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Review for Exams

1 Numerical Linear Algebra

1.1 Convergence of Matrix Splits

Common Problems

- prove that a given matrix split Converges

- prove the gershgorin disc theorem

- Matrix norms (Prove 1s or infinity norm of a matrix)

$$\sum_{k=0}^{n} k^2 \tag{1}$$

$$\sum_{k=0}^{n} k^2 3 \tag{2}$$

Common Methods/tools:

- Gershgorin Disc theorem

- Perron Frobenius theorem

- irreducible diagonally dominant matrices are invertible

- write out the cholesky decomposition

- write out definition (even the stupid supremums or limits)

1.2 Decompositions

Common Problems

- Perform Householder Decompositions

- Prove Multiplication by unitary matrix is backwards stable

- growth factors of LU with pivoting

- Positive definite matrices have nonzero diagonals

- prove existence of SVD

Common tools:

- Write out exact algorithm

- Unitary matrices are preserved under 2 and F norms.

- Definition of Backwards stability

1.3 Error Analysis

Common Problems

- Prove something is stable

- Prove something is backwards stable

- Find the relative error (forward error)

- find the absolute error

- find the (relative) condition number of a problem

Common Tools

2 Numerical Methods for ODEs/PDEs

2.1 Convergence for IVPs

Common Problems:

- Prove that given multi-step method is consistent

- Prove the given multi-step method is stable

- Give an example of why being unstable could ruin things

- prove that something is Lax-richetmeyer stable

Common methods for solving such problems

- Taylor expansion

- $p(s) - z\sigma(s)$

- Neumann Analysis

- equivalence theorems

- Consistency + Lax Richtemeyer stablity $\implies$ Convergence

- Dhalquist Stability theorem

- Prove zero stability

2.2   BVP

Common Problems

- Give the extrapolation method

- Describe method for solving nonlinear equations

- Give error formula

- Write out what the matrices might look like

Tools for solving problems

- Taylor Series

2.3   Advection Equation stuff

Common Problems

- Give the Lax-freidericks, Wendroff, or upwind method

- Determine what the particular method is equivalent to.

- Talk about why the upwind method is the way it is.