Appendix – Project 2

Coefficient of Drag (3D), C_D – Dimensionless quantity relating drag to the lifting bodies shape and free flow

$$C_D = \frac{D}{q_\infty S_{ref}}$$, where S_{ref} represents the reference area

Coefficient of Lift (3D), C_L – Dimensionless quantity relating lift to the lifting bodies shape and free flow

$$C_L = \frac{L}{q_\infty S_{ref}}$$, where S_{ref} represents the reference area

Coefficient of Moment (3D), C_M – Dimensionless quantity relating lift to the lifting bodies shape and free flow

$$C_M = \frac{M}{q_\infty S_{ref}}$$, where S_{ref} represents the reference area

Mean Geometric Chord, $\bar{c}$ – $\bar{c} = \frac{S_{ref}}{b}$

Wing Mean Aerodynamic Chord, c_{mac} – Chord-weighted average chord. Used in stability and control calculations, to normalize pitching moment and to compute static margin.

$$c_{mac} = \frac{2}{S}\int_0^{b/2} c^2 dy$$

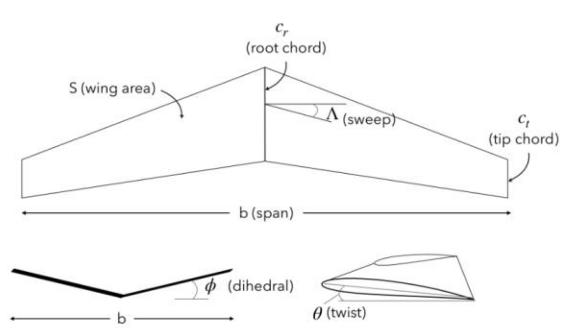
- For a linearly tapered wing - $c_{mac} = \frac{2}{3}(c_r + c_t - \frac{c_r c_t}{c_r + c_t})$

Wing Span, b – Planar projection of the lateral extent of the wing

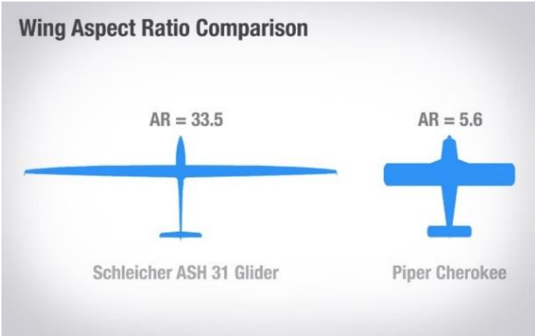
Wing Dihedral, ϕ – Rotation angle from the horizontal. Does not need to be constant along a wing, can be a distribution

Wing Twist, θ – Change in angle of attack along wingspan. Used to adjust lift distribution along a wing. Often used to cause stall to occur at the root before the wing tip.

Wing Sweep, Λ – Measured from quarter chord



Wing Aspect Ratio, AR – $AR = \frac{b^2}{S}$



Taper Ratio, $\boldsymbol{\lambda}$ – $\lambda = \frac{c_t}{c_r}$

Tail Volume Coefficient (Horizontal) – Often used in preliminary sizing of the wing, also known as tail volume ratio (vertical)

