WENTWORTH INSTITUTE OF TECHNOLOGY
ELECTRONICS AND MECHANICAL ENGINEERING DEPARTMENT
ELEC 870 ELECTROMECHANICAL SYSTEMS II
SPRING SEMESTER
LABORATORY # 12
FIR FILTER II
Z -TRANSFORM AND FIR FILTER DESIGN BY MATLAB

PURPOSE

The objective for this lab is to build an intuitive understanding of the relationship between the location of poles and zeros of H(z) (the z-domain), the impulse response h[n] (in the n domain), and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$ domain). Also, we will learn filter design by **MATLAB FDA Tool** and **MATLAB SPA Tool**. Following are the outlines of this experiment:

- Z transform
- Pole and Zero analysis by MATLAB
- FIR Filter design in **FDA Tool**
- FIR Filter design and analysis in **SPA Tool**

REFERENCES:
a. Laboratory experiment # 11
b. Class note # 11,12,13

EQUIPMENT:
(1) Computer with sound card.
(2) MATLAB Software with **SP Tool and FDA Tool.**

PROCEDURE:

1. Read the Appendix of this lab (attached) for FIR Filter Z-Transform and Poles and Zeros and their relationship with frequency response. Also look at the MATLAB code for drawing the poles and zeros on the unit circle.
2. Review the Lab#11 for generating the frequency response of FIR Filter from filter coefficients.

3. Frequency Response:

   - Plot the frequency response of two averaging filters by MATLAB
   - F_s = 300 Hz

   (a) y(n) = 0.5 x(n) + 0.5 x(n-1)

   (b) y(n) = 0.5 x(n) – 0.5 x(n-1)

4. From the frequency response, indicate the types of filters as indicated above.

5. Write the expressions for H(z) for the two filters given above.

6. Draw the poles and zeros for the filters given above.

7. Draw the poles and zeros by MATLAB. Refer to appendix for the MATLAB code for generating poles and zeros. Compare the MATLAB plot with your hand calculations and comment.

8. Can you draw any correlation between the type of filter and the location of zeros?

9. A DSP system filter is described by the difference equation

   y[n] = 1/4 (x[n] + x[n-1] + x[n-2]+x[n-2])

   (a) Determine the filter coefficients.
   (b) What is h[n], the impulse response of this system?
   (c) Determine the system function H(z) for this system.
   (d) Plot the poles and zeros of H(z) in the complex z plane.

10. For the filter given above, plot the frequency response and poles and zeros by MATLAB. What type of filter it is? Can you draw any correlation between the type of filter and the location of zeros?

11. Importing Filter into SP Tool:

12.

From Lab#11 you know how to use SP Tool for import signal from the work space and perform the frequency analysis. In this lab you will lean how to import filter coefficients into SP Tool and to draw the frequency response and poles and zeros.

Suppose we would like to import the filter b=[1/3,1/3,1/3] in to the SP Tool and generate the frequency response and draw poles and zeros without generating any MATLAB codes you have used previously.

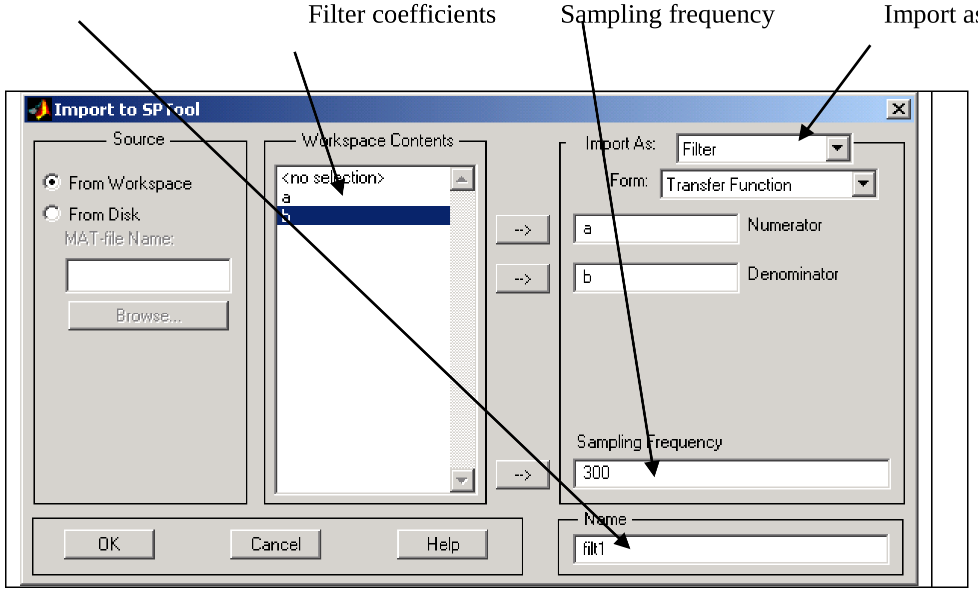
In the MATLAB command window (workspace) define the filter as follows:

```
>> b=[1/3,1/3,1/3 1/4; % FIR filter coefficients (numerator)
>> a=[1,0,0 0]; %for FIR Filter this is defined as [1,0,0] (denominator
>> sptool
```

