HASKELL SYNTAX AND STATIC SEMANTICS WRITTEN IN K-FRAMEWORK

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BY

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THESIS

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Abstract

For this thesis, I introduce a static semantics for Haskell by utilizing the K-Framework. This implementation includes support for the module system of Haskell but not for type classes. There are many layers that have to be implemented in K before type inference can be performed. The first part of the implementation is the entire context free syntax of Haskell in K. Since all the syntax is included, any program written in Haskell extended syntax can be parsed into an abstract syntax tree. However, this includes only the Haskell extended syntax but does not include the syntactic short-cuts such as treating tabs as syntactic sugar for grouping constructs such as curly braces. Programs that include multiple modules can be parsed, but the multiple modules must be written in a single file. This is unlike how the Glasgow Haskell Compiler allows for module imports, where each module must be kept in separate files. The multiple modules are then made as nodes in a directed acyclic graph. A directed edge in the graph represents a module importing another module. This graph is used for importing the user defined types from one module into another module. Context sensitive checks and type inference are then performed on modules. The static semantics specifies that, at each node in the graph, assuming all child modules are already checked and inferred, the user defined types of each of the child modules are imported into the module at the given node. All rules of the Haskell type system must take mutual recursion into account. There is repeated layering of inferences in Haskell. Due to being written in K, my semantics is mathematically precise and executable. Since the semantics is executable, the semantics can be tested against test sets to validate the correctness of the semantics. I utilized the executability of the semantics to test both positive inferences and exceptional inferences. This is part of a larger project to give a formal semantics to Haskell.

Subject Keywords: Haskell; Type-System

To my parents, for their love and support.

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Chapter 1

Introduction

One of the inherent problems of engineered systems in general is that the design of the system is not proven to be working. The system could be non-functional at design time. The designer may not fully understand the system environment or may have not considered the behavior of the system in rare circumstances. Then once the design is made, there may be issues introduced by implementing the design incorrectly. Within the context of computer programs, the current way that programs are created is by making a design or a formal specification of the program, implementing it, and testing the program against unit tests or verifying the behavior of the program after the fact. Formal methods are ways to mathematically prove correctness of a system. Without formal methods, the only way to reason about a system is by testing it against different edge cases. Within the context of a programming language, one way a programming language can be formally specified is by defining a syntax and semantics for that language. The operational semantics of a programming language can be thought of as a transition system upon an abstract syntax tree, which is the program itself written in the language, and a state, which is a function from the variables in the tree to the current values of those variables. This way, real and complex programs written in natural looking programming languages can be interpreted as strings written in formal languages. Once a programming language is defined in this way, certain properties and behavior of the language and programs written in the language can be proven. K is a framework for creating the formal specification of a programming language. It then can interpret programs written in the language by running only the rules of the formal operational semantics of the programming language. This allows programs to be run and analyzed formally. This way the formal specification of the complex programming language can be tested and analyzed with the use of a machine. A K-configuration defines the memory structure of the programming language, made up of cells. The program state can be thought of as the current values of the K-configuration at a certain point in time. Grammar can be written in K using the constructor *syntax*, and a semantic rule can be written in K using the constructor *rule*. Haskell is a purely functional programming language with strong static typing. Purely functional means that the language only allows the user to make functions whose output is only dependent on the function input. Strong static typing means that before a program is run, a type inference algorithm infers the type of the program and ensures that all functions and function applications are allowed with regards to the types of the inputs and outputs. Static refers to the fact that type inference is performed before the code is run, and will not run during the runtime of the code. Strong typing refers to the fact that the compiler will not allow the user to perform workarounds like typecasting. This project details the syntax of Haskell and the type system of Haskell in K.

The Haskell 2010 report gives a cursory description of the type system as a Standard Hindley Milner polymorphic type system, but gives no further indication of how this applies to the specifics of the Haskell syntax. In the specification in this paper, a more complete specification of parts of the type system is presented as a family of mutually inductive rules. Such a presentation is not only the basis of an executable semantics but can later provide the basis of formal and rigorous proofs.

By studying the standard Haskell compiler, GHC, one can see that there must be additional context sensitive checks that must be made prior to type inference.

Chapter 2

Context Free Syntax

This chapter details the first part of the static semantics. In order for any context sensitive checks or type inference can be done, the test programs first need to be parsed into an abstract syntax tree. The context sensitive checks and type inference can then be performed upon the tree. Some difficulties with implementing a grammar into K is that the grammar originally is written in sort descending order in a document. The goal was to build a grammar that can parse actual programs and ensure there was no bugs. To do this, I started with small example programs, wrote out the example abstract syntax tree, and included the sorts necessary to parse them. Then if they didn't parse correctly I could then debug. I then wrote bigger and bigger example programs and included more and more sorts until all the grammar was included.

The Haskell 2010 report is the current official specification of the Haskell language. The grammar specified in section 10.5 of the Haskell 2010 report is a specification of the expanded syntax of Haskell. As specified in section 2.7, the expanded syntax of Haskell specifies Haskell programs when written using semicolons and braces. However, these can be omitted in a real Haskell program. The compiler will then utilize layout rules for certain grammar structures instead. These are specified in section 10.3. The parser for this project does not implement these layout rules and instead only can parse the expanded, layout insensitive syntax of Haskell. It would require another script to convert a program written using the layout sensitive syntax into the expanded syntax in order to parse the program. Section 10.1 specifies the notation used in the grammar. The notation of 10.1 are always in bold in the grammar. So an example production in the document looks like

`qvarid -> ` **`[`** `modid .` **`]`** ` varid`

This means that

```
modid .
```

is optional, and the brackets are not terminals, but the period is a terminal. Any symbol that is not in bold needs to be written in the program in order to parse correctly.

2.1 Syntax Explanation

There are a lot of parts of the grammar that proposed many challenges to implement in K. For instance, a sort definition that includes an option could be just written using a pipe in the K syntax.

So the example production

`qvarid -> [ modid . ] varid`

Is written in my K syntax as split into two options.

```
syntax QVarId ::= VarId | ModId "." VarId [klabel('qVarIdCon)]
```

However, an issue arises when you have something written on the right hand side of the production like

`data [ context => ] simpletype [ = constrs ] [ deriving ]`

for the topdecl sort. If each optional sort were split into two options, then a production that includes n optional sorts would require 2^n options. This would create an unnecessarily large syntax in K.

Instead, for each optional sort in the original grammar, I replaced the optional sort with a new sort. For instance, I replaced

`[ context => ]`

in the original grammar with a new sort called OptContext. Then I just specified

```
syntax OptContext ::= Context "=>" | "" [onlyLabel, klabel('
    emptyContext)]
```

and so the right hand side of the production would now look like

```
"data" OptContext SimpleType OptConstrs OptDeriving  [klabel('
    data)]
```

This is acceptable because Haskell is not an order sorted algebra, so introducing new sorts that are not originally in the grammar is okay.

In K, semantic rules are called K rules. The tag

```
[klabel('exampleLabel)]
```

means that in the abstract syntax tree created by K, a term can be referred to using that klabel in a K rule.

2.2 Implementation of Section 10.2

The following introduces the syntax for the keywords, constants, special symbols, and variables that comprise the terminals for the remaining context free grammar as presented in section 10.2 of the 2010 Haskell report.

```
// Syntax from haskell 2010 Report
// https://www.haskell.org/onlinereport/haskell2010/haskellch10.html#x17-17500010

module HASKELL-SYNTAX

    syntax Integer ::= Token{([0-9]+)
                | (([0]|[o]|[0][O])[0-7]+)
                | (([0][x] | [0][X])[0-9a-fA-F]+)}   [onlyLabel]

    syntax CusFloat ::= Token{([0-9]+[\.][0-9]+([e E][\+\-]?[0-9]+)?)
                                 |([0-9]+[e E][\+\-]?[0-9]+)} [onlyLabel]
    syntax CusChar ::= Token{[\'](~[\'\\\&])[\']} [onlyLabel]
    syntax CusString ::= Token{[\"](~[\"\\]*)[\"]} [onlyLabel]
```

I ran into issue where a program with a variable called size did not parse. I found out that this is because size is a K keyword. So I just specified that a variable could be a variable token, or size. This reveals an issue in the version of K I was using, where K keywords that appear in example parsed programs won't become tokens.

```
    syntax VarId ::= Token{[a-z\_][a-z A-Z\_0-9\']*} [onlyLabel] | "size" [onlyLabel]
```

```
    syntax ConId ::= Token{[A-Z][a-zA-Z \_0-9\']*} [onlyLabel]
    syntax VarSym ::= Token{
```

```
 ([\! \# \$ \% \& \* \+ \/ \> \? \^][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
|[\-] | [\.]
|([\.][\! \# \$ \% \& \* \+ \/ \< \= \> \? \@ \\ \^ \| \- \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
| ([\-][\! \# \$ \% \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~ \:]*)
| ([\@][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
| ([\~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
| ([\\][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
| ([\|][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
| ([\:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
| ([\<][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
| ([\=][\! \# \$ \% \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)} [onlyLabel]
  syntax ConSym ::= Token{[\:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*}   [onlyLabel]
```

I ran into an issue where floats and integers did not parse correctly. They caused parsing errors due to ambiguity of parsing. For example the number 123.45 had ambiguity where the parser did not know if 1, 12, or 123 where integers, and if 5 was an integer, or if the entire thing was one float. Normally in K, different tokens are separated with whitespaces. However, for some reason the parser had difficulty here. Initially, I added a workaround by requiring parentheses around each integer and floating point. This fixed the issue.

```

  syntax IntFloat ::= "(" Integer ")"  [bracket] //NOT OFFICIAL SYNTAX
```

```
                          | "(" CusFloat ")" [bracket]
```

```
    syntax Literal ::= IntFloat | CusChar | CusString
    syntax TyCon ::= ConId
    syntax ModId ::= ConId | ConId "." ModId  [klabel('conModId)
       ]
    syntax QTyCon ::= TyCon | ModId "." TyCon [klabel('conTyCon)
       ]
    syntax QVarId ::= VarId | ModId "." VarId [klabel('qVarIdCon
       )]
    syntax QVarSym ::= VarSym | ModId "." VarSym  [klabel('
       qVarSymCon)]
    syntax QConSym ::= ConSym | ModId "." ConSym  [klabel('
       qConSymCon)]
/*    syntax QTyCls ::= QTyCon
    syntax TyCls ::= ConId
*/
    syntax TyVars ::= List{TyVar, ""} [klabel('typeVars)] //used
        in SimpleType syntax
    syntax TyVar ::= VarId
    syntax TyVarTuple ::= TyVar "," TyVar      [klabel('
       twoTypeVarTuple)]
                        | TyVar "," TyVarTuple [klabel('
                           typeVarTupleCon)]

    syntax Con ::= ConId | "(" ConSym ")"   [klabel('
       conSymBracket)]
    syntax Var ::= VarId | "(" VarSym ")"   [klabel('
       varSymBracket)]
    syntax QVar ::= QVarId | "(" QConSym ")" [klabel('
       qVarBracket)]
    syntax QCon ::= QTyCon | "(" GConSym ")"  [klabel('
       gConBracket)]

    syntax QConOp ::= GConSym | "`" QTyCon "`"    [klabel('
       qTyConQuote)]
    syntax QVarOp ::= QVarSym | "`" QVarId "`"    [klabel('
       qVarIdQuote)]
    syntax VarOp ::= VarSym   | "`" VarId "`"     [klabel('
       varIdQuote)]
    syntax ConOp ::= ConSym   | "`" ConId "`"     [klabel('
       conIdQuote)]

```

```
    syntax GConSym ::= ":" | QConSym
    syntax Vars ::= Var
                  | Var "," Vars  [klabel('varCon)]
    syntax VarsType ::= Vars "::" Type  [klabel('varAssign)]
    syntax Ops ::= Op
                 | Op "," Ops  [klabel('opCon)]
    syntax Fixity ::= "infixl" | "infixr" | "infix"
    syntax Op ::= VarOp | ConOp
    syntax CQName ::= Var | Con | QVar

    /* syntax QConId ::= ConId | ModId "." ConId */

    syntax QOp ::= QVarOp | QConOp
```

2.3 Implementation of Section 10.5

The following introduces the sorts of the context free grammar of the Haskell extended syntax.

2.3.1 Modules

We start with modules.

```
    syntax ModuleName ::= "module" ModId [klabel('moduleName)]

    syntax Module ::= ModuleName "where" Body          [klabel('
       module)]
                    | ModuleName Exports "where" Body  [klabel('
                       moduleExp)]
                    | Body                             [klabel('
                       moduleBody)]

    syntax Body ::= "{" ImpDecls ";" TopDecls "}" [klabel('
       bodyimpandtop)]
                  | "{" ImpDecls "}" [klabel('bodyimpdecls)]
                  | "{" TopDecls "}" [klabel('bodytopdecls)]
```

The sort that contains all the other sorts is a module. A module represents one complete Haskell program. It can have either a name and a body, a name and a body with exports, or just a body.

2.3.2 ImpDecls

An ImpDecl is an import declaration. An import is another module that this module depends on. The definition of ImpDecl is the following

```
syntax ImpDecls ::= List{ImpDecl, ";"} [klabel('impDecls)]
syntax ImpDecl ::= "import" OptQualified ModId OptAsModId OptImpSpec [klabel('impDecl)]
                  | "" [onlyLabel, klabel('emptyImpDecl)]
syntax OptQualified ::= "qualified"
                       | ""  [onlyLabel, klabel('emptyQualified)]
syntax OptAsModId ::= "as" ModId
                     | ""    [onlyLabel, klabel('emptyOptAsModId)]

syntax OptImpSpec ::= ImpSpec
                     | ""    [onlyLabel, klabel('emptyOptImpSpec)]

syntax ImpSpecKey ::= "(" ImportList OptComma ")"
syntax ImpSpec ::= ImpSpecKey
                  | "hiding" ImpSpecKey

syntax ImportList ::= List{Import, ","}

syntax Import ::= Var
                 | TyCon CQList
```

The following example program has a Module with a name and a body of only ImpDecls. It has one ImpDecl.

```
module Foo where
{import Bar
}
```

The following example program has a Module with no name and and a body of only ImpDecls. It has one ImpDecl.

```
import Bar
```

Another example program is

```
module Simp1 where
{import Test
}
```

The corresponding abstract syntax tree in K is

```
`'module`(`'moduleName`(#token("Simp1","ConId")), `'bodyimpdecls
  `(`'impDecls`(`'impDecl`(`'emptyQualified`( .::KList), #token
  ("Test","ConId"), `'emptyOptAsModId`(.::KList), `'
  emptyOptImpSpec`(.::KList)), `'.List{"'impDecls"}`(.::KList))
  ))
```

Module contains two children. The first child of module is moduleName which contains the token Simp1. Simp1 is a constructor ID because it starts with a capital letter. The second child of module is bodyimpdecls. This contains the sort impDecls which is a list of impDecls. This program has only one impDecl. Since there is no qualified, the impDecl contains the child emptyQualified, followed by the token Test. Since there is no AsModId or ImpSpec, the last two children are emptyOptAsModId and emptyOptImpSpec.

2.3.3 TopDecls

The main types of expressions in Haskell are TopDecls - Top Declarations. A top declaration can either be a type, a data, a newtype, a class, an instance, a default, a foreign, or an arbitrary declaration.

Any sort that starts with 'Opt' means that this is optional. In K something can be made optional by declaring the necessary constructors or nothing.

```
    syntax TopDecls ::= List{TopDecl, ";"} [klabel('topdeclslist
       )]

    syntax TopDecl ::= Decl [klabel('topdecldecl)]
                     > "type" SimpleType "=" Type [klabel('type)
                        ]
                     | "data" OptContext SimpleType OptConstrs
                        OptDeriving  [klabel('data)]
                     | "newtype" OptContext SimpleType "="
                        NewConstr OptDeriving [klabel('newtype)]
                     | "class" OptContext ConId TyVar OptCDecls
                        [klabel('class)]
                     | "instance" OptContext QTyCon Inst
                        OptIDecls [klabel('instance)]
                     | "default" Types [klabel('default)]
                     | "foreign" FDecl [klabel('foreign)]
```

2.3.4 Decls

A Decl is any general declaration. So something like

```
f x = x + 2
```

is a Decl.

The following is the definition of Decl and related sorts.

```
  syntax OptDecls ::= "where" Decls | "" [onlyLabel, klabel('emptyOptDecls)]
  syntax Decls ::= "{" DeclsList "}" [klabel('decls)]
  syntax DeclsList ::= List{Decl, ";"} [klabel('declsList)]

  syntax Decl ::= GenDecl
                | FunLhs Rhs [klabel('declFunLhsRhs)]
                | Pat Rhs [klabel('declPatRhs)]
```

This section introduces the sorts whose parent is Decl. This contains general declarations, function left hand side and right hand side, function guards, and expressions.

```
  syntax GenDecl ::= VarsType
                   | Vars "::" Context "=>" Type   [klabel('genAssignContext)]
                   | Fixity Ops
                   | Fixity Integer Ops
                   | "" [onlyLabel, klabel('emptyGenDecl)]

  syntax FunLhs ::= Var APatList [klabel('varApatList)]
                  | Pat VarOp Pat [klabel('patVarOpPat)]
                  | "(" FunLhs ")" APatList [klabel('funlhsApatList)]

  syntax Rhs ::= "=" Exp OptDecls [klabel('eqExpOptDecls)]
               | GdRhs OptDecls [klabel('gdRhsOptDecls)]

  syntax GdRhs ::= Guards "=" Exp
                 | Guards "=" Exp GdRhs
  syntax Guards ::= "|" GuardList
  syntax GuardList ::= Guard | Guard "," GuardList  [klabel('guardListCon)]
  syntax Guard ::= Pat "<-" InfixExp
                 | "let" Decls
                 | InfixExp
```

```

    //definition of exp
    syntax Exp ::= InfixExp
                 > InfixExp "::" Type  [klabel('expAssign)]
                 | InfixExp "::" Context "=>" Type  [klabel('
                   expAssignContext)]

    syntax InfixExp ::= LExp
                      > "-" InfixExp   [klabel('minusInfix)]
                      > LExp QOp InfixExp
```

2.3.5 LExp

LExp is an important sort for the type inference function. This is because LExp defines the different expression types which the type inference function has specific rules for.

The different LExp types are a lambda expression, a 'let-in' expression, an 'if' statement, a case statement, and a 'do' block.

```
    syntax LExp ::= AExp
                  > "\\" APatList "->" Exp [klabel('lambdaFun)]
                  | "let" Decls "in" Exp [klabel('letIn)]
                  | "if" Exp OptSemicolon "then" Exp
                    OptSemicolon "else" Exp [klabel('ifThenElse
                    )]
                  | "case" Exp "of" "{" Alts "}" [klabel('caseOf
                    )]
                  | "do" "{" Stmts "}" [klabel('doBlock)]
```

2.3.6 AExp

AExp is also an import sort for the type inference function. The main parts of AExp that the inference function cares about is QVar and GCon.

QVar is a qualified variable and GCon is a general constructor.

```

    syntax OptSemicolon ::= ";" | "" [onlyLabel, klabel('
      emptySemicolon)]
    syntax OptComma ::= "," | ""     [onlyLabel, klabel('
      emptyComma)]
```

```

  syntax AExp ::= QVar [klabel('aexpQVar)]
                | GCon [klabel('aexpGCon)]
                | Literal [klabel('aexpLiteral)]
                > AExp AExp [left, klabel('funApp)]
                > QCon "{" FBindList "}"
                | AExp "{" FBindList "}" //aexp cannot be qcon UNFINISHED
                        //Liyi: first, does not understand the syntax, it is the Qcon {FBindlist}
                        //or QCon? Second, place a check in preprosssing.
                        //and also check the Fbindlist here must be at least one argument
                > "(" Exp ")"             [bracket]
                | "(" ExpTuple ")"
                | "[" ExpList "]"
                | "[" Exp OptExpComma ".." OptExp "]"
                | "[" Exp "|" Quals "]"
                | "(" InfixExp QOp ")"
                | "(" QOp InfixExp ")" //qop cannot be - (minus) UNFINISHED
                         //Liyi: place a check here to check if QOp is a minus
```

2.4 Example Test Programs

Chapter 3

Configuration

K is used for defining a state machine and the K rules define the transition rules for the state machine. The configuration of the state machine is made up of K cells. The K cells contain the syntax data structure representing the code of the example program. They also contain the memory of the state machine. An actual state of the state machine in K is when the cells each have some term inside of them.

The following is the configuration of my Haskell semantics.

```
requires "haskell-syntax.k"

module HASKELL-CONFIGURATION
    imports HASKELL-SYNTAX

    syntax KItem ::= "startImportRecursion"
    syntax KItem ::= callInit(K)
    //syntax KItem ::= initPreModule(K) [function]
    //syntax KItem ::= tChecker(K) [function]

    configuration
        <T>
            <k> $PGM:ModuleList ~> startImportRecursion </k>
            <tempModule> .K </tempModule>
            <tempCode> .K </tempCode>
```

The $< k >$ cell is the cell that computation takes place in. The abstract syntax tree is initially placed into the $< k >$ cell. The command

```
$PGM:ModuleList
```

means that the parsed tree appears in this cell and the sort that contains all other sorts is ModuleList.

$.K$ means that the cell is initially empty.

tempModule is the name of the current module. tempCode is the current code.

typeIterator is used for creating a fresh type variable for the inference algorithm. It has the current count of how many fresh type variables that were created.

```
            <typeIterator> 1 </typeIterator>
```

3.1 Alpha

Alpha is a map of type renamings. So if a user declares

```
data MyBool = TTrue
;type MyBooltwo = MyBool
```

Then MyBooltwo is a renaming of MyBool. In tempAlpha, an AObject is made. An AObject is a KItem with two children. One can be thought of as a Key and the other is the Value for a map. So MyBool $- >$ MyBooltwo. However, we want to check and reject programs that have multiple renamings, so we cannot use a K Map which has idempotence. However, once we make this check, we can then use a K Map. This is what tempAlphaMap is.