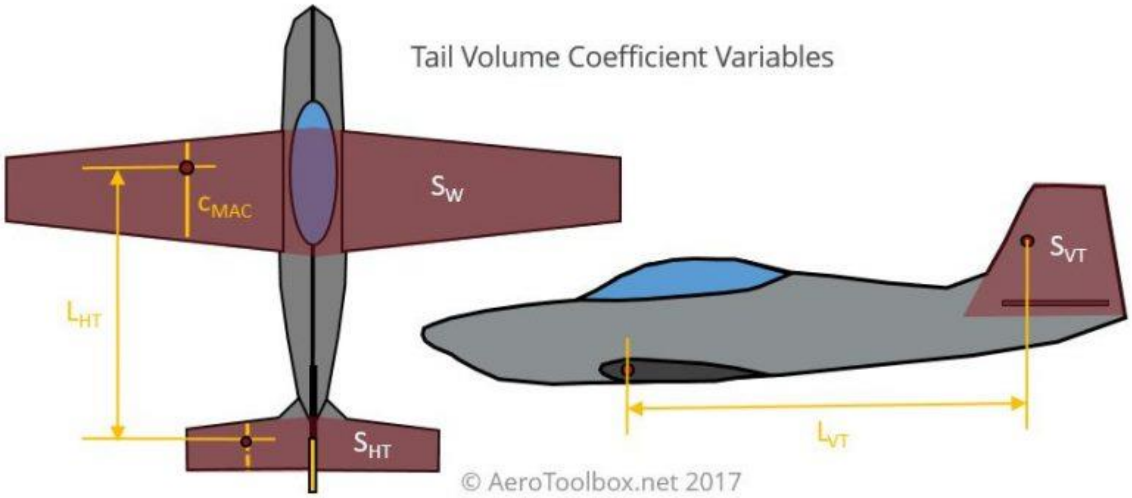
$V_h = \frac{S_h x_h}{S_w c_w}$, where S_h is the horizontal tail area, x_h is the length between the aerodynamic centers of the wing and horizontal tail, S_w is the wing area, and c_w is the mean aerodynamic chord

- Typical values are 0.5 - 0.7

Tail Volume Coefficient (Vertical) – Often used in preliminary sizing of the wing, also known as tail volume ratio (vertical)

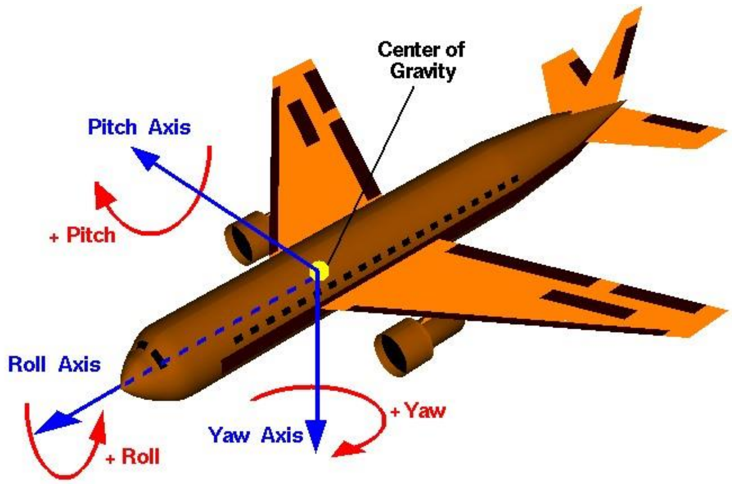
$V_v = \frac{S_v x_v}{S_w b_w}$, where S_v is the vertical tail area, x_v is the length between the aerodynamic centers of the wing and vertical tail, S_w is the wing area, and b_w is the wing span

- Typical values are 0.02 - 0.04



Airframe Stability Derivatives – Measures of how particular forces and moments on an aircraft change as other parameters related to stability change.

Pitch, Roll, Yaw (Visual) –



Longitudinal Static Stability Derivative –

- We want to ensure that this is negative because it prevents the aircraft from pitching upwards/stalling
- Goal is to return the plane to trim (true of all stability derivatives)
- Larger wing can lead to increased stability

$\frac{\partial C_{mcg}}{\partial \alpha} < 0$ or $C_{m,\alpha} < 0$

Yaw Stability Derivative – Also known as directional stability

- This is accomplished via a rudder and a vertical stabilizer
- Not utilized to turn aircraft, utilized to prevent adverse yaw during roll

$\frac{dC_n}{d\beta} > 0$ or $C_{n,\beta} > 0$

- Positive β is in direction of negative yawing moment
- A bigger is ideal, provides more handling

Roll Stability Derivative –

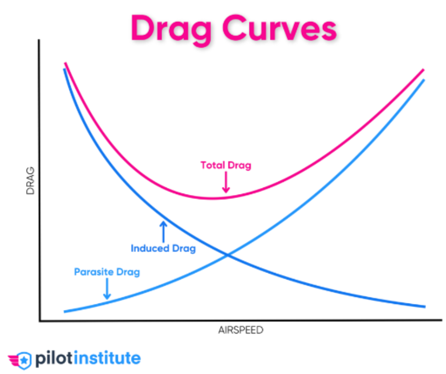
$\frac{dC_{roll}}{d\beta} < 0$ or $C_{roll,\beta} < 0$

- Also known as effective dihedral, dihedral increases roll stability
- Dihedral will create a moment that will want to counteract the given roll through a difference in lift.

