Maths 1 - Week-1

L1 Natural Nos. & their operations

- it helps to keep count of objects → 1, 2, 3, 4..
- 0 represents no obj. at all.
- 0 ... ∞ ⇒ N_0 (natural nos.)
  ↳ do add, subtract, multiply, divide
- $-\infty$ ... 0 ... ∞ ⇒ (Integers) ⇒ Z
  ↳ represent them on a no. line.
- $m \times n \rightarrow \underbrace{m + \ldots + m}_{n \text{ no. of times}}$
- repeated multiplicaⁿ → exponentiation.
- multiplicaⁿ is repeated addition, exponentiaⁿ is repeated multiplicaⁿ.
- Division is repeated subtracⁿ.

  19 % 5 = 3 (quotient)
  19 mod 5 = 4 (remainder)
- factor → a divides b if b mod a = 0 (a|b) or b is a multiple of a.
  a is a factor b if a|b. Therefore, factor occur in pairs. Unless, the number is a perfect square.
- Prime nos. → no factor other than 1 & p. 1 is not a prime.
  2 is smallest prime. We can use

sieve of eratosthenes → remove the multiples of p.

- Every no. can be decomposed uniquely into prime factors. ⇒ prime factorizaⁿ.

1.2 Rational Nos. ($\mathbb{Q}$)

- ↳ $\frac{p}{q}$, p, q are integers, $q \neq 0$
- the same rational no. can be written is many ways possible. It is useful to add, substract, or comparing the rationals.
- ∴, the representaⁿ is not unique. Reduced form, $\frac{p}{q}$ where p, q have no common factors.

  gcd (greatest common divisor) → solve it using prime factorization method.
- For each integer, we have next and prev. integer. There is no integer btw. m & $m+1$. This is not possible for rationals ∴, rationals are dense, integers are discrete.

L3 Real & Complex Nos.

$m \times m \Rightarrow m^2$

$\sqrt{m} = x$, such that $x \times x = m$

- but what about $\sqrt{10}$ (not perfect sq.)
- $\sqrt{2}$ can't be written as p/q. but it has a real quantity. It is measurable. ∴, it is irrational
- Real Nos. (R) = Irrational + Rational Nos.
  ↳ they are dense.

Eg. π, e

But what about $\sqrt{-1}$? → It is a complex no.

L4 Set Theory

- A set is a collec^n of items {...}. It may be infinite. Eg. N, Z, Q, R. No req. that members of a set have uniform type.
- Set is unordered. Duplicates don't matter.
- Cardinality → no. of items in a set. We can't define is Q bigger than Z.
- Finite sets, we can list out the items of a set. ∞ sets can't be listed.

- Russell's Paradox → Collecⁿ of all sets is not a set.
- Items in a set is c/d element. $x \in X$.
- X is a subset of Y. If every element of X is present in Y. $X \subseteq Y$.
- Extent of a set can be explained by Venn diagrams.
- $X \subseteq X$. Every set is a subset of itself.
  $X = Y$ iff. $X \subseteq Y$ & $Y \subseteq X$.
- Proper subset : $X \subseteq Y$ but $X \neq Y$
- The empty set has no elements. ϕ
  $\phi \subseteq X$ for every set X. A set can have other sets.
- Powerset → set of subsets of set.
  $X = \{a, b\}$
  Powerset ⇒ $\{\phi, \{a\}, \{b\}, \{a, b\}\}$
  Powerset of ϕ is $\{\phi\}$.
- Set with n elements will have 2^n subsets. It can be represented using binary nos. as well.

