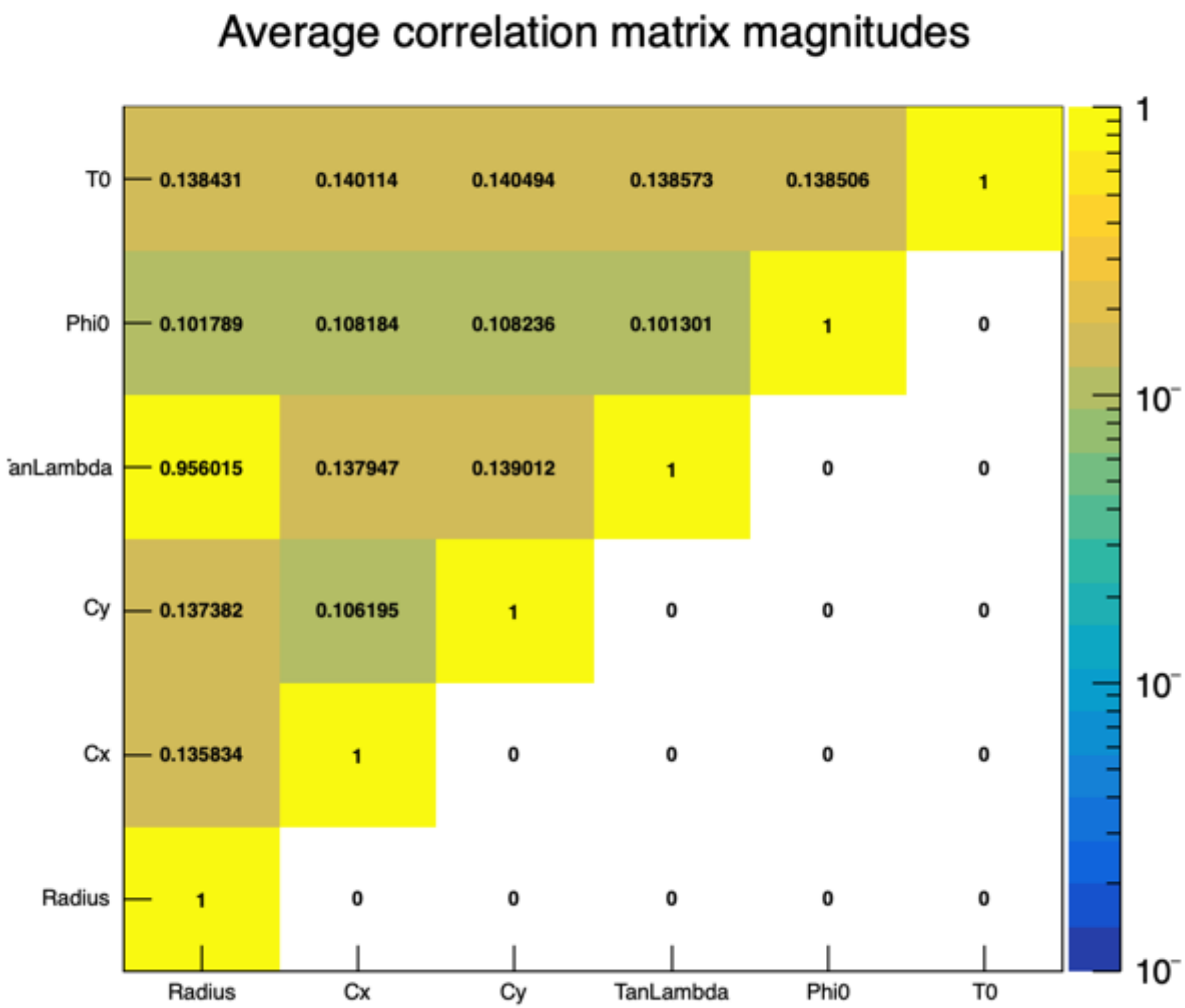
- Toy MC Simulated 105 MeV/c electron + KF fit
- Ensemble of 10K tests, random initial parameters
  - Randomized seed (10σ smearing) to KF, which is iterated
- Parameter pulls, chisq probability as expected

Parameter Correlations



- Correlations between 6 and 13%

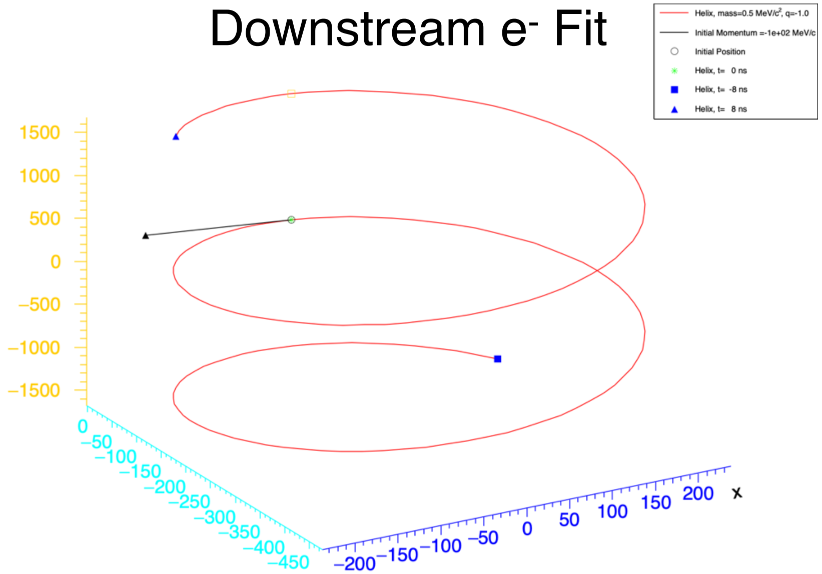
Parameter Correlations, Λ→tan(λ)