.Map means that the cell starts with an empty map.

```
            <tempAlpha> .K </tempAlpha>
            <tempAlphaMap> .Map </tempAlphaMap>
```

3.2 Beta

tempT contains all user defined datatypes. tempT is organized in such a way that makes context sensitive checks easy to perform. tempBeta contains all user defined datatypes organized so that type inference is easy to perform. More is explained in chapter 5.

```
            <tempBeta> .Map </tempBeta>
            <tempT> .K </tempT>
```

3.2.1 Example

If the user makes the data type $CusBool$ in module $Simp5$, and declares it with this example...

```
data CusBool = True2 | False2
```

Then the corresponding $tempBeta$ should look like this. Note how the monomorphic datatype just has an empty $forAll$.

```
    <tempBeta>
        ModPlusType ( Simp5 , False2 ) |-> forAll ( .Set ,
           CusBool .TyVars )
        ModPlusType ( Simp5 , True2 ) |-> forAll ( .Set ,
           CusBool .TyVars )
    </tempBeta>
```

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
data CusBool a b = True2 a | False2 b
```

Then the corresponding tempBeta should look like this.

```
    <tempBeta>
        ModPlusType ( Simp5 , False2 ) |-> forAll ( b , funtype
           ( b , CusBool a b ) )
        ModPlusType ( Simp5 , True2 ) |-> forAll ( a , funtype (
            a , CusBool a b ) )
    </tempBeta>
```

3.3 Delta

tempDelta contains the arity of the user defined dataTypes. So if a user defined datatype takes in two parameters, tempDelta will contain the number 2.

```
            <tempDelta> .Map </tempDelta>
```

3.3.1 Example

If the user makes the data type $CusBool$ in module $Simp5$, and declares it with this example...

```
data CusBool a b = True2 a | False2 b
```

Then the corresponding *tempDelta* should look like this.

```
<tempDelta>
    ModPlusType ( Simp5 , CusBool ) |-> 2
</tempDelta>
```

3.4 Import Data Structure

importTree, recurImportTree, and impTreeVMap contain the data necessary for the directed acyclic graph representing imports.

```
            <importTree> .List </importTree>
            <recurImportTree> .List </recurImportTree>
            <impTreeVMap> .Map </impTreeVMap>
```

3.5 Modules

The modules cell contains all modules that were checked and inferred already. Multiplicity means that there can be multiple module cells.

```
            <modules> //static information about a module
                <module multiplicity="*">
                    <moduleName> .K </moduleName>
                    <moduleAlphaStar> .K </moduleAlphaStar>
                    <moduleBetaStar> .K </moduleBetaStar>
                    <moduleImpAlphas> .List </moduleImpAlphas>
                    <moduleImpBetas> .List </moduleImpBetas>
                    <moduleCompCode> .K </moduleCompCode>
                    <moduleTempCode> .K </moduleTempCode>
                    <imports> .Set </imports>
                    <classes> //static information about a
                       module
                        <class multiplicity="*">
                            <className> .K </className>
                        </class>
                    </classes>
                </module>
            </modules>
```

```
        </T>

endmodule
```

Chapter 4

Context Sensitive Checks

The standard Haskell Compiler is called the Glascow Haskell Compiler. It is otherwise known as GHC. By testing GHC, it is apparent that the compiler will check and reject programs that are syntactically valid, but have additional issues that make the programs invalid. This means that GHC does in fact make additional context sensitive checks that are not part of the context free grammar. In the semantics in this paper, there are also context sensitive checks that are made prior to type inference to reject programs that GHC rejects.

4.1 Data Types

Within the context of a computer program, a data type is a property of data that tells the compiler or interpreter more about how the data is supposed to be used within the context of the program. This is useful for having the compiler find bugs that occur from the programmer misusing data.

`https://en.wikipedia.org/wiki/Data_type`

In Haskell, a user can create a custom data type. This is referred to as a user defined type. Then when the user creates functions or any other expression, they can operate on their own data type.

4.1.1 Polymorphim

A polymorphic data type is a data type that can generalize over other data types. A monomorphic data type is a data type that does not do this.

4.2 Datatype Constructors

In order to perform context sensitive checks to make sure that the user did not have errors when creating types, the types are placed into data structures that make context sensitive checks easy to perform.

Section 4 of the Haskell 2010 report specifies the haskell type system. `https://www.haskell.org/onlinereport/haskell2010/haskellch4.html#x10-620004`

In the `TopDecl` sort, there are three type constructors that are used to create user defined data types. These are `data`, `type`, and `newtype`.

```
  syntax TopDecl ::= Decl [klabel('topdecldecl)]
                   > "type" SimpleType "=" Type [klabel('type)
                     ]
                   | "data" OptContext SimpleType OptConstrs
                     OptDeriving  [klabel('data)]
                   | "newtype" OptContext SimpleType "="
                     NewConstr OptDeriving [klabel('newtype)]
```

These three type constructors are used to create user defined types.

4.2.1 Type

The first one is *type*,

```
type simpletype = type
```

This is used in a Haskell program to declare a new type as a single type. In effect, it renames the type where both names now can be used to refer to the type.

```
type Username = String
```

Is one such example usage of `type`, it creates a new type `Username`, which is defined as just a string. Now the programmer can refer to `Username` or `String` to make a string.

4.2.2 Data

The second one is *data*,

```
data [context =>] simpletype [= constrs] [deriving]
```

This allows a user to declare a new type that may include many fields and polymorphic types.

For instance:

```
data Date = Date Int Int Int
```

This is a new type that includes the type constructor *Date* followed by three integers.

```
data Poly a = Number a
```

This is a new polymorphic type with polymorphic parameter a, that has the type constructor *Number*.

4.2.3 Newtype

The third one is *newtype*,

```
newtype [context =>] simpletype = newconstr [deriving]
```

This is very similar to data except it only parses when the newtype has only one typecon and one field.

4.2.4 Context

Another thing to note is that the

```
[context =>]
```

part of the syntax for the types is deprecated. https://stackoverflow.com/questions/9345589/guards-vs-if-then-else-vs-cases-in-haskell

4.3 Initial Data Structures

The semantics includes a map, called alpha, of new type names as the keys and their declared types as the entries. It collects all appearances of the type constructor `type` in the program, and puts `simpletype` $\mapsto$ `type` in the alpha map. However, one of the checks that is made, is whether a user declared multiple definitions with texttttype, but maps in K only allow for unique keys. So initially the semantics creates a set of tuples, and after checking for multiple type declarations, it then changes the alpha set to a map.

The second data structure that the semantics creates is called T. T holds the user defined types created using `data` and `newtype`.

```
syntax KItem ::= TList(K) //list of T objects for every new type
    introduced by data and newtype
syntax KItem ::= TObject(K,K,K,K) //(module name, type name,
   entire list of poly type vars, list of inner T pieces)
syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type constructor,
   poly type vars, argument sorts, entire constr block, type
   name)
```

The structure, `T`, is a list of TObjects. Each TObject represents a single user defined datatype. It holds the name, the list of polymorphic parameters, and a list of inner T pieces. An inner T piece represents an option of what a type could be. It consists of a type constructor, a list of polymorphic parameters required for this option, the fields for this option, the entire subtree of the AST for this option unedited, and the type name again. The semantics then uses these data structures to perform these checks, and afterwards uses different data structures to perform type inference.

4.3.1 Example T

If the user makes the data type *CusBool* in module *Simp*5, and declares it with this example...

```
data CusBool = True2 | False2
```

Then the corresponding *tempT* should look like this.

```
    <tempT>
        TList ( ListItem ( TObject ( Simp5 , CusBool , .TyVars ,
            ListItem ( InnerTPiece ( True2 , .List , .List ,
                True2
              .OptBangATypes , CusBool ) )
            ListItem ( InnerTPiece ( False2 , .List , .List ,
                False2
              .OptBangATypes , CusBool ) ) ) ) )
```

```
</tempT>
```

4.3.2 K Code

The following parses the tree and searches for the user defined types to place into the data structures.

```
//get alpha and beta
syntax KItem ::= Module(K, K)
syntax KItem ::= preModule(K,K) //(alpha, T)

// STEP 1 CONSTRUCT T AND ALPHA
// alpha = type
// T = newtype and data, temporary data structure

syntax KItem ::= initPreModule(K) [function]
syntax KItem ::= getPreModule(K, K) [function] //(Current
    term, premodule)
syntax KItem ::= makeT (K,K,K,K)

syntax KItem ::= fetchTypes (K,K,K,K)

syntax List ::= makeInnerT (K,K,K) [function] //LIST
syntax List ::= getTypeVars(K) [function] //LIST

syntax KItem ::= getCon(K) [function]
syntax List ::= getArgSorts(K) [function] //LIST

syntax KItem ::= AList(K)
syntax KItem ::= AObject(K,K) //(1st -> 2nd) map without
    idempotency
syntax KItem ::= ModPlusType(K,K)

syntax KItem ::= TList(K) //list of T objects for every new
    type introduced by data and newtype
syntax KItem ::= TObject(K,K,K,K) //(module name, type name,
     entire list of poly type vars, list of inner T pieces)
syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type constructor,
     poly type vars, argument sorts, entire constr block,
    type name)

```

```
    rule initPreModule(J:K) => getPreModule(J,preModule(AList(.List),TList(.List)))

    rule getPreModule(’bodytopdecls(I:K), J:K) => getPreModule(I,J)
    rule getPreModule(’topdeclslist(’type(A:K,, B:K),, Rest:K),J:K) => fetchTypes(A,B,Rest,J) //constructalpha


    rule getPreModule(’topdeclslist(’data(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => makeT(B,C,Rest,J)
    rule getPreModule(’topdeclslist(’newtype(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => makeT(B,C,Rest,J)


    rule getPreModule(’topdeclslist(’topdecldecl(A:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule(’topdeclslist(’class(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule(’topdeclslist(’instance(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule(’topdeclslist(’default(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule(’topdeclslist(’foreign(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule(.TopDecls,J:K) => J

    rule <k> fetchTypes(’simpleTypeCon(I:TyCon,, H:TyVars), ’atypeGTyCon(C:K), Rest:K, preModule(AList(M:List), L:K)) => getPreModule(Rest,preModule(AList(ListItem(AObject(ModPlusType(ModName,I),C)) M), L)) ...</k>
      <tempModule> ModName:KItem </tempModule>

    rule <k> makeT(’simpleTypeCon(I:TyCon,, H:TyVars), D:K, Rest:K, preModule(AList(M:List), TList(ListInside:List))) => getPreModule(Rest,preModule(AList(M),TList(ListItem(TObject(ModName,I,H,makeInnerT(I,H,D))) ListInside))) ...</k>
      <tempModule> ModName:KItem </tempModule>

    rule makeInnerT(A:K,B:K,’nonemptyConstrs(C:K)) => makeInnerT(A,B,C)
```

```
    rule makeInnerT(A:K,B:K,'singleConstr(C:K)) => ListItem(
       InnerTPiece(getCon(C),getTypeVars(C),getArgSorts(C),C,A))
    rule makeInnerT(A:K,B:K,'multConstr(C:K,, D:K)) => ListItem(
       InnerTPiece(getCon(C),getTypeVars(C),getArgSorts(C),C,A))
        makeInnerT(A,B,D)

    rule getTypeVars('constrCon(A:K,, B:K)) => getTypeVars(B)
    rule getTypeVars('optBangATypes(A:K,, Rest:K)) =>
       getTypeVars(A) getTypeVars(Rest)
    rule getTypeVars('optBangAType('emptyBang(.KList),, Rest:K))
        => getTypeVars(Rest)
    rule getTypeVars('atypeGTyCon(A:K)) => .List
    rule getTypeVars('atypeTyVar(A:K)) => ListItem(A)
    rule getTypeVars(.OptBangATypes) => .List

    rule getCon('constrCon(A:K,, B:K)) => A

    rule getArgSorts('constrCon(A:K,, B:K)) => getArgSorts(B)
    rule getArgSorts('optBangATypes(A:K,, Rest:K)) =>
       getArgSorts(A) getArgSorts(Rest)
    rule getArgSorts('optBangAType('emptyBang(.KList),, Rest:K))
        => getArgSorts(Rest)
    rule getArgSorts('atypeGTyCon(A:K)) => ListItem(A)
    rule getArgSorts('atypeTyVar(A:K)) => .List
    rule getArgSorts(.OptBangATypes) => .List

////////////////////////////////////////////////////////////////////////////////////

    rule <k> preModule(A:K,T:K) => startTTransform ...</k>
         <tempAlpha> OldAlpha:K => A </tempAlpha>
         <tempT> OldT:K => T </tempT>
```

4.4 Context Sensitive Checks

For the context sensitive checks, the semantics performs several checks that are not inclusive to all context sensitive checks that need to be made in a complete Haskell semantics, but enough for some sanity of the type inference function.

K Code

The following code introduces all checks that need to be made.

```
    // STEP 2 PERFORM CHECKS

    syntax KItem ::= "error"

    syntax KItem ::= "startChecks"
    syntax KItem ::= "checkNoSameKey"
        //Keys of alpha and keys of T should be unique
    syntax KItem ::= "checkTypeConsDontCollide"
        //Make sure typeconstructors do not collide in T
    syntax KItem ::= "makeAlphaMap"
        //make map for alpha
    syntax KItem ::= "checkAlphaNoLoops"
        //alpha check for no loops
        //check alpha to make sure that everything points to a T
    syntax KItem ::= "checkArgSortsAreTargets"
           //Make sure argument sorts [U] [W,V] are in the set of keys of alpha and targets of T, (keys of T)
    syntax KItem ::= "checkParUsed"
//NEED TO CHECK all the polymorphic parameters from right appear on left. RIGHT SIDE ONLY
//NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY

    rule <k> startChecks
             => checkNoSameKey
                ~> (checkTypeConsDontCollide
                ~> (makeAlphaMap
                ~> (checkAlphaNoLoops
                ~> (checkArgSortsAreTargets
                ~> (checkParUsed))))) ...</k>
```

4.4.1 Name Collision

The programmer should not be able to make two user defined datatypes with the same name, even if one is created using the constructor *data* and another is created using the constructor *type* for instance.

The following example should not be allowed.

```
data Date = Date Int
```

```
;type Date = Contwo Int
```

K Code

The following is the code for the check.

```
    rule <k> checkTypeConsDontCollide
             => tyConCollCheck(T,.List,.Set) ...</k>
         <tempT> T:K </tempT>

    syntax KItem ::= tyConCollCheck(K,K,K) [function] //(TList,
       List of Tycons,Set of Tycons)
    syntax KItem ::= lengthCheck(K,K) [function]

    rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:
       K,ListItem(InnerTPiece(Ty:K,E:K,F:K,H:K,G:K)) Inners:List
       )) Rest:List),J:List,D:Set) =>
                   tyConCollCheck(TList(ListItem(TObject(
                      ModName,A,B,Inners)) Rest),ListItem(Ty) J
                      , SetItem(Ty) D)
    rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:
       K,.List)) Rest:List),J:List,D:Set) =>
                   tyConCollCheck(TList(Rest),J,D)
    rule tyConCollCheck(TList(.List),J:List,D:Set) =>
                   lengthCheck(size(J),size(D))

    rule lengthCheck(A:Int, B:Int) => .K
                   requires A ==Int B

    rule lengthCheck(A:Int, B:Int) => error
                   requires A =/=Int B
```

4.4.2 Type Constructor Collision

The programmer should not be able to use the same type constructors when making different options for their types or use the same type constructors for different types.

The following example should not be allowed.

```
data Date = Date Int | Date Bool
```

The following example should also not be allowed.

```
data Date = Date Int
;data Datetwo = Date Int
```

K Code

The following is the code for the check.

```
    syntax KItem ::= keyCheck(K,K,K,K) [function] //(Alpha, T, List of names, Set of names)

    rule <k> checkNoSameKey
             => keyCheck(A, T, .Set, .List) ...</k>
         <tempAlpha> A:K </tempAlpha>
         <tempT> T:K </tempT>

    rule keyCheck(AList(ListItem(AObject(A:K,B:K)) C:List), T:K, D:Set, G:List) => keyCheck(AList(C), T, SetItem(A) D, ListItem(A) G)
    rule keyCheck(AList(.List), TList(ListItem(TObject(ModName:K, A:K,B:K,C:K)) Rest:List), D:Set, G:List) => keyCheck(AList(.List), TList(Rest), SetItem(A) D, ListItem(A) G)
    rule keyCheck(AList(.List), TList(.List), D:Set, G:List) => lengthCheck(size(G),size(D))


    syntax KItem ::= makeAlphaM(K,K) [function] //(Alpha, AlphaMap)
    syntax KItem ::= tAlphaMap(K) //(AlphaMap) temp alphamap

    rule <k> makeAlphaMap
             => makeAlphaM(A, .Map) ...</k>
         <tempAlpha> A:K </tempAlpha>

    rule makeAlphaM(AList(ListItem(AObject(A:K,B:K)) C:List), M:Map) => makeAlphaM(AList(C), M[A <- B])
    rule makeAlphaM(AList(.List), M:Map) => tAlphaMap(M)

    rule <k> tAlphaMap(M:K) => .K ...</k>
         <tempAlphaMap> OldAlphaMap:K => M </tempAlphaMap>
```

4.4.3 Alpha Cycle Check

There should be no cycles in type renaming using *type*, and the type renaming chains using *type* should terminate with a type defined with *data* or *newtype*.

The following example should not be allowed.

```
type Birthday = Date
;type Date = Birthday
```

The following example should also not be allowed if Date is not defined anywhere.

```
type Birthday = Date
```

K Code

The following is the code for the check.

```
    syntax KItem ::= aloopCheck(K,K,K,K,K,K,K) [function] //(
       Alpha,List of Alpha,Set of Alpha,CurrNode,lengthcheck,T,
       BigSet)

    rule <k> checkAlphaNoLoops
             => aloopCheck(A,.List,.Set,.K,.K,T,.Set) ...</k>
        <tempAlphaMap> A:K </tempAlphaMap>
        <tempT> T:K </tempT>

    //aloopCheck set and list to check cycles
    rule aloopCheck(Alpha:Map (A:KItem |-> B:KItem), D:List, G:
       Set, .K, .K,T:K,S:Set) => aloopCheck(Alpha, ListItem(B)
       ListItem(A) D, SetItem(B) SetItem(A) G, B, .K,T,S)
    rule aloopCheck(Alpha:Map (H |-> B:KItem), D:List, G:Set, H:
       KItem, .K,T:K,S:Set) => aloopCheck(Alpha, ListItem(B) D,
       SetItem(B) G, B, .K,T,S)

    rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
       Set) => aloopCheck(Alpha, .List, .Set, .K, lengthCheck(
       size(G),size(D)),T,G S) //type rename loop ERROR
        requires (notBool H in keys(Alpha)) andBool (H in
           typeSet(T, .Set) orBool H in S)

    rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
       Set) => error //terminal alpha rename is not in T ERROR
```

```
        requires (notBool H in keys(Alpha)) andBool (notBool (H in typeSet(T, .Set) orBool H in S))


    syntax Set ::= typeSet(K,K) [function] //(K, KSet)
    rule typeSet(TList(ListItem(TObject(ModName:K, A:K,B:K,C:K)) Rest:List), D:Set) => typeSet(TList(Rest), SetItem(A) D)
    rule typeSet(TList(.List), D:Set) => D

    rule aloopCheck(.Map, .List, .Set, .K, .K,T:K, S:Set) => .K
```

4.4.4 Argument Sort Check

The argument sorts for types defined using the *data* keyword or the *newtype* keyword should be types that exist.

The following example should not be allowed if *Date* is not defined anywhere.

```
Data Birthday = Birthday Date
```

K Code

The following is the code for the check.

```
//Make sure argument sorts [U] [W,V] are in the set of keys of alpha and targets of T, (keys of T)

    syntax KItem ::= argSortCheck(K,K,K) [function] //(T, AlphaMap)

    rule <k> checkArgSortsAreTargets
              => argSortCheck(T,A,typeSet(T,.Set)) ...</k>
         <tempAlphaMap> A:K </tempAlphaMap>
         <tempT> T:K </tempT>

    rule argSortCheck(TList(ListItem(TObject(ModName:K, A:K,B:K, ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest:List,E:K,F:K)) InnerRest:List)) TListRest:List),AlphaMap:Map,Tset:Set) => argSortCheck(TList(ListItem(TObject(ModName,A,B,ListItem(InnerTPiece(C,D,ArgsRest,E,F)) InnerRest)) TListRest),AlphaMap,Tset)
```

```
        requires ((Arg in keys(AlphaMap)) orBool (Arg in Tset))

    rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
        ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest
        :List,E:K,F:K)) InnerRest:List)) TListRest:List),AlphaMap
        :Map,Tset:Set) => error
        requires (notBool ((Arg in keys(AlphaMap)) orBool (Arg
            in Tset)))

    rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
        ListItem(InnerTPiece(C:K,D:K,.List,E:K,F:K)) InnerRest:
        List)) TListRest:List),AlphaMap:Map,Tset:Set) =>
        argSortCheck(TList(ListItem(TObject(ModName,A,B,InnerRest
        )) TListRest),AlphaMap,Tset)

    rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.
        List)) TListRest:List),AlphaMap:Map,Tset:Set) =>
        argSortCheck(TList(TListRest),AlphaMap,Tset)

    rule argSortCheck(TList(.List),AlphaMap:Map,Tset:Set) => .K
```

4.4.5 Polymorphic Parameter Check

The polymorphic parameters that appear on the right hand side of a *data* declaration need to appear on the left hand side as well.

The following example should not be allowed.

```
Data Newtype = New a b
```

Also, The polymorphic parameters that appear on the left hand side of a *data* declaration need to be unique. However, the parameters that appear on the right hand side do not need to be unique.

The following example should not be allowed.

```
Data Newtype a a = New a
```

However, the following example should be allowed.

```
Data Newtype a = New a a
```

K Code

The following is the code for the check.

