

$$C = (A_2 - Y)^2$$

chain rule

$$A_2 = \text{sigmoid}(z_2)$$

$$\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx}$$

$$z_2 = w_2 A_1 + B_2$$

$$A_1 = \text{ReLU}(z_1)$$

$$\frac{dC}{dw_2} = \frac{dz_2}{dw_2} \times \frac{dA_2}{dz_2} \times \frac{dC}{dA_2}$$

$$z_1 = w_1 x + B_1$$

$$\frac{dC}{dw_2} = (A_1) \left(\sigma(x) (1 - \sigma(x)) \right) (2A_2 - 2Y)$$

sigmoid prime
delta
sigmoid
cost derivative

$$\frac{dC}{dB_2} = \frac{dz_2}{dB_2} \times \frac{dA_2}{dz_2} \times \frac{dC}{dA_2} = (1) \times (\sigma(z_2) (1 - \sigma(z_2))) (2A_2 - 2Y)$$

delta

$$\frac{dC}{dw_1} = \frac{dz_1}{dw_1} \times \frac{dA_1}{dz_1} \times \frac{dz_2}{dA_1} \times \frac{dA_2}{dz_2} \times \frac{dC}{dA_2}$$

$$= (x) (z_1 > 0) \cdot w_2 \cdot \sigma(z_2) (1 - \sigma(z_2)) \cdot (2A_2 - 2Y)$$

$$\frac{dC}{dB_1} = \frac{dz_1}{dB_1} \times \frac{dA_1}{dz_1} \times \frac{dz_2}{dA_1} \times \frac{dA_2}{dz_2} \times \frac{dC}{dA_2}$$

$$= (1) (z_1 > 0) \cdot w_2 \cdot \sigma(z_2) (1 - \sigma(z_2)) \cdot (2A_2 - 2Y)$$

sigmoid prime

cost derivative

delta

previous layer

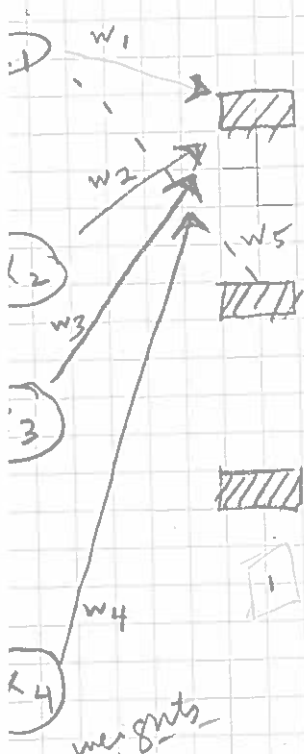
Equation depicts simple two layer neural network, A_1 is the output of the first layer and input to the second layer. W and B are the weights (determining inputs importance) and bias (which shifts the activation function output). For a single neuron, this is the sum of each input multiplied by its weight plus bias and for entire layer of neurons this is done via matrix multiplication $Z = WX + B$

Z is the Weighted Sum, or output for a single Neuron, before Activation function

A is the result of the activation function which introduces non-linearity, allowing the network to learn complicated patterns

C is the Loss function which measures how far the network's prediction is from the target

dC/db and dC/dw are the gradient descent updates used to adjust weights and biases



$$X_1 w_1 + X_2 w_2 + X_3 w_3 + X_4 w_4$$

$$X_1 w_5 + X_2 w_6 + X_3 w_7 + X_4 w_8$$

$$X_1 w_9 + X_2 w_{10} + X_3 w_{11} + X_4 w_{12}$$

x_1
 x_2
 x_3
 x_4

$[4 \times 1]$

$[x_1, x_2, x_3, x_4]$
 $[1 \times 4]$

transpose

weights

w_1	w_2	w_3	w_4
w_5	w_6	w_7	w_8
w_9	w_{10}	w_{11}	w_{12}

$[3 \times 4]$

$[3 \times 1]$

(hidden size, input size)

1.48
 2.48
 3.18
 4.19
 315
 315
 325
 325
 325
 325

$$[10, 20, 30, 40]_{(1,4)} \quad \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \leftarrow [4,1] \quad \text{b. int. shape } (4,1)$$

$$10 \times 0 + 20 \times 1 + 30 \times 2 + 40 \times 3$$

$$0 + 20 + 60 + 120$$

$$80 + 120$$

$$200$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 \end{bmatrix} \quad (1 \times 4)$$

$$\begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \end{bmatrix} \begin{bmatrix} 0 & 10 & 20 & 30 \\ 0 & 20 & 40 & 60 \\ 0 & 30 & 60 & 90 \\ 0 & 40 & 80 & 120 \end{bmatrix}$$

$$(4 \times 1)$$

$$Z_1 = W_{\text{hidden}} X + B_{\text{hidden}}$$

$$\begin{bmatrix} 0.05629 & 0.596 & 0.238 \\ 0.303 & 0.421 & 0.855 \\ 0.655 & 0.457 & 0.618 \end{bmatrix}$$

$$W_{\text{hidden}} \quad 3 \times 3$$