# HASKELL SYNTAX AND STATIC SEMANTICS WRITTEN IN K-FRAMEWORK

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BY

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#### THESIS

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## Abstract

For this thesis, I introduce a static semantics for Haskell by utilizing the K-Framework. This implementation includes support for the module system of Haskell but not for type classes. There are many layers that have to be implemented in K before type inference can be performed. The first part of the implementation is the entire context free syntax of Haskell in K. Since all the syntax is included, any program written in Haskell extended syntax can be parsed into an abstract syntax tree. However, this includes only the Haskell extended syntax but does not include the syntactic short-cuts such as treating tabs as syntactic sugar for grouping constructs such as curly braces. Programs that include multiple modules can be parsed, but the multiple modules must be written in a single file. This is unlike how the Glasgow Haskell Compiler allows for module imports, where each module must be kept in separate files. The multiple modules are then made as nodes in a directed acyclic graph. A directed edge in the graph represents a module importing another module. This graph is used for importing the user defined types from one module into another module. Context sensitive checks and type inference are then performed on modules. The static semantics specifies that, at each node in the graph, assuming all child modules are already checked and inferred, the user defined types of each of the child modules are imported into the module at the given node. All rules of the Haskell type system must take mutual recursion into account. There is repeated layering of inferences in Haskell. Due to being written in K, my semantics is mathematically precise and executable. Since the semantics is executable, the semantics can be tested against test sets to validate the correctness of the semantics. I utilized the executability of the semantics to test both positive inferences and exceptional inferences. This is part of a larger project to give a formal semantics to Haskell.

Subject Keywords: Haskell; Type-System

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To my parents, for their love and support.

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# Chapter 1

## Introduction

One of the inherent problems of engineered systems in general is that the design of the system is not proven to be working. The system could be nonfunctional at design time. The designer may not fully understand the system environment or may have not considered the behavior of the system in rare circumstances. Then once the design is made, there may be issues introduced by implementing the design incorrectly. Within the context of computer programs, the current way that programs are created is by making a design or a formal specification of the program, implementing it, and testing the program against unit tests or verifying the behavior of the program after the fact. Formal methods are ways to mathematically prove correctness of a system. Without formal methods, the only way to reason about a system is by testing it against different edge cases. Within the context of a programming language, one way a programming language can be formally specified is by defining a syntax and semantics for that language. The operational semantics of a programming language can be thought of as a transition system upon an abstract syntax tree, which is the program itself written in the language, and a state, which is a function from the variables in the tree to the current values of those variables. This way, real and complex programs written in natural looking programming languages can be interpreted as strings written in formal languages. Once a programming language is defined in this way, certain properties and behavior of the language and programs written in the language can be proven. K is a framework for creating the formal specification of a programming language. It then can interpret programs written in the language by running only the rules of the formal operational semantics of the programming language. This allows programs to be run and analyzed formally. This way the formal specification of the complex programming language can be tested and analyzed with the use of a machine. A K-configuration defines the memory structure of the programming language, made up of cells. The program state can be thought of as the current values of the K-configuration at a certain point in time. Grammar can be written in K using the constructor syntax, and a semantic rule can be written in K using the constructor rule. Haskell is a purely functional programming language with strong static typing. Purely functional means that the language only allows the user to make functions whose output is only dependent on the function input. Strong static typing means that before a program is run, a type inference algorithm infers the type of the program and ensures that all functions and function applications are allowed with regards to the types of the inputs and outputs. Static refers to the fact that type inference is performed before the code is run, and will not run during the runtime of the code. Strong typing refers to the fact that the compiler will not allow the user to perform workarounds like typecasting. This project details the syntax of Haskell and the type system of Haskell in K.