```
//NEED TO CHECK all the polymorphic parameters from right appear on left. RIGHT SIDE ONLY
//NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY

    syntax KItem ::= parCheck(K,K) [function] //(T,AlphaMap)
    syntax KItem ::= makeTyVarList(K,K,K) [function] //(TyVars,NewList)
    syntax KItem ::= lengthRet(K,K,K) [function]

    rule <k> checkParUsed
             => parCheck(T,.K) ...</k>
         <tempT> T:K </tempT>

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,C:K)) Rest:List),.K) => parCheck(TList(ListItem(TObject(ModName,A:K,B:K,C:K)) Rest:List),makeTyVarList(B,.List,.Set))

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
         parCheck(TList(ListItem(TObject(ModName,A,B,ListItem(InnerTPiece(C,ParRest,D,E,F)) InnerRest)) Rest),NewSet)
            requires Par in NewSet

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) => error
            requires notBool (Par in NewSet)

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,ListItem(InnerTPiece(C:K,.List,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
         parCheck(TList(ListItem(TObject(ModName,A,B,InnerRest)) Rest),NewSet)

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.List)) Rest:List),NewSet:Set) =>
```

```
        parCheck(TList(Rest),NewSet)

   rule parCheck(TList(.List),NewSet:Set) => .K

   rule makeTyVarList('typeVars(A:K,,Rest:K),NewList:List,
      NewSet:Set) => makeTyVarList(Rest, ListItem(A) NewList,
      SetItem(A) NewSet)

   rule makeTyVarList(.TyVars,NewList:List,NewSet:Set) =>
      lengthRet(size(NewList),size(NewSet),NewSet)

   rule lengthRet(A:Int, B:Int, C:K) => C
                   requires A ==Int B

   rule lengthRet(A:Int, B:Int, C:K) => error
                   requires A =/=Int B
```

Chapter 5

Inferencing

Haskell's type system is a Hindley-Milner polymorphic type system that has been extended with type classes to account for overloaded functions.

[haskell 2010 report]

A type system is a set of rules that assign a property to various constructs in a programming language called type. A type is a property that allows the programmer to add constraints to programs.

`https://en.wikipedia.org/wiki/Type_system`

The type inference function infers the type of the expression that is fed into it. A purpose of it is to ensure that all functions and function applications are allowed with regards to the types of the inputs and outputs.

5.1 Type theory

Type theory was created by Bertrand Russell to prevent Russell's Paradox for set theory, introduced by Georg Cantor. The issue was that not specifying a certain property for sets allowed sets to contain themselves in Naive Set Theory. So Bertrand Russell prevented this problem by specifying a property called type for objects, and objects cannot contain their own type.

`https://plato.stanford.edu/entries/type-theory/`

5.2 Data Structures

A monomorphic data type looks like $(a \to b) \to c$

In my K semantics, the equivalent data type looks like

```
forAll ( .Set , funtype ( funtype ( a , b ) , c ) )
```

A polymorphic data type looks like $\forall abc \,.\, (a \to b) \to c$

In my K semantics, the equivalent data type looks like

```
forAll ( a b c , funtype ( funtype ( a , b ) , c ) )
```

I needed to create a syntax for polymorphic types that may contain monomorphic type variables and polymorphic type variables.

Then I made a map from type constructor names to arities, called Delta.

Then I made a map from data constructors and term identifiers to their most general polymorphic types, called beta.

The first part was converting the T data structure into beta, which is more suited for type inference.

5.2.1 Transform T into Beta

```
    // STEP 3 Transform T into beta

    syntax KItem ::= "startTTransform"
    syntax KItem ::= "constructDelta"
    syntax KItem ::= "constructBeta"

    rule <k> startTTransform
             => constructDelta
                ~> (constructBeta) ...</k>

    rule <k> constructDelta
             => makeDelta(T,.Map) ...</k>
         <tempT> T:K </tempT>

    syntax KItem ::= makeDelta(K,Map) [function] //(T,Delta)
    syntax KItem ::= newDelta(Map) //Delta
    syntax KItem ::= newBeta(Map) //beta
    syntax List ::= retPolyList(K,List) [function] //(T,Delta)
```

5.2.2 Construct Beta

Beta is a map from the type constructor to its corresponding type.

Example

If the user makes the data type *CusBool* in module *Simp*5, and declares it with this example...

```
data CusBool = True2 | False2
```

Then the corresponding *tempBeta* should look like this. Note how the monomorphic datatype just has an empty *forAll*.

```
    <tempBeta>
        ModPlusType ( Simp5 , False2 ) |-> forAll ( .Set ,
           CusBool .TyVars )
        ModPlusType ( Simp5 , True2 ) |-> forAll ( .Set ,
           CusBool .TyVars )
    </tempBeta>
```

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
data CusBool a b = True2 a | False2 b
```

Then the corresponding tempBeta should look like this.

```
    <tempBeta>
        ModPlusType ( Simp5 , False2 ) |-> forAll ( b , funtype
            ( b , CusBool a b ) )
        ModPlusType ( Simp5 , True2 ) |-> forAll ( a , funtype (
             a , CusBool a b ) )
    </tempBeta>
```

K Code

The following is the K Code.

```
    rule <k> constructBeta
             => makeBeta(T,.Map) ...</k>
         <tempT> T:K </tempT>

    syntax KItem ::= makeBeta(K,Map) [function] //(T,Beta,Delta)

    rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,
       ListItem(InnerTPiece(Con:K,H:K,D:K,E:K,F:K)) InnerRest:
       List)) Rest:List),Beta:Map) =>
```

```
         makeBeta(TList(ListItem(TObject(ModName,A,B,InnerRest)) Rest),Beta[ModPlusType(ModName,Con) <- betaParser(E,B,A)])
    rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,.List)) Rest:List),Beta:Map) =>
         makeBeta(TList(Rest),Beta)
    rule makeBeta(TList(.List),Beta:Map) =>
         newBeta(Beta)

    syntax KItem ::= betaParser(K,K,K) [function] //(Tree Piece, NewSyntax,Parameters,Constr)
    syntax Set ::= getTyVarsRHS(K,List) [function]

    syntax KItem ::= forAll(Set,K)
    syntax KItem ::= funtype(K,K)

    syntax Set ::= listToSet(List, Set) [function]

    rule listToSet(ListItem(A:KItem) L:List, S:Set) => listToSet(L, SetItem(A) S)
    rule listToSet(.List, S:Set) => S


//if optbangAtypes, need to see if first variable is a typecon
//if its a typecon then need to go into Delta and see the amount of parameters it has
//then count the number of optbangAtypes after the typecon
    rule betaParser('constrCon(A:K,, B:K), Par:K, Con:K) => forAll(getTyVarsRHS(B,.List), betaParser(B, Par, Con))
    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'atypeTyVar(Tyv:K)),, Rest:K), Par:K, Con:K) => funtype(Tyv, betaParser(Rest, Par, Con))
    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'baTypeCon(A:K,, B:K)),, Rest:K), Par:K, Con:K) => funtype('baTypeCon(A:K,, B:K), betaParser(Rest, Par, Con))
    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'atypeGTyCon(Tyc:K)),, Rest:K), Par:K, Con:K) => funtype(Tyc, betaParser(Rest, Par, Con))
    rule betaParser(.OptBangATypes, Par:K, Con:K) => 'simpleTypeCon(Con,, Par)

```

```
    rule getTyVarsRHS(.OptBangATypes,Tylist:List) => listToSet(
        Tylist, .Set)

    rule <k> newBeta(M:Map)
              => .K ...</k>
          <tempBeta> OldBeta:K => M </tempBeta>
```

5.2.3 Construct Delta

Delta contains the arity of the user defined dataTypes. So if a user defined datatype takes in two parameters, tempDelta will contain the number 2.

Example

If the user makes the data type *CusBool* in module *Simp*5, and declares it with this example...

```
data CusBool a b = True2 a | False2 b
```

Then the corresponding *tempDelta* should look like this.

```
<tempDelta>
     ModPlusType ( Simp5 , CusBool ) |-> 2
</tempDelta>
```

```
    rule makeDelta(TList(ListItem(TObject(ModName:K,A:K,Polys:K,
        C:K)) Rest:List),M:Map) =>
          makeDelta(TList(Rest),M[ModPlusType(ModName,A) <- size(
              retPolyList(Polys,.List))])
    rule makeDelta(TList(.List),M:Map) => newDelta(M)

    rule retPolyList('typeVars(A:K,,Rest:K),NewList:List) =>
        retPolyList(Rest, ListItem(A) NewList)
    rule retPolyList(.TyVars,L:List) => L

    rule <k> newDelta(M:Map)
              => .K ...</k>
          <tempDelta> OldDelta:K => M </tempDelta>
```

5.3 Definition of Substitution

A substitution is a set of variables and their replacements. Applying a substitution to an expression means to simultaneously replace each variable in the expression with the replacement term.

`http://www.mathcs.duq.edu/simon/Fall04/notes-7-4/node3.html`

5.4 Inference Algorithm

Much of the inference algorithm is the same as the one introduced in cs 421.

[cs 421 gather exp ty substitution mp]

```
requires "haskell-syntax.k"
requires "haskell-configuration.k"
requires "haskell-preprocessing.k"

module HASKELL-TYPE-INFERENCING
    imports HASKELL-SYNTAX
    imports HASKELL-CONFIGURATION
    imports HASKELL-PREPROCESSING

    syntax KItem ::= "Bool" //Boolean

    // STEP 4 Type Inferencing
    syntax KItem ::= inferenceShell(K) [function]//Input, AlphaMap, Beta, Delta, Gamma
    //syntax KItem ::= typeInferenceFun(K,Map,Map,Map,Map,K,K) [function]//Input, Alpha, Beta, Delta, Gamma
    //syntax KItem ::= typeInferenceFun(Map,K,K) //Gamma, Expression, Guessed Type
    syntax Map ::= genGamma(K,Map,K) [function] //Apatlist, Gamma Type
    syntax KItem ::= genLambda(K,K) [function]
    syntax KItem ::= guessType(Int)
    syntax KItem ::= freshInstance(K, Int) [function]
    syntax Int ::= paramSize(K) [function]


    syntax KItem ::= mapBag(Map)
    syntax KResult ::= mapBagResult(Map)
```

```

    syntax Map ::= gammaSub(Map,Map,Map) [function]//
        substitution, gamma

    rule <k> performIndividualInferencing => inferenceShell(Code
        ) ...</k>
          <tempModule> Mod:KItem </tempModule>

          <moduleName> 'moduleName(Mod) </moduleName>
          <moduleTempCode> Code:KItem </moduleTempCode>

    rule inferenceShell('topdeclslist('type(A:K,, B:K),, Rest:K)
        ) =>
          inferenceShell(Rest) //constructalpha
    rule inferenceShell('topdeclslist('data(A:K,, B:K,, C:K,, D:
        K),, Rest:K)) =>
          inferenceShell(Rest)
    rule inferenceShell('topdeclslist('newtype(A:K,, B:K,, C:K,,
         D:K),, Rest:K)) =>
          inferenceShell(Rest)
    rule inferenceShell('topdeclslist('class(A:K,, B:K,, C:K,, D
        :K),, Rest:K)) =>
          inferenceShell(Rest)
    rule inferenceShell('topdeclslist('instance(A:K,, B:K,, C:K
        ,, D:K),, Rest:K)) =>
          inferenceShell(Rest)
    rule inferenceShell('topdeclslist('default(A:K,, B:K,, C:K,,
         D:K),, Rest:K)) =>
          inferenceShell(Rest)
    rule inferenceShell('topdeclslist('foreign(A:K,, B:K,, C:K,,
         D:K),, Rest:K)) =>
          inferenceShell(Rest)

    rule inferenceShell('topdeclslist('topdecldecl(A:K),, Rest:K
        )) =>
          typeInferenceFun(.ElemList, .Map,A,guessType(0)) ~>
              inferenceShell(Rest)


    rule <k> typeInferenceFun(.ElemList, Gamma:Map, '
        declFunLhsRhs(Fn:K,, Lhsrhs:K), Guess:K) =>
          typeInferenceFun(.ElemList, Gamma, Lhsrhs, Guess) ...</
              k>
```

```
rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'eqExpOptDecls(Ex:K,, Optdecls:K), Guess:K) =>
    typeInferenceFun(.ElemList, Gamma, Ex, Guess) ...</k>
```

5.4.1 Variable Rule

$$\frac{}{\Gamma \vdash x : \tau \mid \text{unify}\{(\tau, \text{freshInstance}(\Gamma(x)))\}}\ \text{Variable}$$

```
rule <k> typeInferenceFun(.ElemList, (Var |-> Type:K) Gamma:Map, 'aexpQVar(Var:K), Guess:KItem)
      => mapBagResult(uniFun(ListItem(uniPair(Guess, freshInstance(Type, TypeIt))))) ...</k> //Variable rule
    <typeIterator> TypeIt:Int => TypeIt +Int paramSize(Type) </typeIterator>
```

5.4.2 Constant Rule

$$\frac{}{\Gamma \vdash c : \tau \mid \text{unify}\{(\tau, \text{freshInstance}(\tau))\}}\ \text{Constant}$$

```
rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'aexpGCon('conTyCon(Mid:K,, Gcon:K)), Guess:KItem)
      => mapBagResult(uniFun(ListItem(uniPair(Guess, freshInstance(Type, TypeIt))))) ...</k> //Constant rule
    <tempBeta> (ModPlusType(Mid,Gcon) |-> Type:K) Beta:Map </tempBeta>
    <typeIterator> TypeIt:Int => TypeIt +Int paramSize(Type) </typeIterator>
```

5.4.3 Lambda Rule

$$\frac{[x : \tau_1] + \Gamma \vdash e : \tau_2 \mid \sigma}{\Gamma \vdash \backslash\ \text{x} \rightarrow\ \text{e} : \tau \mid \text{unify}\{(\sigma(\tau), \sigma(\tau_1 \rightarrow \tau_2))\} \circ \sigma}\ \text{Lambda}$$

```

   syntax KItem ::= typeInferenceFun(ElemList, Map, K, K) [strict(1)]
   syntax KItem ::= typeInferenceFunLambda(ElemList, K, K, K) [strict(1)]
/* automatically generated by the strict(1) in typeInferenceFun or typeInferenceFunAux
   rule typeInferenceFunAux(Es:ElemList, C:K, A:K, B:K) => Es ~> typeInferenceFun(HOLE, C, A, B)
        requires notBool isKResult(Es)
   rule Es:KResult ~> typeInferenceFunAux(HOLE, C:K,A:K, B:K) => typeInferenceFun(Es, C, A, B)
*/

    //lambda rule
    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'lambdaFun(Apatlist:K,, Ex:K), Guess:KItem)
          => typeInferenceFunLambda(val(typeInferenceFun(.ElemList, genGamma(Apatlist,Gamma,guessType(TypeIt)), genLambda(Apatlist,Ex), guessType(TypeIt +Int 1))), .ElemList, Guess, guessType(TypeIt),guessType(TypeIt +Int 1)) ...</k>
         <typeIterator> TypeIt:Int => TypeIt +Int 2 </typeIterator>

    rule <k> typeInferenceFunLambda(valValue(mapBagResult(Sigma:Map)), .ElemList, Tau:K, Tauone:K, Tautwo:K)
         => mapBagResult(compose(uniFun(ListItem(uniPair(typeSub(Sigma,Tau),typeSub(Sigma,funtype(Tauone,Tautwo))))),Sigma)) ...</k>
```

5.4.4 Application Rule

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau \,|\, \sigma_1 \qquad \sigma_1(\Gamma) \vdash e_2 : \sigma_1(\tau_1) \,|\, \sigma_2}{\Gamma \vdash e_1 e_2 : \tau \,|\, \sigma_2 \circ \sigma_1} \text{Application}$$

```

    syntax KItem ::= typeInferenceFunAppli(ElemList, Map, K, K, Map) [strict(1)]

    //application rule
```

```
rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'funApp(Eone:K,, Etwo:K), Guess:KItem)
    => typeInferenceFunAppli(val(typeInferenceFun(.ElemList, Gamma, Eone, funtype(guessType(TypeIt), Guess))), .ElemList, Gamma, Etwo, guessType(TypeIt), .Map) ...</k>
   <typeIterator> TypeIt:Int => TypeIt +Int 1 </typeIterator>

rule <k> typeInferenceFunAppli(valValue(mapBagResult(Sigmaone:Map)), .ElemList, Gamma:Map, Etwo:KItem, guessType(TypeIt:Int), .Map)
    => typeInferenceFunAppli(val(typeInferenceFun(.ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo, typeSub(Sigmaone, guessType(TypeIt)))), .ElemList, .Map, .K, .K, Sigmaone) ...</k>

rule <k> typeInferenceFunAppli(valValue(mapBagResult(Sigmatwo:Map)), .ElemList, .Map, .K, .K, Sigmaone:Map)
    => mapBagResult(compose(Sigmatwo, Sigmaone)) ...</k>
```

5.4.5 IfThenElse Rule

For functions, function guards, cases, and if-then-else are all equivalent.

$$\frac{\Gamma \vdash e_1 : \mathrm{bool} \,|\, \sigma_1 \qquad \sigma_1(\Gamma) \vdash e_2 : \sigma_1(\tau) \,|\, \sigma_2 \qquad \sigma_2 \circ \sigma_1(\Gamma) \vdash e_3 : \sigma_2 \circ \sigma_1(\tau) \,|\, \sigma_3}{\Gamma \vdash \mathrm{if}\, e_1 \,\mathrm{then}\, e_2 \,\mathrm{else}\, e_3 : \tau \,|\, \sigma_3 \circ \sigma_2 \circ \sigma_1}\ \mathrm{IfThenElse}$$

```
syntax KItem ::= typeInferenceFunIfThen(ElemList, Map, K, K, K, Map, Map) [strict(1)]

//if_then_else rule
rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'ifThenElse(Eone:K,, Optsem:K,, Etwo:K,, Optsemtwo:K,, Ethree:K), Guess:KItem)
    => typeInferenceFunIfThen(val(typeInferenceFun(.ElemList, Gamma, Eone, Bool)), .ElemList, Gamma, Etwo, Ethree, Guess, .Map, .Map) ...</k>

```

```
    rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
        Sigmaone:Map)), .ElemList, Gamma:Map, Etwo:KItem, Ethree:
        KItem, Guess:KItem, .Map, .Map)
            => typeInferenceFunIfThen(val(typeInferenceFun(.
                ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo,
                typeSub(Sigmaone, Guess))), .ElemList, Gamma, .K,
                Ethree, Guess, Sigmaone, .Map) ...</k>

    rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
        Sigmatwo:Map)), .ElemList, Gamma:Map, .K, Ethree:KItem,
        Guess:KItem, Sigmaone:Map, .Map)
            => typeInferenceFunIfThen(val(typeInferenceFun(.
                ElemList, gammaSub(compose(Sigmatwo, Sigmaone),
                Gamma, .Map), Ethree, typeSub(compose(Sigmatwo,
                Sigmaone), Guess))), .ElemList, .Map, .K, .K, .K,
                Sigmaone, Sigmatwo) ...</k>

    rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
        Sigmathree:Map)), .ElemList, .Map, .K, .K, .K, Sigmaone:
        Map, Sigmatwo:Map)
            => mapBagResult(compose(compose(Sigmathree, Sigmatwo),
                 Sigmaone)) ...</k>
```

5.5 Mutual Recursion

Something to note is that mutually recursive functions are allowed in Haskell. For example:

$$fx = yx$$

$$yx = fx$$

This set of functions is allowed to compile. When run, the function just simply runs forever. This is unlike the OCaml semantics, which does not allow for mutually recursive functions.

The LetIn Rule must allow for mutual recursion. This means that an example program that must be inferred is

```
let {f = \x -> y x; y = \x -> f x} in (f 2)
```

In this example, the f is an expression that refers to y and y is an expression that refers to f.