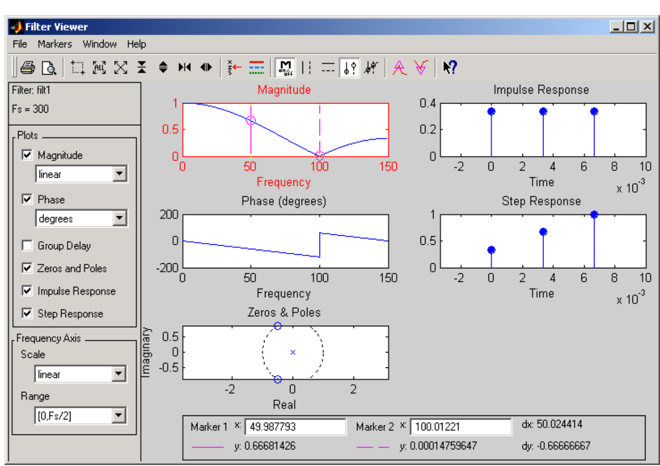
In order to import the filter into SP Tool : **File→ Import from the SPTool main menu.**

Name used within SPTool

Filter coefficients   Sampling frequency   Import as filter



After importing the filter in SP Tool, you can view the frequency response by clicking on **View.** You can view the poles and zero plot, impulse response, step response by clicking on appropriate boxes on this screen. You can edit poles and zeros (by clicking on **Edit**) and look at the effect frequency response.



13. Applying a Filter in SP Tool

- Select the signal to be filtered.
- Select the filter to be applied.
- Click the **Apply** button under the filter list.

To use the SP Tool for filtering a signal, we need to import the signal into SP Tool and then import the filter.

Let us take an example:

- Generate a signal in MATLAB workspace with the following specification:

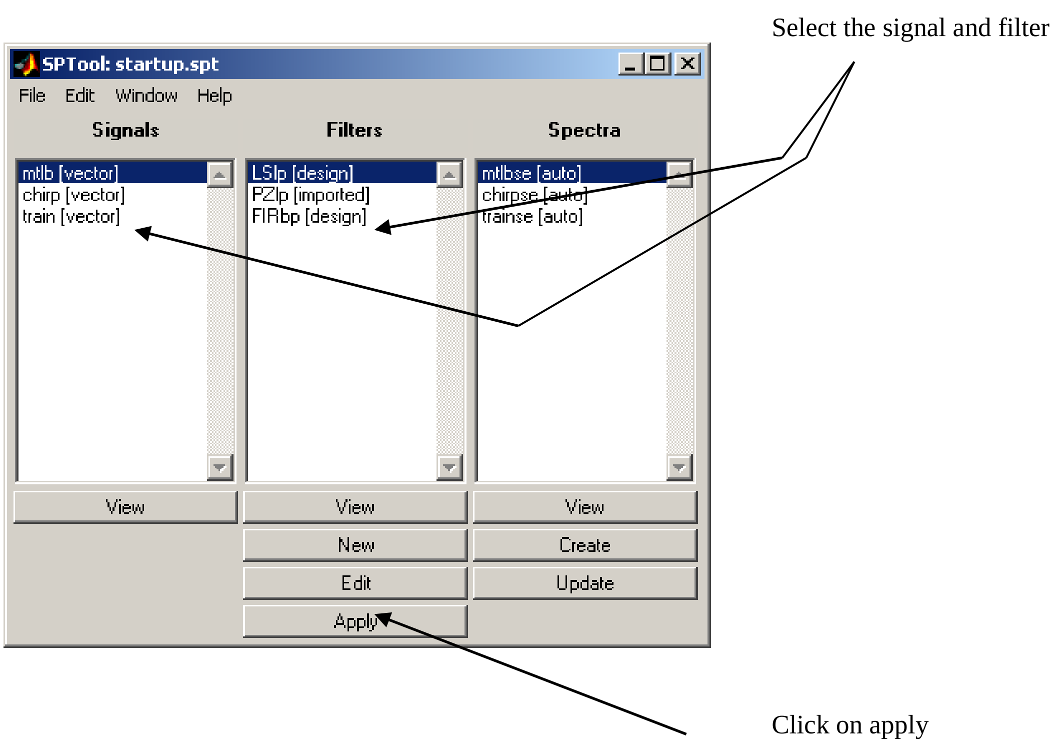
  $x(t) = 8 \cos(2\pi\, 250\, t + \pi/3)$ (Refer to LAB# 11 for generating the signal)

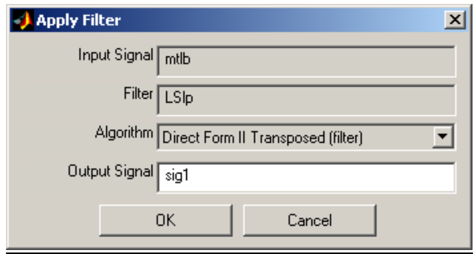
The filter system is defined by the following equation:

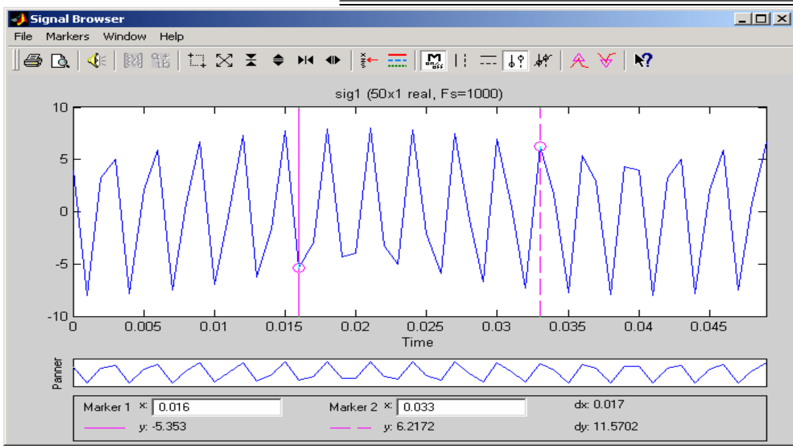
$$y[n] = 1/4\, x[n] + 1/4\, x[n-1] + 1/4\, x[n-2] + 1/4 x[n-3]$$