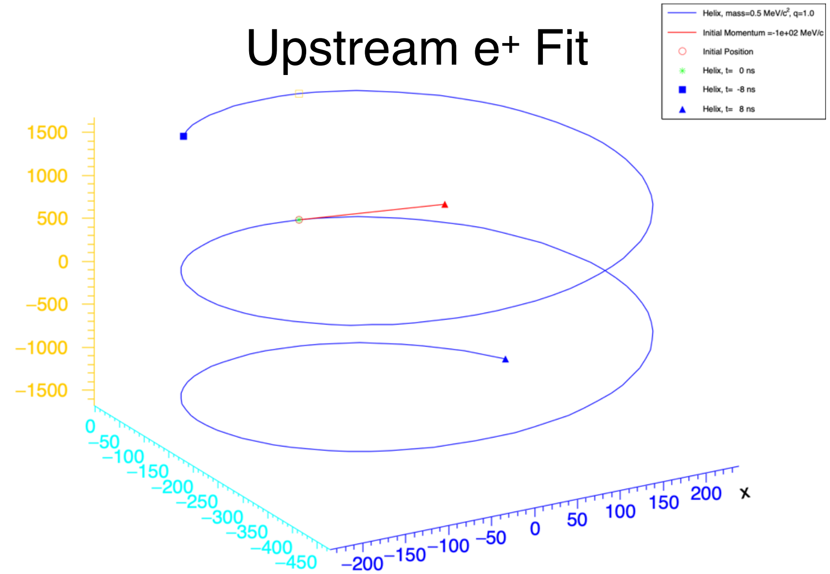
- Large R, tan(λ) correlation

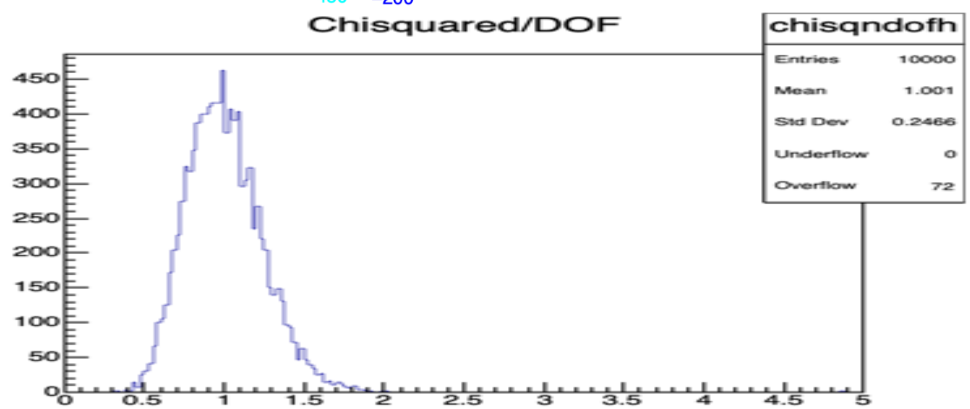
Particle Direction Sensitivity

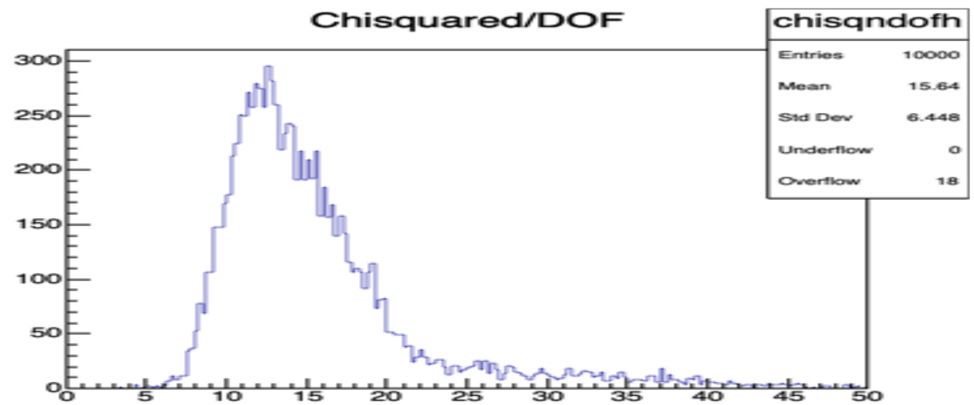
Downstream e^- Fit



Upstream e^+ Fit







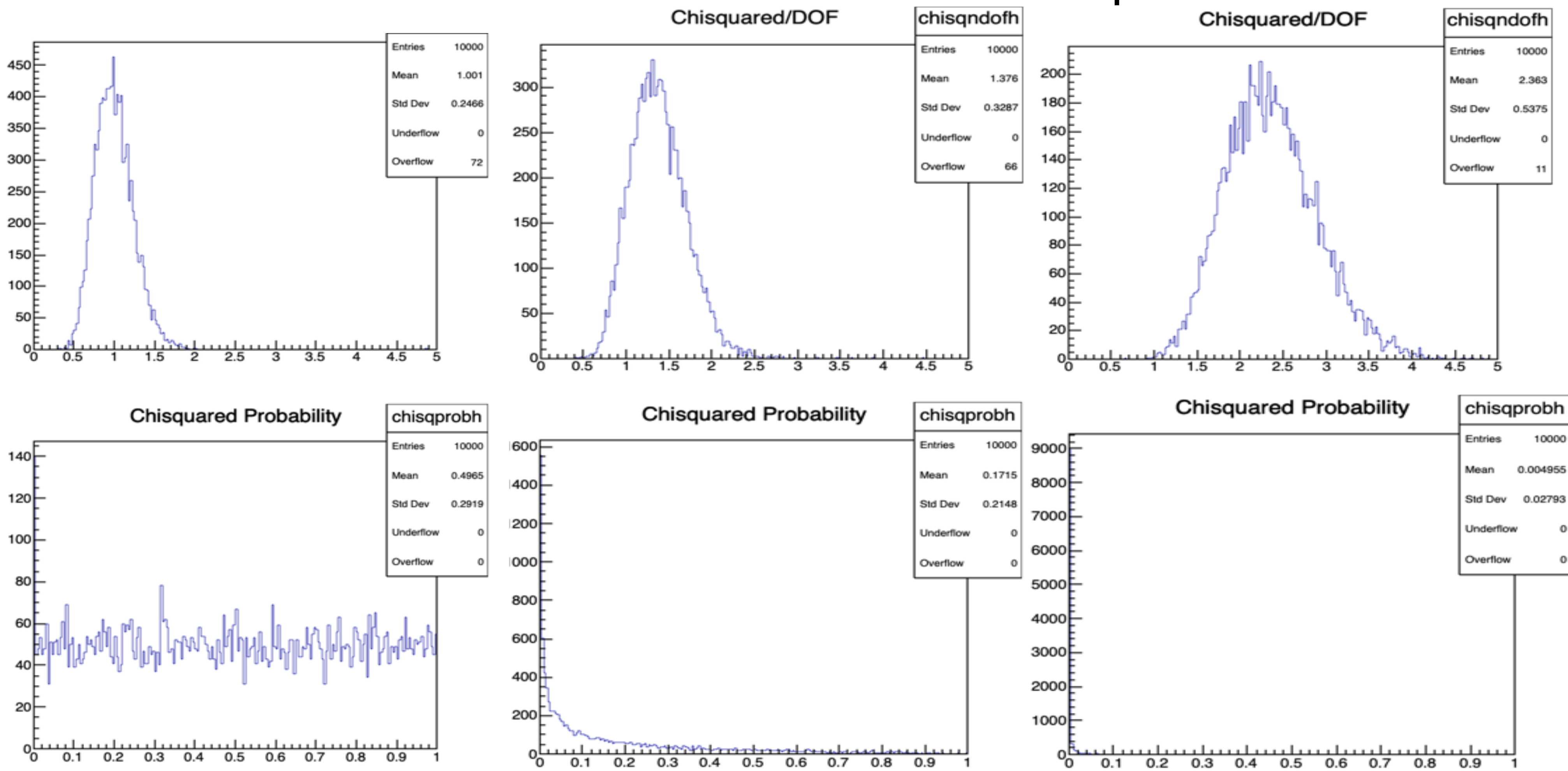
- Hits generated as downstream e^-
- Fit as downstream e^- or upstream e^+ (same helicity, opposite time order)
- Kinematic fit resolves direction ‘degeneracy’
  - energy loss in KF is 2ndary effect

Kinematic PID

μ^- Fit

e^- fit without TrkCaloHit

e^- fit with $\sigma_t = 300$ psec TrkCaloHit



- Generate 10K 105 MeV/c downstream μ^- particles
- Fit as either downstream μ^- or e^- (different $\bar{\mathbf{m}}$)

Other Topics

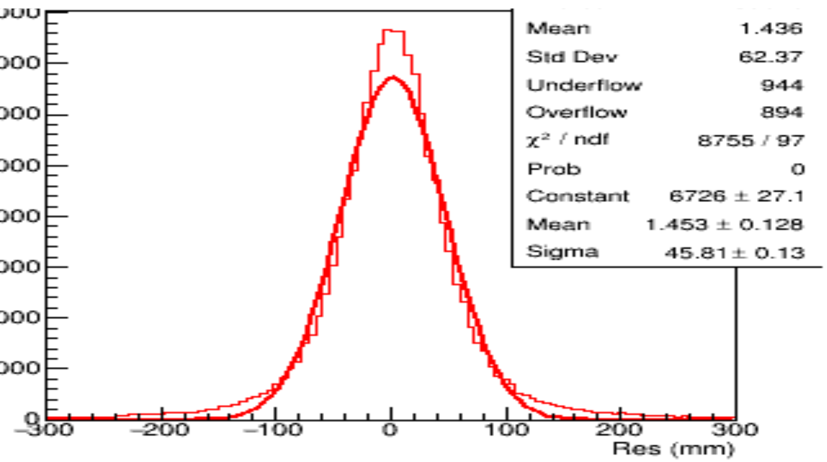
- Time difference position reconstruction
- Signal propagation time slewing
- Time-over-Threshold drift time estimate
- Left-right ambiguity resolution
- Material effect modeling
- Track Quality selection MVA
- Drift modeling
- Hit combining for Pat. Rec.

