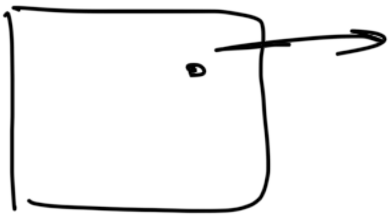
Maths 1 Week -2

L1 Rectangular Coordinate System

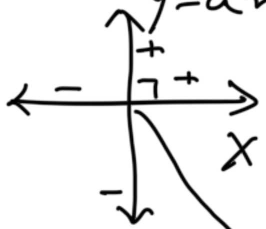


→ how will you describe this pt.

grid lines

we can say this is present in Quadrant I.

(2, 2)



origin ↓ pt. of intersection

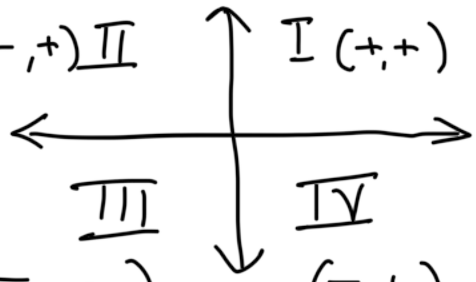
↳ coordinate plane

horizontal line → X-axis

vertical line → Y-axis

any pt. on a horizontal plane is given by (x, y).

Quadrants



→ move in anticlockwise direc^n.

$(5,0) \rightarrow$ on X-axis $(\pm, 0)$ total 6 parts.

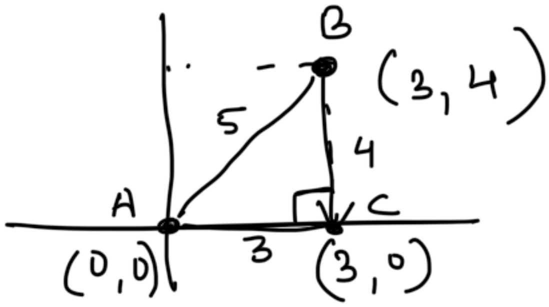
$(0,5) \rightarrow$ on Y-axis $(0, \pm)$

$(0,0) \rightarrow$ origin

L2 Distance Formula

- how to find the dist. of pt. P from origin.

Eg. (3, 4)



from pythagoreus theorem.

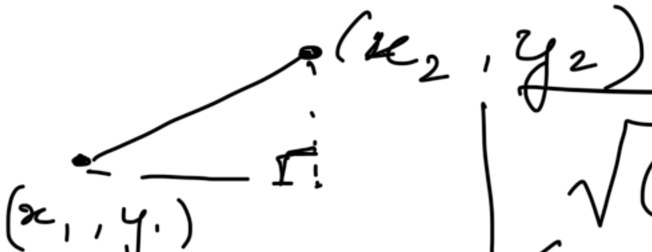
$$AB^2 = BC^2 + AC^2$$

$$= 3^2 + 4^2 = 9 + 16 = 25$$

$$AB = \sqrt{25} = 5$$

- dist. btw. any 2 points.

$P(x_1, y_1)$ and $R(x_2, y_2)$



$$\boxed{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

→ dist. btw. any 2 points. also c/d the distance formula.

L3 Section Formula

- a point P cuts line segment AB in ratio m:n. What is P's coordinates.

