

```

1  import numpy as np
2  from matplotlib import pyplot as plt
3  from scipy.integrate import solve_bvp
4
5  # set up the right hand side of the ODE, with the parameter
6  # tf (the final time rescaled)
7  def chemo_finite_time(t,y,p):
8      alpha = 1
9      tf=p[0]
10     return np.vstack((tf*alpha*y[0] + tf*0.5*y[1], -tf*alpha*y[1]))
11
12 # set up the endpoint conditions. The final one is  $H(tf) = 2a(tf-1)$ 
13 def bc_new(ya,yb,p):
14     x0 = 2
15     a = 1
16     alpha = 1
17     m = 2
18     tf = p[0]
19     return np.array([ya[0]-x0, yb[1]+m, alpha*yb[0]*yb[1]+.25*yb[1]**2-2*a*(tf-1)])
20
21 # construct the solution using solve_bvp. We use an initial guess
22 # of tf=1 as that is the value we are forcing the solution to be close to.
23 t=np.linspace(0,1,10)
24 y = np.zeros((2, t.size))
25 res = solve_bvp(chemo_finite_time, bc_new, t, y, p=[1])
26
27 # plot the results and print out the optimal final time t_f
28 t_plot = np.linspace(0, 1, 100)
29 x_plot = res.sol(t_plot)[0]
30 p_plot = res.sol(t_plot)[1]
31 plt.plot(t_plot, x_plot)
32 plt.plot(t_plot, -.5*p_plot)
33 plt.legend(['cancer cells', 'chemo concentration'])
34 print('t_f = ' + str(res.p[0]))

```

**Algorithm 21.2:** Algorithm for computing the solution of the Chemotherapy problem in Example 21.3.2 for specific values of the relevant parameters.