5.5.1 LetIn Rule

This let rule, unlike the let rule introduced in cs 421, allows for mutual recursion.

$$\frac{\Gamma \vdash \{\,\text{decls}\,\} : \Delta \,|\, \sigma \qquad \sigma(\Delta + \Gamma) \vdash e : \tau \,|\, \sigma'}{\Gamma \vdash \text{let}\,\{\,\text{decls}\,\}\,\text{in}\,\text{exp} : \tau \,|\, \sigma' \circ \sigma}\,\text{LetIn}$$

$$\frac{\Gamma \vdash \{\,\text{decls}\,\} : \Delta \,|\, \sigma}{\Gamma \vdash \text{module}\,\{\,\text{decls}\,\} : \Delta \,|\, \sigma}\,\text{Module}$$

$$\frac{\sigma'_0 = \{\} \qquad \text{dom}(\Psi) = \{\,x_1, \cdots, x_n\,\} \qquad \sigma_{i+1}(\Psi + \Gamma) \vdash e_i : \Psi(x_i) \,|\, \sigma_j \qquad \sigma'_i = \sigma_i \circ \sigma'_{j+1}}{\Gamma \vdash \{\,x_1 = e_1;\, \cdots\, x_i = e_i;\, \cdots\, x_n = e_n\,\}\,\text{in}\,\text{exp} : \bigcup_{i=1}^{n} \{\,x_i \mapsto\ \text{GEN}\,(\sigma'_n(\Gamma), \sigma'_n(\psi_i))\,\} \,|\, \sigma'_n}\,\text{LetIn}$$

K Code

```

syntax KItem ::= typeInferenceFunLetIn(ElemList, Map, Map, K, K, K, Int, Int, Map, Map) [strict(1)]
syntax KItem ::= grabLetDeclName(K, Int) [function]
syntax KItem ::= grabLetDeclExp(K, Int) [function]
syntax KItem ::= mapLookup(Map, K) [function]
syntax Map ::= makeDeclMap(K, Int, Map) [function]
syntax Map ::= applyGEN(Map, Map, Map, Map) [function]

//Haskell let in rule (let rec in exp + let in rule combined)
//gamma |- let rec f1 = e1 and f2 = e2 and f3 = e3 .... in e =>
//beta, [f1 -> tau1, f2 -> tau2, f3 -> tau3,....] + gamma |- e1 : tau1 | sigma1,  [f1 -> simga1(tau1), f2 -> sigma1(tau2), f3 -> sigma1(tau3),....] + sigma1(gamma) |- e2 : sigma1(tau2) | sigma2  [f1 -> sigma2 o sigma1(tau1), f2 -> sigma2 o sigma1(tau2), f3 -> sigma2 o sigma1(tau3),....] + sigma2 o sigma1(gamma) |- e3 : sigma2 o sigma1(tau3) .....  [f1 -> gen(sigma_n o sigma2 o sigma1(tau1), sigma_n o sigma2 o sigma1(Gamma)), f2 -> gen(tau2), f3 -> gen(tau3),....] + gamma |- e : something
rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'letIn(D:K,, E:K), Guess:KItem)
       => typeInferenceFunLetIn(.ElemList, Gamma, makeDeclMap(D, TypeIt, .Map), D, E, Guess, 0, TypeIt, .Map, Beta) ...</k>
```

```
        <typeIterator> TypeIt:Int => TypeIt +Int size(makeDeclMap(D, TypeIt, .Map)) </typeIterator>
        <tempBeta> Beta:Map </tempBeta>

    rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map, Beta:Map)
          => typeInferenceFunLetIn(val(typeInferenceFun(.ElemList, Gamma DeclMap, grabLetDeclExp(D, Iter), mapLookup(DeclMap, grabLetDeclName(D, Iter)))), .ElemList, Gamma, DeclMap, D, E, Guess, Iter, TypeIt, OldSigma, Beta) ...</k>
         //=> typeInferenceFunLetIn(val(typeInferenceFun(DeclMap, grabLetDeclExp(D, Iter +Int TypeIt), Guess)), .ElemList, Gamma, DeclMap, D, E, Guess, Iter, TypeIt, OldSigma) ...</k>
         requires Iter <Int (size(DeclMap))

    rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:Map)), .ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map, Beta:Map)
         => typeInferenceFunLetIn(.ElemList, gammaSub(Sigma, Gamma,.Map), gammaSub(Sigma, DeclMap,.Map), D, E, typeSub(Sigma, Guess), Iter +Int 1, TypeIt, compose(Sigma,OldSigma), Beta) ...</k>
         requires Iter <Int (size(DeclMap))

    rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map, Beta:Map)
         => typeInferenceFunLetIn(val(typeInferenceFun(.ElemList, Gamma applyGEN(Gamma, DeclMap, .Map, Beta), E, Guess)), .ElemList, Gamma, DeclMap, D, E, Guess, Iter, TypeIt, OldSigma, Beta) ...</k>
         requires Iter >=Int (size(DeclMap))

    rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:Map)), .ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map, Beta:Map)
         => mapBagResult(compose(Sigma, OldSigma))...</k>
         requires Iter >=Int (size(DeclMap))
```

```

rule mapLookup((Name |-> Type:KItem) DeclMap:Map, Name:KItem
   ) => Type
rule mapLookup(DeclMap:Map, Name:KItem) => Name
     requires notBool(Name in keys(DeclMap))

rule makeDeclMap('decls(Dec:K), TypeIt:Int, NewMap:Map) =>
   makeDeclMap(Dec, TypeIt, NewMap)
rule makeDeclMap('declsList('declPatRhs('apatVar(Var:K),,
   Righthand:K),, Rest:K), TypeIt:Int, NewMap:Map) =>
   makeDeclMap('decls(Rest), TypeIt +Int 1, NewMap[Var <-
   guessType(TypeIt)])
rule makeDeclMap(.DeclsList, TypeIt:Int, NewMap:Map) =>
   NewMap

rule grabLetDeclName('decls(Dec:K), Iter:Int) =>
   grabLetDeclName(Dec, Iter)
rule grabLetDeclName('declsList(Dec:K,, Rest:K), Iter:Int)
   => grabLetDeclName(Rest, Iter -Int 1)
     requires Iter >Int 0
rule grabLetDeclName('declsList('declPatRhs('apatVar(Var:K)
   ,, Righthand:K),, Rest:K), Iter:Int) => Var
     requires Iter <=Int 0


rule grabLetDeclExp('decls(Dec:K), Iter:Int) =>
   grabLetDeclExp(Dec, Iter)
rule grabLetDeclExp('declsList(Dec:K,, Rest:K), Iter:Int) =>
    grabLetDeclExp(Rest, Iter -Int 1)
     requires Iter >Int 0
rule grabLetDeclExp('declsList('declPatRhs('apatVar(Var:K),,
    Righthand:K),, Rest:K), Iter:Int) => grabLetDeclExp(
   Righthand, Iter)
     requires Iter <=Int 0
rule grabLetDeclExp('eqExpOptDecls(Righthand:K,, Opt:K),
   Iter:Int) => 'eqExpOptDecls(Righthand,, Opt)

rule genGamma('apatVar(Vari:K), Gamma:Map, Guess:K) => Gamma
   [Vari <- Guess]
rule genGamma('apatCon(Vari:K,, Pattwo:K), Gamma:Map, Guess:
   K) => Gamma[Vari <- Guess]

rule genLambda('apatVar(Vari:K), Ex:K) => Ex
```

```
rule genLambda('apatCon(Vari:K,, Pattwo:K), Ex:K) => '
    lambdaFun(Pattwo,, Ex)



rule gammaSub(Sigma:Map, (Key:KItem |-> Type:KItem) Gamma:
    Map, Newgamma:Map)
     => gammaSub(Sigma, Gamma, Newgamma[Key <- typeSub(Sigma,
          Type) ] )

rule gammaSub(Sigma:Map, .Map, Newgamma:Map)
  => Newgamma
```

5.6 Fresh Instance

```
rule freshInstance(guessType(TypeIt:Int), Iter:Int) =>
    guessType(TypeIt)
rule freshInstance(forAll(.Set, B:K), Iter:Int) => B
rule freshInstance(forAll(SetItem(C:KItem) A:Set, B:K), Iter
    :Int) => freshInstance(forAll(A, freshInstanceInner(C, B,
     Iter)), Iter +Int 1)

syntax KItem ::= freshInstanceInner(K,K,Int) [function]

rule freshInstanceInner(Repl:KItem, funtype(A:K, B:K), Iter:
    Int) => funtype(freshInstanceInner(Repl,A,Iter),
    freshInstanceInner(Repl,B,Iter))
rule freshInstanceInner(Repl:KItem, Repl, Iter:Int) =>
    guessType(Iter)
rule freshInstanceInner(Repl:KItem, Target:KItem, Iter:Int)
    => Target [owise]

rule paramSize(forAll(A:Set, B:K)) => size(A)
rule paramSize(A:K) => 0 [owise]
```

5.7 GEN

$$\mathrm{GEN}(\Gamma, \tau) = \forall \alpha_1, \cdots, \alpha_n \,.\, \tau$$

where

$$\{\, \alpha_1, \cdots, \alpha_n \,\} = \mathrm{freevarsty}(\tau) - \mathrm{freevarsenv}(\Gamma)$$

$$\mathrm{freevarsty}('\alpha) = \{\, '\alpha \,\}$$

$$\mathrm{freevarsty}(c) = \{\}$$

where c is a type such as Int

$$\mathrm{freevarsty}(c(\tau_1, \cdots, \tau_n)) = \bigcup_{i=1}^{n} \mathrm{freevarsty}(\tau_i)$$

$$\mathrm{freevarsty}(\forall \alpha_1, \cdots, \alpha_n \,.\, \tau) = \mathrm{freevarsty}(\tau) - \{\, \alpha_1, \cdots, \alpha_n \,\}$$

$$\mathrm{freevarsenv}(\Gamma) = \bigcup_{x \in \mathrm{dom}(\Gamma)} \mathrm{freevarsty}(\Gamma(x))$$

```
 rule applyGEN(Gamma:Map, (Key:KItem |-> Type:KItem) DeclMap
     :Map, NewMap:Map, Beta:Map)
   => applyGEN(Gamma, DeclMap, NewMap[Key <- gen(Gamma, Type
       , Beta)], Beta)

 rule applyGEN(Gamma:Map, .Map, NewMap:Map, Beta:Map)
   => NewMap

//GEN
//GEN(Gamma, Tau) => Forall alpha

syntax KItem ::= gen(Map, K, Map) [function]
syntax Set ::= freeVarsTy(K, Map) [function]
syntax Set ::= freeVarsEnv(Map, Map) [function]


rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map)
    => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set
    freeVarsEnv(Gamma, Beta), Tau)
rule gen(Gamma:Map, Tau:KItem, Beta:Map) => forAll(
    freeVarsTy(Tau, Beta) -Set freeVarsEnv(Gamma, Beta), Tau)
     [owise]

//rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map)
     => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set
    freeVarsEnv(Gamma, Beta), Tau)

rule freeVarsTy(guessType(TypeIt:Int), Beta:Map) => SetItem(
    guessType(TypeIt:Int))
```

```
    rule freeVarsTy(funtype(Tauone:KItem, Tautwo:KItem), Beta:
       Map) => freeVarsTy(Tauone, Beta) freeVarsTy(Tautwo, Beta)
    rule freeVarsTy(Tau:KItem, Beta:Map) => .Set
         requires (forAll(.Set, Tau)) in values(Beta)
    rule freeVarsTy(forAll(Para:Set, Tau:KItem), Beta:Map) =>
       freeVarsTy(Tau, Beta) -Set Para
    rule freeVarsEnv(Gamma:Map, Beta:Map) => listToSet(values(
       Beta), .Set)
```

5.8 Unification Algorithm

Let $S = \{(s_1, t_1), (s_2, t_2), \cdots, (s_n, t_n)\}$ be a unification problem.

Case $S = \{\}$: Unif(S) = Identity function (ie no substitution)

Case $S = (s, t) \cup S'$): Four main steps

Delete: if $s = t$ (they are the same term) then $Unif(S) = Unif(S')$

Decompose: if $s = f(q_1, \cdots, q_m)$ and $t = f(r_1, \cdots, r_m)$ (same f, same m!), then Unif(S) = Unif($\{(q_1, r_1), ..., (q_m, r_m)\} \cup S'$)

Orient: if $t = x$ is a variable, and s is not a variable, Unif(S) = Unif($\{(x, s)\} \cup S'$)

Eliminate: if s = x is a variable, and x does not occur in t (the occurs check), then

Let $\phi = x \mapsto t$

Let ψ = Unif($\phi(S')$)

Unif(S) = $\{x \mapsto \psi(t)\} \circ \psi$

Note:

$$\{\, x \mapsto a \,\} \circ \{\, y \mapsto b \,\} = \{\, y \mapsto (\, \{\, x \mapsto a \,\}(b) \,\} \circ \{\, x \mapsto a \,\}$$

if y not in a

[cs 421 class notes]

```
    //Unification

    syntax Map ::= uniFun(List) [function]
    syntax Bool ::= isVarType(K) [function]
    syntax Bool ::= notChildVar(K,K) [function]
    syntax KItem ::= uniPair(K,K)

```

```
    syntax List ::= uniSub(Map,K) [function] //apply substitution to unification

    syntax KItem ::= typeSub(Map,K) [function] //apply substitution to type
    syntax Map ::= compose(Map,Map) [function]

    rule uniFun(.List) => .Map //substi(.K,.K) is id substitution

    rule uniFun(ListItem(uniPair(S:K,S)) Rest:List) => uniFun(Rest)  //delete rule

    rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List) => uniFun(ListItem(uniPair(T,S)) Rest) //orient rule
         requires isVarType(T) andBool (notBool isVarType(S))

    rule uniFun(ListItem(uniPair(funtype(A:K, B:K), funtype(C:K, D:K))) Rest:List) => uniFun(ListItem(uniPair(A, C)) ListItem(uniPair(B, D)) Rest:List) //decompose rule function type

    rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List)
      => compose((S |-> typeSub(uniFun(uniSub((S |-> T),Rest)),T)),uniFun(uniSub((S |-> T),Rest))) //eliminate rule
    //  => compose(uniFun(uniSub((S |-> T),Rest)),(S |-> typeSub(uniFun(uniSub((S |-> T),Rest)),T))) //eliminate rule
         requires isVarType(S) andBool notChildVar(S,T)

    rule isVarType(S:K) => true
         requires getKLabel(S) ==KLabel 'guessType
    rule isVarType(S:K) => false [owise]

    rule notChildVar(S:K,T:K) => true

    rule uniSub(Sigma:Map,.List) => .List
    rule uniSub(.Map,L:List) => L
    rule uniSub(Sigma:Map, Rest:List ListItem(uniPair(A:K, B:K))) => uniSub(Sigma, Rest) ListItem(uniPair(typeSub(Sigma, A), typeSub(Sigma, B)))

    rule typeSub(Sigma:Map (Tau |-> Newtau:KItem),Tau:KItem) => typeSub(Sigma (Tau |-> Newtau),Newtau)
```

```
    rule typeSub(Sigma:Map,funtype(Tauone:KItem,Tautwo:KItem))
       => funtype(typeSub(Sigma,Tauone),typeSub(Sigma,Tautwo))
    rule typeSub(Sigma:Map,Tau:KItem) => Tau [owise]
```

5.9 Composition of Substitutions

Composition of substitutions means that if the composition was applied to a type, it would first apply to the rightmost substitution and then afterwards apply to the substitution to the left of it, and so on.

```
    syntax Map ::= composeIn(Map, Map, Map, K, K) [function]

    rule compose(Sigmaone:Map, Sigmatwo:Map) => composeIn(
       Sigmaone, Sigmatwo, .Map, .K, .K)

    rule composeIn(Sigmaone:Map, (Key:KItem |-> Type:KItem)
       Sigmatwo:Map, NewMap:Map, .K, .K) => composeIn(Sigmaone,
       Sigmatwo, NewMap, Key, Type)

    rule composeIn((Keyone |-> Typetwo:KItem) Sigmaone:Map,
       Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
        composeIn(Sigmaone, Sigmatwo, NewMap, Keyone, Typeone)

    rule composeIn((Typeone |-> Typetwo:KItem) Sigmaone:Map,
       Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
        composeIn((Typeone |-> Typetwo) Sigmaone, Sigmatwo,
       NewMap[Keyone <- Typetwo], .K, .K)
         requires notBool(Keyone in keys(Sigmaone))

    rule composeIn(Sigmaone:Map, Sigmatwo:Map, NewMap:Map,
       Keyone:KItem, Typeone:KItem) => composeIn(Sigmaone,
       Sigmatwo, NewMap[Keyone <- Typeone], .K, .K) [owise]

    rule composeIn(Sigmaone:Map, .Map, NewMap:Map, .K, .K) =>
       Sigmaone NewMap
endmodule
```

5.10 Examples

Chapter 6

Multiple Module Support

The next step is to implement multiple modules into the Haskell semantics. Similar to including files or objects in other programming languages, Haskell modules can include other modules and use functions, types, and typeclasses declared in the module. There are a several considerations and additional checks that need to be made. Modules need to include other modules There cannot be inclusion cycles Modules need to be able to access user defined types from the other modules that are included When referencing types from other modules, there is a scope where if the user makes another type of the same name in the current module then when the user references the type, it refers to the type from the current module. For instance:

```
File 1:
module File1 where
{data A = B
}
File 2:
module File2 where
{data A = B
;data C = B A
}
```

This will compile. The A used in data C is File2.A However, If there are multiple types with the same name declared outside of the current module. If you try to refer to the type without the parent module, there will be a compiler error because there is ambiguity. For instance:

```
File 1:
module File1 where
{data A = B
}
File 2:
module File2 where
{data A = B
```

```
}
File 3:
module File3 where
{import File1
;import File2
;data C = B A
}
```

This will not compile because in File 3, type A is ambiguous and can mean File1.A or File2.A Type synonyms need to include polymorphism For instance: The user can write type A a = B a

Since we need to check for module inclusion cycles and also build the set of user defined types for each module and included modules, I decided to use a tree. The plan for the algorithm is as follows 1. Construct tree for module inclusion 2. Check tree for cycles 3. Go to each leaf and recursively go up the tree and build alpha* and beta* for the types of the module and the children and desugar the scope so that each type specifies the scope.

Where alpha is the map of type synonyms declared in the current module and alpha* is the map of type synonyms declared in the current module and all the included modules. Beta is the set of user defined types from using data and newtype declared in the current module. Beta* is the set of user defined types from using data and newtype declared in the current module and all the included modules. Desugar the scope means that when the user references a type, desugar the reference to also include the parent module at all times. The syntax also needed to be changed to allow for multiple modules. The new syntax added is

```
//  CUSTOM SYNTAX NOT PART OF OFFICAL HASKELL

    syntax ModuleList ::= Module [klabel('modListSingle)] |
       Module "<NEXTMODULE>" ModuleList [klabel('modList)]
```

This is because K cannot read mutiple files. So instead all the included modules for a program are dumped into one file and are seperated by the keyword ¡NEXTMODULE¿ This creates a list of modules called ModuleList.

```
//
requires "haskell-syntax.k"
requires "haskell-configuration.k"

module HASKELL-PREPROCESSING
```

```
    imports HASKELL-SYNTAX
    imports HASKELL-CONFIGURATION

    //USER DEFINED LIST
    //definition of ElemList

    //syntax KItem ::= ElemList
    syntax ElemList ::= List{Element,","} [strict]
//    syntax Int ::= lengthOfList(ElemList) [function]

//    rule lengthOfList(.ElemList) => 0
//    rule lengthOfList(val(K:K),L:ElemList) => 1 +Int lengthOfList(L)
//    rule lengthOfList(valValue(K:K),L:ElemList) => 1 +Int lengthOfList(L)

    syntax Element ::= val(K) [strict]
    syntax ElementResult ::= valValue(K)
    syntax Element ::= ElementResult
    syntax KResult ::= ElementResult
    rule val(K:KResult) => valValue(K) [structural]

    //form ElemList
//    syntax ElemList ::= formElemList(K) [function]

    //CONVERT ~> TO List
    //list convert
//    syntax List ::= convertToList(K)  [function]
//    rule convertToList(.K) => .List
//    rule convertToList(A:KItem ~> B:K) => ListItem(A) convertToList(B)


    syntax KItem ::= dealWithImports(K,K)

    rule <k> 'modListSingle('module(A:K,, B:K)) => dealWithImports(A,B) ...</k>

    (.Bag =>
          <module>...   //DOT DOT DOT MEANS OVERWRITE ONLY SOME OF THE DEFAULTS
            <moduleName> A </moduleName>
       ...</module>
```

```
    )

    rule <k> 'modList('module(A:K,, B:K),, C:K) => dealWithImports(A,B) ~> C ...</k>

    (.Bag =>
          <module>...   //DOT DOT DOT MEANS OVERWRITE ONLY SOME OF THE DEFAULTS
             <moduleName> A </moduleName>
        ...</module>
    )

//    rule dealWithImports(Mod:K, A:K) => callInit(A)

//    rule <k> dealWithImports(Mod:K, A:K) => callInit(A) ...</k>

    rule <k> dealWithImports(Mod:K, 'bodyimpandtop(A:K,, B:K)) => .K ...</k>
        <importTree> L:List => L importListConvert(Mod, A) </importTree>
        <recurImportTree> L:List => L importListConvert(Mod, A) </recurImportTree>

        <moduleName> Mod </moduleName>
        <imports> S:Set (.Set => SetItem(A)) </imports>
        <moduleTempCode> OldTemp:K => B </moduleTempCode>

    rule <k> dealWithImports(Mod:K, 'bodyimpdecls(A:K)) => .K ...</k>
        <importTree> L:List => L importListConvert(Mod, A) </importTree>
        <recurImportTree> L:List => L importListConvert(Mod, A) </recurImportTree>

        <moduleName> Mod </moduleName>
        <imports> S:Set (.Set => SetItem(A)) </imports>

//    rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) => callInit(A) ...</k>
    rule <k> dealWithImports(Mod:K, 'bodytopdecls(B:K)) => .K ...</k>

```

```
        <moduleName> Mod </moduleName>
        <moduleTempCode> OldTemp:K => B </moduleTempCode>

    //importlist convert
    syntax List ::= importListConvert(K,K) [function]
    syntax KItem ::= impObject(K,K)

    rule importListConvert(Name:K, 'impDecls(A:K,, Rest:K)) =>
       importListConvert(Name, A) importListConvert(Name, Rest)
    rule importListConvert('moduleName(Name:K), 'impDecl(A:K,,
       Modid:K,, C:K,, D:K)) => ListItem(impObject(Name, Modid))
    rule importListConvert(Name:K, .ImpDecls) => .List
```

6.1 Module Inclusion Tree

```
    /*NEW TODO ALGORITHM
1. Construct tree for module inclusion
2. Check tree for cycles
3. Go to each leaf and recursively go up the tree and build
   alpha* and beta* for the types of the module and the children
(and specify scoping) (desugar the scope so that each type
   specifies the scope) */

    syntax KItem ::= "checkImportCycle"
    syntax KItem ::= "recurseImportTree"

/*    rule <k> performNextChecks
             => checkUseVars
                ~> (checkLabelUses
                ~> (checkBlockAddress(.K)
                ~> (checkNoNormalBlocksHavingLandingpad(.K, TNS
                   -Set TES)
                ~> (checkAllExpBlocksHavingLandingpad(.K, TES)
                ~> (checkAllExpInFromInvoke(.K, TES)
                ~> (checkLandingpad
                ~> checkLandingDomResumes)))))) ...</k> */

    rule <k> startImportRecursion => checkImportCycle
                                     ~> (recurseImportTree)...</
                                        k>

```

```
    syntax KItem ::= cycleCheck(K,Map,List,List) [function] //
        current node, map of all nodes to visited or not, stack,
        graph
    syntax Map ::= createVisitMap(List,Map) [function] //graph,
        visitmap
    syntax KItem ::= getUnvisitedNode(K,K, Map) [function] //
        visitmap
    syntax List ::= getNodeNeighbors(K,List) [function] //
        visitmap

    rule <k> checkImportCycle
             => cycleCheck(.K,createVisitMap(I, .Map),.List,I)
                ...</k>
         <importTree> I:List </importTree>
         <impTreeVMap> .Map => createVisitMap(I, .Map) </
            impTreeVMap>

    syntax KItem ::= "visited"
    syntax KItem ::= "unvisited"
    syntax KItem ::= "none"

    rule createVisitMap(ListItem(impObject(A:K,B:K)) Rest:List,
        M:Map)
             => createVisitMap(Rest, M[A <- unvisited][B <-
                unvisited])
    rule createVisitMap(.List, M:Map) => M

    rule getUnvisitedNode(.K, .K, .Map) => none
    rule getUnvisitedNode(.K, .K, (A:K |-> B:K) M:Map)
           => getUnvisitedNode(A, B, M)
    rule getUnvisitedNode(A:KItem, unvisited, M:Map) => A
    rule getUnvisitedNode(A:KItem, visited, M:Map)
            =>  getUnvisitedNode(.K, .K, M)



    rule getNodeNeighbors(Node:K,.List) => .List
    rule getNodeNeighbors(.K,Rest:List) => .List

    rule getNodeNeighbors(Node:KItem,ListItem(impObject(Node,B:
        KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
        ListItem(B)
```

```
    rule getNodeNeighbors(Node:KItem,ListItem(impObject(A:KItem,
       B:KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
         requires Node =/=K A


    rule cycleCheck(none, M:Map, .List, L:List) => .K
    rule cycleCheck(.K, M:Map, .List, I:List) => cycleCheck(
       getUnvisitedNode(.K, .K, M), M, .List, I)
    rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:List)
       => cycleCheck(Node, M, S, I)
    rule cycleCheck(Node:K, M:Map, S:List, I:List)
              => cycleCheck(.K, M[Node <- visited],
                   getNodeNeighbors(Node,I) S, I)
         requires Node =/=K .K andBool Node =/=K none
    rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) =>
       cycleCheck(Node, M, S, I)
         requires S =/=K .List

/*
    rule cycleCheck(A:K,.K,.K,I:K) => cycleCheck(A,
       createVisitMap(I,.Map),.List,I)



    rule cycleCheck(Node:K, M:Map, S:List, I:K) => cycleCheck(.K
       , M[Node <- visited], getNodeNeighbors(Node,I) S, I)

    rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) =>
       cycleCheck(Node, M, S, I)
    //rule cycleCheck(.K, M:Map, .K, ListItem(impObject(A:K,B:K)
       ) Rest:List) => cycleCheck(ListItem(impObject(A:K,B:K))
       Rest:List)
*/
```