# Chapter 2

## Context Free Syntax

This chapter details the first part of the static semantics. In order for any context sensitive checks or type inference can be done, the test programs first need to be parsed into an abstract syntax tree. The context sensitive checks and type inference can then be performed upon the tree. Some difficulties with implementing a grammar into K is that the grammar originally is written in sort descending order in a document. The goal was to build a grammar that can parse actual programs and ensure there was no bugs. To do this, I started with small example programs, wrote out the example abstract syntax tree, and included the sorts necessary to parse them. Then if they didn't parse correctly I could then debug. I then wrote bigger and bigger example programs and included more and more sorts until all the grammar was included.

The Haskell 2010 report is the current official specification of the Haskell language. The grammar specified in section 10.5 of the Haskell 2010 report is a specification of the expanded syntax of Haskell. As specified in section 2.7, the expanded syntax of Haskell specifies Haskell programs when written using semicolons and braces. However, these can be omitted in a real Haskell program. The compiler will then utilize layout rules for certain grammar structures instead. These are specified in section 10.3. The parser for this project does not implement these layout rules and instead only can parse the expanded, layout insensitive syntax of Haskell. It would require another script to convert a program written using the layout sensitive syntax into the expanded syntax in order to parse the program. Section 10.1 specifies the notation used in the grammar. The notation of 10.1 are always in bold in the grammar. So an example production in the document looks like

```
qvarid -> [ modid . ] varid
This means that
```

 $<sup>1 \</sup>mod id$  .

is optional, and the brackets are not terminals, but the period is a terminal. Any symbol that is not in bold needs to be written in the program in order to parse correctly.

## 2.1 Syntax Explanation

There are a lot of parts of the grammar that proposed many challenges to implement in K. For instance, a sort definition that includes an option could be just written using a pipe in the K syntax.

So the example production

```
qvarid -> [ modid . ] varid
```

Is written in my K syntax as split into two options.

```
1 syntax QVarId ::= VarId | ModId "." VarId [klabel('qVarIdCon)]
```

However, an issue arises when you have something written on the right hand side of the production like

```
data [ context => ] simpletype [ = constrs ] [ deriving
]
```

for the topdecl sort. If each optional sort were split into two options, then a production that includes n optional sorts would require  $2^n$  options. This would create an unnecessarily large syntax in K.

Instead, for each optional sort in the original grammar, I replaced the optional sort with a new sort. For instance, I replaced

```
[ context => ]
```

in the original grammar with a new sort called OptContext. Then I just specified

and so the right hand side of the production would now look like

This is acceptable because Haskell is not an order sorted algebra, so introducing new sorts that are not originally in the grammar is okay.

In K, semantic rules are called K rules. The tag

```
1 [klabel('exampleLabel)]
```

means that in the abstract syntax tree created by K, a term can be referred to using that klabel in a K rule.

## 2.2 Implementation of Section 10.2

The following introduces the syntax for the keywords, constants, special symbols, and variables that comprise the terminals for the remaining context free grammar as presented in section 10.2 of the 2010 Haskell report.

```
1 // Syntax from haskell 2010 Report
  // https://www.haskell.org/onlinereport/haskell2010/haskellch10.
     html#x17-17500010
4
  module HASKELL-SYNTAX
5
6
     syntax Integer ::= Token\{([0-9]+)\}
7
               |(([0][0][0][0])[0-7]+)
8
               |(([0][x] | [0][X])[0-9a-fA-F]+)|
                                           [onlyLabel]
9
10
     syntax CusFloat ::= Token\{([0-9]+[\.][0-9]+([e\ E
        | | + - | ? [0-9] + ) ? |
11
                             |([0-9]+[e E][+-]?[0-9]+)|
                               onlyLabel]
12
     13
```

