Homework 1 (CS 345, Summer 2016). Due at the start of class on June 1.

Turn in a single document with your solutions, either as a paper document (in class) or electronic document (to D2L dropbox). Solutions should be clear and neat, whether handwritten or typeset. Show your work, write clear sentences, and include clear diagrams if necessary.

1. (15 points.) Indicate whether each proposition below is true or false, and explain your answer.

(a) $\exists\, m, n \in \mathbb{N}\ (n^2 + m^2 = 5)$

(b) $\forall\, n \in \mathbb{N}\ \exists\, m \in \mathbb{N}\ (n < m^2)$

(c) $\exists\, n \in \mathbb{N}\ \forall\, m \in \mathbb{N}\ (n^2 < m)$

(d) $\forall\, x \in \mathbb{R}\ \exists\, y \in \mathbb{R}\ (x = y^2)$

(e) $\forall\, x \in \mathbb{R}\ \exists\, y \in \mathbb{R}\ (x < 0 \ \vee\ x = y^2)$

2. (10 points.) Suppose we want to rewrite the Java-style pseudocode in Box A to flip the positions of *then* and *else* clauses. A partial answer is already shown in Box B – just provide the contents for the blank line. Use DeMorgan's Law. The final result in box B should be logically identical to the pseudocode in box A.

Box A	Box B

`if (dog.is_barking() || window.glass.is_broken())`
`{`
`    call(police); // be alert`
`}`
`else`
`{`
`    watch(netflix); // relax`
`}`
	`if ( ________?________ )`
	`{`
	`    watch(netflix); // relax`
	`}`
	`else`
	`{`
	`    call(police); // be alert`
	`}`

3. (15 points.) We define sets A, B, E using set-former notation like so. (If you need to review set notation and terminology, see Shaffer §2.2.1.)

$$A = \{\, m^2 \mid m \in \mathbb{Z} \ \wedge\ 0 \le m \le 5 \,\}$$
$$B = \{\, 3n{+}7 \mid n \in \mathbb{N} \ \wedge\ n^2 < 20 \,\}$$
$$E = \{\, n \in \mathbb{Z} \mid \exists\, m \in \mathbb{Z}\ (n = 2m) \,\}$$

(a) What is the common name for set E?

(b) Enumerate the members of set $A \cup B$.

(c) Enumerate the members of set $A \cap E$.

(d) Enumerate the members of set $(A - E) \times (B - E)$.

(e) Define any proper subset of E using set-former notation.

4. (20 points.) Prove the following claims using mathematical induction.

(a) $\forall\, n \in \mathbb{N}\ (n^2 + n$ is even).

(b) $\forall\, n \in \mathbb{N}\ (3^n - 1$ is even).

(c) $\forall\, n \in \mathbb{Z}^+, \left(\begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix} \right)$, where a denotes some nonzero real number.

(d) $\forall\, n \in \mathbb{N}\ (n \le 8 \ \vee\ n! > 4^n)$.

5. (20 points.) Recall that a k-ary tree is a rooted tree where each node has up to k children (for some positive integer k).

(a) Write a recursive characterization of a k-ary tree with height $h \geq 0$. (Hint: generalize from the root+subtrees recursive characterization of binary trees.)

(b) A <u>complete k-ary tree</u> is a k-ary tree where all the tree's leaves have the same depth, and each internal node has k children. Write a conjecture about how many leaves are present in a recursive characterization of a complete k-ary tree with height $h \geq 0$.

(c) Prove your conjecture using structural induction.

6. (20 points.) Below is pseudocode for a sorting algorithm called BubbleSort, with time-cost coefficients shown to the left of each line.

(a) Use step-counting to find how much time $T(n)$ is used by BubbleSort to sort an array of size n. Assume best-case input (i.e., whatever input data that would use the least time).

(b) (+5 points extra credit). Repeat, but assume worst-case input.

	`BubbleSort(A[1..n])`
c_1	`for i ← 1 to n-1`
c_2	`   for j ← 1 to n - i`
c_3	`      if A[n-k] > A[n-k+1]`
c_4	`         temp ← A[n-k]`
c_5	`         A[n-k] ← A[n-k+1]`
c_6	`         A[n-k+1] ← temp`