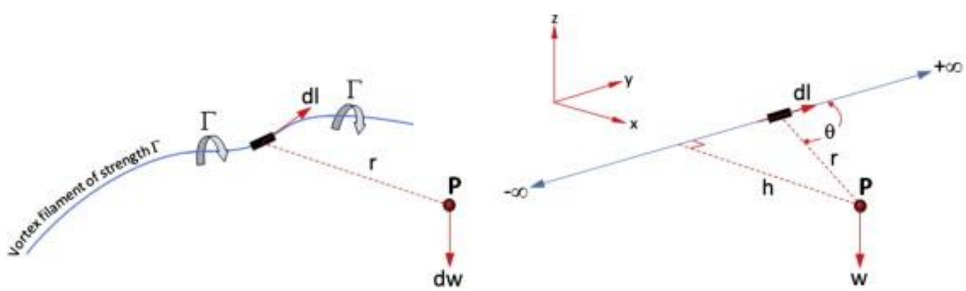
Wake Vortex – Disturbance in the air that forms behind an aircraft

Downwash – Downward moving air behind a wing. Consequence of Newton's 3rd Law. Lift means the air is pushing the body up, therefore the body must also be pushing the air downwards. Reduces the effective angle of attack

Induced Drag – Drag produced by downwash; energy being left behind in the wake. To eliminate this drag, we would need to eliminate the downwash and therefore eliminate lift. Lifting bodies must have induced drag. Also known as *vortex drag.*



Vortex Filament – Arbitrary vortex line segment. Magnitude given by: $|V| = \frac{\Gamma}{4\pi r}$



Biot-Savart Law – Used to compute the velocity induced by a filament of constant vorticity.

Path Integral form: $\vec{V} = \frac{\Gamma}{4\pi}\int \frac{d\vec{l} \times \vec{r}}{|r|^3}$

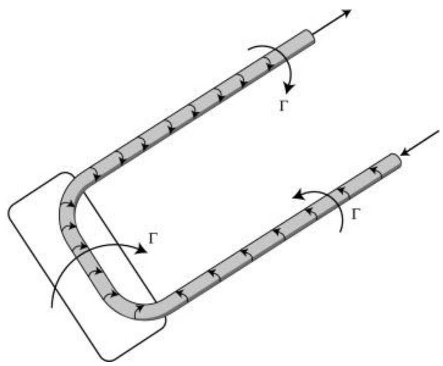
For a straight line segment: $V_\theta = \frac{\Gamma}{4\pi h}(cos\theta_1 - cos\theta_2)$, where θ_1= 0 and $\theta_2 = \pi$ for a infinite vortex and $\theta_2 = \frac{\pi}{2}$ for a semi-infinite vortex

Circulation along a vortex filament must be constant

<u>Helmholtz's vortex theorems</u> –

1) Circulation along a vortex filament must be constant
2) Filament cannot just suddenly end in the fluid
3) The segments must form a closed path, or extend to infinity

<u>Horseshoe Vortex</u> – Simplified representation of vortex system present in the flow of air around a wing. Contains bound vortex and two trailing vortices



<u>Lifting Line Theory</u> – Mathematical model used to approximate lift distribution and aerodynamic behavior of finite wings. Utilizes infinite horseshoe vortices. Vorticity is the derivative of the circulation along the lifting line (the line along which vorticity is distributed). Results in a vortex sheet of continuous varying vorticity.