Sampling frequency is $f_s = 1000$ samples per second

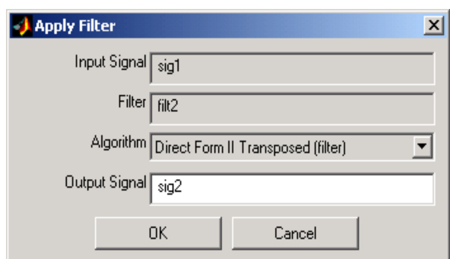
- Import the signal and filter in SP Tool.
- Apply the signal to the filter and view the resultant signal.

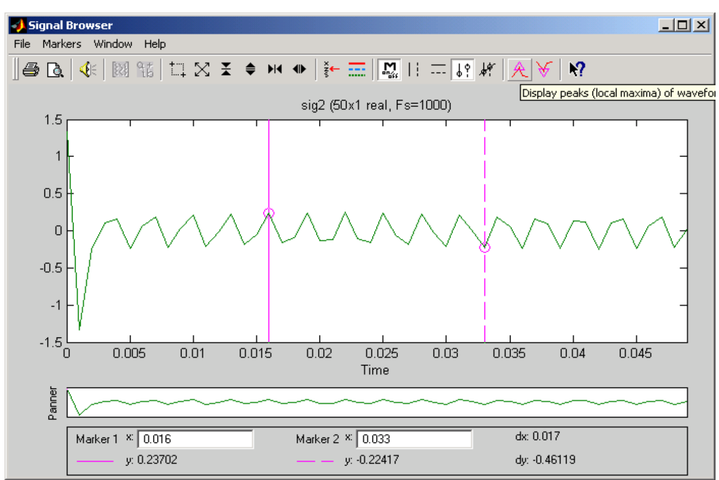






Imported signal prior to applying the filter.





Signal after processing through the filter in SP Tool

13. **Filter Design with FDA Tool**

In this section we will learn how to design a FIR Filter. After designing a filter we can import the filter in SP Tool and bring a signal from the workspace and apply the designed filter.

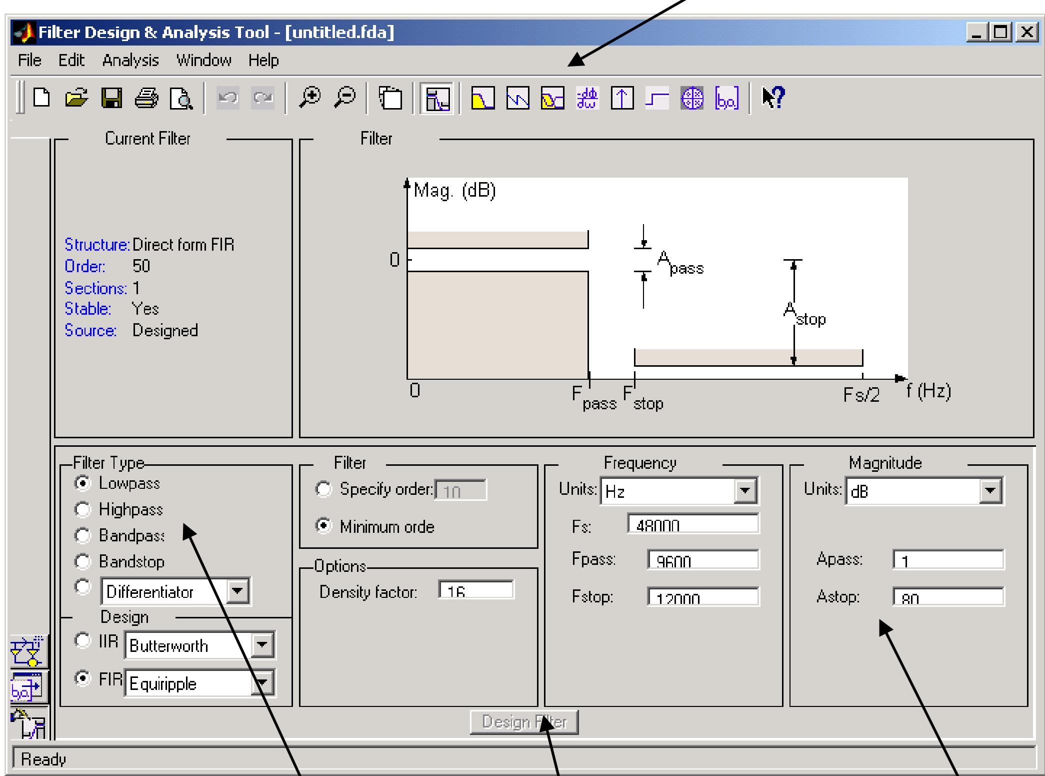
The Filter Design Toolbox is a collection of tools built on top of the MATLAB computing environment and the Signal Processing Toolbox. The toolbox includes a number of advanced filter design techniques that support designing, simulating, and analyzing fixed point and custom floating point filters for a wide range of precisions. The FDA Tool is a graphical user interface for the analysis and design of digital filters.