Backup

Other Helix Equations

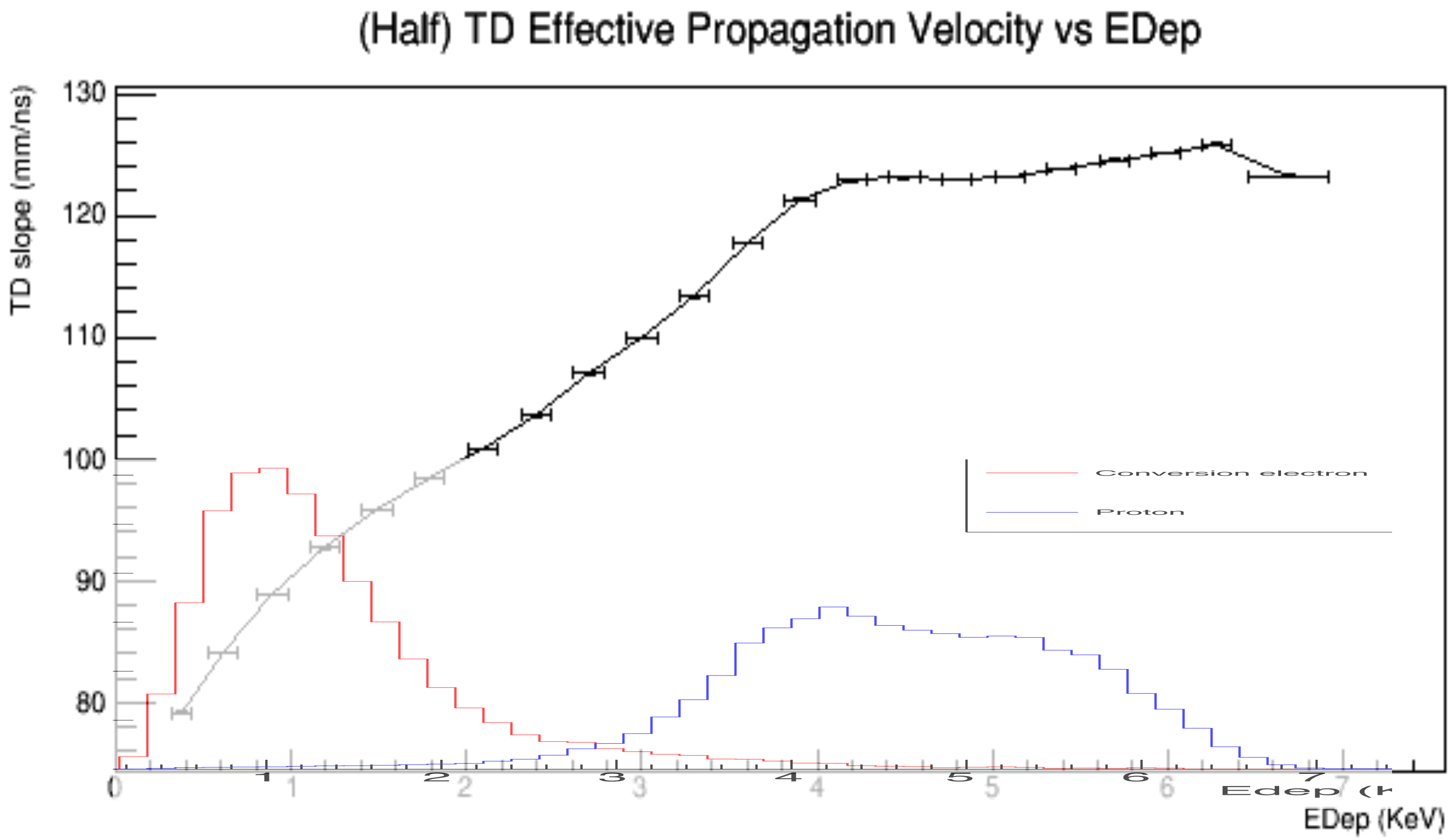
- Consider wire $\perp$ to z at W, azimuth = η
- $\bar{\phi} \equiv \phi_0 + W_z/(R \cdot \Lambda) - \eta$
- $\Delta \equiv -\sin\eta(C_x - W_x) + \cos\eta(C_y - W_y)$
- DOCA = $\Lambda(R-\cos(\bar{\phi}) - \Delta)/\mathrm{sqrt}(\Lambda^2 + R^2\sin^2(\bar{\phi})) + O(r^2_{straw}/R)$

Logitudinal Hit Position



- Time readout at both ends
  - 10 n-sec coincidence in readout reduces randoms
- Δt gives rough position along straw
  - Important for pattern recognition
  - Must calibrate for slewing effects
- Earliest time is used for drift calculations
  - Must calibration for slewing effects

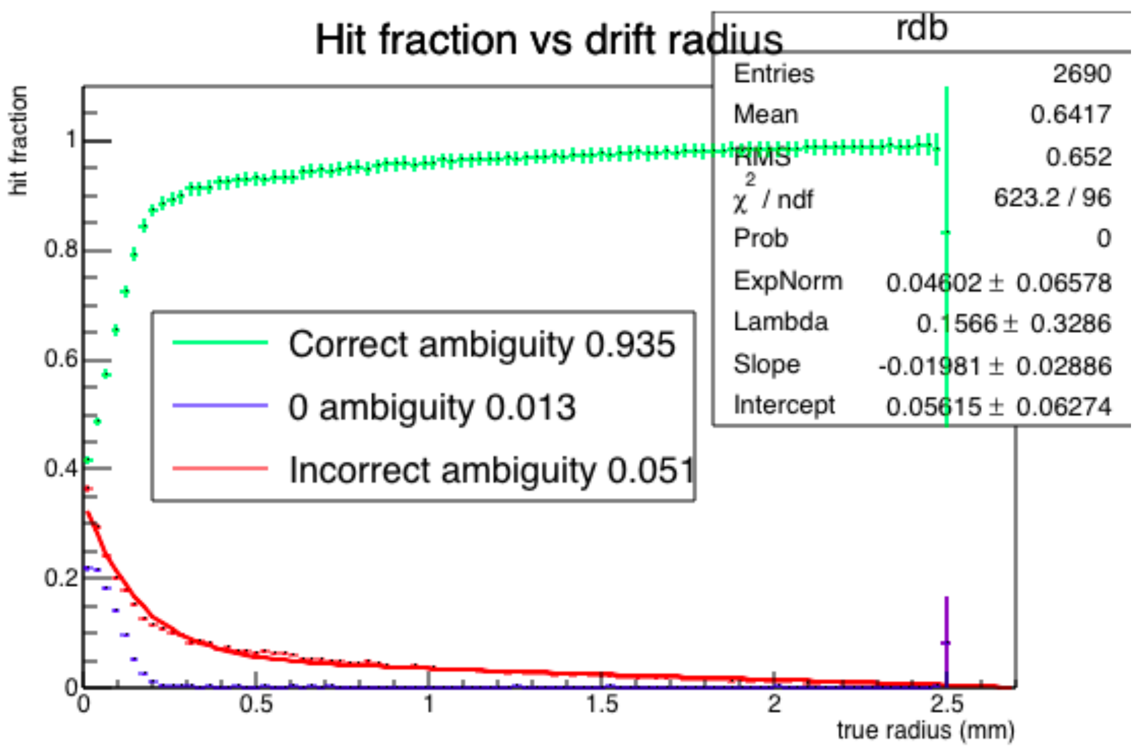
Wire Signal Speed (Slewing)