I ran into issue where a program with a variable called size did not parse. I found out that this is because size is a K keyword. So I just specified that a variable could be a variable token, or size. This reveals an issue in the version of K I was using, where K keywords that appear in example parsed programs won't become tokens.

```
3
      ([\! \# \$ \& \* \+ \/ \> \? \^][\! \# \$ \& \* \+ \.
         \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
4
     |[\-] | [\.]
5
     |([\.][\! \# \$ \& \* \+ \/ \< \= \> \? \@ \\ \^ \| \- \~
        \:][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~ \:]*)
     | ([\-][\! \# \$ \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~
6
        \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~
        \:]*)
7
     | ([\@][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~\:]+)
     | ([\~][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
8
        \~ \:]+)
9
     | ([\\][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~\:]+)
10
     | ([\|][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~ \:]+)
11
     | ([\:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~][\!\#\$\%\&\*\+\.\/\<\=\>\?\@\\\^\|\-
        \~\:]*)
12
     | ([\<][\! \# \$ \% \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~
        \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~ \:]*)
13
     | ([\=][\! \# \$ \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~
        \:][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~ \:]*)} [onlyLabel]
14
      \= \> \? \@ \\ \^ \| \- \~][\! \# \$ \& \* \+ \. \/ \<
           \= \> \? \@ \\ \^ \| \- \~ \:]*}
                                           [onlyLabel]
```

I ran into an issue where floats and integers did not parse correctly. They caused parsing errors due to ambiguity of parsing. For example the number 123.45 had ambiguity where the parser did not know if 1, 12, or 123 where integers, and if 5 was an integer, or if the entire thing was one float. Normally in K, different tokens are separated with whitespaces. However, for some reason the parser had difficulty here. Initially, I added a workaround by requiring parentheses around each integer and floating point. This fixed the issue.

```
3
                         | "(" CusFloat ")" [bracket]
1
       syntax Literal ::= IntFloat | CusChar | CusString
2
       syntax TyCon ::= ConId
3
       syntax ModId ::= ConId | ConId "." ModId [klabel('conModId)
          ]
       syntax QTyCon ::= TyCon | ModId "." TyCon [klabel('conTyCon)
          ]
       syntax QVarId ::= VarId | ModId "." VarId [klabel('qVarIdCon
5
          ) ]
       syntax QVarSym ::= VarSym | ModId "." VarSym [klabel('
6
          qVarSymCon)]
       syntax QConSym ::= ConSym | ModId "." ConSym [klabel('
7
          qConSymCon)]
        syntax QTyCls ::= QTyCon
       syntax TyCls ::= ConId
9
10 */
11
       syntax TyVars ::= List{TyVar, ""} [klabel('typeVars)] //used
           in SimpleType syntax
       syntax TyVar ::= VarId
12
13
       syntax TyVarTuple ::= TyVar "," TyVar [klabel('
          twoTypeVarTuple)]
14
                           | TyVar "," TyVarTuple [klabel('
                              typeVarTupleCon)]
15
16
       syntax Con ::= ConId | "(" ConSym ")" [klabel('
          conSymBracket)]
17
       syntax Var ::= VarId | "(" VarSym ")" [klabel('
          varSymBracket)]
18
       syntax QVar ::= QVarId | "(" QConSym ")" [klabel('
          qVarBracket)]
       syntax QCon ::= QTyCon | "(" GConSym ")" [klabel('
19
          gConBracket)]
20
       syntax QConOp ::= GConSym | "'" QTyCon "'"
21
                                                     [klabel('
          qTyConQuote)]
22
       syntax QVarOp ::= QVarSym | "'" QVarId "'"
                                                     [klabel('
          qVarIdQuote)]
       syntax VarOp ::= VarSym | "'" VarId "'"
23
                                                     [klabel('
          varIdQuote)]
24
       syntax ConOp ::= ConSym | "\" ConId "\"
                                                     [klabel('
          conIdQuote)]
25
```

```
26
       syntax GConSym ::= ":" | QConSym
27
       syntax Vars ::= Var
28
                      | Var ", " Vars [klabel('varCon)]
29
       syntax VarsType ::= Vars "::" Type [klabel('varAssign)]
30
       syntax Ops ::= Op
31
                     | Op ", " Ops [klabel('opCon)]
32
       syntax Fixity ::= "infixl" | "infixr" | "infix"
33
       syntax Op ::= VarOp | ConOp
34
       syntax CQName ::= Var | Con | QVar
35
36
       /* syntax QConId ::= ConId | ModId "." ConId */
37
38
       syntax QOp ::= QVarOp | QConOp
```

## 2.3 Implementation of Section 10.5

The following introduces the sorts of the context free grammar of the Haskell extended syntax.