The FDA Tool operates in different modes:

a. Design mode
b. Import mode
c. Quantization mode
d. Realization mode
e. Transformation mode.

>>fdatool

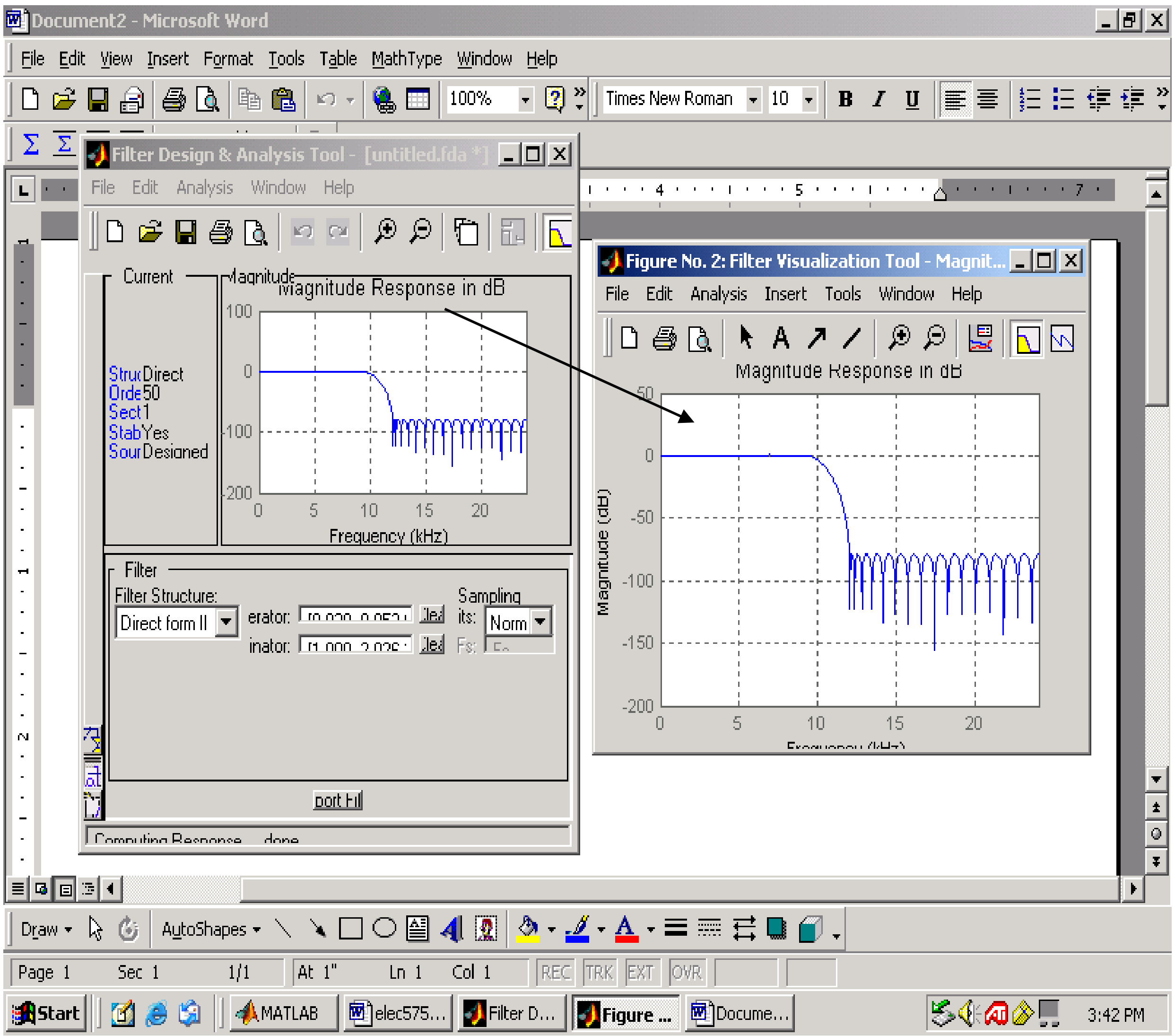
Analysis methods



Type of filter Click here for design filter Filter Specifications

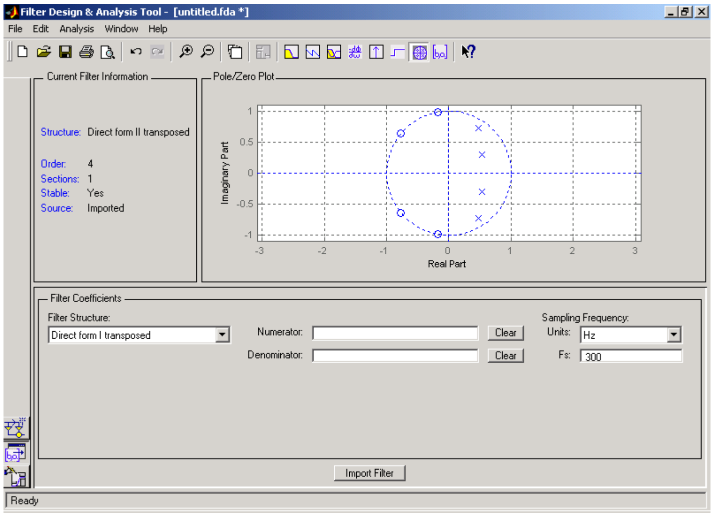
13. Analyze Filters with FDA Tool

The Filter Visualization Tool **(fvtool)** can be accessed by the **Full View Analysis** option in the **Analysis** menu by clicking on the **Full View Analysis** button on the tool bar.



