M T W T F S S

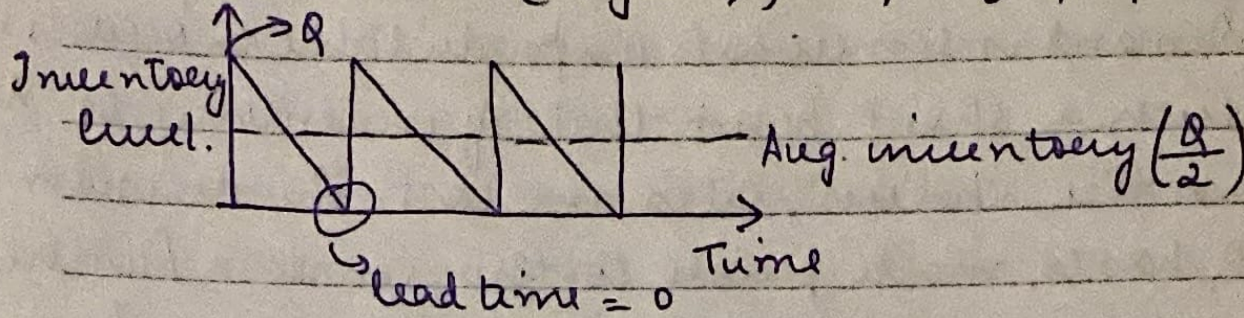
Week-5

L1 Deterministic Inventory - EOQ

- A decision theoritic problem abt. stocking a particular product. The decision maker observes the level of stock available & decides whether to order more. Inventory → is the level of stock. Inventory problem is about when & how much to order. It's tradeoff btw. carrying cost & ordering cost. → EOQ
- Items bought from vendors ——
  (i) cost of product
  (ii) ordering cost per order (fixed)
  (iii) carrying cost for holding the items.
  (iv) shortage cost (backorder costs)
- For now, consider deterministic multi-period inventory models → annual demand & various costs listed are known with certainity.
- Impt. decision is how much to order? → Q
- Cost of the prod. (C Rs/unit) → no role
- Order cost (C_o Rs/order)
- Carrying / Holding cost (C_c Rs/unit/yr)
  ↳ $C_c = iC$ (interest rate - %/yr)
- Shortage cost (C_s Rs/unit/yr) → backorders indicating any unfulfilled demand will be met subsequently.
  → loss of customer goodwill.
  → cost of losing customer
  → loss of profit associated with prod.

- Economic order quantity

Annual demand (D/year), C_o, C_c, Q



- No. of orders/yr. = D/Q

Annual order cost = DC_o/Q

Avg. inventory in sys. = Q/2

Annual " C_c = $QC_c/2$

$$\boxed{T = \frac{DC_o}{Q} + \frac{QC_c}{2}}$$

$$\boxed{Q^* = \sqrt{2DC_o/C_c}}$$

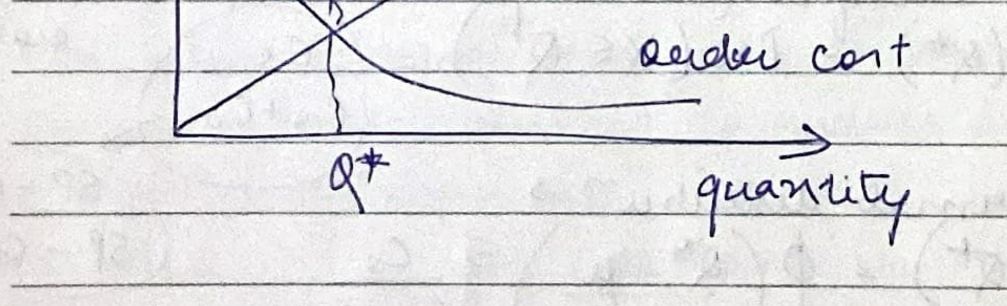
$$TC^* = \sqrt{2DC_oC_c}$$

Eg. D = 10000/yr, C_o = ₹300/order, C_c = ₹4/unit/yr.

$Q^* = 1224.75$ → EOQ

$TC^* = 4898.98$ → doesn't include cost item.

N = D/Q = 10000/1224.75 = 8.17 orders/yr.



L2 Probabilistic Inventory (News Vendor Prob.)

- Consider a retailer who sells Diwali lights. Demand is somewhat unpredictable & occurs in such a short burst that if inventory is not in the shelves, sales are lost. The decision of Q has to made prior to the season. Further, cost of unsold inventory is too high. The unsold items are sold at a steep discount. A newspaper vendor faces the same problem.
- 2 essential info → anticipated demand, cost of producing too much or too little.
- Assumpⁿ → products are separable, planning is done for single period, demand is random, deliveries are made before the demand occurs, cost of shortage or excess are linear.
- X demand, $G(x)$ → cdf, $g(x)$ → pdf, μ, σ, C_o (left over items) (salvage), C_s (shortage cost)

Q

TC exp.

$$Y(Q) = C_o \int_0^Q (Q-x)\, g(x)\, dx + C_s \int_Q^\infty (x-Q)\, g(x)\, dx$$

To min. total cost, we should choose Q^* that satisfies (→

$$G(Q^*) = Pr(X \le Q^*) = \frac{C_s}{C_s + C_o}$$

→ critical ratio

$$\frac{SP - Cost}{SP - Salv.\ Value}$$

If Normal distribuⁿ →

$$G(Q^*) = \phi\left(\frac{Q^* - \mu}{\sigma}\right) = \frac{C_s}{C_s + C_o}$$

$$\phi(z) = \frac{C_s}{C_s + C_o} \Rightarrow \boxed{Q^* = \mu + z\sigma}$$

- In uncertainity, the app. producⁿ or order quantity depends on both, the distribuⁿ of demand & the relative costs of over producing under producing.
- If demand is normall distributed, then increasing the variability of demand will increase the producⁿ quantity if the critical ratio is greater than 0.5.

L3/4 → Deterministic Dynamic Inventory (Bellman eqⁿ)

- EOQ model is a model of deterministic demand scenario where decision is taken once. The newsvendor problem is stochastic but still time horizon is 1 day. Now we have deter. demand, but extended time horizon. This org. periodic monitoring the inventory at various times whether to orders & how much
  ↳ DP (Bellman Eqⁿ)
- $t = 1, 2 \dots T$, D_t (demand) → dynamic but known.
- At each beginning, the org. decides whether & how much to order. The orders are delivered instantaneously. Lead Time = 0.

- When products are ordered, there is a fixed ordering cost and variable product cost. At the end, cost of holding inventory gets added.
- There is ltd. storage capacity & ltd. ordering capa-

$t = 1, 2 \ldots T$, $S_t \rightarrow$ inventory level at time t.

a_t (actn set) $\rightarrow$ order quantity

- $f_t(S_t) \rightarrow$ total cost of meeting the demand at stage t, when state is S_t.

$C(S_t, a_t) \rightarrow$ immediate cost of ordering a_t, inventory when the state is S_t

- State updatn —

$$S_{t+1} = \text{Max}(S_t + a_t - D_t, 0)$$

- Bellman eqn,

$$f_t(S_t) = \min_{a_t} \left(C(S_t, a_t) + f_{t+1}(S_{t+1}) \right)$$

- Solve it by the process of backward inductn.