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Explanation 1

So what we are trying to do is justify going from:

$$\sqrt{\sum_{j=1}^{n}(\sum_{k=1}^{n} c_{jk}(\xi_k - \zeta_k))^2} \tag{1}$$

$$\tag{2}$$

to:

$$\leq \sum_{k} \sqrt{\sum_{j=1}^{n}(c_{jk}(\xi_k - \zeta_k))^2} \tag{3}$$

in other words why can we take the sum out of the square root? I'll justify this right here. So remember if we have a vector x indexed by x_j then the two norm of this vector is given by:

$$\|x\| = \sqrt{\sum_{j=1}^{n}(x_j)^2} \tag{4}$$

if we set x_j equal to $\sum_{k=1}^{n} c_{jk}(\xi_k - \zeta_k)$ then notice the whole vector x looks like:

$$x = \begin{bmatrix} \sum_{k=1}^{n} c_{1k}(\xi_k - \zeta_k) \\ \vdots \\ \sum_{k=1}^{n} c_{nk}(\xi_k - \zeta_k) \end{bmatrix} \tag{5}$$

rewriting this by factoring out the sum we get:

$$x = \sum_{k=1}^{n} \begin{bmatrix} c_{1k}(\xi_k - \zeta_k) \\ \vdots \\ c_{nk}(\xi_k - \zeta_k) \end{bmatrix} \tag{6}$$

So x is in reality a linear combination of these c vectors. So in total we have:

$$\sqrt{\sum_{j=1}^{n}(\sum_{k=1}^{n} c_{jk}(\xi_k - \zeta_k))^2} \tag{7}$$

$$= \sqrt{\sum_{j=1}^{n}(x_j)^2} \text{ by definition of } x_j \tag{8}$$

$$= \|x\|_2 \tag{9}$$

$$= \left\| \sum_{k=1}^{n} \begin{bmatrix} c_{1k}(\xi_k - \zeta_k) \\ \vdots \\ c_{nk}(\xi_k - \zeta_k) \end{bmatrix} \right\| \text{ by what we did in (6)} \tag{10}$$

From here we can now apply the triangle inequality because of the two norm here:

$$\leq \sum_{k=1}^{n} \left\| \begin{bmatrix} c_{1k}(\xi_k - \zeta_k) \\ \vdots \\ c_{nk}(\xi_k - \zeta_k) \end{bmatrix} \right\| \tag{11}$$

and then by definition of the two norm:

$$= \sum_{k=1}^{n} \sqrt{\sum_{j=1}^{n} (c_j(\xi_j - \zeta_k))^2} \tag{12}$$

So that is how we were able to pull out the sum. Is because there was a hidden two norm.