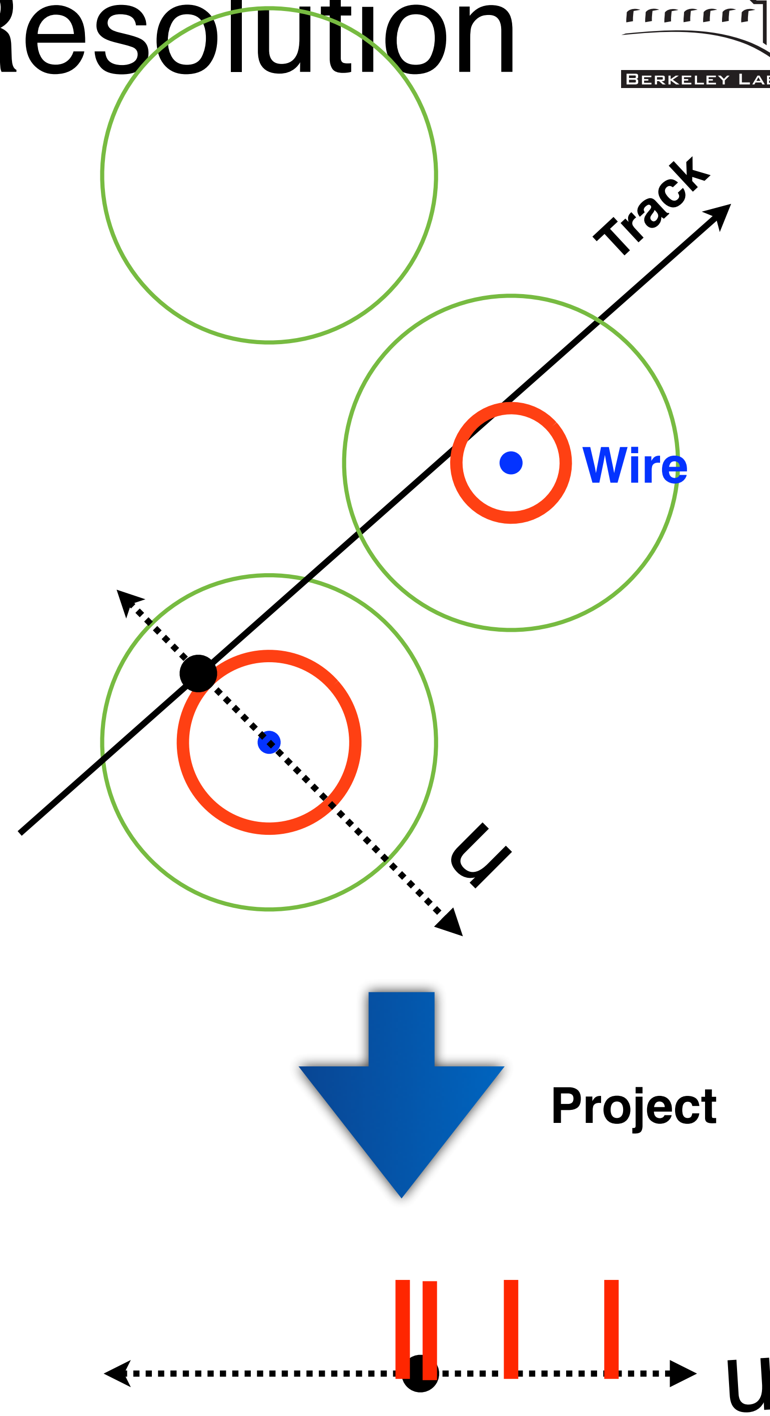


- Effective signal speed varies from 0.5 c to 0.9 c
  - Caused by slewing effects
  - Measured in prototypes
- Corrected using pulseheight (ADC) in timing propagation

Left-Right Ambiguity Resolution

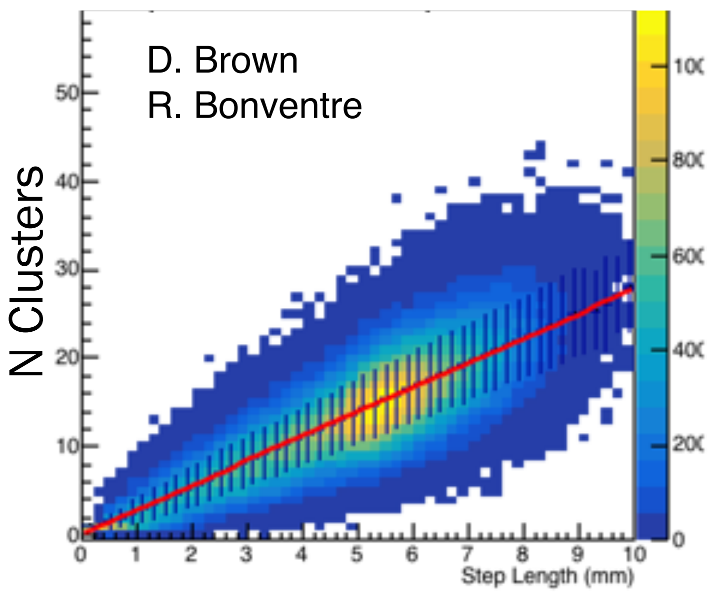
- Iterated with KF fit
- Determined for sets of hits in the same panel
  - negligible propagation errors
- Compute projection χ^2 for all possible hit ambiguity states (including inactive)
  - penalty term for inactive hits
- Set LR hit errors according to best χ^2



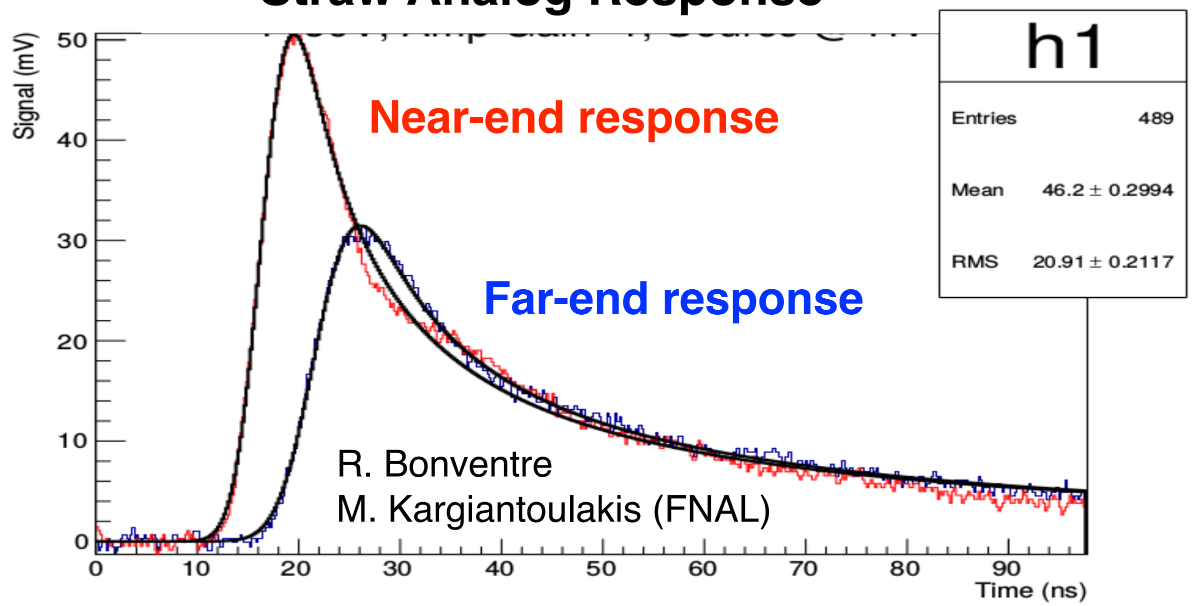


Straw Response Simulation

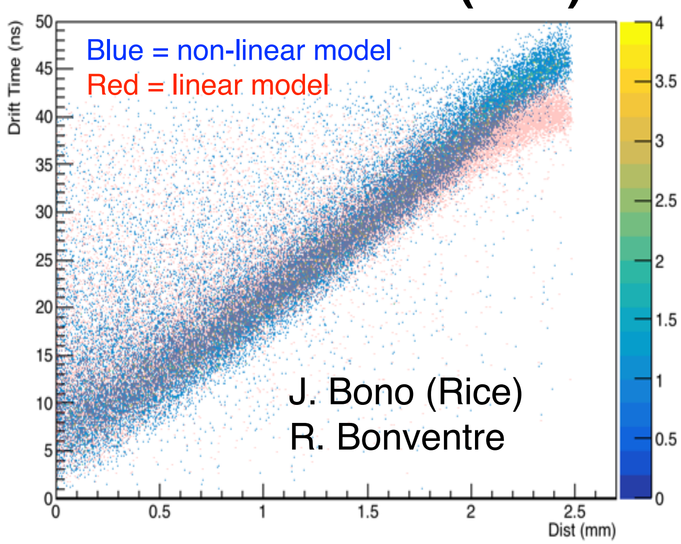
Cluster Creation



Straw Analog Response



Distance-To-Time (D2T)



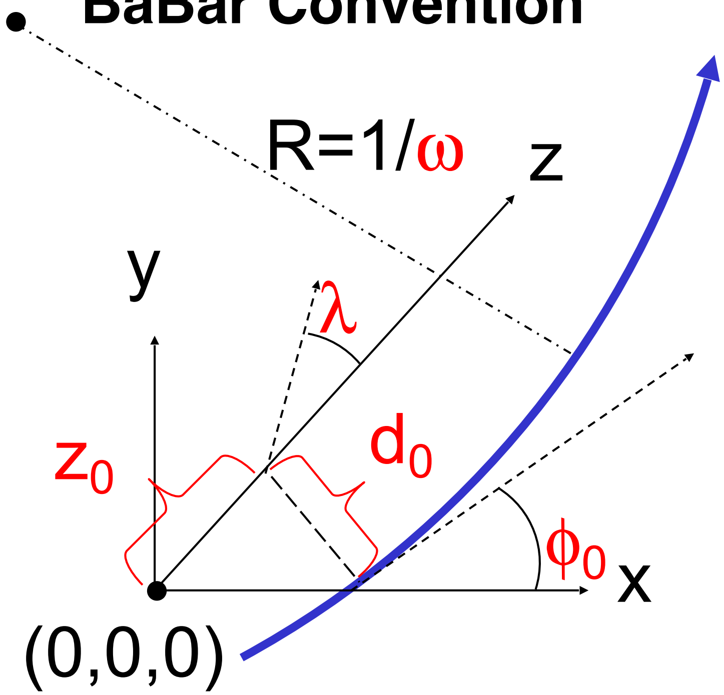
- Start with G4 energy deposits in straw gas
- Model ion cluster creation, electron statistics, electrostatic gain and drift, waveguide attenuation and dispersion, amplifier transfer function, …
- Tuned to prototype and literature data