<u>Fundamental Equation of Lifting Line Theory</u> –

$$\frac{2\Gamma(y)}{V(y)c(y)} = m(y)[\alpha(y) - \alpha_0(y) - tan^{-1}(\frac{1}{4\pi V(y)}\int_{-b/2}^{b/2}\frac{\gamma(y')dy'}{(y-y')})]$$

Allows for solving for the resulting lift distribution

<u>Elliptical Lift Distribution</u> – Efficient design from an induced drag standpoint. Produces constant downwash. Elliptic lift distribution is the minimum induced drag solution for a fixed lift and span

Induced drag – $D_i = \frac{L^2}{q_\infty \pi b^2}$

<u>Method of Restricted Variations</u> – Begin with an arbitrary lift distribution and add differential amounts of lift at arbitrary locations. Allows for best wing shape by small adjustments within set limitations

Inviscid Span Efficiency, e_{inv} – Measure of how close the lift distribution is to elliptic ($e_{inv} \leq 1$)

$$e_{inv} = \left(\sum_{n=1}^{N} n \left(\frac{A_n}{A_1}\right)^2\right)^{-1}$$

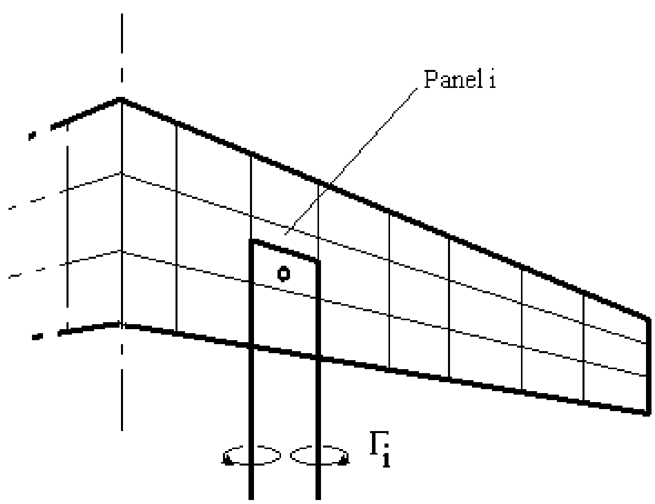
General Induced Drag Equation –

Dimensional – $D_i = \frac{L^2}{q\pi b^2 e_{inv}}$ Non-Dimensional – $C_{Di} = \frac{C_l^2}{\pi AR e_{inv}}$

For an elliptical distribution, $e_{inv} = 1$

Vortex Lattice Method (VLM) – Essentially a 3D extension of the thin airfoil theorem

- Lifting surface is divided into panels, each with a horseshoe vortex
- Uses thin airfoil assumption
  - Lifting surfaces are flat
  - Changes in twist are handled in boundary condition, not the geometry
- Each horseshoe vortex has constant strength
- Uses a lumped vortex method – distributed vorticity along each panel is all lumped into one vortex
- Flow tangency applied at control points



Trimmed – An aircraft is considered trimmed if the sum of its forces and moment are zero; it is in static equilibrium

Center of Pressure – Location where the resultant moment is zero. Sometimes thought of as where the resultant pressure field acts. Moves around as aircraft changes angle of attack

Aerodynamic Center – Location where the moment does not change with angle of attack. Typically, near or at quarter chord.

$$\frac{dC_m}{d\alpha} = 0$$

Longitudinal Static Stability – Center of gravity must be ahead of the aircraft's aerodynamic center

Static Margin – Measure of the stability of an aircraft. A typical static margin is 10–15% for general aviation, 5–10% for commercial/military transports, and < 0 for modern fighters

$$\frac{x}{c} = -\left(\frac{dC_m}{dC_L}\right)_{cg}$$

Neutral Point – Cg location on the aircraft where the static margin is zero

Reference Area – Flat projected area that scales with the wing. Primarily used for normalization purposes. Typically uses a trapezoidal shape

Resources

- Twist – https://www.fzt.haw-hamburg.de/pers/Scholz/HOOU/AircraftDesign_7_WingDesign.pdf
- Tail volume coefficient – https://aerotoolbox.com/design-aircraft-tail/