14. Filter Quantization

You can get poles and zero plot by FDA Tool.



15. Exporting from FDA Tool

In order to use the filters designed in the FDA Tool, they must be exported.

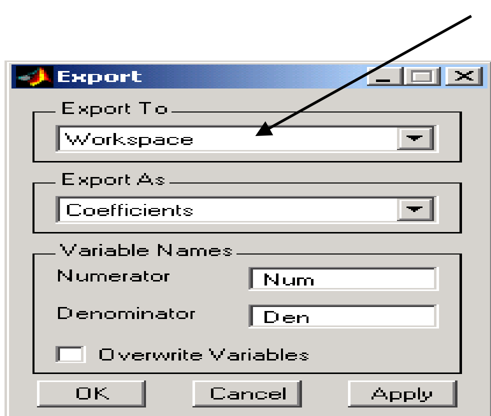
Filters can be exported to

- Workspace - coefficients can be
  - Vectors
  - Objects
- MAT file or text file.
- SP Tool
- C-Header file.

To export a filter

- To the workspace go to **File → Export**
  - Select workspace
  - Select coefficients or objects

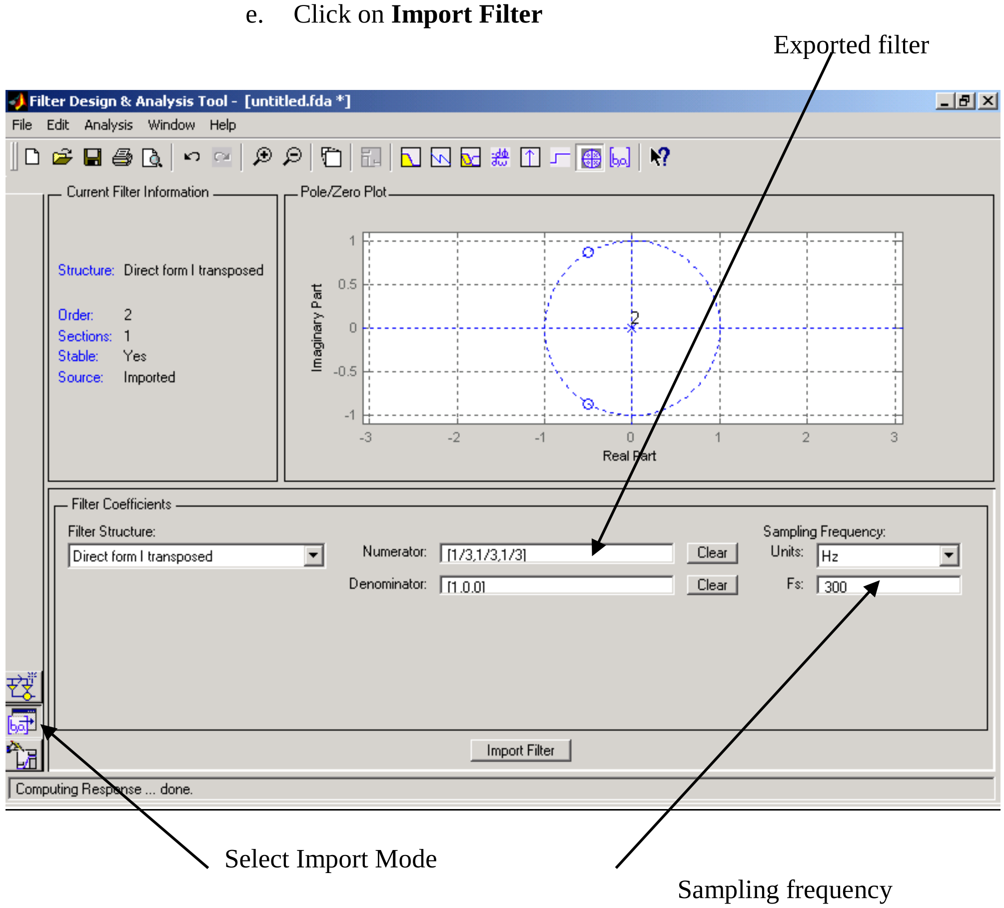
- To the SPTOOL go to **File → Export to SP Tool**



16. Importing Existing Filters

The FDA Tool can be used to edit and analyze already existing filters by importing them from the MATLAB workspace:

a. Select **Import Mode**
b. Choose desired **Filter Structure**
c. Enter the coefficients
d. Enter the **Sampling Frequency**
e. Click on **Import Filter**



17. Putting it altogether: Follow the following steps:

- **Design a filter by FDA Tool according to the desired specification.**
- **Generate a signal to be filtered in MATLAB workspace.**
- **Import the generated signal in workspace to SP Tool.**
- **Export the designed filter in SP Tool.**
- **Apply the designed filter to the signal.**
- **View the filtered signal in SP Tool**
- **Export the filtered signal to the workspace.**

