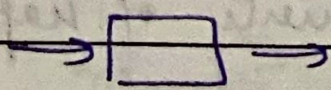
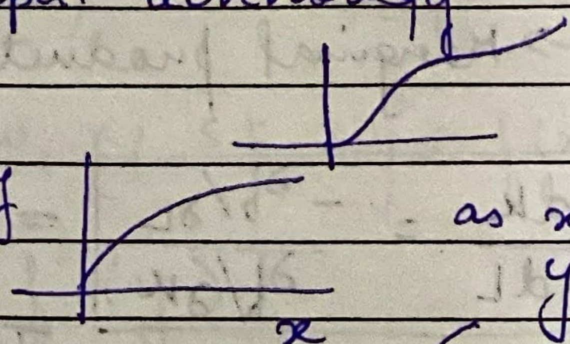
Week-3

L1 <u>Marginal Rate of Technical Substitution</u>

- 1 input 1 output technology



$y = f(x)$ y



as $x\uparrow$
$y\uparrow$ at decreasing rate

diminishing marginal returns

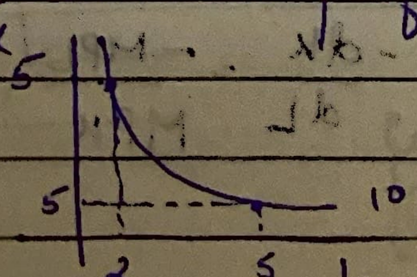
marginal productivity decreases as amt. of input increases.

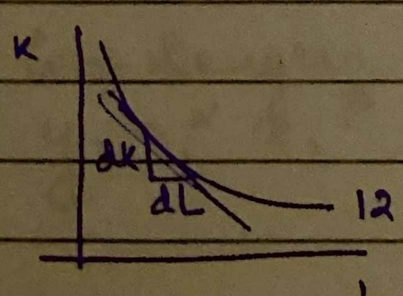
- 2 input 1 output technology

$y = f(L, K)$ — $L \rightarrow$ M $\xrightarrow{x} y$, $K \rightarrow$

if we keep ~~on~~ $K = K_0$ & keep on increasing L, marginal productivity starts declining beyond a pt. & vice-versa.

→ Substitutability of 1 input for other



- 

$\frac{\partial f}{\partial L}$ ⇒ partial derivative of f w.r.t. L

↳ Marginal productivity of labour

$\frac{\partial f}{\partial k} \Rightarrow$ partial derivative of f w.r.t. k

$\hookrightarrow$ Marginal productivity of capital

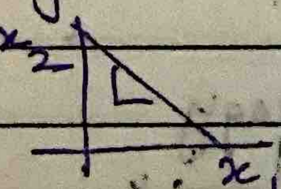
$$\boxed{\frac{dk}{dL} = \frac{-\partial f/\partial L}{\partial f/\partial K}} = MRTS$$

Marginal Rate of Technical Substitution

- Linear Technology

$$y = ax_1 + bx_2 \qquad a>0,\ b>0$$

$$\frac{\Delta x_2}{\Delta x_1} = \frac{a}{b}$$

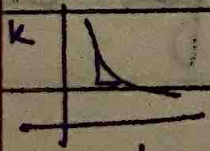


$$\boxed{MRTS = -\frac{MP_1}{MP_2} = -\frac{a}{b}}$$

- Cobb Douglas Tech.

$$y = KL \qquad MRTS = -1$$

$$MPL = \frac{\partial y}{\partial L} = K \qquad MPK = \frac{\partial y}{\partial K} = L$$



$$\boxed{MRTS = -\frac{dK}{dL} = \frac{-MPL}{MPK} = -\frac{K}{L}}$$

L2 Cost Minimization

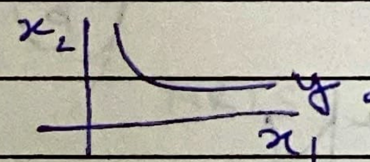
$y = f(x_1 \dots x_n) \to w_i$ per unit

$$\min \sum_{i=1}^{n} w_i x_i \quad \text{s.t.} \quad y \le f(x_1 \dots x_n)$$

$$\boxed{TC = w \cdot f^{-1}(y)}$$

- Linear technology

$y = 2x_1 + x_2$



↳ $y = f(x_1, x_2)$

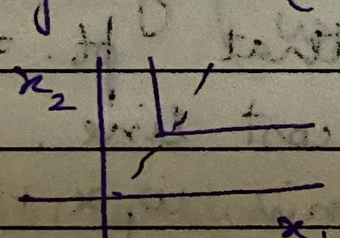
↳ any combination of input 1 & 2 s.t. $2x_1 + x_2 = y$

