Maths - 1 Week - 3

L1 Quaderatic Functions

- A quaderatic func^n $\Rightarrow$ $f(x) = ax^2 + bx + c$, where $a \neq 0$.

necessary condition to be quadratic.

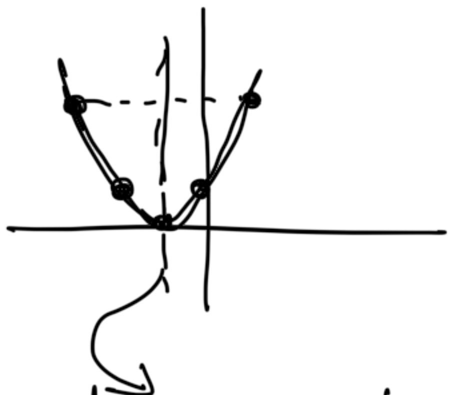
$ax^2 \Rightarrow$ quaderatic
$bx \Rightarrow$ liniar
$c \Rightarrow$ constant

- The graph of quaderatic func^n is called a parabola. How to plot? Use ordered pairs on the coordinate plane.

- Eg. $x^2 + 2x + 1 \Longrightarrow$

1. Table of pairs
2. Plot them
3. connect them

x	y
-2	1
-1	0
0	1
1	4



- all parabolas have an axis of symmetry. ✓ The point at which axis of symmetry intersects the parabola is c/d vertex.
The y-intercept is c.

axis of sym. eq^n $\Rightarrow$ $x = \frac{-b}{2a}$

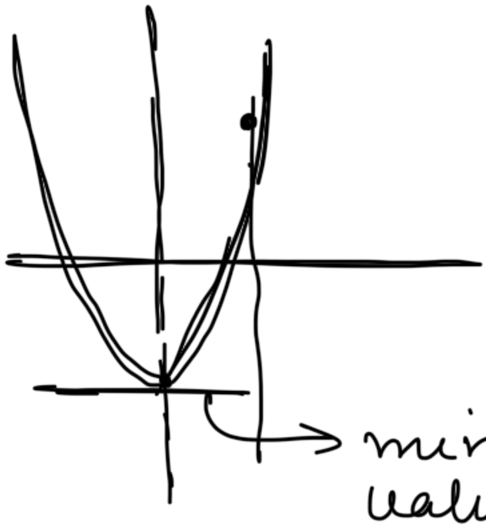
x coordinate of vertex $= \frac{-b}{2a}$.

Eg. $x^2 + 8x + 9$

y intercept $= 9 \Rightarrow 0, 9$

axis $= \frac{-8}{2} = -4$

vertex $= (-4, -7)$

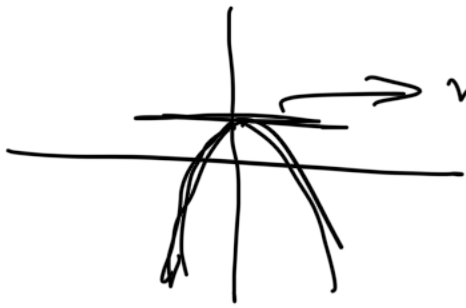


Eg. $-x^2 + 1$

y int $= 1$

axis $= -0 = 0$

vertex $= (0, 1)$



- The y coordinate of the vertex of a particular quadratic func^n will give the max or min.

$a > 0$, opens up & has min value

$a < 0$, opens down & has max value

Range $\Rightarrow R \cap \{ f(x) / f(x) \geq f_{min} \}$ or

$R \cap \{ f(x) / f(x) \leq f_{max} \}$

<u>L2 Eqs. of Quadratic Equations</u>

Eg. $f(x) = x^2 - 6x + 9$

$a > 0$, min value.

axis $\Rightarrow \frac{-b}{2a} = -\frac{-6}{2} = 3$

$f(3) = 9 - 18 + 9 = 0$

$(3, 0)$ Range $= \mathbb{R} \cap \{f(x) / f(x) \geq 0\}$

Eg. 500 customers, ₹40. For each ₹4 hike, you loose 10 customers.

price $= 40 + 4x$

pass $= 500 - 10x$

$I = (500 - 10x)(40 + 4x)$

$= -40x^2 + 1600x + 20000$

max value, $a < 0 \Rightarrow$

x vertex $= \frac{-b}{2a} = 20$

$I(20) = 36000 \rightarrow$ max income possible.

new fare $\Rightarrow 40 + 4x = ₹120$

- Slope of a quadratic func^n

$f(x) = ax^2 + bx + c$, where $a \neq 0$

$y = mx + c$

$\hookrightarrow$ m is the slope.