18. Exercise:

a. Generate a Chirp signal with the following specification: (refer to lab#4&5)

f1 = 200 Hz
f2 =2000Hz
T= 10 sec
fs=8000Hz

b. Generate this signal as y vector
c. Listen to this signal by sound command
d. Design a filter which will filter signals with frequencies from 500 Hz to 1200 Hz. Design the filter by FDA Tool.
e. Generate the frequency response for the designed filter to see whether it meets your need.
f. Generate pole and zero plot.
g. Modify the filter if it is necessary.
h. Export the filter in SP Tool .
i. Import the chirp signal to SP Tool.
j. Apply the filter to chirp signal.
k. View the filtered signal in SP Tool.
l. Export the filter coefficients from FDA Tool to MATLAB Workspace.
m. Convolute the chirp signal with the filter coefficients.
n. Listen to the filtered signal by sound command.
o. Comment on the filtered signal.

APPENDIX TO LAB # 12

WENTWORTH INSTITUTE OF TECHNOLOGY
ELECTRONICS AND MECHANICAL ENGINEERING DEPARTMENT
ELEC 575 DIGITAL SIGNAL PROCESSING
NOTES ON CHAPTER #7

Z TRANSFORMS

1. Definition of Z Transform

A finite length signal x[n] can be represented by the relations:

$$x[n] = \sum_{k=0}^{N} x[k]\delta[n-k]$$

and the Z transform of such a signal is defined by the formula:

$$X(z) = \sum_{k=0}^{N} x[k] z^{-k}$$

X(z) can be written in the form:

$$X(z) = \sum_{k=0}^{N} x[k](z^{-1})^{k}$$

n Domain	⇔	z Domain
$x[n] = \sum_{k=0}^{N} x[k]\delta[n-k]$	⇔	$X(z) = \sum_{k=0}^{N} x[k] z^{-k}$
x[n]	⇔	X(z)
$x[n] = \delta[n-n_0]$	⇔	$X(z) = z^{-n_0}$

2. The z transform of an FIR Filter

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

y[n] = x[n] * h[n]

$$h[n]=\sum_{k=0}^{M} b_k\delta[n-k]$$

$$H(z)=\sum_{k=0}^{M} b_k z^{-k}=\sum_{k=0}^{M} h[k]z^{-k}$$

$$h[n]=\sum_{k=0}^{M} b_k\delta[n-k] \quad \Leftrightarrow \quad H(z)=\sum_{k=0}^{M} b_k z^{-k}=\sum_{k=0}^{M} h[k]z^{-k}$$

The system function H(z) is a function of the complex variable z. As we have already noted, H(z) in the equation shown above is the z transform of the impulse response and for the FIR case, it is an M th-degree polynomial in the variable z^{-1}. Therefore H(z) will have M zeros.

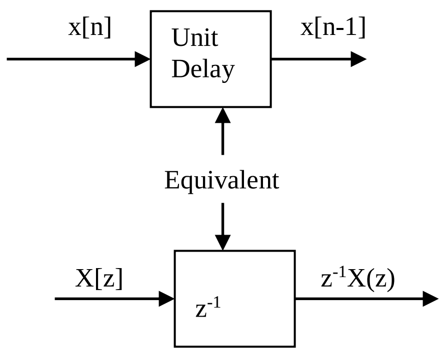
3. The Time Delay Property of the z Transform

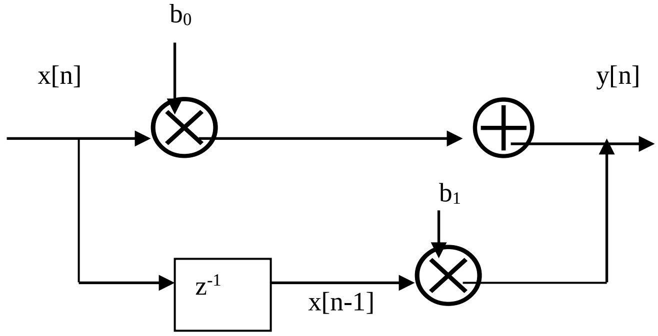
A delay of one sample multiplies the z transform by z^{-1}.

$x[n-1] \Leftrightarrow z^{-1}X(z)$

Time delay of n_0 samples multiplies the z transform by z^{-1}.

$$x[n-n_0] \Leftrightarrow z^{-n_0}X(z)$$





4. Convolution and the z Transform

$$y[n] = x[n]*h[n] \quad \Leftrightarrow \quad Y(z) = H(z)\,X(z)$$

5. Cascading System

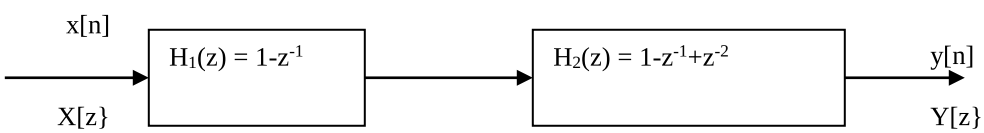
$$h[n] = h_1[n]*h_2[n] \quad \Leftrightarrow \quad H(z) = H_1(z)\,H_2(z)$$

6. Factoring z polynomials

$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$

One of the roots H(z) is z=1 as $H(z) = (1-z^{-1})(1-z^{-1}+z^{-2})$

$1-z^{-1}$ is a factor of H(z)