BaBar Track Fit Parameterization

L ≡ transverse flight
P ≡ $\{d_0, \phi_0, \omega, z_0, \tan\lambda\}$

- Based on seeing a small segment of a helix arc
- Geometric description
  - with kinematic interpretation
- Arbitrary parametric variable

BaBar Convention



$$x(L) = 1/\omega \cdot \sin(\phi_0+\omega L)-(1/\omega+d_0)\sin\phi_0$$

$$y(L) = -1/\omega \cdot \cos(\phi_0+\omega L)+(1/\omega+d_0)\cos\phi_0$$

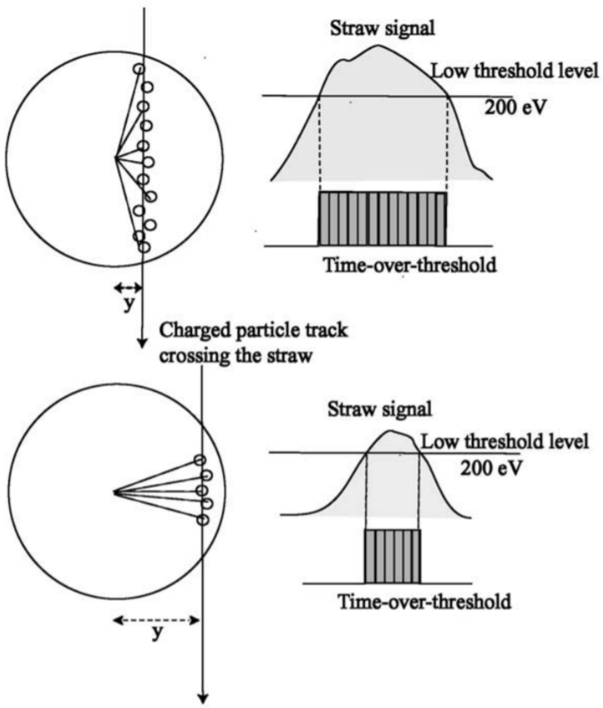
$$z(L) = z_0+L \cdot \tan\lambda$$

Natural description of low-curvature tracks coming from a known point

Material Effects

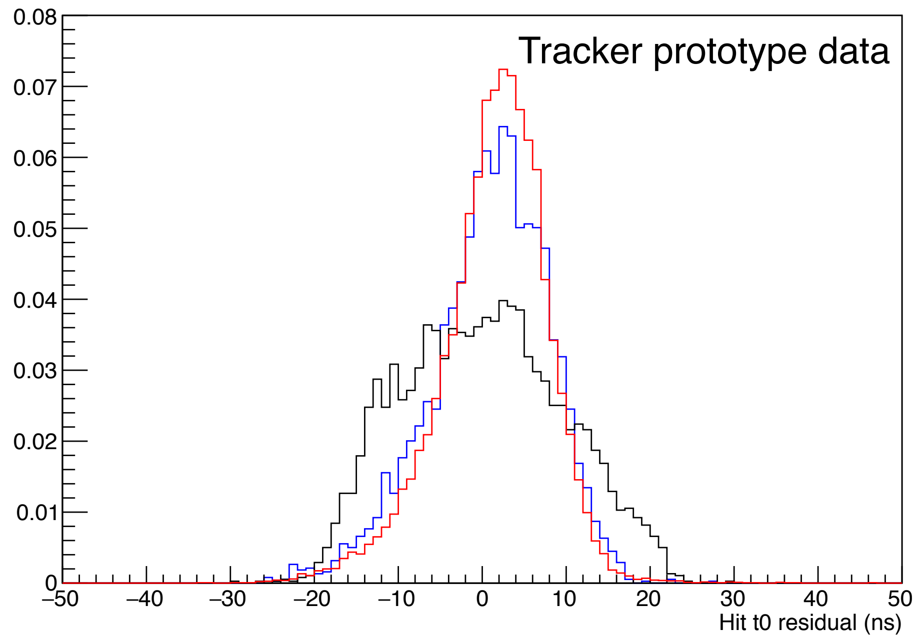
- Lynch-Dahl model (NIM B58 (1991))
  - Screened Rutherford cross-section
  - Parameterized by tail truncation factor
    - Can be tuned to model reconstruction truncation
    - Mu2e straw wall truncation value: 0.999
- Most probable value for energy loss, straggling
  - Sternheimer parameterization
  - Mean is biased towards tail
  - Landau tails are ‘self-truncated’ by pat. rec.

Time Over Threshold



DOCA and path length vs. TOT

(Akesson et al., 2001)



