

$$C = (A_2 - Y)^2$$

chain rule

$$A_2 = \text{sigmoid}(z_2)$$

$$\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx}$$

$$z_2 = w_2 A_1 + b_2$$

$$A_1 = \text{relu}(z_1)$$

$$\frac{dc}{dw_2} = \frac{dz_2}{dw_2} \times \frac{dA_2}{dz_2} \times \frac{dC}{dA_2}$$

$$z_1 = w_1 x + b_1$$

$$\frac{dc}{dw_2} = (A_1) (\sigma(x)(1 - \sigma(x))(2A_2 - 2y))$$

sigmoid prime delta wrt derivative
sigmoid

$$\frac{dc}{dB_2} = \frac{dz_2}{dB_2} \times \frac{dA_2}{dz_2} \times \frac{dc}{dA_2} = (1) \times (\sigma(z_2)(1 - \sigma(z_2))) \times (2A_2 - 2y)$$

$$\frac{dc}{dw_1} = \frac{dz_1}{dw_1} \times \frac{dA_1}{dz_1} \times \frac{dA_2}{dz_1} \times \frac{dc}{dA_2} =$$

$$= (x) (z_1 > 0) \cdot w_2 \cdot \sigma(z_2)(1 - \sigma(z_2)) \cdot (2A_2 - 2y)$$

$$\frac{dc}{dB_1} = \frac{dz_1}{dB_1} \times \frac{dA_1}{dz_1} \times \frac{dz_2}{dA_1} \times \frac{dA_2}{dz_2} \times \left(\frac{dc}{dA_2} \right) =$$

$$= (1) (z_1 > 0) \cdot w_2 \cdot \sigma(z_2)(1 - \sigma(z_2)) \cdot (2A_2 - 2y)$$