L5 Construcⁿ of Subsets & Set Operations

- set comprehension → for ∞ sets.

subset of even integers

$\{x \mid x \in \mathbb{Z}, \cancel{x} \bmod 2 = 0\}$

collect the relevant from existing set.

begin with a existing set

apply a condi^n

- Intervals $\Rightarrow \{z \mid z \in \mathbb{Z}, \underbrace{-6 \leq z \leq +6}_{\text{interval from } -6 \text{ to } +6}\}$

Closed interval $\Rightarrow [0,1] \Rightarrow 0 \leq x \leq 1$

Open interval $\Rightarrow (0,1) \Rightarrow 0 < x < 1$

- Union $\rightarrow$ combine $\Rightarrow X \cup Y$. 
- Intersect^n $\rightarrow$ common elements $\Rightarrow X \cap Y$. 
- Set Difference $\Rightarrow$ elements in X not in Y. $\Rightarrow X \setminus Y$ or $X - Y$ 

  $X - Y \neq Y - X$.
- Complement $\rightarrow$ not in $X \Rightarrow \bar{X}$ or X^c.

  always defined relative to a larger set called universe.

L6 Set Examples

- Recap of set theory.
- Set comprehension $\rightarrow$ generate, filter,

and finally transform.

Eg. reals in interval $[-1, 2)$

$\{x \mid x \in \mathbb{R},\ -1 \leq x < 2\}$

Eg. perfect sq. $\Rightarrow \{n^2 \mid n \in \mathbb{N}\}$

- Choose the generator wisely.

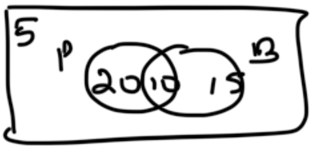
L7 Egs. of Set Operations

Eg. 1 30 $\to$ Phys.

25 $\to$ Bio

10 $\to$ both

5 $\to$ neither



$5 + 20 + 10 + 15 = 50$

$\Downarrow$

Total.

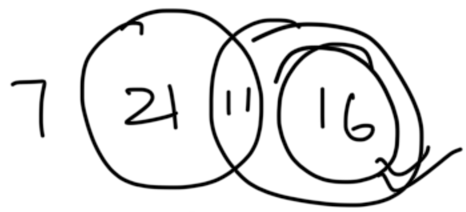
Eg. 2 Total $\to$ 55

P $\longrightarrow$ 32

Both $\to$ 11

Neither $\to$ 7

B not P $\longrightarrow$ 16



$55 - (7 + 21 + 11)$

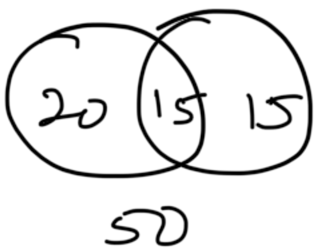
Eg. 3 T $= 60$

35 $=$ Phy

30 $=$ Bio

10 $=$ Took neith

common $\longrightarrow$ 15



2.8 Relations

- Recall sets → $\cup$, $\cap$, $\setminus$, ^{c}, collecⁿ of items, set comprehension.
- Cartesian product

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

→ order is impt.

→ for set of nos., you can visualize product as 2D space.

- you can select some pairs from the cartesian product ⇒ Relaⁿ.

Eg. $\{(m,n) \mid (m,n) \in N \times N,\ n = m+1\}$

- Binary Relaⁿ $R \subseteq A \times B$

Notaⁿ → $(a,b) \in R$, $a\,R\,b$

Eg. Teachers & Courses

$$A = \{(t,c) \mid (t,c) \in T \times C,\ t \text{ teaches } c\}$$

you can represent it as a graph

Eg. Mother & Child

$$M = \{(m,c) \mid (m,c) \in P \times P,\ m \text{ is the mother of } c\}$$

Eg. $\{(a,b) \mid (a,b) \in R \times R,\ \sqrt{a^2+b^2} = 5\}$

- Cartesian prod. of more than 2 sets.

$\{(a,b,c) \mid (a,b,c) \in N \times N \times N,\ a,b,c > 0,$

$a^2 + b^2 = c^2 \}$

- Sq. $\subset \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$
- Identity Rela^n $\Rightarrow$ $I \subseteq A \times A$

  $I = \{(a,b) \mid (a,b) \in A \times A,\ a = b\}$

  or $I = \{(a,a) \mid a \in A\}$
- Reflexive Rela^n $\Rightarrow$ $R \subseteq A \times A,\ I \subseteq R$

  $\{(a,b) \mid (a,b) \in N \times N,\ a, b > 0,\ a \mid b\}$
- Symmetric $\Rightarrow$ $(a,b) \in R$ iff. $(b,a) \in R$
- Transitive $\Rightarrow$ $(a,b) \in R,\ (b,c) \in R$

$$\begin{tikzcd} & b \arrow[dr] & \\ a \arrow[ur] \arrow[rr, dashed] & & c \end{tikzcd}$$

  then $(a,c) \in R$.
- Antisym. $\Rightarrow$ $(a, b \in R)$ & $a \neq b$, then $(b,a) \notin R$.
- Equivalence Rela^n $\Rightarrow$ Reflexive + symmetric + transitive.