L3 Slope of a Quadratic Eq^n

$y = x^2 \Rightarrow ax^2 + bx + c$

$a = 1, \; b = 0, \; c = 0$

$\boxed{2ax + b} \Rightarrow$ slope of quad. eq^n

slope $= 2 \times 1 \, x + 0 = 2x$

L4 Solu^n of Quadratic Eq^n using graph

- $ax^2 + bx + c = 0$ → set to a value becomes quad. eq^n.

  $a, b, c \Rightarrow$ integers

  $a \neq 0$ → then this is standard form of equation.

- zeros of eq^n (x - intercept of graph) → are the roots of the quad. eq^n.

- $x^2 + 6x + 8 = 0$

  y int. $= 8$, x vertex $= -3$

  $x^2 + 2x + 4x + 8 = 0$

  $x(x+2) + 4(x+2) = 0$

  $(x+4)(x+2) = 0 \Rightarrow x = -2, -4$

  2 real roots.

- $x^2 + 2x + 1 = 0$

  $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

  $b^2 - 4ac$

  $4 - 4 = 0$

1 real root $\Rightarrow$ $-\frac{2}{2}$ $\Rightarrow$ -1

- $x^2+1=0$ $\Rightarrow$ no real roots.
  ↳ it never intersects the x-axis.

L5 Slope: Line & Parabola

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1} \quad \left(\frac{\text{rise}}{\text{run}}\right)$$

↳ it defines the sensitivity of how the Δ of x will Δ in y.

for $y=mx+c$, m is the slope (linear equaⁿ) which is constant.

- for $y=x^2 \Rightarrow 2x$

for all quadratic equations of the form $ax^2+bx+c=0$

$$\boxed{\text{slope} = 2ax+b}$$

slope $=0 \Rightarrow 2ax+b=0 \Rightarrow x=-\frac{b}{2a}$

when slope becomes 0, the quad. eqⁿ achieves its max. or min. value.

L6 Summary: Quad. Functions

linear eq^n $\Rightarrow$ $y = mx + c$, $m \neq 0$

quad eq^n $\Rightarrow$ $f(x) = ax^2 + bx + c$, $a \neq 0$

- how to plot it?
  - find axis of sym $\Rightarrow$ x vertex
  - calculate y for x vertex
  - find y intercept by $x = 0$.

- $f(x) = x^2 + 4x + 4$

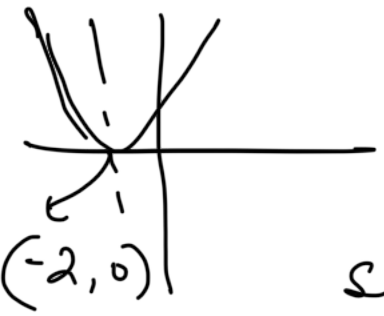
  y int $= (0, 4)$

  x vertex $= \dfrac{-b}{2a} = -\dfrac{4}{2} = -2$

  axis pt. $= (-2)^2 + (-8) + 4$

  $= 0$

  $\Rightarrow (-2, 0)$ only 1 root $\Rightarrow -2$



slope $= 2ax + b$

$= 2 \times 1\, x + 4$

$= 2x + 4$

if we put $-(x^2 + 4x + 4)$

we get this

L7 Solutions of Quadratic Eq^ns using Factorization

$y = f(x) = a(x-p)(x-q)$

p & q are x-intercepts

is c/d intercept form.

Eg. $y = 3(x-1)(x-5)$

$3 \Rightarrow$ axis of sym.

$3(2)(-2) = -12$

$(1,0), (5,0), (3,-12)$

- How to convert intercept into std. form?

we use the FOIL method.

$(ax+b)(cx+d) = ax.cx + ax.d + b.cx + b.d$

Eg. roots $\Rightarrow$ $2/3, -4 \Rightarrow$ std. form

$(x - \frac{2}{3})(x+4) = 0$

$x^2 - \frac{2}{3}x + 4x - \frac{8}{3}$

$3x^2 - 2x + 12x - 8 = 0$

$3x^2 + 10x - 8 = 0$

- Std. form to intercept form

$f(x) = 5x^2 - 13x + 6$

$5x^2 - 10x - 3x + 6$

$5x(x-2) - 3(x-2) = 0$

$(5x-3)(x-2) = 0$

$\underbrace{5(x - 3/5)(x-2) = 0}_{\text{intercept form}}$

Eg. $x^2 = 8x \Rightarrow x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0, 8$

