Week 9 Notes

L3 - Mode Selection

Selecting Mode of Transportation

Considerations

- When selecting a mode of transportation the following must be considered
  - Cost of Transportation (CFT)
  - Cost for Ordering (CFO)
  - Cost for Inventory (CFI)
- We choose the mode of transportation which has the minimal total cost while satisfying the demand
- For this lecture we will focus on deterministic environment (no uncertainties)

Considerations: Cost for Transportation

- Different modes of transport charge differently based on the item and/or the distance
- Hence, based on the total quantity shipped, the total transportation cost will vary

Say,

C_T → Cost to transport a single unit of the item for a single unit of distance (Rs./unit-distance)

D → Demand for the item in the planning horizon (Units)

D_T → Total distance to transport an item (Distance)

Cost For Transportation (CFT) = C_T * D * D_T (Rs.)

* $\boxed{CFT = C_T D D_T}$

Considerations: Cost for Ordering

- "Lot size" refers to the number of units of an item present in a single order
- Hence, the number of times an item is ordered is affected by the lot-size

Number of Orders = Demand/ Lot-Size

$\boxed{N = D/Q}$

- Furthermore, a mode of transport may specify the quantity that needs to be shipped in every lot.
- Hence, the mode of transport will affect the number of orders

Say,

C_O → Cost for a single order (Rs./order)

D → Demand for the item in the planning horizon (Units)

Q → Lot size (Units/ order)

Number of orders (N) = D/Q

Cost For Ordering (CFO) = C_o * N (Rs.)

* $\boxed{CFO = C_O \frac{D}{Q}}$

Considerations: Cost for Inventory

- The inventory held in the system gets affected by the mode of transport
- This is because,
  - The lot size affects the cycle inventory (= lot size/2)
  - The lead time (time between when the order is shipped and when it is received) affects the pipeline inventory (= demand* lead time)
- Hence, based on the mode of transportation, the total inventory cost incurred will vary

New 2 comp.

Say, → holding cost

C_H → Cost to hold an item (in inventory) for a given amount of time (Rs./unit-time)

D → Demand for the item in the planning horizon (Units)

Q → Lot size (Units/ order)

L → Lead time (Time)

$\boxed{CFI = CI + PI}$

Cost For Inventory (CFI) = Cycle Inventory + Pipeline Inventory = (Q/2)*C_H + (D*L*C_H)

* $\boxed{CI = \frac{Q}{2} \times C_H}$

* $\boxed{PI = DLC_H}$

What is the best "Lot Size"

- The total cost for a given mode of transport is given as
- Total Transportation Cost (TTC) = CFT + CFO + CFI

TTC = {C_T * D * D_T} + {C_o * (D/Q)} + {(Q/2)*C_H + (D*L*C_H)}

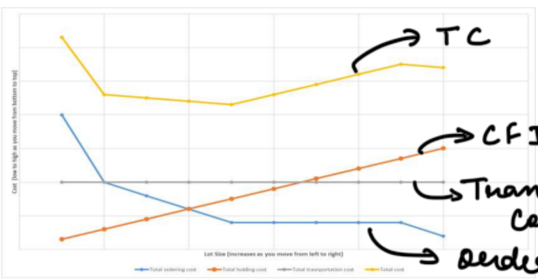
- Now, in the above equation, everything except for "Lot-Size (Q)" is known
- Hence, the question is, what is the "Best Q"
- To answer this question, use the same approach as the one in the EOQ model
- If the approach is followed, the identified Q is

$$\boxed{Q = \sqrt{\frac{2 * D * C_o}{C_H}}}$$

→ same as EOQ model but TTC will differ due to lot size

- The formula indicates that Q is independent of the transport mode choice.
- Nevertheless, in many cases, the mode specifies the lot size and hence it affects costs

Behaviour of Cost Components



→ TC

→ CFI

→ Transport. Cost

→ ordering cost

$$R_Q = \sqrt{\frac{2 \times 1000 \times 200}{3}} = 366$$

Say a company wants to transport a product (P), from its factory in A to a retail store in B. Two modes of transport, "Rail" and "Road" are available. Rail charges Rs. 2/unit-Km and takes 2 days to transport, road also charges Rs. 2/unit-Km but takes only 1 day to transport. The distance between the factory in A and the store in B is 32km via road, and 30 Km via rail. The total demand for the product is 10000 units every year. The cost of holding inventory is Rs. 3/unit-day. Every order sent via rail will cost the company Rs. 200 in paperwork, while for road it is 750. Then

1. Which mode of transport should be chosen by the company?
2. What is the difference in cost between the two modes of transportation?