6.2 Leaf DFS

```
//COPY IMPORT GRAPH, NEED SECOND GRAPH FOR RECURSING, ADDITIONAL
    GRAPH FOR SELECTING IMPORTS FOR ALPHA* AND BETA*
//DFS for leaf
//acquire alpha and beta for leaf
//merge alpha and beta with imports to produce alpha* and beta*
//perform checks
```

```
//perform inferencing
//insert alpha* and beta* into importing modules
//remove all edges pointing to leaf

    syntax KItem ::= "leafDFS"
    syntax KItem ::= "getAlphaAndBeta"
    syntax KItem ::= "getAlphaBetaStar"
    syntax KItem ::= "performIndividualChecks"
    syntax KItem ::= "performIndividualInferencing"
    syntax KItem ::= "insertAlphaBetaStar"
    syntax KItem ::= "removeAllEdges"
    syntax KItem ::= "seeIfFinished"

    rule <k> recurseImportTree => leafDFS
                                  ~> (getAlphaAndBeta
                                  //~> (getAlphaBetaStar
                                  ~> ( performIndividualInferencing ))...</k>

//rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) => callInit(A) ...</k>

//    rule <k> leaf
//              => cycleCheck(.K,createVisitMap(I, .Map),.List,I) ...</k>
//          <importTree> I:List </importTree>

//////////////////////////////////////////////////////////////////////////////////////


    syntax KItem ::= returnLeafDFS(K,List,Map) [function] // current node, map of all nodes to visited or not, stack, graph
    syntax KItem ::= innerLeafDFS(K,List) [function]
    syntax KItem ::= loadModule(K)

    rule <k> leafDFS
              => returnLeafDFS(.K,I,M) ...</k>
         <recurImportTree> I:List </recurImportTree>
         <impTreeVMap> M:Map </impTreeVMap>

```

```
    rule returnLeafDFS(.K,ListItem(impObject(Node:KItem,B:KItem)) I:List,M:Map) => returnLeafDFS(B,I,M)
    rule returnLeafDFS(Node:KItem,I:List,M:Map) => returnLeafDFS(innerLeafDFS(Node,I),I,M)
         requires innerLeafDFS(Node,I) =/=K none
    rule returnLeafDFS(Node:KItem,I:List,M:Map) => loadModule(Node)
         requires innerLeafDFS(Node,I) ==K none

    rule innerLeafDFS(Node:KItem,ListItem(impObject(Node,B:KItem)) I:List) => B
    rule innerLeafDFS(Node:KItem,ListItem(impObject(A:KItem,B:KItem)) I:List) => innerLeafDFS(Node,I)
         requires Node =/=K A
    rule innerLeafDFS(Node:KItem,.List) => none
//    returnLeafDFS(Node:KItem,ListItem(impObject(Node,B:KItem)) I:List,M:Map) => returnLeafDFS(B,I,M)


//////////////////////////////////////////////////////////////////////////////////////

    //call before Checker Code
//    rule <k> callInit(S:K) => initPreModule(S) ...</k>
//         <tempModule> A:K => S </tempModule>

    rule <k> loadModule(S:KItem) => .K ...</k>
         <tempModule> A:K => S </tempModule>

    rule <k> getAlphaAndBeta => initPreModule(Code) ...</k>
         <tempModule> Mod:KItem </tempModule>

         <moduleName> 'moduleName(Mod) </moduleName>
         <moduleTempCode> Code:KItem </moduleTempCode>
```

6.3 Insert AlphaBetaStar

```
//    syntax KItem ::= "insertAlphaBetaStar"

    syntax KItem ::= insertABRec(K,List)
    syntax KItem ::= insertAB(K)

```

```
    rule <k> insertAlphaBetaStar => insertABRec(Mod, Imp) ...</k>
         <tempModule> Mod:KItem </tempModule>
         <importTree> Imp:List </importTree>

    rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem, Node)) I:List) => insertAB(B) ~> insertABRec(Node, I) ...</k>

    rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem, C:KItem)) I:List) => insertABRec(Node, I) ...</k>
           requires Node =/=K C

    rule <k> insertAB(B) => .K ...</k>

         <tempAlphaStar> Alph:KItem </tempAlphaStar>
         <tempBetaStar> Bet:KItem </tempBetaStar>

         <moduleName> 'moduleName(B) </moduleName>
         <moduleImpAlphas> ImpAlphas:List => ListItem(Alph) ImpAlphas </moduleImpAlphas>
         <moduleImpBetas> ImpBetas:List => ListItem(Bet) ImpBetas </moduleImpBetas>


endmodule
```

Chapter 7

Conclusion

This project helped me with my K knowledge. I also learned a lot about how compilers work and how programming languages are defined. Also, defining yet another real word language in K allows the K-framework to become more popular as another community can learn about it and utilize it. The more that real world languages are defined in K, the more potential K has to become a popular language in industry.

Appendix A

haskell-syntax.k

```
// Syntax from haskell 2010 Report
// https://www.haskell.org/onlinereport/haskell2010/haskellch10.html#x17-17500010

module HASKELL-SYNTAX

    syntax Integer ::= Token{([0-9]+)
                   | (([0][o]|[0][O])[0-7]+)
                   | (([0][x] | [0][X])[0-9a-fA-F]+)}   [onlyLabel]

    syntax CusFloat ::= Token{([0-9]+[\.][0-9]+([e E][\+\-]?[0-9]+)?)
                                    |([0-9]+[e E][\+\-]?[0-9]+)} [onlyLabel]
    syntax CusChar ::= Token{[\'](~[\'\\\&])[\']} [onlyLabel]
    syntax CusString ::= Token{[\"](~[\"\\]*)[\"]} [onlyLabel]
    syntax VarId ::= Token{[a-z\_][a-z A-Z\_0-9\']*} [onlyLabel] | "size" [onlyLabel]
    syntax ConId ::= Token{[A-Z][a-zA-Z \_0-9\']*} [onlyLabel]
    syntax VarSym ::= Token{
   ([\! \# \$ \% \& \* \+ \/ \> \? \^][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
  |[\-] | [\.]
  |([\.][\! \# \$ \% \& \* \+ \/ \< \= \> \? \@ \\ \^ \| \- \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
  | ([\-][\! \# \$ \% \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~ \:]*)
  | ([\@][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
  | ([\~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
```

```
    | ([\\][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
    | ([\|][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]+)
    | ([\:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
    | ([\<][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
    | ([\=][\! \# \$ \% \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~ \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)} [onlyLabel]
  syntax ConSym ::= Token{[\:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*}   [onlyLabel]

  syntax IntFloat ::= "(" Integer ")"  [bracket] //NOT OFFICIAL SYNTAX
                    | "(" CusFloat ")" [bracket]
  syntax Literal ::= IntFloat | CusChar | CusString
  syntax TyCon ::= ConId
  syntax ModId ::= ConId | ConId "." ModId  [klabel('conModId)]
  syntax QTyCon ::= TyCon | ModId "." TyCon [klabel('conTyCon)]
  syntax QVarId ::= VarId | ModId "." VarId [klabel('qVarIdCon)]
  syntax QVarSym ::= VarSym | ModId "." VarSym  [klabel('qVarSymCon)]
  syntax QConSym ::= ConSym | ModId "." ConSym  [klabel('qConSymCon)]
  syntax TyVars ::= List{TyVar, ""} [klabel('typeVars)] //used in SimpleType syntax
  syntax TyVar ::= VarId
  syntax TyVarTuple ::= TyVar "," TyVar      [klabel('twoTypeVarTuple)]
                      | TyVar "," TyVarTuple [klabel('typeVarTupleCon)]

  syntax Con ::= ConId | "(" ConSym ")"   [klabel('conSymBracket)]
```

```
syntax Var ::= VarId | "(" VarSym ")"   [klabel('varSymBracket)]
syntax QVar ::= QVarId | "(" QConSym ")" [klabel('qVarBracket)]
syntax QCon ::= QTyCon | "(" GConSym ")"  [klabel('gConBracket)]

syntax QConOp ::= GConSym | "`" QTyCon "`"    [klabel('qTyConQuote)]
syntax QVarOp ::= QVarSym | "`" QVarId "`"    [klabel('qVarIdQuote)]
syntax VarOp ::= VarSym   | "`" VarId "`"     [klabel('varIdQuote)]
syntax ConOp ::= ConSym   | "`" ConId "`"     [klabel('conIdQuote)]

syntax GConSym ::= ":" | QConSym
syntax Vars ::= Var
              | Var "," Vars  [klabel('varCon)]
syntax VarsType ::= Vars "::" Type  [klabel('varAssign)]
syntax Ops ::= Op
             | Op "," Ops  [klabel('opCon)]
syntax Fixity ::= "infixl" | "infixr" | "infix"
syntax Op ::= VarOp | ConOp
syntax CQName ::= Var | Con | QVar

syntax QOp ::= QVarOp | QConOp

syntax ModuleName ::= "module" ModId [klabel('moduleName)]

syntax Module ::= ModuleName "where" Body          [klabel('module)]
                | ModuleName Exports "where" Body  [klabel('moduleExp)]
                | Body                             [klabel('moduleBody)]

syntax Body ::= "{" ImpDecls ";" TopDecls "}" [klabel('bodyimpandtop)]
              | "{" ImpDecls "}" [klabel('bodyimpdecls)]
              | "{" TopDecls "}" [klabel('bodytopdecls)]

syntax ImpDecls ::= List{ImpDecl, ";"} [klabel('impDecls)]
```

```
syntax Exports ::= "(" ExportList OptComma ")"
syntax ExportList ::= List{Export, ","}

syntax Export ::= QVar
                | QTyCon OptCQList
                | ModuleName

//optional cname list
syntax OptCQList ::= "(..)"
                   | "(" CQList ")"   [klabel('cqListBracket)]
                     //Liyi: a check needs to place in preprocessing to check
                     //if the CQList is a cname list or a qvar list.
                   | "" [onlyLabel, klabel('emptyOptCNameList)]
syntax CQList ::= List{CQName, ","}

syntax ImpDecl ::= "import" OptQualified ModId OptAsModId OptImpSpec [klabel('impDecl)]
                 | "" [onlyLabel, klabel('emptyImpDecl)]
syntax OptQualified ::= "qualified"
                      | ""  [onlyLabel, klabel('emptyQualified)]
syntax OptAsModId ::= "as" ModId
                    | ""    [onlyLabel, klabel('emptyOptAsModId)]

syntax OptImpSpec ::= ImpSpec
                    | ""    [onlyLabel, klabel('emptyOptImpSpec)]

syntax ImpSpecKey ::= "(" ImportList OptComma ")"
syntax ImpSpec ::= ImpSpecKey
                 | "hiding" ImpSpecKey

syntax ImportList ::= List{Import, ","}

syntax Import ::= Var
                | TyCon CQList
syntax TopDecls ::= List{TopDecl, ";"} [klabel('topdeclslist)]
```

```

syntax TopDecl ::= Decl [klabel('topdecldecl)]
                 > "type" SimpleType "=" Type [klabel('type)]
                 | "data" OptContext SimpleType OptConstrs OptDeriving  [klabel('data)]
                 | "newtype" OptContext SimpleType "=" NewConstr OptDeriving [klabel('newtype)]
                 | "class" OptContext ConId TyVar OptCDecls [klabel('class)]
                 | "instance" OptContext QTyCon Inst OptIDecls [klabel('instance)]
                 | "default" Types [klabel('default)]
                 | "foreign" FDecl [klabel('foreign)]

syntax FDecl ::= "import" CallConv CusString Var "::" FType
               | "import" CallConv Safety CusString Var "::" FType
               | "export" CallConv Safety CusString Var "::" FType
//Liyi: fdecl needs to use special function in preprocessing
// to get the actually elements from the impent and expent from the CusString
//did string analysis

syntax Safety ::= "unsafe" | "safe"

syntax CallConv ::= "ccall" | "stdcall" | "cplusplus" | "jvm" | "dotnet"
syntax FType ::= FrType
               | FaType "->" FType // unsure about this one syntax is ambiguous UNFINISHED

syntax FrType ::= FaType
                | "()"

syntax FaType ::= QTyCon ATypeList

//define declaration.
syntax OptDecls ::= "where" Decls | "" [onlyLabel, klabel('emptyOptDecls)]
syntax Decls ::= "{" DeclsList "}" [klabel('decls)]
syntax DeclsList ::= List{Decl, ";"} [klabel('declsList)]
```

```

syntax Decl ::= GenDecl
              | FunLhs Rhs [klabel('declFunLhsRhs)]
              | Pat Rhs [klabel('declPatRhs)]

syntax OptCDecls ::= "where" CDecls | "" [onlyLabel, klabel('emptyOptCDecls)]
syntax CDecls ::= "{" CDeclsList "}"
syntax CDeclsList ::= List{CDecl, ";"}

syntax CDecl ::= GenDecl
               | FunLhs Rhs
               | Var Rhs

syntax OptIDecls ::= "where" IDecls | "" [onlyLabel, klabel('emptyOptIDecls)]
syntax IDecls ::= "{" IDeclsList "}"
syntax IDeclsList ::= List{IDecl, ";"} [klabel('ideclslist)]

syntax IDecl ::= FunLhs Rhs [klabel('cdeclFunLhsRhs)]
               | Var Rhs [klabel('cdeclVarRhs)]
               | "" [onlyLabel, klabel('emptyIDecl)]

syntax GenDecl ::= VarsType
                 | Vars "::" Context "=>" Type   [klabel('genAssignContext)]
                 | Fixity Ops
                 | Fixity Integer Ops
                 | "" [onlyLabel, klabel('emptyGenDecl)]

//three optional data type for the TopDecl data operator.
//deriving data type
syntax OptDeriving ::= Deriving | "" [onlyLabel, klabel('emptyDeriving)]
syntax Deriving ::= "deriving" DClass
                  | "deriving" "(" DClassList ")"
syntax DClassList ::= List{DClass, ","}
syntax DClass ::= QTyCon

syntax FunLhs ::= Var APatList [klabel('varApatList)]
                | Pat VarOp Pat [klabel('patVarOpPat)]
                | "(" FunLhs ")" APatList [klabel('funlhsApatList)]
```

```
    syntax Rhs ::= "=" Exp OptDecls [klabel('eqExpOptDecls)]
                 | GdRhs OptDecls [klabel('gdRhsOptDecls)]

    syntax GdRhs ::= Guards "=" Exp
                   | Guards "=" Exp GdRhs
    syntax Guards ::= "|" GuardList
    syntax GuardList ::= Guard | Guard "," GuardList  [klabel('guardListCon)]
    syntax Guard ::= Pat "<-" InfixExp
                   | "let" Decls
                   | InfixExp

    //definition of exp
    syntax Exp ::= InfixExp
                 > InfixExp "::" Type  [klabel('expAssign)]
                 | InfixExp "::" Context "=>" Type  [klabel('expAssignContext)]

    syntax InfixExp ::= LExp
                      > "-" InfixExp   [klabel('minusInfix)]
                      > LExp QOp InfixExp

    syntax LExp ::= AExp
                  > "\\" APatList "->" Exp [klabel('lambdaFun)]
                  | "let" Decls "in" Exp [klabel('letIn)]
                  | "if" Exp OptSemicolon "then" Exp OptSemicolon "else" Exp [klabel('ifThenElse)]
                  | "case" Exp "of" "{" Alts "}" [klabel('caseOf)]
                  | "do" "{" Stmts "}" [klabel('doBlock)]
```

LExp is an important sort for the inference function. This is because LExp defines the different expression types which the inference function has specific rules for.

```
    syntax OptSemicolon ::= ";" | "" [onlyLabel, klabel('emptySemicolon)]
    syntax OptComma ::= "," | ""     [onlyLabel, klabel('emptyComma)]
```

```
syntax AExp ::= QVar [klabel('aexpQVar)]
              | GCon [klabel('aexpGCon)]
              | Literal [klabel('aexpLiteral)]
              > AExp AExp [left, klabel('funApp)]
              > QCon "{" FBindList "}"
              | AExp "{" FBindList "}" //aexp cannot be qcon UNFINISHED
                      //Liyi: first, does not understand the syntax, it is the Qcon {FBindlist}
                      //or QCon? Second, place a check in preprosssing.
                      //and also check the Fbindlist here must be at least one argument
              > "(" Exp ")"            [bracket]
              | "(" ExpTuple ")"
              | "[" ExpList "]"
              | "[" Exp OptExpComma ".." OptExp "]"
              | "[" Exp "|" Quals "]"
              | "(" InfixExp QOp ")"
              | "(" QOp InfixExp ")" //qop cannot be - (minus) UNFINISHED
                       //Liyi: place a check here to check if QOp is a minus


syntax OptExpComma ::= "," Exp | "" [onlyLabel, klabel('emptyExpComma)]
syntax OptExp ::= Exp | "" [onlyLabel, klabel('emptyExp)]

syntax ExpList ::= Exp | Exp "," ExpList  [right]
syntax ExpTuple ::= Exp "," Exp           [right, klabel('twoExpTuple)]
                  | Exp "," ExpTuple      [right, klabel('expTupleCon)]

//constr datatypes
syntax OptConstrs  ::= "=" Constrs [klabel('nonemptyConstrs)] | "" [onlyLabel, klabel('emptyConstrs)]
syntax Constrs     ::= Constr [klabel('singleConstr)] | Constr "|" Constrs [klabel('multConstr)]
syntax Constr      ::= Con OptBangATypes [klabel('constrCon)] // (arity con  =  k, k     0) UNFINISHED
                     | SubConstr ConOp SubConstr
```

```
                         | Con "{" FieldDeclList "}"

    syntax NewConstr   ::= Con AType [klabel('newConstrCon)]
                         | Con "{" Var "::" Type "}"

    syntax SubConstr   ::= BType | "!" AType
    syntax FieldDeclList ::= List{FieldDecl, ","}
    syntax FieldDecl ::= VarsType
                       | Vars "::" "!" AType


    syntax OptBangATypes ::= List{OptBangAType, " "} [klabel('optBangATypes)]
    syntax OptBangAType ::= OptBang AType [klabel('optBangAType)]
    syntax OptBang ::= "!" | "" [onlyLabel, klabel('emptyBang)]

    syntax OptContext ::= Context "=>" | "" [onlyLabel, klabel('emptyContext)]
    syntax Context ::= Class
                     | "(" Classes ")"

    syntax Classes ::= List{Class, ","}

    syntax SimpleClass ::= QTyCon TyVar  [klabel('classCon)]

    syntax Class       ::= SimpleClass
                         | QTyCon "(" TyVar ATypeList ")"
                              //Liyi: a check in preprossing to check if the Atype list is empty
                              //it must have at least one item

    //define type and simple type
    syntax SimpleType  ::= TyCon TyVars  [klabel('simpleTypeCon)]
    syntax Type ::= BType
                  | BType "->" Type  [klabel('typeArrow)]
    syntax BType ::= AType
                   | BType AType [klabel('baTypeCon)]

    syntax ATypeList ::= List{AType, ""} [klabel('atypeList)]

```

```
syntax AType ::= GTyCon                      [klabel('atypeGTyCon)]
               | TyVar                       [klabel('atypeTyVar)]
               | "(" TypeTuple ")"           [klabel('atypeTuple)]
               | "[" Type "]"                [klabel('tyList)]
               | "(" Type ")"                [bracket]
syntax TypeTuple ::= Type "," Type           [right,klabel('twoTypeTuple)]
                   | Type "," TypeTuple      [klabel('typeTupleCon)]
syntax Types ::= List{Type, ","}

syntax GConCommas ::= "," | "," GConCommas
syntax GConCommon ::= "()" | "[]" | "(" GConCommas ")" //was incorrect syntax
syntax GTyCon ::= QTyCon
                | GConCommon
                | "(->)"

syntax GCon ::= GConCommon
              | QCon

//inst definition
syntax Inst  ::= GTyCon
               | "(" GTyCon TyVars ")" //TyVars must be distinct UNFINISHED
               | "(" TyVarTuple ")" //TyVars must be distinct
               | "[" TyVar "]"  [klabel('tyVarList)]
               | "(" TyVar "->" TyVar ")" //TyVars must be distinct
//pat definition
syntax Pat ::= LPat QConOp Pat
             | LPat

syntax LPat ::= APat
              | "-" IntFloat    [klabel('minusPat)]
              | GCon APatList  [klabel('lpatCon)]//arity gcon = k UNFINISHED

```

```
    syntax APatList ::= APat | APat APatList [klabel('apatCon)]

    syntax APat ::= Var [klabel('apatVar)]
                  | Var "@" APat
                  | GCon
                  | QCon "{" FPats "}"
                  | Literal [klabel('apatLiteral)]
                  | "_"
                  | "(" Pat ")"   [bracket]
                  | "(" PatTuple ")"
                  | "[" PatList "]"
                  | "~" APat

    syntax PatTuple ::= Pat "," Pat         [klabel('twoPatTuple)]
                      | Pat "," PatTuple    [klabel('patTupleCon)]
    syntax PatList ::= Pat
                      | Pat "," PatList     [klabel('patListCon)]

    syntax FPats ::= List{FPat, ","}
    syntax FPat ::= QVar "=" Pat

    //definition of quals
    syntax Quals ::= Qual | Qual "," Quals  [klabel('qualCon)]

    syntax Qual ::= Pat "<-" Exp
                  | "let" Decls
                  | Exp

    //definition of alts
    syntax Alts ::= Alt | Alt ";" Alts

    syntax Alt ::= Pat "->" Exp  [klabel('altArrow)]
                 | Pat "->" Exp "where" Decls
                 | "" [onlyLabel, klabel('emptyAlt)]

    //definition of stmts
    syntax Stmts ::= StmtList Exp OptSemicolon
    syntax StmtList ::= List{Stmt, ""}
    syntax Stmt ::= Exp ";"
                  | Pat "<-" Exp ";"
```

```
                  | "let" Decls ";"
                  | ";"

  //definition of fbind
 syntax FBindList ::= List{FBind, ","}
 syntax FBind ::= QVar "=" Exp
```