$A\ (x_1, y_1)$ $\quad P(x, y)$

$B\ (x_2, y_2)$

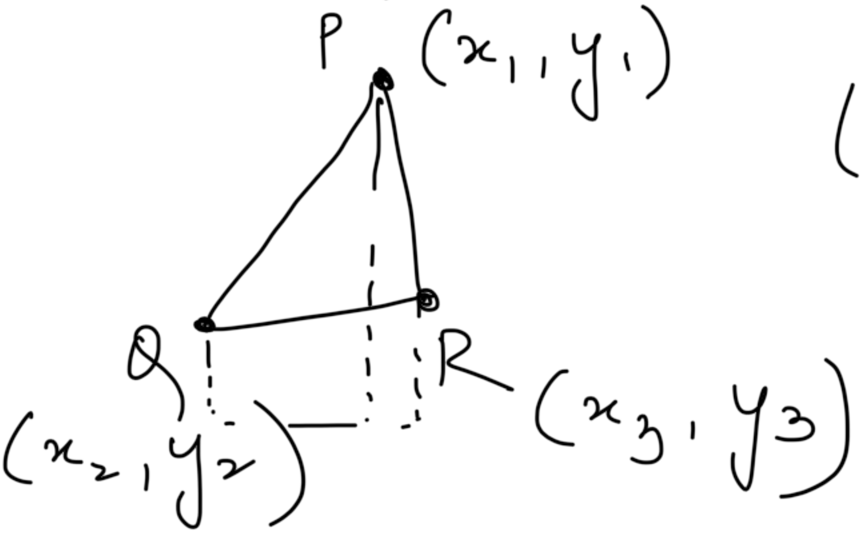
these 2 Δs are similar.

$$\frac{m}{n} = \frac{AP}{PB}$$

$$\frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$$

$$\boxed{x = \frac{m x_2 + n x_1}{m+n}}, \quad \boxed{y = \frac{m y_2 + n y_1}{m+n}}$$

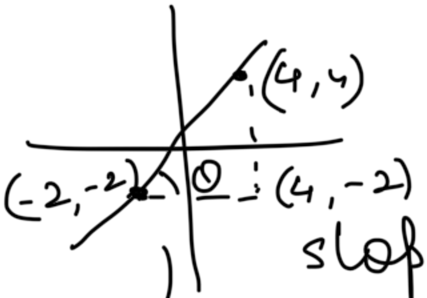
L4 Area of a Δ



(always go in anticlockwise direct^n)

$$\boxed{\text{Area of } \Delta \text{ given by Coordinates} = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|}$$

L5 Slope of a line



2 pts. uniquely determine a line.

slope = $\dfrac{\Delta \text{ in } y \text{ direc}^n}{\Delta \text{ in } x \text{ direc}^n}$

$$\boxed{\tan\theta = m = \frac{y_1 - y_2}{x_1 - x_2}}$$

$m \rightarrow$ slope of the line

$\theta \rightarrow$ inclina^n of the line with $+x$-axis, in anticlockwise direc^n.

$0^\circ \leq \theta < 180^\circ$

- line is parallel, $m = \tan 0^\circ = 0$
- line is $\perp$, $m = \tan 90^\circ$ = undefined
- If θ is the inclina^n of line l, then $\tan\theta$ is the slope or gradient of line l.

$\theta \neq 90^\circ$, then $m = \tan\theta$.

for obtuse θ, $m = \tan(180 - \theta) = -\tan\theta$

$$= \frac{y_1 - y_2}{x_1 - x_2}$$

L6 Parallel & $\perp$ Lines

- can a slope of a line uniquely determine a line?

  No, you can't uniquely determine the line from the slope.

- l_1 & l_2 be non-vertical lines, with slopes m_1 & m_2 with inclinaⁿ α & β.

  $l_1 \parallel l_2 \Rightarrow \alpha = \beta$

  $\Rightarrow \tan\alpha = \tan\beta$

  $\Rightarrow \boxed{m_1 = m_2}$

  If $m_1 = m_2 \Rightarrow \tan\alpha = \tan\beta$

  $0^\circ \le \alpha, \beta \le 180^\circ \Rightarrow \alpha = \beta$

  Two non vertical lines are parallel if slopes are equal.

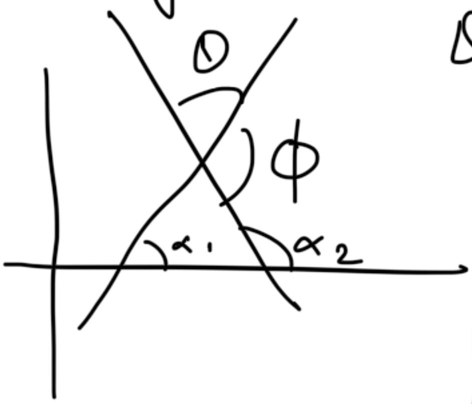
- l_1 & l_2 are 2 non-verticals, with slope m_1 & m_2 with inclinatⁿ α & β

  $l_1 \perp l_2 \Rightarrow 90^\circ + \alpha = \beta$

  $\tan\beta = \tan(90 + \alpha) = -\cot\alpha = \frac{-1}{\tan\alpha}$

  $\boxed{m_1 m_2 = -1}$

- same condⁿ with inclinaⁿ α_1 & α_2. they intersect.