#### 2.3.1 Modules

We start with modules.

```
1
      syntax ModuleName ::= "module" ModId [klabel('moduleName)]
3
      syntax Module ::= ModuleName "where" Body
                                                            [klabel('
          module)]
4
                       | ModuleName Exports "where" Body
                                                            [klabel('
                           moduleExp)]
                                                            [klabel('
5
                       | Body
                           moduleBody) ]
6
7
      syntax Body ::= "{" ImpDecls ";" TopDecls "}" [klabel('
          bodyimpandtop) ]
8
                     | "{" ImpDecls "}" [klabel('bodyimpdecls)]
9
                     | "{" TopDecls "}" [klabel('bodytopdecls)]
```

The sort that contains all the other sorts is a module. A module represents one complete Haskell program. It can have either a name and a body, a name and a body with exports, or just a body.

#### 2.3.2 ImpDecls

3 }

An ImpDecl is an import declaration. An import is another module that this module depends on. The definition of ImpDecl is the following

```
syntax ImpDecls ::= List{ImpDecl, ";"} [klabel('impDecls)]
 1
       syntax ImpDecl ::= "import" OptQualified ModId OptAsModId
2
           OptImpSpec [klabel('impDecl)]
                         "" [onlyLabel, klabel('emptyImpDecl)]
3
4
       syntax OptQualified ::= "qualified"
                               | "" [onlyLabel, klabel('
5
                                  emptyQualified)]
6
       syntax OptAsModId ::= "as" ModId
 7
                            | ""
                                     [onlyLabel, klabel('
                                emptyOptAsModId) ]
8
9
       syntax OptImpSpec ::= ImpSpec
10
                            | ""
                                     [onlyLabel, klabel('
                                emptyOptImpSpec) ]
11
12
       syntax ImpSpecKey ::= "(" ImportList OptComma ")"
13
       syntax ImpSpec ::= ImpSpecKey
14
                         | "hiding" ImpSpecKey
15
16
       syntax ImportList ::= List{Import, ","}
17
18
       syntax Import ::= Var
19
                        | TyCon CQList
```

The following example program has a Module with a name and a body of only ImpDecls. It has one ImpDecl.

```
1 module Foo where
2 {import Bar
3 }
```

The following example program has a Module with no name and and a body of only ImpDecls. It has one ImpDecl.

```
1 import Bar

Another example program is
1 module Simp1 where
2 {import Test
```

The corresponding abstract syntax tree in K is

Module contains two children. The first child of module is moduleName which contains the token Simp1. Simp1 is a constructor ID because it starts with a capital letter. The second child of module is bodyimpdecls. This contains the sort impDecls which is a list of impDecls. This program has only one impDecl. Since there is no qualified, the impDecl contains the child emptyQualified, followed by the token Test. Since there is no AsModId or ImpSpec, the last two children are emptyOptAsModId and emptyOptImpSpec.

## 2.3.3 TopDecls

The main types of expressions in Haskell are TopDecls - Top Declarations. A top declaration can either be a type, a data, a newtype, a class, an instance, a default, a foreign, or an arbitrary declaration.