7. **Relationship between the z domain and the $\hat{\omega}$ domain.**

$\hat{\omega}$ Domain $\Leftrightarrow$ z Domain

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \quad \Leftrightarrow \quad H(Z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$H(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{w}}}$$

The relationship between z domain and the $\hat{\omega}$ domain hinges on the important formula:

$$z = e^{j\hat{\omega}}$$

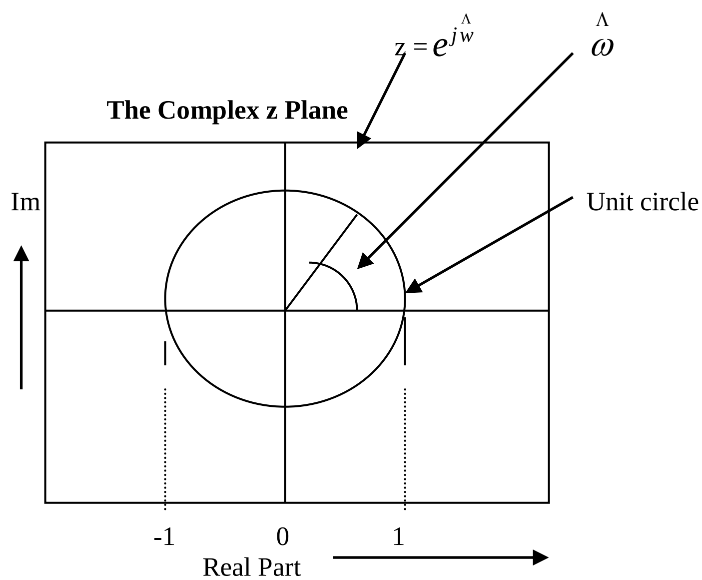
If a signal z^n is the input to an LTI filter, the resulting output is

$$y[n] = H(z)e^{j\hat{w}n}$$
$$y[n] = H(e^{j\hat{w}})e^{j\hat{w}n}$$

8. The z plane and Unit Circle

From the above we can see that the frequency response $H(e^{j\hat{w}})$ can be obtained from the system function H(z) by evaluating H(z) for a specific value of z. From the complex algebra, we can see that z = $e^{j\hat{w}}$ lie on a circle of radius 1 and the angle of $\hat{\omega}$ varies between $-\pi < \hat{\omega} < \pi$.

The graphical representation of figure below gives us a convenient way of visualizing the relationship between $\hat{\omega}$ domain and the z domain. Because the $\hat{\omega}$ domain lies on specials part of the z domain – the unit circle-many properties of the frequency response are evident from plots of system function properties in the z plane. The normalized frequency $\hat{\omega}$ is equivalent to angle in the z plane.



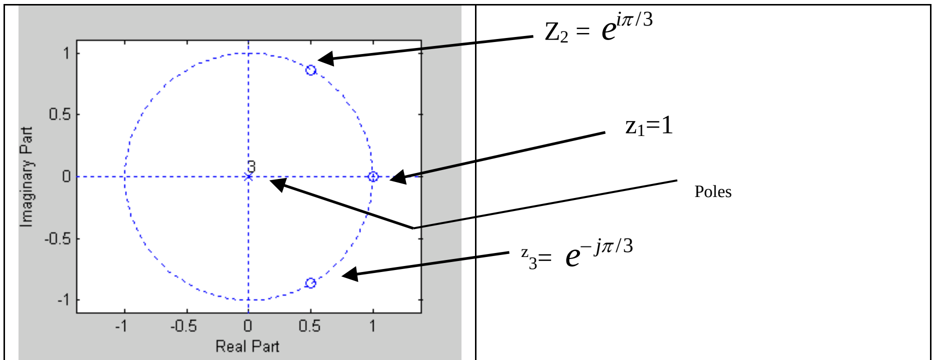
9. The Zeros and Poles of H(z)

We have already seen that the system function for an FIR system is essentially determined by its zeros.

Consider the system function H(z) = $1-2z^{-1}+2z^{-2}-z^{-3}$
We cab rewrite the above expression as :

$$H(z) = \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$
$$= \frac{(z-1)(z-e^{j\pi/3})(z-e^{-j\pi/3})}{z^3}$$

Zeros of H(z) are $z_1 = 1$ $\quad z_2 = e^{i\pi/3}$ $\quad z_3 = e^{-j\pi/3}$
Poles of H(z) are 0,0,0
The poles and zeros can be drawn on a z-plane that includes the unit circle as shown below:



Zeros and Poles of H(z) marked in a z plane that includes the unit circle.

10. Significance of Zeros of H(z)

Zeros of polynomial system function are sufficient to determine H(z) except for a constant multiplier. That means if we know the zeros we can write the filter equation for the FIR filter H(z).

The roots of the above system function:

$H(z) = 1-2z^{-1}+2z^{-2}-z^{-3}$ are $z_1=1$ $\quad z_2 = e^{i\pi/3}$ $\quad z_3 = e^{-j\pi/3}$

As shown in the figure above , these zeros are all on the unit circle, so complex sinusoids with frequencies 0, $\pi/3$ and $-\pi/3$ will be set to zero by the system.

As illustrated by this example, the zeros of the system function that lie on the unit circle correspond to frequencies at which the gain of the system is zero. Thus complex sinusoids at those frequencies are blocked or ‘nulled’ by the system.

11. Nulling Filters (Designing of FIR filters).

We have just shown that if the zeros of H(z) lie on the unit circle, then certain sinusoidal input signals are removed or nulled by the filter. Therefore it should be possible to use the result in designing an FIR filter that can null a particular sinusoidal input.