- No TOT (black): 9.4 ns
- 1D TOT (blue): 7.1 ns
- 2D TOT (red): 6.4 ns

- TOT is sensitive to drift time
- Useful to improve initial hit time resolution

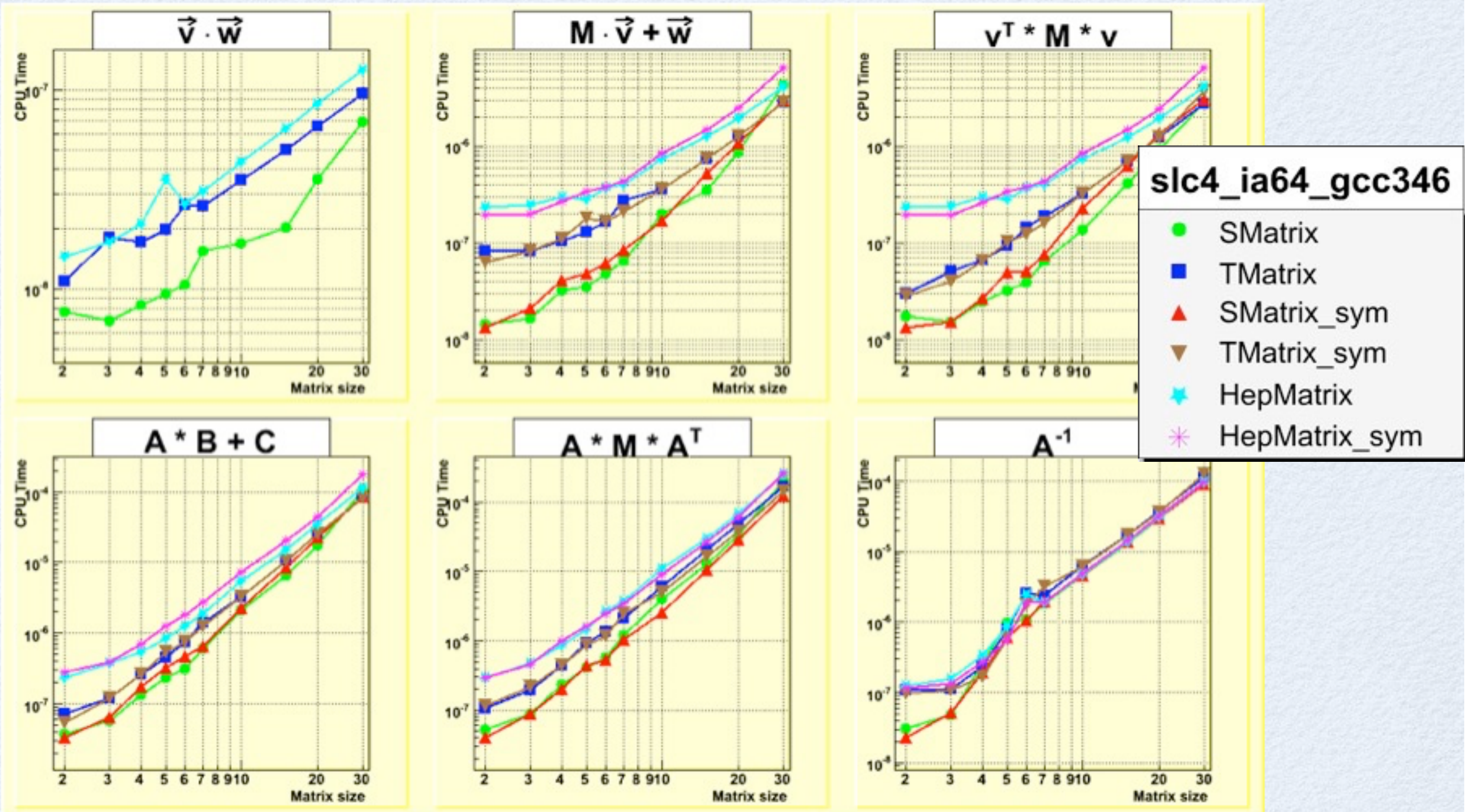
Matrix Feature Comparison

	SMatrix	Eigen	CLHEP
Native Symmetric Matrix Support?	✔	✕	✔
Inversion Error Testing?	✔	✕	✔
CLHEP Interface Compatibility?	✔	✕	✔
6-Dimensional Optimization?	✔	?	✕
Interface to Spacetime Vectors?	2,3, and 4D (GenVector)	2 and 3D only	✕
Native Root Persistence?	✔	✕	✕
Linear Algebra Support?	multiply, add, invert, similarity, …	Basic + Decompose, solve, sparse, …	multiply, add, invert, similarity, …
Parallelization support?	✔	?	✕
Indexing style?	C (start from 0)	C (start from 0)	Mixed C + Fortran

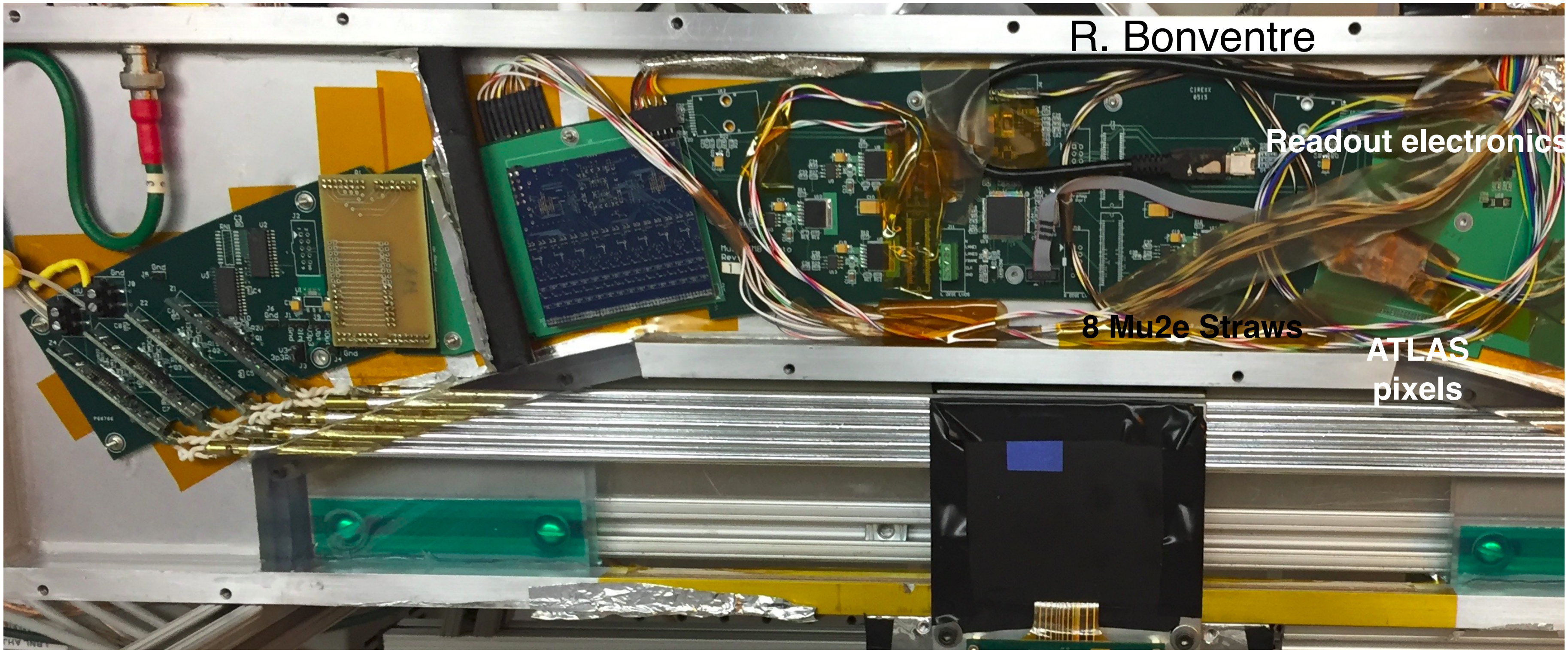
SMatrix Performance

Lorenzo Moneta (CERN)