Any sort that starts with 'Opt' means that this is optional. In K something can be made optional by declaring the necessary constructors or nothing.

```
syntax TopDecls ::= List{TopDecl, ";"} [klabel('topdeclslist
1
           ) ]
 2
 3
       syntax TopDecl ::= Decl [klabel('topdecldecl)]
                         > "type" SimpleType "=" Type [klabel('type)
 4
                             1
5
                         | "data" OptContext SimpleType OptConstrs
                            OptDeriving [klabel('data)]
                         | "newtype" OptContext SimpleType "="
 6
                            NewConstr OptDeriving [klabel('newtype)]
                         | "class" OptContext ConId TyVar OptCDecls
 7
                             [klabel('class)]
 8
                         | "instance" OptContext QTyCon Inst
                            OptIDecls [klabel('instance)]
9
                         | "default" Types [klabel('default)]
10
                         | "foreign" FDecl [klabel('foreign)]
```

#### 2.3.4 Decls

A Decl is any general declaration. So something like

```
1 \quad \text{f } x = x + 2
```

is a Decl.

The following is the definition of Decl and related sorts.

This section introduces the sorts whose parent is Decl. This contains general declarations, function left hand side and right hand side, function guards, and expressions.

```
1
       syntax GenDecl ::= VarsType
2
                         | Vars "::" Context "=>" Type [klabel('
                             genAssignContext)]
 3
                         | Fixity Ops
4
                         | Fixity Integer Ops
                         | "" [onlyLabel, klabel('emptyGenDecl)]
5
6
7
       syntax FunLhs ::= Var APatList [klabel('varApatList)]
8
                        | Pat VarOp Pat [klabel('patVarOpPat)]
9
                        | "(" FunLhs ")" APatList [klabel('
                            funlhsApatList)]
10
11
       syntax Rhs ::= "=" Exp OptDecls [klabel('eqExpOptDecls)]
12
                     | GdRhs OptDecls [klabel('gdRhsOptDecls)]
13
14
       syntax GdRhs ::= Guards "=" Exp
15
                       | Guards "=" Exp GdRhs
       syntax Guards ::= "|" GuardList
16
17
       syntax GuardList ::= Guard | Guard "," GuardList [klabel('
           guardListCon) ]
18
       syntax Guard ::= Pat "<-" InfixExp</pre>
19
                       | "let" Decls
20
                       | InfixExp
```

```
21
22
       //definition of exp
23
       syntax Exp ::= InfixExp
24
                     > InfixExp "::" Type [klabel('expAssign)]
25
                     | InfixExp "::" Context "=>" Type [klabel('
                        expAssignContext) ]
26
27
       syntax InfixExp ::= LExp
28
                          > "-" InfixExp
                                            [klabel('minusInfix)]
29
                          > LExp QOp InfixExp
```

#### 2.3.5 LExp

LExp is an important sort for the type inference function. This is because LExp defines the different expression types which the type inference function has specific rules for.

The different LExp types are a lambda expression, a 'let-in' expression, an 'if' statement, a case statement, and a 'do' block.

```
syntax LExp ::= AExp

"\\" APatList "->" Exp [klabel('lambdaFun)]

"let" Decls "in" Exp [klabel('letIn)]

"if" Exp OptSemicolon "then" Exp
OptSemicolon "else" Exp [klabel('ifThenElse)]

"case" Exp "of" "{" Alts "}" [klabel('caseOf)]

"do" "{" Stmts "}" [klabel('doBlock)]
```

## 2.3.6 AExp

AExp is also an import sort for the type inference function. The main parts of AExp that the inference function cares about is QVar and GCon.

QVar is a qualified variable and GCon is a general constructor.

```
4
5
       syntax AExp ::= QVar [klabel('aexpQVar)]
6
                     | GCon [klabel('aexpGCon)]
7
                      | Literal [klabel('aexpLiteral)]
                      > AExp AExp [left, klabel('funApp)]
9
                      > QCon "{" FBindList "}"
10
                      | AExp "{" FBindList "}" //aexp cannot be qcon
                          UNFINISHED
11
                              //Liyi: first, does not understand the
                                   syntax, it is the Qcon {FBindlist}
12
                              //or QCon? Second, place a check in
                                 preprosssing.
13
                              //and also check the Fbindlist here
                                 must be at least one argument
14
                      > "(" Exp ")"
                                                 [bracket]
                      | "(" ExpTuple ")"
15
16
                      | "[" ExpList "]"
17
                      | "[" Exp OptExpComma ".." OptExp "]"
                      | "[" Exp "|" Quals "]"
18
                      | "(" InfixExp QOp ")"
19
20
                      | "(" QOp InfixExp ")" //qop cannot be - (
                         minus) UNFINISHED
21
                               //Liyi: place a check here to check
                                   if QOp is a minus
```