Appendix B

haskell-configuration.k

```
requires "haskell-syntax.k"

module HASKELL-CONFIGURATION
    imports HASKELL-SYNTAX

    syntax KItem ::= "startImportRecursion"
    syntax KItem ::= callInit(K)
    //syntax KItem ::= initPreModule(K) [function]
    //syntax KItem ::= tChecker(K) [function]

    configuration
        <T>
            <k> $PGM:ModuleList ~> startImportRecursion </k>
            <tempModule> .K </tempModule>
            <tempCode> .K </tempCode>
            <typeIterator> 1 </typeIterator>
            <tempAlpha> .K </tempAlpha>
            <tempAlphaMap> .Map </tempAlphaMap>
            <tempBeta> .Map </tempBeta>
            <tempT> .K </tempT>
            <tempDelta> .Map </tempDelta>
            <tempAlphaStar> .K </tempAlphaStar>
            <tempBetaStar> .K </tempBetaStar>
            <importTree> .List </importTree>
            <recurImportTree> .List </recurImportTree>
            <impTreeVMap> .Map </impTreeVMap>
            <modules> //static information about a module
                <module multiplicity="*">
                    <moduleName> .K </moduleName>
                    <moduleAlphaStar> .K </moduleAlphaStar>
                    <moduleBetaStar> .K </moduleBetaStar>
                    <moduleImpAlphas> .List </moduleImpAlphas>
                    <moduleImpBetas> .List </moduleImpBetas>
```

```
                    <moduleCompCode> .K </moduleCompCode>
                    <moduleTempCode> .K </moduleTempCode>
                    <imports> .Set </imports>
                    <classes> //static information about a module
                        <class multiplicity="*">
                            <className> .K </className>
                        </class>
                    </classes>
                </module>
            </modules>
        </T>

endmodule
```

Appendix C

haskell-preprocessing.k

```
//
requires "haskell-syntax.k"
requires "haskell-configuration.k"

module HASKELL-PREPROCESSING
    imports HASKELL-SYNTAX
    imports HASKELL-CONFIGURATION

    //USER DEFINED LIST
    //definition of ElemList

    //syntax KItem ::= ElemList
    syntax ElemList ::= List{Element,","} [strict]
//    syntax Int ::= lengthOfList(ElemList) [function]

//    rule lengthOfList(.ElemList) => 0
//    rule lengthOfList(val(K:K),L:ElemList) => 1 +Int lengthOfList(L)
//    rule lengthOfList(valValue(K:K),L:ElemList) => 1 +Int lengthOfList(L)

    syntax Element ::= val(K) [strict]
    syntax ElementResult ::= valValue(K)
    syntax Element ::= ElementResult
    syntax KResult ::= ElementResult
    rule val(K:KResult) => valValue(K) [structural]

    //form ElemList
//    syntax ElemList ::= formElemList(K) [function]

    //CONVERT ~> TO List
    //list convert
//    syntax List ::= convertToList(K)  [function]
```

```
//    rule convertToList(.K) => .List
//    rule convertToList(A:KItem ~> B:K) => ListItem(A) convertToList(B)


     syntax KItem ::= dealWithImports(K,K)

     rule <k> 'modListSingle('module(A:K,, B:K)) => dealWithImports(A,B) ...</k>

     (.Bag =>
           <module>...   //DOT DOT DOT MEANS OVERWRITE ONLY SOME OF THE DEFAULTS
             <moduleName> A </moduleName>
        ...</module>
     )

     rule <k> 'modList('module(A:K,, B:K),, C:K) => dealWithImports(A,B) ~> C ...</k>

     (.Bag =>
           <module>...   //DOT DOT DOT MEANS OVERWRITE ONLY SOME OF THE DEFAULTS
             <moduleName> A </moduleName>
        ...</module>
     )

//    rule dealWithImports(Mod:K, A:K) => callInit(A)

//    rule <k> dealWithImports(Mod:K, A:K) => callInit(A) ...</k>

     rule <k> dealWithImports(Mod:K, 'bodyimpandtop(A:K,, B:K)) => .K ...</k>
         <importTree> L:List => L importListConvert(Mod, A) </importTree>
         <recurImportTree> L:List => L importListConvert(Mod, A) </recurImportTree>

         <moduleName> Mod </moduleName>
         <imports> S:Set (.Set => SetItem(A)) </imports>
         <moduleTempCode> OldTemp:K => B </moduleTempCode>

```

```
    rule <k> dealWithImports(Mod:K, 'bodyimpdecls(A:K)) => .K ...</k>
        <importTree> L:List => L importListConvert(Mod, A) </importTree>
        <recurImportTree> L:List => L importListConvert(Mod, A) </recurImportTree>

        <moduleName> Mod </moduleName>
        <imports> S:Set (.Set => SetItem(A)) </imports>

//    rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) => callInit(A) ...</k>
    rule <k> dealWithImports(Mod:K, 'bodytopdecls(B:K)) => .K ...</k>

        <moduleName> Mod </moduleName>
        <moduleTempCode> OldTemp:K => B </moduleTempCode>

    //importlist convert
    syntax List ::= importListConvert(K,K) [function]
    syntax KItem ::= impObject(K,K)

    rule importListConvert(Name:K, 'impDecls(A:K,, Rest:K)) => importListConvert(Name, A) importListConvert(Name, Rest)
    rule importListConvert('moduleName(Name:K), 'impDecl(A:K,, Modid:K,, C:K,, D:K)) => ListItem(impObject(Name, Modid))
    rule importListConvert(Name:K, .ImpDecls) => .List

////////////////////////////////////////////////////////////////////////////////////

    /*NEW TODO ALGORITHM
1. Construct tree for module inclusion
2. Check tree for cycles
3. Go to each leaf and recursively go up the tree and build alpha* and beta* for the types of the module and the children
(and specify scoping) (desugar the scope so that each type specifies the scope) */

    syntax KItem ::= "checkImportCycle"
    syntax KItem ::= "recurseImportTree"

/*    rule <k> performNextChecks
```

```
            => checkUseVars
               ~> (checkLabelUses
               ~> (checkBlockAddress(.K)
               ~> (checkNoNormalBlocksHavingLandingpad(.K, TNS -Set TES)
               ~> (checkAllExpBlocksHavingLandingpad(.K, TES)
               ~> (checkAllExpInFromInvoke(.K, TES)
               ~> (checkLandingpad
               ~> checkLandingDomResumes)))))) ...</k> */

    rule <k> startImportRecursion => checkImportCycle
                                      ~> (recurseImportTree)...</k>

    syntax KItem ::= cycleCheck(K,Map,List,List) [function] // current node, map of all nodes to visited or not, stack, graph
    syntax Map ::= createVisitMap(List,Map) [function] //graph, visitmap
    syntax KItem ::= getUnvisitedNode(K,K, Map) [function] // visitmap
    syntax List ::= getNodeNeighbors(K,List) [function] // visitmap

    rule <k> checkImportCycle
            => cycleCheck(.K,createVisitMap(I, .Map),.List,I) ...</k>
         <importTree> I:List </importTree>
         <impTreeVMap> .Map => createVisitMap(I, .Map) </impTreeVMap>

    syntax KItem ::= "visited"
    syntax KItem ::= "unvisited"
    syntax KItem ::= "none"

    rule createVisitMap(ListItem(impObject(A:K,B:K)) Rest:List, M:Map)
            => createVisitMap(Rest, M[A <- unvisited][B <- unvisited])
    rule createVisitMap(.List, M:Map) => M

    rule getUnvisitedNode(.K, .K, .Map) => none
    rule getUnvisitedNode(.K, .K, (A:K |-> B:K) M:Map)
```

```
          => getUnvisitedNode(A, B, M)
   rule getUnvisitedNode(A:KItem, unvisited, M:Map) => A
   rule getUnvisitedNode(A:KItem, visited, M:Map)
           =>  getUnvisitedNode(.K, .K, M)



   rule getNodeNeighbors(Node:K,.List) => .List
   rule getNodeNeighbors(.K,Rest:List) => .List

   rule getNodeNeighbors(Node:KItem,ListItem(impObject(Node,B:KItem)) Rest:List) => getNodeNeighbors(Node, Rest) ListItem(B)
   rule getNodeNeighbors(Node:KItem,ListItem(impObject(A:KItem,B:KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
        requires Node =/=K A


   rule cycleCheck(none, M:Map, .List, L:List) => .K
   rule cycleCheck(.K, M:Map, .List, I:List) => cycleCheck(getUnvisitedNode(.K, .K, M), M, .List, I)
   rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:List) => cycleCheck(Node, M, S, I)
   rule cycleCheck(Node:K, M:Map, S:List, I:List)
               => cycleCheck(.K, M[Node <- visited], getNodeNeighbors(Node,I) S, I)
        requires Node =/=K .K andBool Node =/=K none
   rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) => cycleCheck(Node, M, S, I)
        requires S =/=K .List

/*
   rule cycleCheck(A:K,.K,.K,I:K) => cycleCheck(A,createVisitMap(I,.Map),.List,I)



   rule cycleCheck(Node:K, M:Map, S:List, I:K) => cycleCheck(.K, M[Node <- visited], getNodeNeighbors(Node,I) S, I)

   rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) => cycleCheck(Node, M, S, I)
```

```
    //rule cycleCheck(.K, M:Map, .K, ListItem(impObject(A:K,B:K)) Rest:List) => cycleCheck(ListItem(impObject(A:K,B:K)) Rest:List)
*/

////////////////////////////////////////////////////////////////////////////////////

//COPY IMPORT GRAPH, NEED SECOND GRAPH FOR RECURSING, ADDITIONAL GRAPH FOR SELECTING IMPORTS FOR ALPHA* AND BETA*
//DFS for leaf
//acquire alpha and beta for leaf
//merge alpha and beta with imports to produce alpha* and beta*
//perform checks
//perform inferencing
//insert alpha* and beta* into importing modules
//remove all edges pointing to leaf

    syntax KItem ::= "leafDFS"
    syntax KItem ::= "getAlphaAndBeta"
    syntax KItem ::= "getAlphaBetaStar"
    syntax KItem ::= "performIndividualChecks"
    syntax KItem ::= "performIndividualInferencing"
    syntax KItem ::= "insertAlphaBetaStar"
    syntax KItem ::= "removeAllEdges"
    syntax KItem ::= "seeIfFinished"

    rule <k> recurseImportTree => leafDFS
                                  ~> (getAlphaAndBeta
                                  //~> (getAlphaBetaStar
                                  ~> (performIndividualInferencing))...</k>

//rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) => callInit(A) ...</k>

//    rule <k> leaf
//              => cycleCheck(.K,createVisitMap(I, .Map),.List,I) ...</k>
//          <importTree> I:List </importTree>

```

```
////////////////////////////////////////////////////////////////////////////////////

    syntax KItem ::= returnLeafDFS(K,List,Map) [function] // current node, map of all nodes to visited or not, stack, graph
    syntax KItem ::= innerLeafDFS(K,List) [function]
    syntax KItem ::= loadModule(K)

    rule <k> leafDFS
             => returnLeafDFS(.K,I,M) ...</k>
         <recurImportTree> I:List </recurImportTree>
         <impTreeVMap> M:Map </impTreeVMap>

    rule returnLeafDFS(.K,ListItem(impObject(Node:KItem,B:KItem)) I:List,M:Map) => returnLeafDFS(B,I,M)
    rule returnLeafDFS(Node:KItem,I:List,M:Map) => returnLeafDFS(innerLeafDFS(Node,I),I,M)
         requires innerLeafDFS(Node,I) =/=K none
    rule returnLeafDFS(Node:KItem,I:List,M:Map) => loadModule(Node)
         requires innerLeafDFS(Node,I) ==K none

    rule innerLeafDFS(Node:KItem,ListItem(impObject(Node,B:KItem)) I:List) => B
    rule innerLeafDFS(Node:KItem,ListItem(impObject(A:KItem,B:KItem)) I:List) => innerLeafDFS(Node,I)
         requires Node =/=K A
    rule innerLeafDFS(Node:KItem,.List) => none
//    returnLeafDFS(Node:KItem,ListItem(impObject(Node,B:KItem)) I:List,M:Map) => returnLeafDFS(B,I,M)


////////////////////////////////////////////////////////////////////////////////////

    //call before Checker Code
//    rule <k> callInit(S:K) => initPreModule(S) ...</k>
//         <tempModule> A:K => S </tempModule>

    rule <k> loadModule(S:KItem) => .K ...</k>
         <tempModule> A:K => S </tempModule>

```

```
    rule <k> getAlphaAndBeta => initPreModule(Code) ...</k>
         <tempModule> Mod:KItem </tempModule>

         <moduleName> 'moduleName(Mod) </moduleName>
         <moduleTempCode> Code:KItem </moduleTempCode>


////////////////////////////////////////////////////////////////////////////////////

    //get alpha and beta
    syntax KItem ::= Module(K, K)
    syntax KItem ::= preModule(K,K) //(alpha, T)

    // STEP 1 CONSTRUCT T AND ALPHA
    // alpha = type
    // T = newtype and data, temporary data structure

    syntax KItem ::= initPreModule(K) [function]
    syntax KItem ::= getPreModule(K, K) [function] //(Current term, premodule)
    syntax KItem ::= makeT (K,K,K,K)

    syntax KItem ::= fetchTypes (K,K,K,K)

    syntax List ::= makeInnerT (K,K,K) [function] //LIST
    syntax List ::= getTypeVars(K) [function] //LIST

    syntax KItem ::= getCon(K) [function]
    syntax List ::= getArgSorts(K) [function] //LIST

    syntax KItem ::= AList(K)
    syntax KItem ::= AObject(K,K) //(1st -> 2nd) map without idempotency
    syntax KItem ::= ModPlusType(K,K)

    syntax KItem ::= TList(K) //list of T objects for every new type introduced by data and newtype
    syntax KItem ::= TObject(K,K,K,K) //(module name, type name, entire list of poly type vars, list of inner T pieces)
    syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type constructor, poly type vars, argument sorts, entire constr block, type name)

```

```
//    rule initPreModule('module(I:ModuleName,, J:K)) => getPreModule(J,preModule(AList(.List),TList(.List)))
//    rule initPreModule('moduleExp(I:ModuleName,, L:K,, J:K)) => getPreModule(J,preModule(AList(.List),TList(.List)))
//    rule initPreModule('moduleBody(J:Body)) => getPreModule(J,preModule(AList(.List),TList(.List)))

    rule initPreModule(J:K) => getPreModule(J,preModule(AList(.List),TList(.List)))

    rule getPreModule('bodytopdecls(I:K), J:K) => getPreModule(I,J)
    rule getPreModule('topdeclslist('type(A:K,, B:K),, Rest:K),J:K) => fetchTypes(A,B,Rest,J) //constructalpha


    rule getPreModule('topdeclslist('data(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => makeT(B,C,Rest,J)
    rule getPreModule('topdeclslist('newtype(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => makeT(B,C,Rest,J)


    rule getPreModule('topdeclslist('topdecldecl(A:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule('topdeclslist('class(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule('topdeclslist('instance(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule('topdeclslist('default(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule('topdeclslist('foreign(A:K,, B:K,, C:K,, D:K),, Rest:K),J:K) => getPreModule(Rest,J)
    rule getPreModule(.TopDecls,J:K) => J

    //rule getPreModule('module(I:ModuleName,L:K, J:K)) => preModule(J)

    rule <k> fetchTypes('simpleTypeCon(I:TyCon,, H:TyVars), 'atypeGTyCon(C:K), Rest:K, preModule(AList(M:List), L:K)) => getPreModule(Rest,preModule(AList(ListItem(AObject(ModPlusType(ModName,I),C)) M), L)) ...</k>
      <tempModule> ModName:KItem </tempModule>

```

```
    rule <k> makeT('simpleTypeCon(I:TyCon,, H:TyVars), D:K, Rest:K, preModule(AList(M:List), TList(ListInside:List))) => getPreModule(Rest,preModule(AList(M),TList(ListItem(TObject(ModName,I,H,makeInnerT(I,H,D))) ListInside))) ...</k>
      <tempModule> ModName:KItem </tempModule>

    rule makeInnerT(A:K,B:K,'nonemptyConstrs(C:K)) => makeInnerT(A,B,C)
    rule makeInnerT(A:K,B:K,'singleConstr(C:K)) => ListItem(InnerTPiece(getCon(C),getTypeVars(C),getArgSorts(C),C,A))
    rule makeInnerT(A:K,B:K,'multConstr(C:K,, D:K)) => ListItem(InnerTPiece(getCon(C),getTypeVars(C),getArgSorts(C),C,A)) makeInnerT(A,B,D)

    rule getTypeVars('constrCon(A:K,, B:K)) => getTypeVars(B)
    rule getTypeVars('optBangATypes(A:K,, Rest:K)) => getTypeVars(A) getTypeVars(Rest)
    rule getTypeVars('optBangAType('emptyBang(.KList),, Rest:K)) => getTypeVars(Rest)
    rule getTypeVars('atypeGTyCon(A:K)) => .List
    rule getTypeVars('atypeTyVar(A:K)) => ListItem(A)
    rule getTypeVars(.OptBangATypes) => .List

    //rule getCon('emptyConstrs()) => .K
    //rule getCon('nonemptyConstrs(A:K)) => getCon(A)
    rule getCon('constrCon(A:K,, B:K)) => A

    //rule getArgSorts('constrCon(A:K,, B:K)) => B
    rule getArgSorts('constrCon(A:K,, B:K)) => getArgSorts(B)
    rule getArgSorts('optBangATypes(A:K,, Rest:K)) => getArgSorts(A) getArgSorts(Rest)
    rule getArgSorts('optBangAType('emptyBang(.KList),, Rest:K)) => getArgSorts(Rest)
    rule getArgSorts('atypeGTyCon(A:K)) => ListItem(A)
    rule getArgSorts('atypeTyVar(A:K)) => .List
    rule getArgSorts(.OptBangATypes) => .List

////////////////////////////////////////////////////////////////////////////////


    rule <k> preModule(A:K,T:K) => startTTransform ...</k>
      <tempAlpha> OldAlpha:K => A </tempAlpha>
```

```
        <tempT> OldT:K => T </tempT>

///////////////////////////////////////////////////////////////////////////////////////

    // STEP 2 PERFORM CHECKS

    syntax KItem ::= "error"

    syntax KItem ::= "startChecks"
    syntax KItem ::= "checkNoSameKey"
        //Keys of alpha and keys of T should be unique
    syntax KItem ::= "checkTypeConsDontCollide"
        //Make sure typeconstructors do not collide in T
    syntax KItem ::= "makeAlphaMap"
        //make map for alpha
    syntax KItem ::= "checkAlphaNoLoops"
        //alpha check for no loops
        //check alpha to make sure that everything points to a T
    syntax KItem ::= "checkArgSortsAreTargets"
           //Make sure argument sorts [U] [W,V] are in the set of keys of alpha and targets of T, (keys of T)
    syntax KItem ::= "checkParUsed"
//NEED TO CHECK all the polymorphic parameters from right appear on left. RIGHT SIDE ONLY
//NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY

//    rule <k> preModule(A:K,T:K) => startChecks ...</k>
//         <tempAlpha> OldAlpha:K => A </tempAlpha>
//         <tempT> OldT:K => T </tempT>


/*    rule <k> performNextChecks
             => checkUseVars
                ~> (checkLabelUses
                ~> (checkBlockAddress(.K)
                ~> (checkNoNormalBlocksHavingLandingpad(.K, TNS -Set TES)
                ~> (checkAllExpBlocksHavingLandingpad(.K, TES)
                ~> (checkAllExpInFromInvoke(.K, TES)
                ~> (checkLandingpad
                ~> checkLandingDomResumes)))))) ...</k> */