sigmoid prime wrt derivative

\leftarrow delta \rightarrow

previous layer

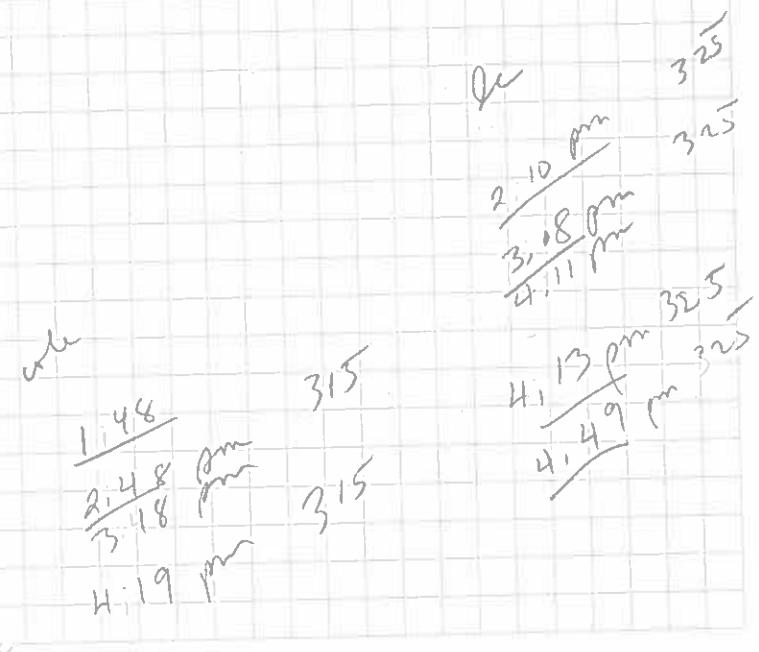
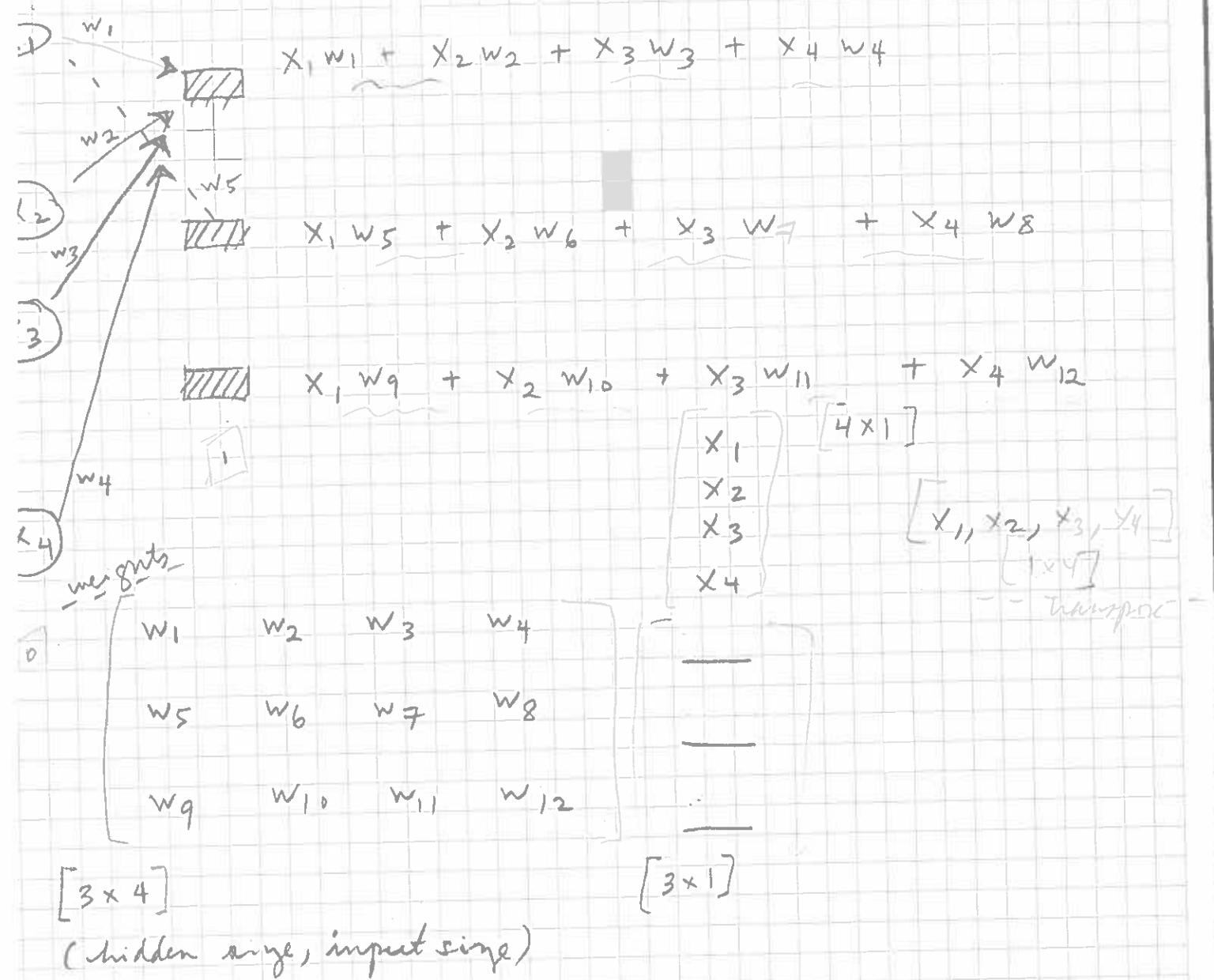
Equation depicts simple two layer neural network, A1 is the output of the first layer and input to the second layer. W and B are the weights (determining inputs importance) and bias (which shifts the activation function output). For a single neuron, this is the sum of each input multiplied by its weight plus bias and for entire layer of neurons this is done via matrix multiplication $Z = WX + B$

Z is the Weighted Sum, or output for a single Neuron, before Activation function

A is the result of the activation function which introduces non-linearity, allowing the network to learn complicated patterns

C is the Loss Function which measures how far the network's prediction is from the target

dC/db and dC/dw are the gradient descent updates used to adjust weights and biases



$$\begin{bmatrix} 10, 20, 30, 40 \end{bmatrix}_{(1,4)} \quad \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad \leftarrow \quad [4,1)$$

$$10 * 0 + 20 * 1 + 30 * 2 + 40 * 3$$

$$0 + 20 + 60 + 120$$

$$80 + 120$$

$$200$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix} \quad (1 \times 4)$$

$$\begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \end{bmatrix} \quad \begin{bmatrix} 0 & 10 & 20 & 30 \\ 0 & 20 & 40 & 60 \\ 0 & 30 & 60 & 90 \\ 0 & 40 & 80 & 120 \end{bmatrix}$$

(4×1)

$$z_1 = w_{\text{hidden}} X + b_{\text{hidden}}$$

$$\begin{bmatrix} 0.05629 & 0.596 & 0.238 \\ 0.303 & 0.421 & 0.855 \\ 0.655 & 0.457 & 0.618 \end{bmatrix}$$

w_{hidden}
 3×3