$\theta = \alpha_2 - \alpha_1$, $\alpha_1, \alpha_2 \neq 90°$

$\tan\theta = \tan(\alpha_2 - \alpha_1)$

$$\boxed{\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}} \quad m_1 m_2 \neq -1$$

$$\tan\phi = \frac{m_1 - m_2}{1 + m_1 m_2}$$

L7 Representaⁿ of a Line - 1

- horizontal line → iff. it is ∥ to x-axis.
  ↪ value it takes on y-axis. $\boxed{y = a}$
  (x, a) → all pts. on this line
- vertical line → iff. it's ∥ to y-axis.
  ↪ value it takes on x-axis. $\boxed{x = b}$
  (b, y) → all pts. on this line.

Ques. exp. of line passing thru $(5, 7)$.
Horizontal line ⇒ $y = 7$
Vertical line ⇒ $x = 5$

- Point Slope Form

line l, slope m, point $P(x_0, y_0)$.

$Q(x, y) \rightarrow$ arbitary point.

$$m = \frac{y - y_0}{x - x_0} \Rightarrow \boxed{(y - y_0) = m(x - x_0)}$$

↳ pt. slope form.

Eg. $(5, 6)$, $m = -2$

$$y - 6 = -2(x - 5)$$

$$y - 6 = -2x + 10$$

$$y = 16 - 2x \qquad \begin{aligned} x &= 3 \\ y &= 10 \end{aligned}$$

- 2 Point Form

$\left.\begin{array}{l} P(x_1, y_1) \ \& \ Q(x_2, y_2) \\ \text{and } R(x, y) \end{array}\right\}$ collinear

slope of PR and slope of PQ are equal

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

Two-point form

Eg. eqⁿ of line, (5, 10) & (−4, −2)

(x, y)

$$\frac{y-10}{x-5} = \frac{-2-10}{-4-5} = \frac{-\cancel{12}\,4}{-\cancel{9}\,3}$$

$$\frac{y-10}{x-5} = \frac{4}{3} \Rightarrow 3y - 30 = 4x - 20$$

$$3y = 4x + 10$$

$$y = \frac{4}{3}x + \frac{10}{3}$$

L8 Representaⁿ of line 2

- Slope-intercept form

if line m cuts y-axis at c. This c become y-intercept (0, c).

$$\boxed{y = mx + c} \longrightarrow \text{for y-intercept}$$

$$\boxed{y = m(x-d)} \longrightarrow \text{for x intercept at d. } (d, 0)$$

Eg. eqⁿ of line $m = \frac{1}{2}$, $y = -3/2$

$$y = mx + c$$

$$y = \frac{1}{2}x - \frac{3}{2} \Rightarrow 2y = x - 3$$

- Intercept form.

x-intercept at a & y-intercept at b. Two pts. $(a, 0)$ & $(0, b)$

$$(y-0) = \frac{b-0}{0-a}(x-a)$$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

x intercept   y-intercept.

L9 General Eq^n of Line

- We have studied four types of line versions. Problem with all those equations. It can't handle the vertical lines. Therefore, we have to introduce a general eq^n of line which includes all above 4 eq^{ns} & vertical lines as well.
- Also, in all 4 of them, x & y are the variables that we have to find.

$$\boxed{Ax + By + C = 0} \rightarrow \text{general } eq^n \text{ of line.}$$

↳ linear eq^n in 2 variables.

A, B simultaneously $\neq 0$.

Eg. $3x - 4y + 12 = 0$

$m \Rightarrow \quad 4y = 3x + 12$

$y = \frac{3}{4}x + 3 \qquad m = \frac{3}{4}$

~~x~~ y-intercept $\Rightarrow$ 3 (put $x = 0$, calculate y)

x-intercept $\Rightarrow$ -4 (put $y = 0$, calculate x)

L10 Eqⁿ of $\parallel$ & $\perp$ lines in general form

- if there are 2 lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$, $b_1, b_2 \neq 0$

$\parallel$ lines $\Rightarrow$ $\boxed{a_1b_2 = a_2b_1}$ $\quad m_1 = m_2$

$\perp$ lines $\Rightarrow$ $\boxed{a_1a_2 + b_1b_2 = 0}$ $\quad m_1m_2 = -1$

L11 Eqⁿ of a $\perp$ line passing thru a pt.