```

```
    rule <k> startChecks
            => checkNoSameKey
               ~> (checkTypeConsDontCollide
               ~> (makeAlphaMap
               ~> (checkAlphaNoLoops
               ~> (checkArgSortsAreTargets
               ~> (checkParUsed))))) ...</k>

    rule <k> checkTypeConsDontCollide
            => tyConCollCheck(T,.List,.Set) ...</k>
         <tempT> T:K </tempT>

    //syntax KItem ::= tChecker(K) [function]
    syntax KItem ::= tyConCollCheck(K,K,K) [function] //(TList, List of Tycons,Set of Tycons)
    syntax KItem ::= lengthCheck(K,K) [function]
    //syntax KItem ::= tyConCollCheck(K,K,K) [function]
    //syntax K ::= innerCollCheck(K) [function]
    //syntax K ::= tyConCollCheckPasser(K, K) [function]

    //rule tChecker(preModule(Alpha:Map,T:K,Mod:K)) => tyConCollCheck(innerCollCheck(T),preModule(Alpha,T,Mod))

    //rule tyConCollCheck(.K,preModule(Alpha:Map,H:K,Mod:K)) => tyConCollCheck(innerCollCheck(H),preModule(Alpha,H,Mod))

    rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:K,ListItem(InnerTPiece(Ty:K,E:K,F:K,H:K,G:K)) Inners:List)) Rest:List),J:List,D:Set) =>
                    tyConCollCheck(TList(ListItem(TObject(ModName,A,B,Inners)) Rest),ListItem(Ty) J, SetItem(Ty) D)
    rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:K,.List)) Rest:List),J:List,D:Set) =>
                    tyConCollCheck(TList(Rest),J,D)
    rule tyConCollCheck(TList(.List),J:List,D:Set) =>
                    lengthCheck(size(J),size(D))

    rule lengthCheck(A:Int, B:Int) => .K
                    requires A ==Int B

    rule lengthCheck(A:Int, B:Int) => error
                    requires A =/=Int B
```

```
    //rule tyConCollCheck(TList(TObject(A:K,B:K,C:K) ~> Rest:K), J:K) => tyConCollCheckPasser(TList(innerCollCheck(TObject(A:K,B:K,C:K)) ~> Rest:K),J:K)
    syntax KItem ::= keyCheck(K,K,K,K) [function] //(Alpha, T, List of names, Set of names)

    rule <k> checkNoSameKey
             => keyCheck(A, T, .Set, .List) ...</k>
         <tempAlpha> A:K </tempAlpha>
         <tempT> T:K </tempT>
    //rule <k> checkAlphaNoSameKey
    //         => akeyCheck(.K, .Set) ...</k>

    rule keyCheck(AList(ListItem(AObject(A:K,B:K)) C:List), T:K, D:Set, G:List) => keyCheck(AList(C), T, SetItem(A) D, ListItem(A) G)
    rule keyCheck(AList(.List), TList(ListItem(TObject(ModName:K, A:K,B:K,C:K)) Rest:List), D:Set, G:List) => keyCheck(AList(.List), TList(Rest), SetItem(A) D, ListItem(A) G)
    rule keyCheck(AList(.List), TList(.List), D:Set, G:List) => lengthCheck(size(G),size(D))


    syntax KItem ::= makeAlphaM(K,K) [function] //(Alpha, AlphaMap)
    syntax KItem ::= tAlphaMap(K) //(AlphaMap) temp alphamap

    rule <k> makeAlphaMap
             => makeAlphaM(A, .Map) ...</k>
         <tempAlpha> A:K </tempAlpha>

    rule makeAlphaM(AList(ListItem(AObject(A:K,B:K)) C:List), M:Map) => makeAlphaM(AList(C), M[A <- B])
    rule makeAlphaM(AList(.List), M:Map) => tAlphaMap(M)

    rule <k> tAlphaMap(M:K) => .K ...</k>
         <tempAlphaMap> OldAlphaMap:K => M </tempAlphaMap>

//    syntax KItem ::= tkeyCheck(K,K,K) [function] //(T,List of T,Set of T)

//    rule <k> checkTNoSameKey
```

```
//              => tkeyCheck(T, .Set, T) ...</k>
//          <tempT> T:K </tempT>

//    rule tkeyCheck(TList(ListItem(TObject(A:K,B:K,C:K)) Rest:
    List), D:Set, G:K) => tkeyCheck(TList(Rest), SetItem(A) D, G)
//    rule tkeyCheck(TList(.List), D:Set, TList(G:List)) =>
    lengthCheck(size(G),size(D))

    syntax KItem ::= aloopCheck(K,K,K,K,K,K,K) [function] //(
        Alpha,List of Alpha,Set of Alpha,CurrNode,lengthcheck,T,
        BigSet)

    rule <k> checkAlphaNoLoops
              => aloopCheck(A,.List,.Set,.K,.K,T,.Set) ...</k>
         <tempAlphaMap> A:K </tempAlphaMap>
         <tempT> T:K </tempT>

    //aloopCheck set and list to check cycles
    rule aloopCheck(Alpha:Map (A:KItem |-> B:KItem), D:List, G:
        Set, .K, .K,T:K,S:Set) => aloopCheck(Alpha, ListItem(B)
        ListItem(A) D, SetItem(B) SetItem(A) G, B, .K,T,S)
    rule aloopCheck(Alpha:Map (H |-> B:KItem), D:List, G:Set, H:
        KItem, .K,T:K,S:Set) => aloopCheck(Alpha, ListItem(B) D,
        SetItem(B) G, B, .K,T,S)

    rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
        Set) => aloopCheck(Alpha, .List, .Set, .K, lengthCheck(
        size(G),size(D)),T,G S) //type rename loop ERROR
         requires (notBool H in keys(Alpha)) andBool (H in
             typeSet(T, .Set) orBool H in S)

    rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
        Set) => error //terminal alpha rename is not in T ERROR
         requires (notBool H in keys(Alpha)) andBool (notBool (H
              in typeSet(T, .Set) orBool H in S))


    syntax Set ::= typeSet(K,K) [function] //(K, KSet)
    rule typeSet(TList(ListItem(TObject(ModName:K, A:K,B:K,C:K))
         Rest:List), D:Set) => typeSet(TList(Rest), SetItem(A) D)
    rule typeSet(TList(.List), D:Set) => D

```

```
//    rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K) =>
    keys(Alpha) ~> H
//         requires notBool H in keys(Alpha)

     rule aloopCheck(.Map, .List, .Set, .K, .K,T:K, S:Set) => .K
//    rule aloopCheck(AList(Front:List ListItem(AObject(H,B:K))
    C:List), D:List, G:Set, H:ConId) => aloopCheck(AList(C:List),
     ListItem(B) D, SetItem(B) G, B)


//    syntax KItem ::= TList(K) //list of T objects for every
    new type introduced by data and newtype
//    syntax KItem ::= TObject(K,K,K) //(type name, entire list
    of poly type vars, list of inner T pieces)
//    syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type
    constructor, poly type vars, argument sorts, entire constr
    block, type name)

//Make sure argument sorts [U] [W,V] are in the set of keys of
    alpha and targets of T, (keys of T)

     syntax KItem ::= argSortCheck(K,K,K) [function] //(T,
        AlphaMap)

     rule <k> checkArgSortsAreTargets
               => argSortCheck(T,A,typeSet(T,.Set)) ...</k>
          <tempAlphaMap> A:K </tempAlphaMap>
          <tempT> T:K </tempT>

     rule argSortCheck(TList(ListItem(TObject(ModName:K, A:K,B:K,
        ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest
        :List,E:K,F:K)) InnerRest:List)) TListRest:List),AlphaMap
        :Map,Tset:Set) => argSortCheck(TList(ListItem(TObject(
        ModName,A,B,ListItem(InnerTPiece(C,D,ArgsRest,E,F))
        InnerRest)) TListRest),AlphaMap,Tset)
          requires ((Arg in keys(AlphaMap)) orBool (Arg in Tset))

     rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
        ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest
        :List,E:K,F:K)) InnerRest:List)) TListRest:List),AlphaMap
        :Map,Tset:Set) => error
          requires (notBool ((Arg in keys(AlphaMap)) orBool (Arg
             in Tset)))
```

```
    rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
       ListItem(InnerTPiece(C:K,D:K,.List,E:K,F:K)) InnerRest:
       List)) TListRest:List),AlphaMap:Map,Tset:Set) =>
       argSortCheck(TList(ListItem(TObject(ModName,A,B,InnerRest
       )) TListRest),AlphaMap,Tset)

    rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.
       List)) TListRest:List),AlphaMap:Map,Tset:Set) =>
       argSortCheck(TList(TListRest),AlphaMap,Tset)

    rule argSortCheck(TList(.List),AlphaMap:Map,Tset:Set) => .K

//NEED TO CHECK all the polymorphic parameters from right appear
     on left. RIGHT SIDE ONLY
//NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY

    syntax KItem ::= parCheck(K,K) [function] //(T,AlphaMap)
    syntax KItem ::= makeTyVarList(K,K,K) [function] //(TyVars,
       NewList)
    syntax KItem ::= lengthRet(K,K,K) [function]

    rule <k> checkParUsed
              => parCheck(T,.K) ...</k>
         <tempT> T:K </tempT>

    //rule makeParLists(TList(ListItem(TObject(A:K,ListItem(Arg:
       KItem) PolyList:List,C:K)) Rest:List),Tlist:List,Tset:Set
       ) => makeParLists(TList(ListItem(TObject(A,PolyList,C))
       Rest),ListItem(Arg) Tlist,SetItem(Arg) Tset)
    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,C:K))
        Rest:List),.K) => parCheck(TList(ListItem(TObject(
       ModName,A:K,B:K,C:K)) Rest:List),makeTyVarList(B,.List,.
       Set))

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
       ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List
       ,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
         parCheck(TList(ListItem(TObject(ModName,A,B,ListItem(
             InnerTPiece(C,ParRest,D,E,F)) InnerRest)) Rest),
             NewSet)
            requires Par in NewSet

```

```
    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
        ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List
        ,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
         error
            requires notBool (Par in NewSet)

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
        ListItem(InnerTPiece(C:K,.List,D:K,E:K,F:K)) InnerRest:
        List)) Rest:List),NewSet:Set) =>
         parCheck(TList(ListItem(TObject(ModName,A,B,InnerRest))
             Rest),NewSet)

    rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.List
        )) Rest:List),NewSet:Set) =>
         parCheck(TList(Rest),NewSet)

    rule parCheck(TList(.List),NewSet:Set) => .K

    rule makeTyVarList('typeVars(A:K,,Rest:K),NewList:List,
        NewSet:Set) => makeTyVarList(Rest, ListItem(A) NewList,
        SetItem(A) NewSet)

    rule makeTyVarList(.TyVars,NewList:List,NewSet:Set) =>
        lengthRet(size(NewList),size(NewSet),NewSet)

    rule lengthRet(A:Int, B:Int, C:K) => C
                    requires A ==Int B

    rule lengthRet(A:Int, B:Int, C:K) => error
                    requires A =/=Int B

    //rule argSortCheck(TList(ListItem(TObject(A:K,B:K,C:K)

////////////////////////////////////////////////////////////////////////////////////////


    // STEP 3 Transform T into beta

    syntax KItem ::= "startTTransform"
    syntax KItem ::= "constructDelta"
    syntax KItem ::= "constructBeta"

    rule <k> startTTransform
```

```
                  => constructDelta
                     ~> (constructBeta) ...</k>

     rule <k> constructDelta
                  => makeDelta(T,.Map) ...</k>
          <tempT> T:K </tempT>

     syntax KItem ::= makeDelta(K,Map) [function] //(T,Delta)
     syntax KItem ::= newDelta(Map) //Delta
     syntax KItem ::= newBeta(Map) //beta
     syntax List ::= retPolyList(K,List) [function] //(T,Delta)

     rule makeDelta(TList(ListItem(TObject(ModName:K,A:K,Polys:K,C:K)) Rest:List),M:Map) =>
          makeDelta(TList(Rest),M[ModPlusType(ModName,A) <- size(retPolyList(Polys,.List))])
     rule makeDelta(TList(.List),M:Map) => newDelta(M)

     rule retPolyList('typeVars(A:K,,Rest:K),NewList:List) => retPolyList(Rest, ListItem(A) NewList)
     rule retPolyList(.TyVars,L:List) => L

     rule <k> newDelta(M:Map)
                  => .K ...</k>
          <tempDelta> OldDelta:K => M </tempDelta>

     rule <k> constructBeta
                  => makeBeta(T,.Map) ...</k>
          <tempT> T:K </tempT>

     syntax KItem ::= makeBeta(K,Map) [function] //(T,Beta,Delta)

     rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,ListItem(InnerTPiece(Con:K,H:K,D:K,E:K,F:K)) InnerRest:List)) Rest:List),Beta:Map) =>
          makeBeta(TList(ListItem(TObject(ModName,A,B,InnerRest)) Rest),Beta[ModPlusType(ModName,Con) <- betaParser(E,B,A)])
     rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,.List)) Rest:List),Beta:Map) =>
          makeBeta(TList(Rest),Beta)
     rule makeBeta(TList(.List),Beta:Map) =>
          newBeta(Beta)
```

```
//    rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K, ListItem(InnerTPiece(C:K,H:K,D:K,E:K,F:K)) InnerRest:List)) Rest:List),Beta:Map) =>
//         makeBeta(TList(ListItem(TObject(ModName,A,B,InnerRest)) Rest),Beta)

    syntax KItem ::= betaParser(K,K,K) [function] //(Tree Piece, NewSyntax,Parameters,Constr)
    syntax Set ::= getTyVarsRHS(K,List) [function]

    syntax KItem ::= forAll(Set,K)
    syntax KItem ::= funtype(K,K)

    syntax Set ::= listToSet(List, Set) [function]

    rule listToSet(ListItem(A:KItem) L:List, S:Set) => listToSet(L, SetItem(A) S)
    rule listToSet(.List, S:Set) => S


//if optbangAtypes, need to see if first variable is a typecon
//if its a typecon then need to go into Delta and see the amount of parameters it has
//then count the number of optbangAtypes after the typecon
    rule betaParser('constrCon(A:K,, B:K), Par:K, Con:K) => forAll(getTyVarsRHS(B,.List), betaParser(B, Par, Con))
    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'atypeTyVar(Tyv:K)),, Rest:K), Par:K, Con:K) => funtype(Tyv, betaParser(Rest, Par, Con))
    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'baTypeCon(A:K,, B:K)),, Rest:K), Par:K, Con:K) => funtype('baTypeCon(A:K,, B:K), betaParser(Rest, Par, Con))
    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'atypeGTyCon(Tyc:K)),, Rest:K), Par:K, Con:K) => funtype(Tyc, betaParser(Rest, Par, Con))
    rule betaParser(.OptBangATypes, Par:K, Con:K) => 'simpleTypeCon(Con,, Par)
//    rule betaParser('optBangATypes('optBangAType('emptyBang(.KList),, 'atypeGTyCon(Tyc:K)),, Rest:KItem)) => getTypeVars(A) getTypeVars(Rest)
//    rule getTypeVars('optBangAType('emptyBang(.KList),, Rest:K)) => getTypeVars(Rest)
```

```
//    rule getTypeVars('atypeGTyCon(A:K)) => .List
//    rule getTypeVars('atypeTyVar(A:K)) => ListItem(A)
//    rule getTypeVars(.OptBangATypes) => .List

    rule getTyVarsRHS(.OptBangATypes,Tylist:List) => listToSet(
       Tylist, .Set)

    rule <k> newBeta(M:Map)
             => .K ...</k>
         <tempBeta> OldBeta:K => M </tempBeta>

//////////////////////////////////////////////////////////////////////////////////


//    syntax KItem ::= "insertAlphaBetaStar"

    syntax KItem ::= insertABRec(K,List)
    syntax KItem ::= insertAB(K)

    rule <k> insertAlphaBetaStar => insertABRec(Mod, Imp) ...</k
       >
         <tempModule> Mod:KItem </tempModule>
         <importTree> Imp:List </importTree>

    rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem,
       Node)) I:List) => insertAB(B) ~> insertABRec(Node, I)
       ...</k>

    rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem,
       C:KItem)) I:List) => insertABRec(Node, I) ...</k>
             requires Node =/=K C

    rule <k> insertAB(B) => .K ...</k>

         <tempAlphaStar> Alph:KItem </tempAlphaStar>
         <tempBetaStar> Bet:KItem </tempBetaStar>

         <moduleName> 'moduleName(B) </moduleName>
         <moduleImpAlphas> ImpAlphas:List => ListItem(Alph)
            ImpAlphas </moduleImpAlphas>
         <moduleImpBetas> ImpBetas:List => ListItem(Bet)
            ImpBetas </moduleImpBetas>

```

```
endmodule
```

Appendix D

haskell-type-inferencing.k

```
requires "haskell-syntax.k"
requires "haskell-configuration.k"
requires "haskell-preprocessing.k"

module HASKELL-TYPE-INFERENCING
    imports HASKELL-SYNTAX
    imports HASKELL-CONFIGURATION
    imports HASKELL-PREPROCESSING

    syntax KItem ::= "Bool" //Boolean

    // STEP 4 Type Inferencing
    syntax KItem ::= inferenceShell(K) [function]//Input, AlphaMap, Beta, Delta, Gamma
    //syntax KItem ::= typeInferenceFun(K,Map,Map,Map,Map,K,K) [function]//Input, Alpha, Beta, Delta, Gamma
    //syntax KItem ::= typeInferenceFun(Map,K,K) //Gamma, Expression, Guessed Type
    syntax Map ::= genGamma(K,Map,K) [function] //Apatlist, Gamma Type
    syntax KItem ::= genLambda(K,K) [function]
    syntax KItem ::= guessType(Int)
//    syntax KItem ::= lambdaReturn(K,K,K)
    syntax KItem ::= freshInstance(K, Int) [function]
    syntax Int ::= paramSize(K) [function]



    syntax KItem ::= mapBag(Map)
    syntax KResult ::= mapBagResult(Map)

    syntax Map ::= gammaSub(Map,Map,Map) [function]//substitution, gamma

```

```
    rule <k> performIndividualInferencing => inferenceShell(Code) ...</k>
      <tempModule> Mod:KItem </tempModule>

      <moduleName> 'moduleName(Mod) </moduleName>
      <moduleTempCode> Code:KItem </moduleTempCode>

    rule inferenceShell('topdeclslist('type(A:K,, B:K),, Rest:K)) =>
      inferenceShell(Rest) //constructalpha
    rule inferenceShell('topdeclslist('data(A:K,, B:K,, C:K,, D:K),, Rest:K)) =>
      inferenceShell(Rest)
    rule inferenceShell('topdeclslist('newtype(A:K,, B:K,, C:K,, D:K),, Rest:K)) =>
      inferenceShell(Rest)
    rule inferenceShell('topdeclslist('class(A:K,, B:K,, C:K,, D:K),, Rest:K)) =>
      inferenceShell(Rest)
    rule inferenceShell('topdeclslist('instance(A:K,, B:K,, C:K,, D:K),, Rest:K)) =>
      inferenceShell(Rest)
    rule inferenceShell('topdeclslist('default(A:K,, B:K,, C:K,, D:K),, Rest:K)) =>
      inferenceShell(Rest)
    rule inferenceShell('topdeclslist('foreign(A:K,, B:K,, C:K,, D:K),, Rest:K)) =>
      inferenceShell(Rest)

    rule inferenceShell('topdeclslist('topdecldecl(A:K),, Rest:K)) =>
      typeInferenceFun(.ElemList, .Map,A,guessType(0)) ~> inferenceShell(Rest)


    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'declFunLhsRhs(Fn:K,, Lhsrhs:K), Guess:K) =>
      typeInferenceFun(.ElemList, Gamma, Lhsrhs, Guess) ...</k>
    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'eqExpOptDecls(Ex:K,, Optdecls:K), Guess:K) =>
      typeInferenceFun(.ElemList, Gamma, Ex, Guess) ...</k>

```

```
    //T-App
    //rule typeInferenceFun('aexpQVar(Var:K), Alpha:Map, Beta:Map, Delta:Map, (Var |-> Sigma:K) Gamma:Map,.K,.K) => Sigma
    //Gamma Proves x:phi(tau) if Gamma(x) = \forall alpha_1, ..., alpha_n . tau
    //where phi replaces all occurrences of alpha_1, ..., alpha_n by monotypes tau_1, ..., tau_n

    rule <k> typeInferenceFun(.ElemList, (Var |-> Type:K) Gamma:Map, 'aexpQVar(Var:K), Guess:KItem)
           => mapBagResult(uniFun(ListItem(uniPair(Guess, freshInstance(Type, TypeIt))))) ...</k> //Variable rule
          <typeIterator> TypeIt:Int => TypeIt +Int paramSize(Type) </typeIterator>

    //rule typeInferenceFun('aexpGCon(Gcon:K), Alpha:Map, (Gcon |-> Sigma:K) Beta:Map, Delta:Map, Gamma:Map,.K,.K) => Sigma //T-App
    //rule typeInferenceFun('aexpGCon(Gcon:K), Alpha:Map, Lol:Map, Delta:Map, Gamma:Map,.K,.K) => Sigma //T-App
    //     <tempBeta> (Gcon |-> Sigma:K) Beta:Map </tempBeta>

    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'aexpGCon('conTyCon(Mid:K,, Gcon:K)), Guess:KItem)
           => mapBagResult(uniFun(ListItem(uniPair(Guess, freshInstance(Type, TypeIt))))) ...</k> //Constant rule
          <tempBeta> (ModPlusType(Mid,Gcon) |-> Type:K) Beta:Map </tempBeta>
          <typeIterator> TypeIt:Int => TypeIt +Int paramSize(Type) </typeIterator>

    //lambda rule
//    rule <k> typeInferenceFun(Gamma:Map, 'lambdaFun(Apatlist:K,, Ex:K), Guess:KItem)
//          => typeInferenceFun(genGamma(Apatlist,Gamma,guessType(TypeIt)), genLambda(Apatlist,Ex), guessType(TypeIt +Int 1))
//          ~> lambdaReturn(Guess,guessType(TypeIt),guessType(TypeIt +Int 1)) ...</k>
```

```
//          <typeIterator> TypeIt:Int => TypeIt +Int 2 </typeIterator>

//     rule <k> Sigma:Map ~> lambdaReturn(Tau:K, Tauone:K, Tautwo:K)
//           => compose(uniFun(ListItem(uniPair(typeSub(Sigma,Tau),typeSub(Sigma,funtype(Tauone,Tautwo))))),Sigma) ...</k>

   syntax KItem ::= typeInferenceFun(ElemList, Map, K, K) [strict(1)]
   syntax KItem ::= typeInferenceFunLambda(ElemList, K, K, K) [strict(1)]
/* automatically generated by the strict(1) in typeInferenceFun or typeInferenceFunAux
   rule typeInferenceFunAux(Es:ElemList, C:K, A:K, B:K) => Es ~> typeInferenceFun(HOLE, C, A, B)
        requires notBool isKResult(Es)
   rule Es:KResult ~> typeInferenceFunAux(HOLE, C:K,A:K, B:K) => typeInferenceFun(Es, C, A, B)
*/

    //lambda rule
    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'lambdaFun(Apatlist:K,, Ex:K), Guess:KItem)
          => typeInferenceFunLambda(val(typeInferenceFun(.ElemList, genGamma(Apatlist,Gamma,guessType(TypeIt)), genLambda(Apatlist,Ex), guessType(TypeIt +Int 1))), .ElemList, Guess, guessType(TypeIt),guessType(TypeIt +Int 1)) ...</k>
         <typeIterator> TypeIt:Int => TypeIt +Int 2 </typeIterator>

    rule <k> typeInferenceFunLambda(valValue(mapBagResult(Sigma:Map)), .ElemList, Tau:K, Tauone:K, Tautwo:K)
         => mapBagResult(compose(uniFun(ListItem(uniPair(typeSub(Sigma,Tau),typeSub(Sigma,funtype(Tauone,Tautwo))))),Sigma)) ...</k>

    //rule <k> substi(S:Map) ~> lambdaReturn(Tau:K, Tauone:K, Tautwo:K)
    //      => S[Tauone] ...</k>