Annual Concurrency Forum Meeting, 4-6 Feb 2013, Fermilab

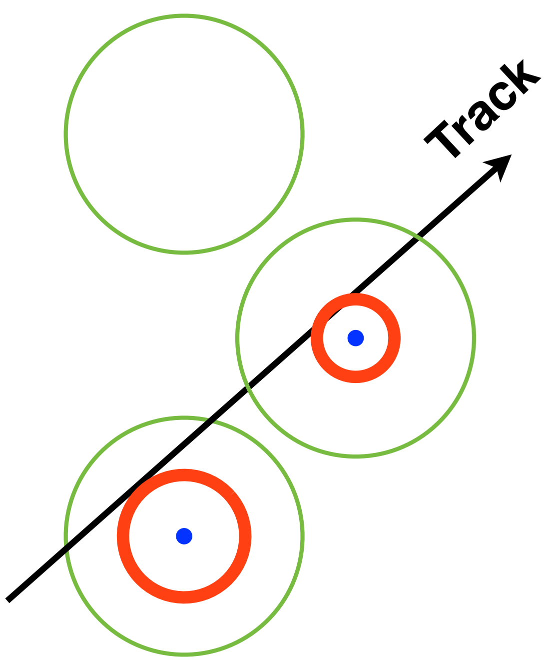


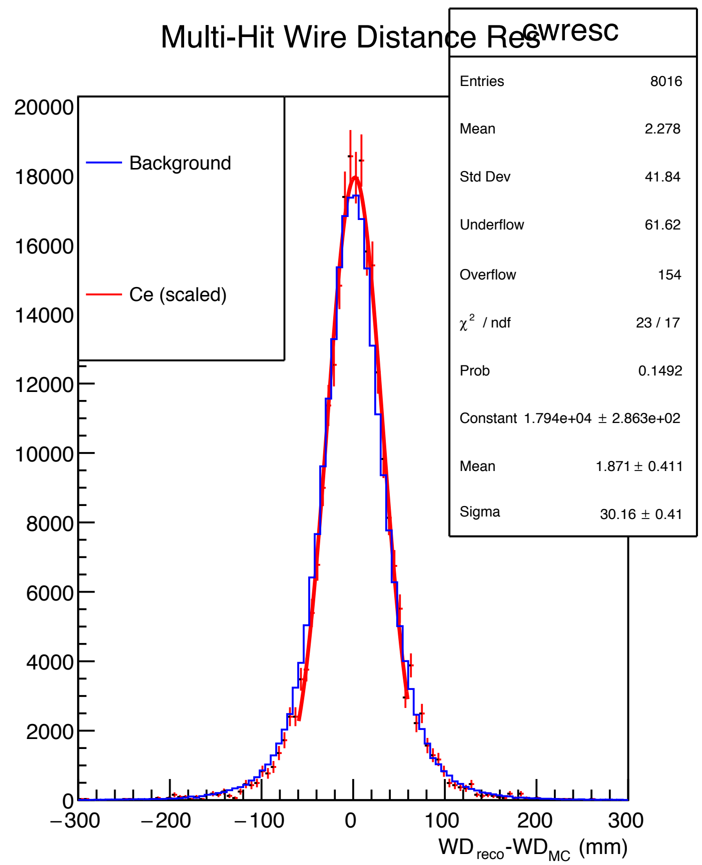
8-Straw Prototype

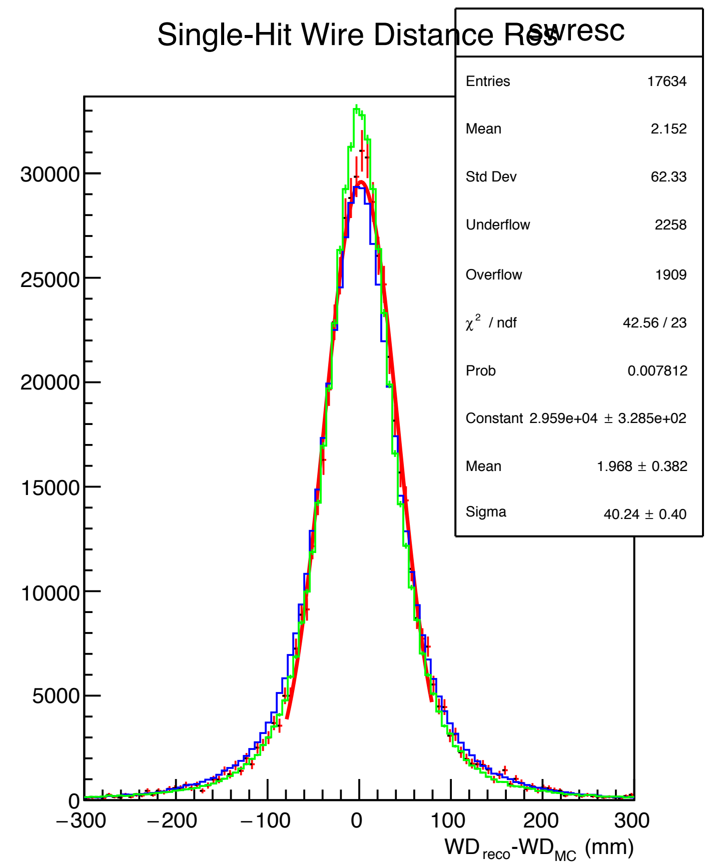


- 2 ATLAS pixel modules
  - ~20 um resolution
- scintillator cosmic-ray trigger
- Measured electronics transfer, resolutions, Drift velocity

Combined Straw Hits



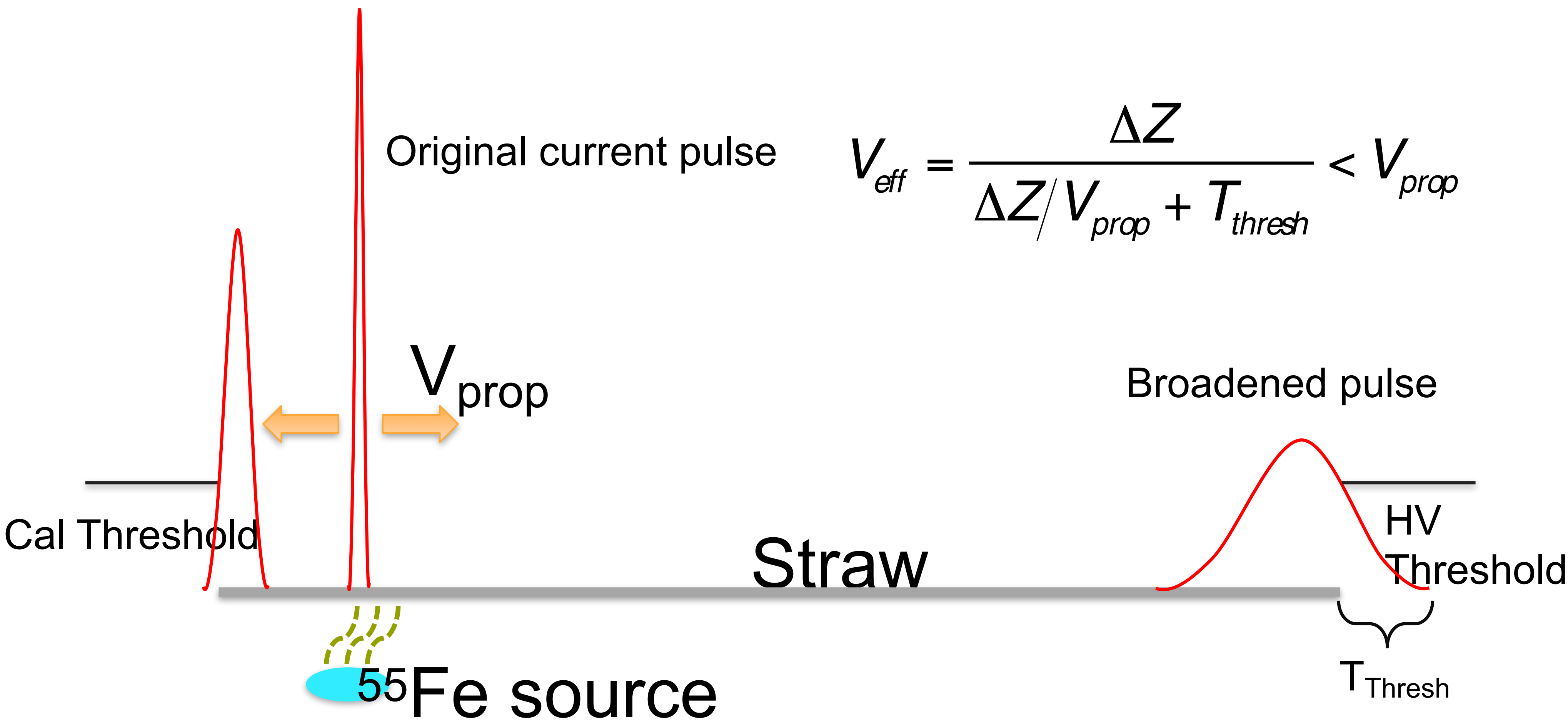




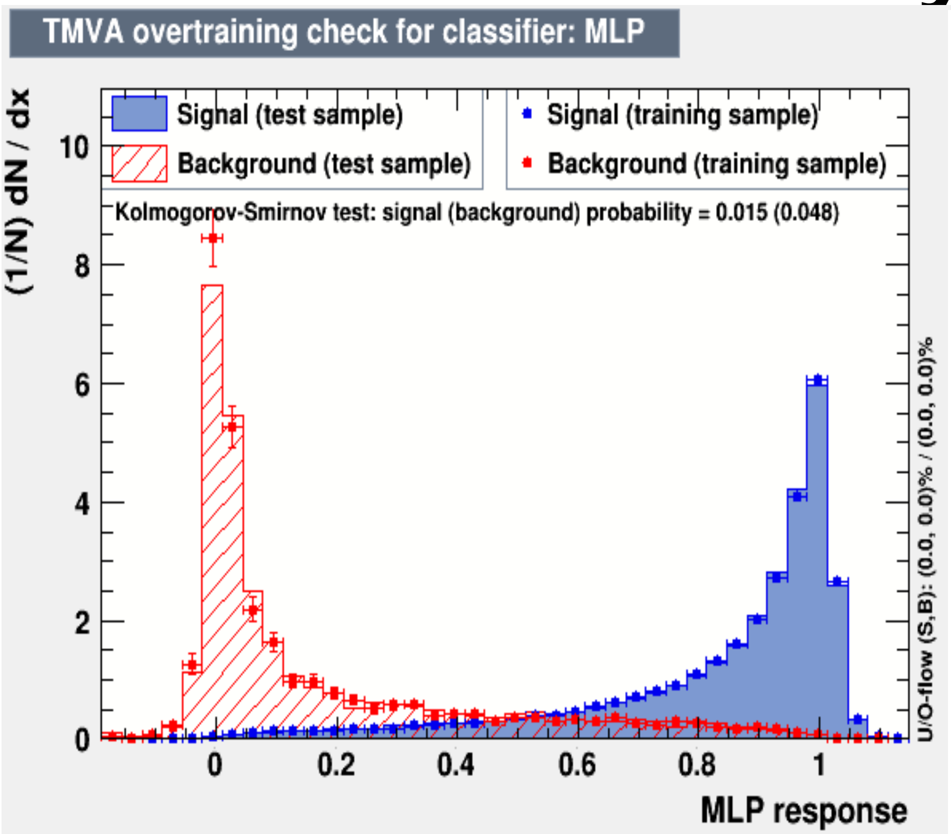
- Combine hits in adjacent straws in a panel
- Improves resolution along wire by ~1/sqrt(2), with large reduction in tails
- Degrades resolution ⊥ to wire (effective size) by a factor of ~2
- Reduces hit combinatorics in Pat. Rec.