  $\rightarrow$ an equivalence rela^n partitions as set. Groups of equivalent relations are c/d equivalence classes, which behaves like equality.

L9 Functions

- converts input to output.

  $x \mapsto x^2$ or $sq(x) = x^2$

  input acts as a parameter here.

we have to specify the domain of I/O.

Domain ⇒ input set

Codomain ⇒ output of possible values.

Range ⇒ actual values of the output can take.

- We can have a Relaⁿ R_f for each function f.

$R_f \subset$ domain (f) × range (f)

→ It is defined on entire domain.

→ single - valued.

→ drawing f should be equivalent to plot the R_f.

- Line $f(x) = 3.5x + 5.7$

↳ slope ↳ intercept.

↳ if you Δ slope & intercept, the lines Δs.

- Injective → 1 to 1 → diff. inputs producing diff. outputs.

- Surjective → onto → range is the codomain.

- Bijective → 1-1 correspondence btw. domain & codomain

↳ iff. it is both injective & surjective

↳ it helps you to count the no. of items in the set.

- for ∞ sets, eg. no. of lines = $R \times R$.
- for every pt. (x_1, y_1) & (x_2, y_2) there is a unique line passing thru it.
  ↳ this doesn't prove the bijection
- the bijection often establishes that domain & codomain have same cardinility.

L10 Relations: Eqs.

- $A \times B$, all pairs (a, b), $a \in A$ & $b \in B$.
  ↳ it extends to $n \times n \times n$. .
- Rela^n picks out some certain tuples in the cartesian product.
- Divisibility

$$D = \{(d, n) \mid (d, n) \in N \times N, d \mid n\}$$

  ↳ it can even extend to integers
- Prime Powers.
  ↳ pair of natural nos. (p, n), s.t. p is prime & $n = p^m$ for some natural no. m.

$$P = \{p \mid p \in N, \text{factors}(p) = \{1, p\}, p \neq 1\}$$

$$PP = \{(p, n) \mid (p, n) \in P \times N, n = p^m, m \in N\}$$

- you can even go beyond the nos.
eg. of flight routes.
- Table is a kind of relations only.
↳ some are unique columns which is called as a key.
→ F → Roll nos. ↦ {Name, DOB}
→ We can join the relation on same columns.

L11 Func^n Eqs.

- $x \mapsto x^2$, $g(x) = x^2$
- domain, codomain, range
- we can associate relations to functions
- what are the range of values that we can achieve?
- $f(x) = x^2$, min at $x = 0$, no max!!
- diff. btw. local max or global max. and same for minima
- what abt. can a func^n grow faster than the other?

L12 Prime Nos. ⇒ used in cryptography.

- A prime no. p has exactly 2 factors → 1, p.
∴, 1 is not a prime.
- It is infinite set. ⇒ Euclid of Alexandria.
- Also, there is no largest prime.

L13 Why Is a Number Irrational

- discovery of irrational nos. $\Rightarrow$ Pythagoreus
- $\sqrt{2}$ is irrational $\Rightarrow$ Proof by mathematical reasoning.

L14 Set vs Collec^{ns}

- Set is a collec^n of items. $j+1 = \{j \{j\}\}$
- Russell's Paradox $\rightarrow$ we cannot have sets of all sets.
  $\hookrightarrow$ not every collec^n is a set.

L15 Degrees of Infinity

- how to count ∞ sets? use bijec^n. Pair elements from the set, so that nothing is left out.
- discrete sets like N & Z are countable. Also, Q is also countable. But R is not countable.