```

```
    //syntax KItem ::= appliReturn(Map, K, K, Map)
    //syntax KItem ::= typeChildSub(Map, K) [function]

    syntax KItem ::= typeInferenceFunAppli(ElemList, Map, K, K, Map) [strict(1)]

    //application rule
    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'funApp(Eone:K,, Etwo:K), Guess:KItem)
         => typeInferenceFunAppli(val(typeInferenceFun(.ElemList, Gamma, Eone, funtype(guessType(TypeIt), Guess))), .ElemList, Gamma, Etwo, guessType(TypeIt), .Map) ...</k>
        <typeIterator> TypeIt:Int => TypeIt +Int 1 </typeIterator>

    rule <k> typeInferenceFunAppli(valValue(mapBagResult(Sigmaone:Map)), .ElemList, Gamma:Map, Etwo:KItem, guessType(TypeIt:Int), .Map)
         => typeInferenceFunAppli(val(typeInferenceFun(.ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo, typeSub(Sigmaone, guessType(TypeIt)))), .ElemList, .Map, .K, .K, Sigmaone) ...</k>

    rule <k> typeInferenceFunAppli(valValue(mapBagResult(Sigmatwo:Map)), .ElemList, .Map, .K, .K, Sigmaone:Map)
         => mapBagResult(compose(Sigmatwo, Sigmaone)) ...</k>

//    rule <k> Sigmaone:Map ~> appliReturn(Gamma:Map, Etwo:KItem, guessType(TypeIt:Int), .Map)
//          => typeInferenceFun(gammaSub(Sigmaone, Gamma, .Map), Etwo, typeSub(Sigmaone, guessType(TypeIt)))
//          ~> appliReturn(.Map, .K, .K, Sigmaone) ...</k>

//    rule <k> Sigmatwo:Map ~> appliReturn(.Map, .K, .K, Sigmaone:Map)
//          => compose(Sigmatwo, Sigmaone) ...</k>

    syntax KItem ::= typeInferenceFunIfThen(ElemList, Map, K, K, K, Map, Map) [strict(1)]

    //if_then_else rule
```

```
rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'ifThenElse(
    Eone:K,, Optsem:K,, Etwo:K,, Optsemtwo:K,, Ethree:K),
    Guess:KItem)
       => typeInferenceFunIfThen(val(typeInferenceFun(.
           ElemList, Gamma, Eone, Bool)), .ElemList, Gamma,
           Etwo, Ethree, Guess, .Map, .Map) ...</k>

rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
    Sigmaone:Map)), .ElemList, Gamma:Map, Etwo:KItem, Ethree:
    KItem, Guess:KItem, .Map, .Map)
       => typeInferenceFunIfThen(val(typeInferenceFun(.
           ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo,
           typeSub(Sigmaone, Guess))), .ElemList, Gamma, .K,
           Ethree, Guess, Sigmaone, .Map) ...</k>

rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
    Sigmatwo:Map)), .ElemList, Gamma:Map, .K, Ethree:KItem,
    Guess:KItem, Sigmaone:Map, .Map)
       => typeInferenceFunIfThen(val(typeInferenceFun(.
           ElemList, gammaSub(compose(Sigmatwo, Sigmaone),
           Gamma, .Map), Ethree, typeSub(compose(Sigmatwo,
           Sigmaone), Guess))), .ElemList, .Map, .K, .K, .K,
           Sigmaone, Sigmatwo) ...</k>

rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
    Sigmathree:Map)), .ElemList, .Map, .K, .K, .K, Sigmaone:
    Map, Sigmatwo:Map)
       => mapBagResult(compose(compose(Sigmathree, Sigmatwo),
            Sigmaone)) ...</k>

syntax KItem ::= typeInferenceFunLetIn(ElemList, Map, Map, K
    , K, K, Int, Int, Map, Map) [strict(1)]
syntax KItem ::= grabLetDeclName(K, Int) [function]
syntax KItem ::= grabLetDeclExp(K, Int) [function]
syntax KItem ::= mapLookup(Map, K) [function]
syntax Map ::= makeDeclMap(K, Int, Map) [function]
syntax Map ::= applyGEN(Map, Map, Map, Map) [function]

//Haskell let in rule (let rec in exp + let in rule combined
    )
//gamma |- let rec f1 = e1 and f2 = e2 and f3 = e3 .... in e
     =>
```

```
    //beta, [f1 -> tau1, f2 -> tau2, f3 -> tau3,....] + gamma |- e1 : tau1 | sigma1,  [f1 -> simga1(tau1), f2 -> sigma1(tau2), f3 -> sigma1(tau3),....] + sigma1(gamma) |- e2 : sigma1(tau2) | sigma2  [f1 -> sigma2 o sigma1(tau1), f2 -> sigma2 o sigma1(tau2), f3 -> sigma2 o sigma1(tau3),....] + sigma2 o sigma1(gamma) |- e3 : sigma2 o sigma1(tau3) .....  [f1 -> gen(sigma_n o sigma2 o sigma1(tau1), sigma_n o sigma2 o sigma1(Gamma)), f2 -> gen(tau2), f3 -> gen(tau3),....] + gamma |- e : something
    rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'letIn(D:K,, E:K), Guess:KItem)
          => typeInferenceFunLetIn(.ElemList, Gamma, makeDeclMap(D, TypeIt, .Map), D, E, Guess, 0, TypeIt, .Map, Beta) ...</k>
         <typeIterator> TypeIt:Int => TypeIt +Int size(makeDeclMap(D, TypeIt, .Map)) </typeIterator>
         <tempBeta> Beta:Map </tempBeta>

    rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map, Beta:Map)
           => typeInferenceFunLetIn(val(typeInferenceFun(.ElemList, Gamma DeclMap, grabLetDeclExp(D, Iter), mapLookup(DeclMap, grabLetDeclName(D, Iter)))), .ElemList, Gamma, DeclMap, D, E, Guess, Iter, TypeIt, OldSigma, Beta) ...</k>
          //=> typeInferenceFunLetIn(val(typeInferenceFun(DeclMap, grabLetDeclExp(D, Iter +Int TypeIt), Guess)), .ElemList, Gamma, DeclMap, D, E, Guess, Iter, TypeIt, OldSigma) ...</k>
          requires Iter <Int (size(DeclMap))

    rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:Map)), .ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map, Beta:Map)
          => typeInferenceFunLetIn(.ElemList, gammaSub(Sigma, Gamma,.Map), gammaSub(Sigma, DeclMap,.Map), D, E, typeSub(Sigma, Guess), Iter +Int 1, TypeIt, compose(Sigma,OldSigma), Beta) ...</k>
          requires Iter <Int (size(DeclMap))

```

```
rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap
    :Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int
    , OldSigma:Map, Beta:Map)
      => typeInferenceFunLetIn(val(typeInferenceFun(.
          ElemList, Gamma applyGEN(Gamma, DeclMap, .Map, Beta
          ), E, Guess)), .ElemList, Gamma, DeclMap, D, E,
          Guess, Iter, TypeIt, OldSigma, Beta) ...</k>
      requires Iter >=Int (size(DeclMap))

rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:
    Map)), .ElemList, Gamma:Map, DeclMap:Map, D:KItem, E:
    KItem, Guess:KItem, Iter:Int, TypeIt:Int, OldSigma:Map,
    Beta:Map)
      => mapBagResult(compose(Sigma, OldSigma))...</k>
      requires Iter >=Int (size(DeclMap))

rule mapLookup((Name |-> Type:KItem) DeclMap:Map, Name:KItem
    ) => Type
rule mapLookup(DeclMap:Map, Name:KItem) => Name
     requires notBool(Name in keys(DeclMap))

//rule makeDeclMap('decls(A:K), TypeIt:Int, NewMap:Map) =>
    makeDeclMap(A, TypeIt +Int 1, NewMap)
rule makeDeclMap('decls(Dec:K), TypeIt:Int, NewMap:Map) =>
    makeDeclMap(Dec, TypeIt, NewMap)
rule makeDeclMap('declsList('declPatRhs('apatVar(Var:K),,
    Righthand:K),, Rest:K), TypeIt:Int, NewMap:Map) =>
    makeDeclMap('decls(Rest), TypeIt +Int 1, NewMap[Var <-
    guessType(TypeIt)])
rule makeDeclMap(.DeclsList, TypeIt:Int, NewMap:Map) =>
    NewMap

rule grabLetDeclName('decls(Dec:K), Iter:Int) =>
    grabLetDeclName(Dec, Iter)
rule grabLetDeclName('declsList(Dec:K,, Rest:K), Iter:Int)
    => grabLetDeclName(Rest, Iter -Int 1)
     requires Iter >Int 0
rule grabLetDeclName('declsList('declPatRhs('apatVar(Var:K)
    ,, Righthand:K),, Rest:K), Iter:Int) => Var
     requires Iter <=Int 0
```

```
    rule grabLetDeclExp('decls(Dec:K), Iter:Int) => grabLetDeclExp(Dec, Iter)
    rule grabLetDeclExp('declsList(Dec:K,, Rest:K), Iter:Int) => grabLetDeclExp(Rest, Iter -Int 1)
         requires Iter >Int 0
    rule grabLetDeclExp('declsList('declPatRhs('apatVar(Var:K),, Righthand:K),, Rest:K), Iter:Int) => grabLetDeclExp(Righthand, Iter)
         requires Iter <=Int 0
    rule grabLetDeclExp('eqExpOptDecls(Righthand:K,, Opt:K), Iter:Int) => 'eqExpOptDecls(Righthand,, Opt)

    rule genGamma('apatVar(Vari:K), Gamma:Map, Guess:K) => Gamma[Vari <- Guess]
    rule genGamma('apatCon(Vari:K,, Pattwo:K), Gamma:Map, Guess:K) => Gamma[Vari <- Guess]

    rule genLambda('apatVar(Vari:K), Ex:K) => Ex
    rule genLambda('apatCon(Vari:K,, Pattwo:K), Ex:K) => 'lambdaFun(Pattwo,, Ex)


    rule gammaSub(Sigma:Map, (Key:KItem |-> Type:KItem) Gamma:Map, Newgamma:Map)
        => gammaSub(Sigma, Gamma, Newgamma[Key <- typeSub(Sigma, Type) ] )
    //  => gammaSub(Sigma, Gamma, Newgamma[Key <- typeChildSub(Sigma, Type) ] )

    rule gammaSub(Sigma:Map, .Map, Newgamma:Map)
      => Newgamma

    //rule typeChildSub((guessType(TypeIt) |-> Type:KItem) Sigma:Map, guessType(TypeIt:Int)) => Type

    //rule typeChildSub(Sigma:Map, guessType(TypeIt:Int)) => guessType(TypeIt)
    //     requires notBool (guessType(TypeIt) in keys(Sigma))

    rule freshInstance(guessType(TypeIt:Int), Iter:Int) => guessType(TypeIt)
    rule freshInstance(forAll(.Set, B:K), Iter:Int) => B
```

```
    rule freshInstance(forAll(SetItem(C:KItem) A:Set, B:K), Iter:Int) => freshInstance(forAll(A, freshInstanceInner(C, B, Iter)), Iter +Int 1)

    syntax KItem ::= freshInstanceInner(K,K,Int) [function]

    rule freshInstanceInner(Repl:KItem, funtype(A:K, B:K), Iter:Int) => funtype(freshInstanceInner(Repl,A,Iter), freshInstanceInner(Repl,B,Iter))
    rule freshInstanceInner(Repl:KItem, Repl, Iter:Int) => guessType(Iter)
    rule freshInstanceInner(Repl:KItem, Target:KItem, Iter:Int) => Target [owise]

    rule paramSize(forAll(A:Set, B:K)) => size(A)
    rule paramSize(A:K) => 0 [owise]


     rule applyGEN(Gamma:Map, (Key:KItem |-> Type:KItem) DeclMap:Map, NewMap:Map, Beta:Map)
       => applyGEN(Gamma, DeclMap, NewMap[Key <- gen(Gamma, Type, Beta)], Beta)

     rule applyGEN(Gamma:Map, .Map, NewMap:Map, Beta:Map)
       => NewMap

    //GEN
    //GEN(Gamma, Tau) => Forall alpha

    syntax KItem ::= gen(Map, K, Map) [function]
    syntax Set ::= freeVarsTy(K, Map) [function]
    syntax Set ::= freeVarsEnv(Map, Map) [function]
    //syntax KItem ::= setBag(Set)
//    syntax Set ::= listToSet(List, Set) [function]


    rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map) => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set freeVarsEnv(Gamma, Beta), Tau)
    rule gen(Gamma:Map, Tau:KItem, Beta:Map) => forAll(freeVarsTy(Tau, Beta) -Set freeVarsEnv(Gamma, Beta), Tau) [owise]

```

```
    //rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map) => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set freeVarsEnv(Gamma, Beta), Tau)

    rule freeVarsTy(guessType(TypeIt:Int), Beta:Map) => SetItem(guessType(TypeIt:Int))
    rule freeVarsTy(funtype(Tauone:KItem, Tautwo:KItem), Beta:Map) => freeVarsTy(Tauone, Beta) freeVarsTy(Tautwo, Beta)
    rule freeVarsTy(Tau:KItem, Beta:Map) => .Set
         requires (forAll(.Set, Tau)) in values(Beta)
    rule freeVarsTy(forAll(Para:Set, Tau:KItem), Beta:Map) => freeVarsTy(Tau, Beta) -Set Para
    rule freeVarsEnv(Gamma:Map, Beta:Map) => listToSet(values(Beta), .Set)


//    rule listToSet(ListItem(A:KItem) L:List, S:Set) => listToSet(L, SetItem(A) S)
//    rule listToSet(.List, S:Set) => S

//////////////////////////////////////////////////////////////////////////////////////

    //Unification

    syntax Map ::= uniFun(List) [function]
    //syntax List ::= uniSub(K,K,K) [function]
    syntax Bool ::= isVarType(K) [function]
    syntax Bool ::= notChildVar(K,K) [function]
    syntax KItem ::= uniPair(K,K)

    syntax List ::= uniSub(Map,K) [function] //apply substitution to unification

    syntax KItem ::= typeSub(Map,K) [function] //apply substitution to type
    syntax Map ::= compose(Map,Map) [function]

    // syntax KItem ::= Map

    rule uniFun(.List) => .Map //substi(.K,.K) is id substitution

```

```
    rule uniFun(ListItem(uniPair(S:K,S)) Rest:List) => uniFun(Rest)  //delete rule

    // rule uniFun(SetItem(I:K)) => .K //uniFun(Rest)  //delete rule

    rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List) => uniFun(ListItem(uniPair(T,S)) Rest) //orient rule
         requires isVarType(T) andBool (notBool isVarType(S))

    //rule uniFun(ListItem(uniPair(forAll(Svars:List,S:K),forAll(.List,T:K))) Rest:List,Sigma:Map) => uniFun(ListItem(uniPair(forAll(.List,T),forAll(Svars,S))) Rest,Sigma) // orient rule
      //   requires Svars =/=K .List

    //rule uniFun(ListItem(uniPair(guessType(S:Int),forAll(.List,T:K))) Rest:List,Sigma:Map) => uniFun(ListItem(uniPair(forAll(.List,T:K),guessType(S))) Rest,Sigma) //orient rule

   // rule uniFun(ListItem(uniPair(forAll(.List,S:K),T:K)) Rest:List, Sigma:Map) => uniFun(uniSub('aexpQVar(Var),T,Rest), Sigma['aexpQVar(Var) <- T]) //eliminate rule
   //      requires notChildVar('aexpQVar(Var:K),T)

    rule uniFun(ListItem(uniPair(funtype(A:K, B:K), funtype(C:K, D:K))) Rest:List) => uniFun(ListItem(uniPair(A, C)) ListItem(uniPair(B, D)) Rest:List) //decompose rule function type

    rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List)
      => compose((S |-> typeSub(uniFun(uniSub((S |-> T),Rest)),T)),uniFun(uniSub((S |-> T),Rest))) //eliminate rule
    //  => compose(uniFun(uniSub((S |-> T),Rest)),(S |-> typeSub(uniFun(uniSub((S |-> T),Rest)),T))) //eliminate rule
         requires isVarType(S) andBool notChildVar(S,T)

    rule isVarType(S:K) => true
         requires getKLabel(S) ==KLabel 'guessType
    rule isVarType(S:K) => false [owise]

    rule notChildVar(S:K,T:K) => true
```

```
rule uniSub(Sigma:Map,.List) => .List
rule uniSub(.Map,L:List) => L
rule uniSub(Sigma:Map, Rest:List ListItem(uniPair(A:K, B:K))
   ) => uniSub(Sigma, Rest) ListItem(uniPair(typeSub(Sigma,
   A), typeSub(Sigma, B)))

//rule typeSub(substi(.Map),Tau:KItem) => Tau
rule typeSub(Sigma:Map (Tau |-> Newtau:KItem),Tau:KItem) =>
   typeSub(Sigma (Tau |-> Newtau),Newtau)
rule typeSub(Sigma:Map,funtype(Tauone:KItem,Tautwo:KItem))
   => funtype(typeSub(Sigma,Tauone),typeSub(Sigma,Tautwo))
rule typeSub(Sigma:Map,Tau:KItem) => Tau [owise]

syntax Map ::= composeIn(Map, Map, Map, K, K) [function]

rule compose(Sigmaone:Map, Sigmatwo:Map) => composeIn(
   Sigmaone, Sigmatwo, .Map, .K, .K)

rule composeIn(Sigmaone:Map, (Key:KItem |-> Type:KItem)
   Sigmatwo:Map, NewMap:Map, .K, .K) => composeIn(Sigmaone,
   Sigmatwo, NewMap, Key, Type)

rule composeIn((Keyone |-> Typetwo:KItem) Sigmaone:Map,
   Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
    composeIn(Sigmaone, Sigmatwo, NewMap, Keyone, Typeone)

rule composeIn((Typeone |-> Typetwo:KItem) Sigmaone:Map,
   Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
    composeIn((Typeone |-> Typetwo) Sigmaone, Sigmatwo,
   NewMap[Keyone <- Typetwo], .K, .K)
     requires notBool(Keyone in keys(Sigmaone))

rule composeIn(Sigmaone:Map, Sigmatwo:Map, NewMap:Map,
   Keyone:KItem, Typeone:KItem) => composeIn(Sigmaone,
   Sigmatwo, NewMap[Keyone <- Typeone], .K, .K) [owise]

rule composeIn(Sigmaone:Map, .Map, NewMap:Map, .K, .K) =>
   Sigmaone NewMap

//rule composeIn(Sigmaone:Map, .Map, .Map, .K, .K) =>
   Sigmaone

```

```
    //rule composeIn((Key:KItem |-> Type:KItem) Sigmaone:Map, .Map, NewMap:Map) => composeIn(Sigmaone, .Map, NewMap[Key <- mapLookup(Sigmaone, Type)])

    //rule compose(Sigmaone:Map,Sigmatwo:Map) => updateMap(Sigmaone,Sigmatwo)
    //rule compose(Sigmaone:Map, (Keytwo:KItem |-> Valtwo:KItem) Sigmatwo:Map) => compose(Sigmaone[Keytwo <- Valtwo][Valtwo <- mapLookup(Sigmaone, Keytwo)], Sigmatwo)
    //rule compose(Sigmaone:Map, (Key:KItem |-> Type:KItem) Sigmatwo:Map, .K) => compose(Sigmaone[Type <- mapLookup(Sigmaone, Key)][Key <- Type], Sigmatwo, mapLookup(Sigmaone, Key))
    //rule compose(Sigmaone:Map, (Key:KItem |-> Type:KItem) Sigmatwo:Map) => composeIn(Sigmaone, Sigmatwo, mapLookup(Sigmaone, Key))
    //     requires (notBool (Type in values(Sigmaone))) andBool (Type =/=K mapLookup(Sigmaone, Key))
    //rule compose(Sigmaone:Map, (Key:KItem |-> Type:KItem) Sigmatwo:Map) => compose(Sigmaone[Key <- Type][Type <- mapLookup(Sigmaone, Key)], Sigmatwo)
    //     requires (notBool (Type in values(Sigmaone))) andBool (Type =/=K mapLookup(Sigmaone, Key))
//    rule compose(Sigmaone:Map, (Keytwo:KItem |-> Valtwo:KItem) Sigmatwo:Map) => compose(Sigmaone[Valtwo <- mapLookup(Sigmaone, Keytwo)], Sigmatwo)
//         requires (Valtwo in values(Sigmaone)) andBool (Valtwo =/=K mapLookup(Sigmaone, Keytwo))
    //rule compose(Sigmaone:Map, (Keytwo:KItem |-> Valtwo:KItem) Sigmatwo:Map) => compose((Keytwo |-> Valtwo) Sigmaone, Sigmatwo)
    // requires notBool (Keytwo in keys(Sigmaone))
    //rule compose(Sigmaone:Map, .Map) => Sigmaone
    // rule compose(substi(Sone:K,Tone:K),substi(Stwo:K,Ttwo:K)) => substi(typeSub(substi(Stwo,Ttwo),Sone),Tone)


   // rule notChildVar('aexpQVar(Var:K),T)


    //T-Var
//    rule typeInferenceFun('funApp(Eone:K,, Etwo:K), Alpha:Map, Beta:Map, Delta:Map, Gamma:Map,.K,.K) =>
```

```
//         typeInferenceFun('funApp(Eone,, Etwo), Alpha, Beta, Delta, Gamma,typeInferenceFun(Eone,Alpha, Beta, Delta, Gamma,.K,.K),typeInferenceFun(Etwo,Alpha, Beta, Delta, Gamma,.K,.K))
//    rule typeInferenceFun('funApp(Eone:K,, Etwo:K), Alpha:Map, Beta:Map, Delta:Map, Gamma:Map, funtype(Tauone:K, Tautwo:K), Tauone) => Tautwo

    //T-Lam
//    rule typeInferenceFun('lambdaFun(Apatlist:K,, Ex:K), Alpha:Map, Beta:Map, Delta:Map, Gamma:Map,.K,.K) =>
//         typeInferenceFun('lambdaFun(Apatlist,, Ex), Alpha, Beta, Delta, Gamma,typeInferenceFun(Ex, Alpha, Beta, Delta, genGamma(Apatlist,Gamma),.K,.K),.K)

//    rule typeInferenceFun('lambdaFun(Apatlist:K,, Ex:K), Alpha:Map, Beta:Map, Delta:Map, Gamma:Map,Tautwo:K,.K) => Tautwo

endmodule
```

References