$$\boxed{C(y) = \min\left(\frac{w_1}{2}, \frac{w_2}{1}\right) \cdot y}$$

- Leontiff Technique

$y = \min(x_1, x_2)$ $\quad x_1 < x_2 \Rightarrow x_1$

$x_2 < x_1 \Rightarrow x_2$



Cost = $w_1 y + w_2 y$

$$\boxed{\text{Cost} = (w_1 + w_2) y}$$

- Cobb Douglas

$y = x_1^{\alpha} x_2^{\beta}$ $\quad \min_{x_1, x_2} w_1 x_1 + w_2 x_2$

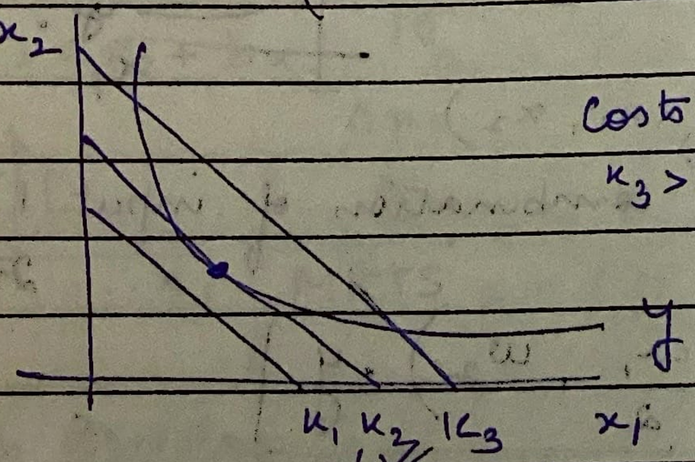
st. $x_1^{\alpha} x_2^{\beta} = y$

$$x_1^* = \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \left(\frac{w_2}{w_1}\right)^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$$

$$x_2^* = \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{w_1}{w_2}\right)^{\frac{\alpha}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$$

$$\text{min. cost} = w_1 x_1^* + w_2 x_2^*$$

$$= \left(\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right) \times w_1^{\frac{\alpha}{\alpha+\beta}} w_2^{\frac{\beta}{\alpha+\beta}} y^{\frac{1}{\alpha+\beta}}$$



$\hookrightarrow$ min. cost (isocost) for this y.

slope of isoquant at that pt. = slope of an isocost line.

$$\boxed{MRTS = -\frac{w_1}{w_2}}$$

$$\min \; w_1 x_1 + w_2 x_2$$
$$\text{s.t. } y = f(x_1, x_2)$$

L3 **Cost Minimization 2**

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \quad \text{s.t.} \quad y \leq f(x_1, x_2)$$

$$w_1 x_1 + w_2 x_2 = k$$

Slope of isoquant = MRTS = slope of isocost $\Rightarrow \frac{MP_1}{MP_2} = \frac{w_1}{w_2} \Rightarrow \boxed{\frac{MP_1}{w_1} = \frac{MP_2}{w_2}}$

Eg $y \leq x_1^{1/3} x_2^{2/3}$

$$MRTS = \frac{MP_1}{MP_2} = \frac{\frac{1}{3} x_1^{-2/3} x_2^{2/3}}{\frac{2}{3} x_1^{1/3} x_2^{-1/3}} = \frac{-x_2}{2x_1}$$

slope of isoquant $= \frac{-w_1}{w_2}$

$$\frac{-w_1}{w_2} = \frac{-x_2}{2x_1} \Rightarrow x_2 = \frac{w_1}{w_2} \cdot 2x_1$$

$$y_0 = x_1^{1/3} \cdot \left(\frac{w_1}{w_2}\right)^{2/3} \times (2x_1)^{2/3}$$

$$= \left(\frac{2w_1}{w_2}\right)^{2/3} x_1$$

$$x_1^* = \frac{y_0}{\left(\frac{2w_1}{w_2}\right)^{2/3}}$$

- be careful about the corner solutions, where tangency criterion is not satisfied.