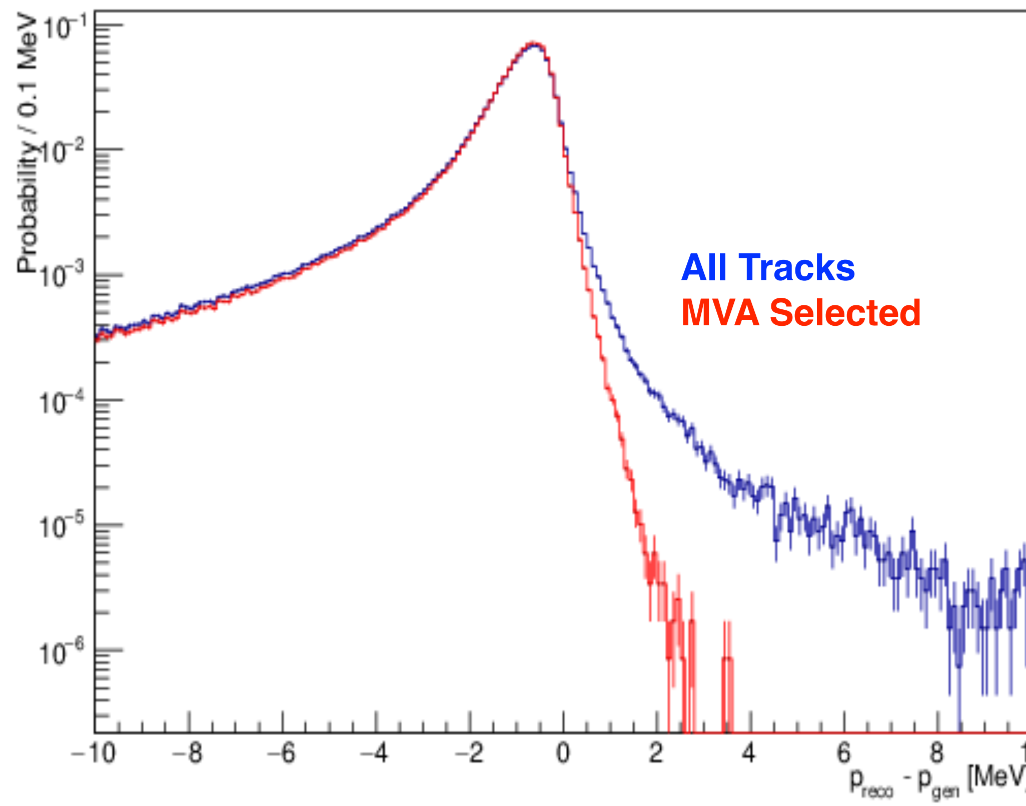
Straw Signal Propagation

$$V_{eff} = \frac{\Delta Z}{\Delta Z / V_{prop} + T_{thresh}} < V_{prop}$$



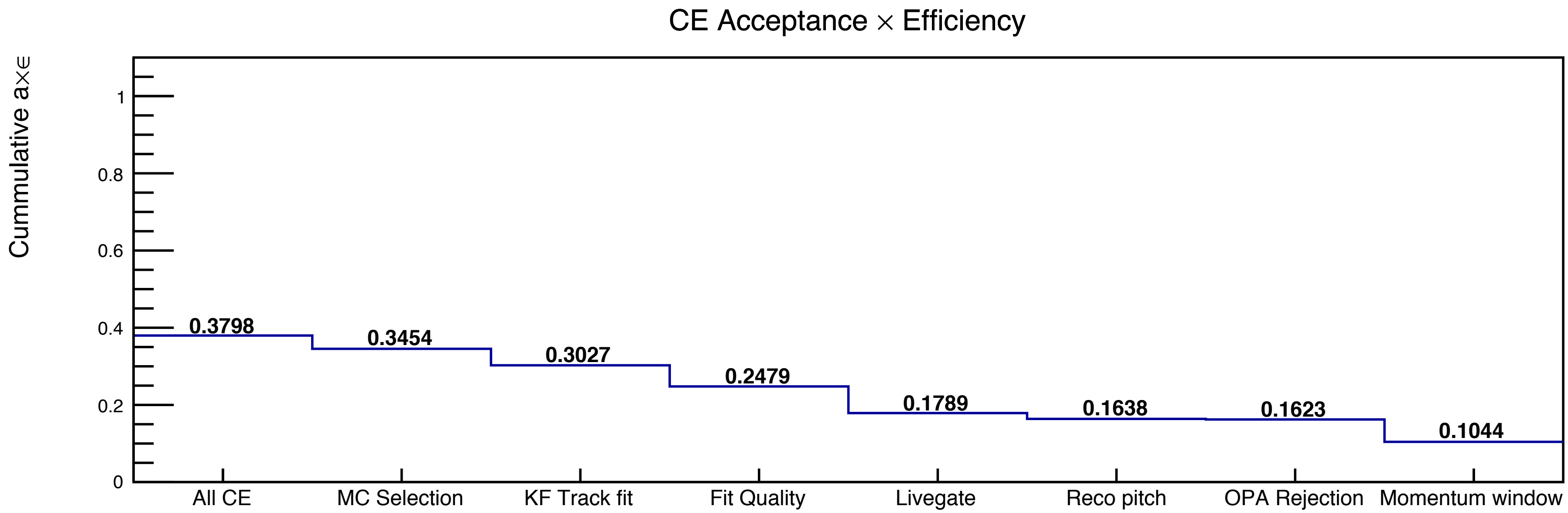
Track Quality Selection

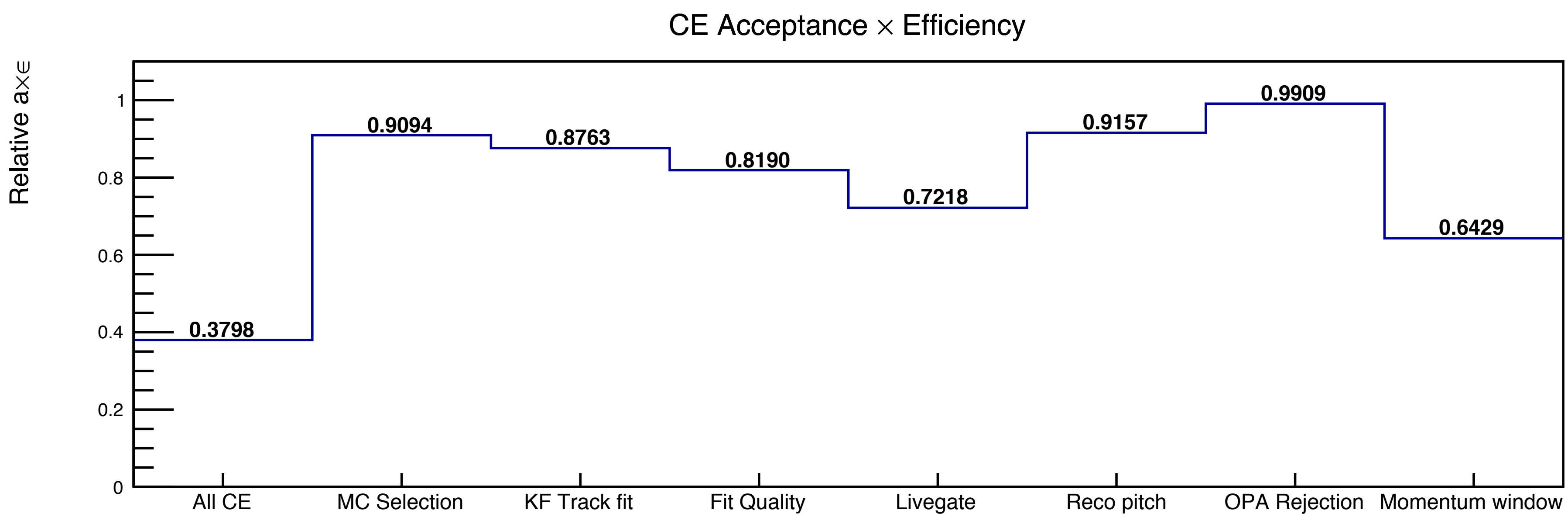




- Momentum resolution crucial for Mu2e physics goals
- ANN used to select poor momentum resolution
  - Fit chisq, Hit count, Hit pattern vs expected, … (8 total)
  - Training emphasizes tail elimination
- 75% signal efficiency, tail reduction 10^{-3}

Tracking Acceptance x Efficiency





Track Trigger

- Track trigger based on time cluster + helix fit
- Meeting time spec
  - Hit multiplicity reduction
  - Optimized data structures
  - Algorithm tweaking
- 2.5 msec/event
  - 64-core node
- Rejection factor ~2000 (physics limit)