Week-3

L1 Facility Locaⁿ

L2 Single Dim. - Single Facility Problem

- 2 ways of measuring dist. btw. 2 pts.

Euclidean → $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
(direct route)

Metropolitan → $d_{ij} = |x_i - x_j| + |y_i - y_j|$
(dist. in x + dist. in y)

- 1 facility locaⁿ problem can be solved easily by analytics. Obj. funcⁿ differs in diff. services.

Max. of utility, profit, social benefit
Min. of travel times, cost.

- Eg. Vendor on beach, fix an arbitrary point on beach as origin.

- $\text{Max } z = \sum_{i=1}^{m} w_i (s - x_i) + \sum_{i=m+1}^{n} w_i (s - x_i)$

w_i → relative demand at locaⁿ i.
x_i → locaⁿ i of demand on beach
s → locaⁿ of vendor

$\frac{\partial z}{\partial s} = 0 \Rightarrow \sum_{i=1}^{m} w_i = \sum_{i=m+1}^{n} w_i$

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L3 2 Dim - Single Facility

- So, locaⁿ should be at the median w.r.t. demand density. ⇒ Cross-Median approach

2D → $\text{Min } z = \sum_{i=1}^{n} w_i \{|x_i - x_s| + |y_i + y_s|\}$

(x_i, y_i) → coordinates of the i^{th} location
(x_s, y_s) → coordinates of the facility
w_i → demand at the i^{th} locaⁿ.

- Since x & y coord. are indep. of each other. Solve 2 sub-problems (one for each axis). Optimal locaⁿ will have, x_s at the median value of w_i ordered in x-direcⁿ, y_s at the median value of w_i ordered in y directⁿ. Now, the optimal locaⁿ may be at a pt., on a line or within the region.

- $\text{Min } z = \sum_{i=1}^{n} w_i \left((x_i - x_s)^2 + (y_i - y_s)^2\right)^{1/2}$

Euclidean metric.

$$x_i = \frac{\sum_{i=1}^{n} \frac{w_i x_i}{d_{is}}}{\sum_{i=1}^{n} \frac{w_i}{d_{is}}} \qquad y_i = \frac{\sum_{i=1}^{n} \frac{w_i y_i}{d_{is}}}{\sum_{i=1}^{n} \frac{w_i}{d_{is}}}$$

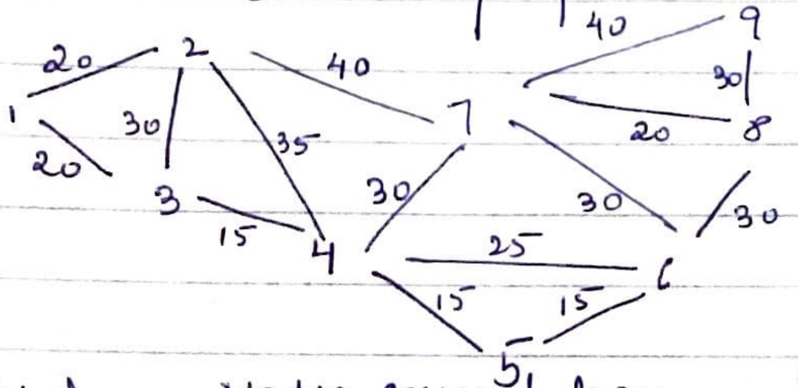
$$d_{is} = \left((x_i - x_s)^2 + (y_i - y_s)^2\right)^{1/2}$$

- Centre of gravity method — Non optimal, as it does not minimize the travel distances from each locaⁿ to the service facility.

$$x_{cg} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum w_i}, \quad y_{cg} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i}$$

L4 Intro. to Multiple Facilities Locaⁿ.

- Locaⁿ set covering problem.



Site within 30 miles could be served as a node.