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L4 Cost Function

- Linear Tech.

$$y = ax_1 + bx_2$$

$$\boxed{\text{min. cost} = \min\left(\frac{w_1}{a}, \frac{w_2}{b}\right) \cdot y}$$

$$x_1^* = \frac{w_1}{a} \cdot y \quad \text{if } \frac{w_1}{a} < \frac{w_2}{b}$$

$$= 0 \quad \text{if } \frac{w_1}{a} > \frac{w_2}{b}$$

$$= y - bx_2^* \quad \text{if } \frac{w_1}{a} = \frac{w_2}{b}$$

$$x_1^* / x_2^* = x_1^* / x_2^* (w_1, w_2, y)$$

↳ conditional input demand funcⁿ.

$$C = C(w_1, w_2, y)$$

- Leontiff

$$y = \min(ax_1, bx_2)$$

$$C^* = C^*(w_1, w_2, y) = \boxed{\left(\frac{w_1}{a} + \frac{w_2}{b}\right) \cdot y}$$

- Cobb Douglas

$$y = x_1^\alpha x_2^\beta$$

$$\boxed{C^* = \left(\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right) w_1^{\frac{\alpha}{\alpha+\beta}} w_2^{\frac{\beta}{\alpha+\beta}} \, y^{\frac{1}{\alpha+\beta}}}$$

$$y = ax_1 + bx_2 \Rightarrow AC(y) = \min\left(\frac{w_1}{a}, \frac{w_2}{b}\right)$$

$$y = \min(ax_1, bx_2) \Rightarrow AC(y) = \left(\frac{w_1}{a} + \frac{w_2}{b}\right)$$

$$y = x_1^\alpha x_2^\beta \Rightarrow AC(y) = \left(\left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\right) w_1^{\frac{\alpha}{\alpha+\beta}} w_2^{\frac{\beta}{\alpha+\beta}}$$

if $\alpha + \beta = 1$

L5 **Scale**

- $y = f(x_1, x_2)$ $\quad (x_{10}, x_{20}) \rightarrow (\lambda x_{10}, \lambda x_{20})$

$$f(\lambda x_{10}, \lambda x_{20}) = \lambda f(x_{10}, x_{20}) \quad \text{CRS}$$
$$> \lambda f(x_{10}, x_{20}) \quad \text{IRS}$$
$$< \lambda f(x_{10}, x_{20}) \quad \text{DRS}$$

- CRS

$$\left.\begin{array}{l} x_{10}, x_{20} \rightarrow y_0 \\ 2x_{10}, 2x_{20} \rightarrow 2y_0 \end{array}\right\} \text{replica}^n$$

IRS

$8\times$ ($1 \times 1 \times 1 \Rightarrow 1\,m^3$ → $2 \times 2 \times 2 \Rightarrow 8\,m^3$) $\quad 4\times$ ($6 \times 1 \times 1 = 6\,m^2$ → $6 \times 2 \times 2 = 24\,m^2$)

- CRS

$$y = ax_1 + bx_2 \qquad\qquad y = \min(ax_1, bx_2)$$
$$= a\lambda x_1 + b\lambda x_2 \qquad\qquad = \min(a\lambda x_1, b\lambda x_2)$$
$$= \lambda(ax_1 + bx_2) \qquad\qquad = \lambda \min(ax_1, bx_2)$$
$$= \lambda y \qquad\qquad = \lambda y$$

- $y = x_1^{\alpha} x_2^{\beta} \Rightarrow \lambda^{\alpha+\beta} x_1^{\alpha} x_2^{\beta}$

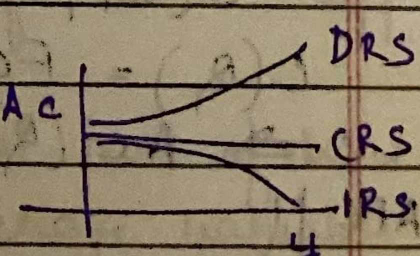
$\alpha + \beta = 1 \Rightarrow$ CRS

$\alpha + \beta > 1 \Rightarrow$ IRS

$\alpha + \beta < 1 \Rightarrow$ DRS

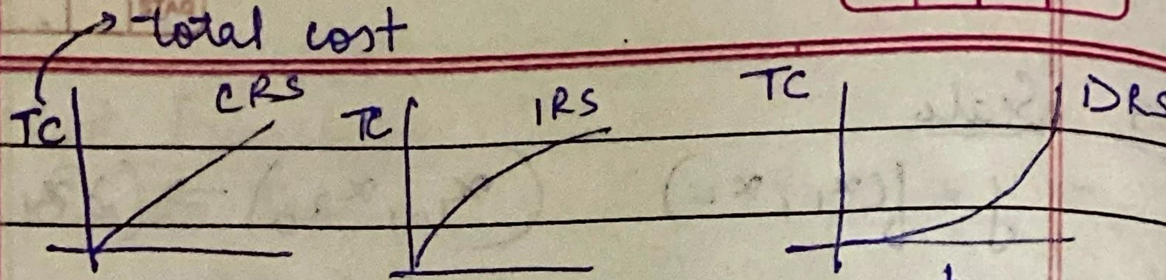
- $AC = \dfrac{C(w_1, w_2, y)}{y}$

→ avg. cost



AC decreases incase of IRS

" increases incase of DRS



increases at decreasing rate

increases at increasing rate

- short run scale – when you can't change at least 1 factor of product^n

long run – all factors can be varied

- Return to scale – high volume for a single produce
- Return to Scope – multiple products using 1 firm