Eg. eqⁿ of line $\perp$ to $x - 2y + 3 = 0$, passing thru pt. $(-1, 2)$

$x - 2y + 3 = 0 \Rightarrow m_1 = \frac{1}{2}$

$$m_1 \times m_2 = -1 \Rightarrow m_2 = -2$$

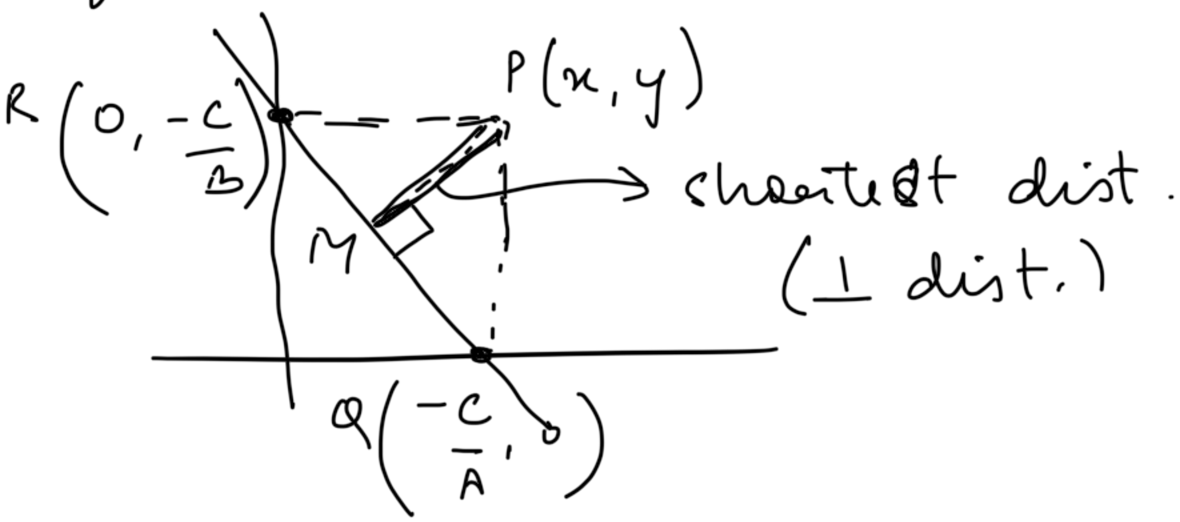
$$y = mx + c \Rightarrow y - 2 = -2(x+1)$$

$$y = -2x$$

4.2 Distance of a line from a given pt.

pt. $P(x_1, y_1)$ from line l $Ax + By + C = 0$

for $A, B \neq 0$,



$$\Delta(PQR) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

or $\Rightarrow$ $$\frac{1}{2}\frac{|C|}{|AB|}\left|Ax_1 + By_1 + C\right|$$

$$QR = \frac{|C|}{|AB|}\sqrt{A^2 + B^2}$$

$$\boxed{PM = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}}$$

- similarly, we can find the dist. btw. any 2 parallel lines.

l_1 & l_2 are || with slopes m.

$y = mx + c_1$

$y = m + c_2$

$$\boxed{\text{dist.} = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}}$$

or $$\boxed{d = \frac{|c_1 - c_2|}{\sqrt{A^2 + B^2}}}$$

Eg. dist. of pt $(3, -5)$ from line $3x - 4y - 26 = 0$

$$d = \frac{|9 + 20 - 26|}{5} = \frac{3}{5}$$

L13 Straight Line Fit

we drop it on the line || to y-axis.

↳ helps to calculate SSE.

SSE ⇒ sum of squared error.

also, with the least SSE, we can conclude which is a better line fit.

$$\boxed{\text{min. } SSE \sum_{i=1}^{n} (y_i - mx_i - c)^2}$$