Given: TC 68 100 ✓ 71500

Component	Rail	Road
Demand (D)	1000 units per year	1000 units per year
Transport distance (D_T)	30 Km	32 Km
Lead time	2 days	1 day
Transport Cost (C_T)	Rs. 2/unit-Km	Rs. 2/unit-Km
Holding cost (C_H)	Rs. 3/unit-day	Rs. 3/unit-day
Order Cost (C_O)	Rs. 200/ order	Rs. 750/ order

$$\sqrt{\frac{2000 \times 750}{3}}$$

$R_Q = 707$

Rail

$CFT = C_T D D_T$

$= 1000 \times 30 \times 2$

$= 60{,}000$

$CFO = \frac{200 \times 1000}{366}$

$= 600$

$CFI = CI + PI = 7500$

$CI = 500 \times 3 = 1500$

$PI = 1000 \times 2 \times 3 = 6000$

Road

$CFT = C_T D D_T = 1000 \times 32 \times 2$

$= 64000$

$CFO = 2 \times 750 = 1500$

$CFI = CI + PI$

$= 500 \times 3$

$= 1500$

$1000 \times 1 \times 3$

3000

4500

Say a company wants to transport a product (P), from its factory in A to a retail store in B. Two modes of transport, "Rail" and "Road" are available. Rail charges Rs. 2/unit-Km and takes 2 days to transport, road also charges Rs. 2/unit-Km but takes only 1 day to transport. The distance between the factory in A and the store in B is 32km via road, and 30 Km via rail. The total demand for the product is 10000 units every year. The cost of holding inventory is Rs. 3/unit-day. Every order sent via rail will cost the company Rs. 200 in paperwork, while for road it is 750. **Furthermore, rail must carry 1000 units in any given lot.** Then

1. Which mode of transport should be chosen by the company?
2. What is the difference in cost between the two modes of transportation?

Rail $= \sqrt{2 \times 1000}$

Given:

Component	Rail	Road ✓
Demand (D)	1000 units per year	1000 units per year
Transport distance (D_T)	30 Km	32 Km
Lead time	2 days	1 day
Transport Cost (C_T)	Rs. 2/unit-Km	Rs. 2/unit-Km
Holding cost (C_H)	Rs. 3/unit-day	Rs. 3/unit-day
Order Cost (C_O)	Rs. 200/ order	Rs. 750/ order
Lot size (Q)	**1000/ order**	*(no restriction)* 707

$CFT = 2 \times 1000 \times 30$

$= 60000$

$CFO = 1 \times 200 = 200$

$CFI = CI + PI$

$CI = 500 \times 3 = 1500$

$PI = 1000 \times 2 \times 3$

$= 6000$

7500

$67{,}700$

↓

$67{,}560$

Thus, you can see how lot size makes a difference in selecting the mode of transport.

L4: TSP Problem

Routing

- When it comes to logistics, an important question is "How should a given set of locations be visited?"
- This is referred to as the routing problem
- The aim of any routing problem is to ensure all the locations are visited (and the demand is satisfied) by attaining
  - Minimal travel distance
  - Minimal use of resources
  - Or both
- In this lecture, we will explore two problems
1. Travelling Salesman Problem (TSP)
2. Vehicle Routing Problem (VRP)
- It is noted that though these problems are NP-Hard, a learner needs to understand the mathematical construct to solve it via any Heuristic technique
- Hence, in this lecture, the focus will be on formulating these problems mathematically

Travelling Salesman Problem (TSP)

- In this problem, a salesman needs to visit "n" nodes and then only return to the starting point
- The salesman can start from any node (from the "n" possible nodes)
- In the TSP, it is assumed that the salesman can go from any node to another node
- The salesman must not perform any sub-tours (form closed paths that does not include all nodes)
- Then, the question is "What route should the salesman take to visit all the cities, such that the total distance travelled is minimal"

TSP – Verbal Formulation

Sets & Parameters:

i, j → set of nodes {1,2,3…n}

D_{ij} → distance between node-i and node-j *(Note: $D_{i,i}$ is set to a large value)*

Decision variable:

X_{ij} = 1 if the salesman visits node-j immediately after visiting node-i

X_{ij} = 0 otherwise

U_i → Auxiliary variable to eliminate sub-tours

Objective Function:

The total distance travelled by the salesman must be minimized

Constraints:

1. The salesman must visit all the nodes exactly once
2. The salesman must not do any sub-tours (sub-tour elimination constraints)
3. Non-Negativity

TSP – Mathematical Formulation

Objective Function:

The total distance travelled by the salesman must be minimized

$$\boxed{Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} D_{i,j} * X_{i,j}}$$

Constraints:

1. *The salesman must visit all the nodes exactly once*

$$\sum_{j=1}^{n} X_{i,j} = 1\ \forall\ i\ \in \{1,\dots,n\}$$

$$\sum_{i=1}^{n} X_{i,j} = 1\ \forall\ j\ \in \{1,\dots,n\}$$

2. *The salesman must not do any sub-tours*

$$U_i - U_j + n * X_{i,j} \le n - 1\ \forall\ i \in \{1,\dots,n-1\}\ j \in \{2,\dots,n\}\ and\ i \ne j$$

3. *Non-Negativity* $U_j \ge 0, X_{i,j} \in \{0,1\} \forall i,j$

L5: Vehicle Routing Problem

Vehicle Routing Problem (VRP) – Introduction

- In the TSP, a single salesman needs to visit all the nodes and return back
- The TSP does not consider
  - Any capacity restrictions
  - Multiple salesmen (or vehicles)
  - Does not consider a specific (set) of starting points
- Thus, the Vehicle Routing Problem (VRP) is an extension of the TSP
- The simplest form of the VRP *(Single depot VRP)* is one where
  - There is a single depot
  - There are "n" nodes (customers)
  - Each node requires a certain quantity that needs to be delivered from the depot
  - There are a fixed number of vehicles available, and the capacity of the vehicles is known
  - The distance (between the nodes, as well as from the depot to the nodes) is known
  - The vehicles must start from the depot and return to the depot after visiting the customer nodes
- There are also Multiple depot VRPs, where the customers can be served by any vehicle from any depot
- In this lecture, the discussion is only on the Single Depot VRP

Single Depot VRP – Types

we use this for now

- In the Single Depot VRP there are three types of problems which are possible

1. All the vehicles have to be used (it is assumed that every vehicle will have at least 1 node attached to it)
2. Minimum number of vehicles must be used
3. Using enough vehicles such that the total distance travelled is minimum

Single Depot VRP – Verbal Formulation

- There are several ways to formulate the VRP
- The following formulation is based on Achuthan and Caccetta (1991), 'Integer linear programming formulation for a vehicle routing problem', *European Journal of Operational Research*, Vol. 52, pp. 86-89.
- In this formulation, the number of vehicles is known and all vehicles must be used

Sets & Parameters:

i, j → set of nodes {1,2,3…n}

D_{ij} → distance between node-i and node-j *(Note: $D_{i,i}$ is set to a large value)*

V → set of vehicles {1,2,…k}

Q_j → Demand at node-j

W → Capacity for a Vehicle

new parameters

Decision variable:

$X_{i,j}$ =1 if vehicle travels from node-i to node-j, 0 otherwise

U_i → Auxiliary variable to eliminate sub-tours

Objective function: Minimize the total travel distance

Constraints:

1. Every node must be reached either from the depot (the source) or another node
2. Every node must end either in another customer node or at the depot (the destination)
3. There are exactly V-routes that start from the depot and that terminate at the depot (ensuring all vehicles are used)
4. The load carried in each route does not exceed the vehicle capacity
5. Non-Negativity (Binary variable)

Constraints:

1. Every node must be reached either from the depot (the source) or another node

$$\sum_{i=1, i\ne j}^{n} X_{i,j} + X_{0,j} = 1\ \forall\ j \in \{1,\dots,n\}\ where\ node\ "0"\ representes\ the\ depot$$

2. Every node must end either in another customer node or at the depot (the destination)

$$\sum_{j=1, j\ne i}^{n} X_{i,j} + X_{i,0} = 1\ \forall\ i\ \in \{1,\dots,n\}\ where\ node\ "0"\ representes\ the\ depot$$

3. There are exactly V-routes that start from the depot and that terminate at the depot (ensuring all vehicles are used)

$$\sum_{i=1}^{n} X_{i,0} = V$$

$$\sum_{j=1}^{n} X_{0,j} = V$$

4. The load carried in each route does not exceed the vehicle capacity (W)

$$U_i - U_j + W * X_{i,j} \le W - Q_j\ \forall\ i,j\ \in \{(0,i),(i,j),(i,0): 1 \le i \ne j \le 0\}$$

$$U_{depot\ is\ destination} - U_{depot\ is\ source} = W$$

In the above "1" represents the first node in N and "0" represents the depot

5. Non-Negativity (Binary Decision Variable): $X_{i,j} \in \{0,1\}\ \forall i,j$