Node	Nodes served from here	sites which can serve this node
1	1, 2, 3	1, 2, 3
:	:	:

Ans. → 3, 8

L5 Multiple Facilities Locaⁿ - I

- Set Covering Problem

Min $Z = \sum_{j=1}^{m} x_j$ $\quad \sum_{j=1}^{m} a_{ij} x_j \geq 1, \; i = 1, 2 .. m$

$x_j \in \{0, 1\}$ $\quad a_{ij} = \begin{cases} 1 & j \in J_i \\ 0 & \text{otherwise} \end{cases}$

$J_q = \{ v_j \mid d(v_j, v_q) \leq S_q \}$

- Partial Set Covering Problem

Max $Z = \sum_{i=1}^{m} \text{Max} \{ a_{ij} x_j \mid j = 1, 2 \dots p \}$

$\sum_{j=1}^{p} x_j \leq k$ $\quad x_j \in \{0, 1\}$

$x_j = \begin{cases} 1 & \text{if vertex } j \text{ is selected as a median} \\ 0 & \text{otherwise} \end{cases}$

$a_{ij} = \begin{cases} 1 & \text{if } j \in J_i \\ 0 & \text{otherwise} \end{cases}$

L6 Multiple Facilities Locaⁿ-2

- p-median problem

$$y_j = \begin{cases} 1 & \text{if vertex } j \text{ is selected as a median} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if vertex } i \text{ is assigned to the median at vertex } j \\ 0 & \text{otherwise} \end{cases}$$

- due to the budget constraint, only p no. of medians (facilities) can be built. Hence, optimizaⁿ problem.

$$\text{Min } z = \sum_{j=1}^{m} \sum_{i=1}^{m} w_i \, d_{ij} \, x_{ij}$$

$$\sum_{j=1}^{m} x_{ij} = 1, \quad \forall i = 1, 2 \dots m \; ; \quad x_{ij} \le y_j \quad \forall i, j$$

$$\sum_{j=1}^{m} y_j \le p, \quad p < m$$

L7 Generalized Median Problem

$a_{ij} = \begin{cases} 1 & \text{if vertex } i \text{ can be assigned to the median at vertex } j \\ 0 & \text{otherwise} \end{cases}$

f_i = fixed cost of having the median at vertex j

c_{ij} = cost of serving i from median at j.

$$\text{Min } z = \sum_{j=1}^{p} \left(f_j y_j + \sum_{i=1}^{m} c_{ij} x_{ij} \right)$$

$$\sum_{j=1}^{m} a_{ij} x_{ij} = 1, \ \forall i = 1, 2, \dots m$$

other constraints are same.

- Maximal covering problem – It minimizes the no. of facilities located under the constraint that all demand locaⁿ needs to be served. However, some budgetary constraints might not allow us to serve all demand locaⁿˢ. In such cases, the problem is to locate the budgeted facilities to max. the serviced demand.
- On the other hand, when the objective is to min. the total wt. dist. travelled from all demand centres to the opened facilities, the problem is c/d a median problem.

$I = 1, 2 \dots n$ —— p_i

$J = 1, 2 \dots m$ —— k (max.)

max. response time R

$t_{ij} \Rightarrow$ time/dist. from j to i.

$a_{ij} = \begin{cases} 1 & t_{ij} \leq R \\ 0 & \text{otherwise} \end{cases}$

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$y_j = \begin{cases} 1 & \text{if facility is built at site } j \\ 0 & \text{otherwise} \end{cases}$

$$\text{Max. } Z = \left\{ \sum_{i \in I} \max_{j \in J} \{a_{ij}\, p_i\, y_j\} \right.$$

$$\sum_{j \in J} y_j \leq k \qquad d_{ij} = \begin{cases} 1 & \text{if } t_{ij} \leq T \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{j \in J} d_{ij}\, y_j \geq 1 \;, \; \forall i \in I$$

L8 Facility Locaⁿ Problem using Python (PULP)

- 4 locaⁿˢ → (L_1, L_2, L_3, L_4) ⇒ (100, 120, 150, 200)
  6 suppliers → ($S_1, S_2, S_3, S_4, S_5, S_6$)

Locaⁿ/Supp.	S_1	S_2	S_3	S_4	S_5	S_6
L_1	9	7	5	6	3	4
L_2	7	8	5	3	3	4
L_3	8	6	2	3	5	6
L_4	2	5	6	7	6	5

- x_i ($i = 1, 2, 3, 4$) ⇒ (1: yes & 2: no)
  y_{ij} ($j = 1, 2, 3, 4, 5, 6$)
  Assume locaⁿˢ have ∞ capacity.
- Coding.