Suppose $x[n] = \cos(\hat{\omega}_0 n) = \frac{1}{2}e^{j\hat{\omega}_0 n} + \frac{1}{2}e^{-j\hat{\omega}_0 n}$

Each complex exponential can be removed with first order FIR filter, and then the two filters would be cascaded to form the second order nulling filter that removes the cosine. The second order FIR filter will have two zeros $z_1 = e^{i\pi/3}$ $\quad z_2 = e^{-j\pi/3}$

So the system function will be

$H_1(z) = 1-z_1z^{-1}$

Because $H_1(z)=0$ at $z=z_1$

Similarly $H_2(z) = 1-z_2z^{-1}$ will remove z_2.

So the second order nulling filter will be the product

$H(z) = H_1(z)\, H_2(z)$

So $H(z) = H_1(z)\, H_2(z) = (1-z_1z^{-1})\,(1-z_2z^{-1})$

$$= 1-(z_1 + z_2)z^{-1} + (z_1z_2)z^{-2}$$

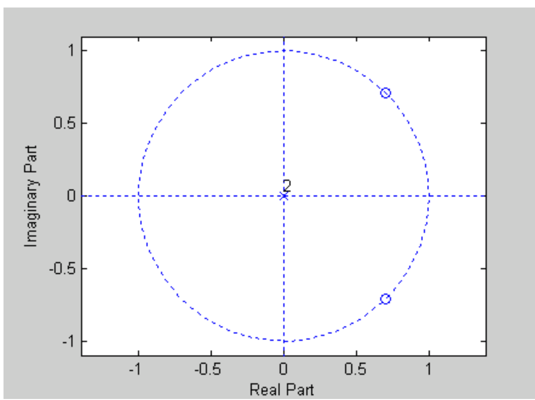
$$= 1-\cos(\hat{\omega}_0)z^{-1} + z^{-2}$$

Figure below shows that two zeros needed to remove a cosine signal with frequency $\hat{\omega}_0 = \pi/4$

So $H(z) = 1-(\cos \pi/4)z^{-1} - z^{-2} = 1-\sqrt{2}z^{-1} + z^{-2}$

Thus the nulling filter that will remove the signal $\cos(0.25\pi n)$ from the input is rthe FIR filter whose difference equation is

$y[n] = x[n] - \sqrt{2}x[n-1] - x[n-2]$.

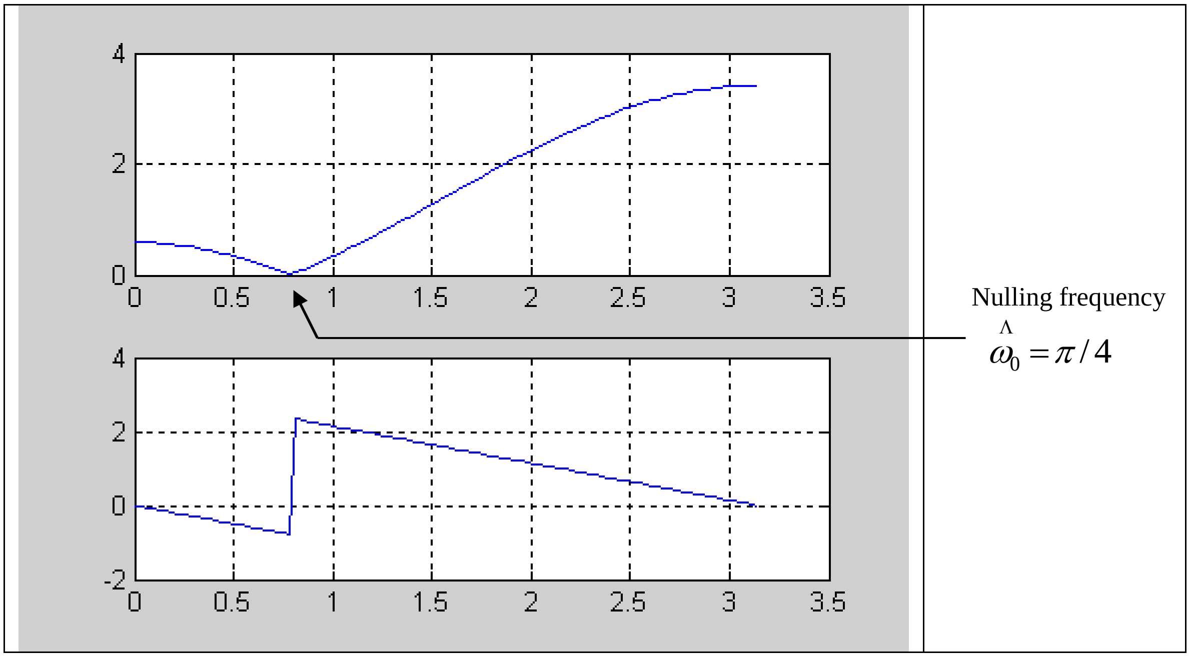


The MATLAB code for drawing the poles and zeros on the unit circle is as follows:

```
b=[1 -1.414 1];
a=[1 0 0];
[z p k]=tf2zp(b,a);
zplane(z,p)
```

Note: tf2zp requires length (b) = length (a)

With the b's are given we can draw the frequency response of the above filter as follows:



Frequency response of the above filter.