Eg. $x^2 - 4x + 4 = 0 \Rightarrow x^2 - 2x - 2x + 4 = 0$

$x(x-2) - 2(x-2) = 0 \Rightarrow x = -2$

Eg. $x^2 - 25 = 0 \Rightarrow x^2 = 25 \Rightarrow x = \sqrt{25}$

$x = \pm 5$

L8 Solution Using Sq. Method

$x^2 + 10x = 24$

$x^2 + 10x + 25 = 24 + 25$

$(x+5)^2 = 49 = (7)^2$

$x \pm 5 = \pm 7 \Rightarrow x + 5 = \pm 7 \Rightarrow 2, -12$

L9 Quadratic Formula

- when RHS is not a perfect square it leads to irrational roots (still real)

Eg. $(x-2)^2 = 32$

$$x - 2 = \pm\sqrt{32} = \pm 4\sqrt{2}$$

$$x = 2 \pm 4\sqrt{2}$$

for $ax^2 + bx + c = 0$, $a \neq 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \rightarrow \text{quadratic formula}$$

$\boxed{b^2 - 4ac} \rightarrow$ discriminant

$b^2 - 4ac$	roots
> 0	2
< 0	0
$= 1$	1

for > 0 ; perfect sq. $\rightarrow$ 2 rationals

, imperfect sq. $\rightarrow$ 2 irrationals

Eg. $9x^2 - 12x + 4 = 0$

$b^2 - 4ac \Rightarrow 144 - 4 \times 36 = 0 \Rightarrow$ 1 root

$$x = \frac{-b}{2a} = \frac{+12}{18} = \frac{2}{3}$$

Eg. $2x^2 - 16x + 33 = 0$

$b^2 - 4ac \Rightarrow 256 - 2\cancel{64} = -8$ no real roots

- graphing → to verify things
- factoring → if constant term is 0 or factors are easy to find.
- comp. the sq. → use when b is even.
- quadratic formula → use when other methods fail.

L10 Summary

Std. form ⇒ $ax^2 + bx + c = 0$

intercept form ⇒ $a(x-p)(x-q) = 0$

- it is not always possible to convert std. form into the intercept form.

Eg. $f(x) = x^2 + 1$

$$\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a^2} \right)$$

→ these are the coordinates of

the vertex of the quadratic eq^n or the axis of symmetry cuts the quadratic eq^n in half.

L11 Additional Lecture

std. $\Rightarrow$ $y = ax^2 + bx + c$

a, b, c are real & $a \neq 0$

$$\text{Vertex} = \left(-\frac{b}{2a} , c - \frac{b^2}{4a}\right)$$

$$(h , k)$$

$$h = \frac{-b}{2a} , \quad k = c - \frac{b^2}{4a}$$

$$y = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \cancel{a} \frac{b^2}{4a^{\cancel{2}}} + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \underbrace{\left(c - \frac{b^2}{4a}\right)}_{k}$$

$$\boxed{y = a(x-h)^2 + k}$$

↳ vertex form of the parabola.

Eg. $y = 3x^2 + 6x + 9$

vertex $= -\frac{\cancel{6}}{\cancel{6}}, \quad 9 - \frac{\cancel{36}\,3}{\cancel{12}}$

$= (-1, 6)$ is the vertex of the given parabola.

Eg. vertex $= (1, 2)$, origin

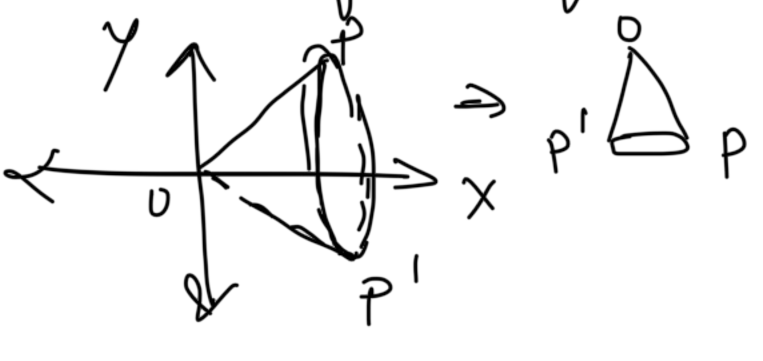
$$y = a(x-1)^2 + 2$$

also, $a = -2$, when $y = 0$

$$y = -2(x-1)^2 + 2$$

$$y = 4x - 2x^2$$

L12 Surface of Revolution



revolution of the line along x-axis creates a cone.