$$c(S_1 + S_2) \leq c(S_1) + c(S_2)$$

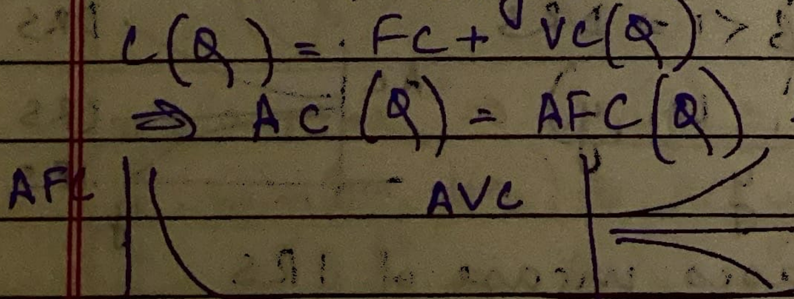
L6 Firm Cost Curves to Supply Func^n

- min $w_1 x_1 + w_2 x_2$ s.t $y \leq f(x_1, x_2)$

$$c(w_1, w_2, y)$$

$$c(Q) = FC + VC(Q)$$

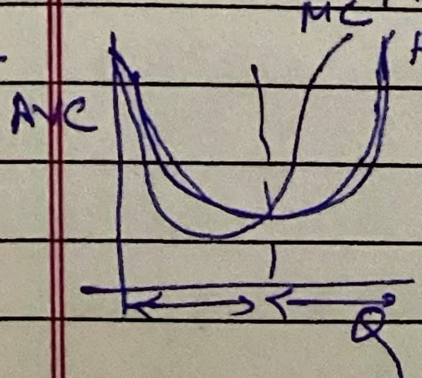
$$\Rightarrow AC(Q) = AFC(Q) + AVC(Q)$$



$\frac{dAC}{dQ} = \frac{1}{Q}\left[MC - AC\right]$. $MC > AC \Rightarrow AC\uparrow$

$MC < AC \Rightarrow AC\downarrow$

$MC = AC$, AC is constant

- 

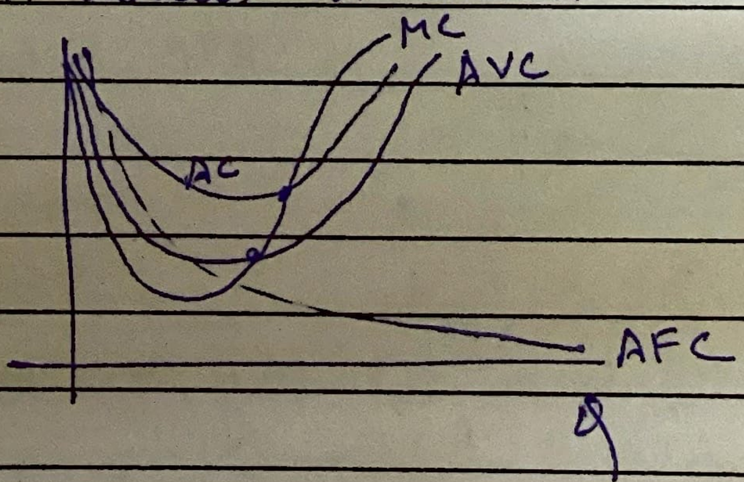
$AVC\downarrow \Rightarrow MC < AVC$

$AVC\uparrow \Rightarrow MC > AVC$

$MC = VC$

$VC(Q) = \int MC \, dQ$

- Cost curves in short run



$P = MC(Q)$

(i) for supply funcⁿ, obtain the marginal cost funcⁿ.

(ii) return only the forward slopping of the supply funcⁿ.

(iii) further, return that part of line that lies above the average variable cost curve.

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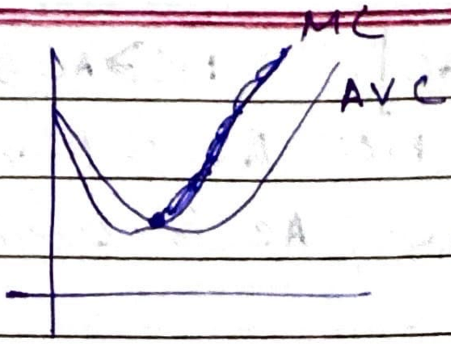

$P \geq AVC(Q)$