## 2.4 Example Test Programs

# Chapter 3

# Configuration

K is used for defining a state machine and the K rules define the transition rules for the state machine. The configuration of the state machine is made up of K cells. The K cells contain the syntax data structure representing the code of the example program. They also contain the memory of the state machine. An actual state of the state machine in K is when the cells each have some term inside of them.

The following is the configuration of my Haskell semantics.

```
In the description below I give incrementally the various
   requires "haskell-syntax.k"
                                         components that make up the configuration used in the
                                         Haskell statics semantics given in this paper, along with
 2
                                         a brief description of what they mane and examples of
   module HASKELL-CONFIGURATION
                                         what they hold and how they are used..
 4
        imports HASKELL-SYNTAX
 5
 6
        syntax KItem ::= "startImportRecursion"
 7
        syntax KItem ::= callInit(K)
 8
       -//syntax KItem ::- initPreModule(K) [function]
 9
        //syntax KItem ::- tChecker(K) [function]
10
11
        configuration
12
             <T>
13
                 <k> $PGM:ModuleList ~> startImportRecursion </k>
14
                 <tempModule> .K </tempModule>
15
                 <tempCode> .K </tempCode>
```

The  $\langle k \rangle$  cell is the cell that computation takes place in. The abstract syntax tree is initially placed into the  $\langle k \rangle$  cell. The command

1 \$PGM:ModuleList

means that the parsed tree appears in this cell and the sort that contains all other sorts is ModuleList.

Putting K means that the cell is initially empty.

tempModule is the name of the current module. tempCode is the current

#### code.

typeIterator is used for creating a fresh type variable for the inference algorithm. It has the current count of how many fresh type variables that were created.

```
1 <typeIterator> 1 </typeIterator>
```

## 3.1 Alpha

Alpha is a map of type renamings. So if a user declares

```
1 data MyBool = TTrue
2 ;type MyBooltwo = MyBool
```

Then MyBooltwo is a renaming of MyBool. In tempAlpha, an AObject is made. An AObject is a KItem with two children. One can be thought of as a Key and the other is the Value for a map. So MyBool -> MyBooltwo. However, we want to check and reject programs that have multiple renamings, so we cannot use a K Map which has idempotence. However, once we make this check, we can then use a K Map. This is what tempAlphaMap is.

Putting .Map means that the cell starts with an empty map.

## 3.2 Beta

tempT contains all user defined datatypes. tempT is organized in such a way that makes context sensitive checks easy to perform. tempBeta contains all user defined datatypes organized so that type inference is easy to perform. More is explained in chapter 5.

#### 3.2.1 Example

If the user makes the data type CusBool in module Simp5, and declares it with this example...:

```
1 data CusBool = True2 | False2
```

t Then the corresponding tempBeta should look and TyVars monomorphic datatype just has an empty forAll.

as below. like this. Note how the For more on monomorphic and polymorphic types, please see Section 5

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
1 data CusBool a b = True2 a | False2 b
```

Then the corresponding tempBeta should look like this.

## 3.3 Delta

tempDelta contains the arity of the user defined dataTypes. So if a user-a mapping from the dataype name to defined datatype takes in two parameters, tempDelta will contain the number

2. Again, it is initially empty.

```
1 <tempDelta> .Map </tempDelta>
```

### 3.3.1 Example

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
l data CusBool a b = True2 a | False2 b
```

Then the corresponding tempDelta should look like this.

```
1 <tempDelta>
2          ModPlusType ( Simp5 , CusBool ) |-> 2
3 </tempDelta>
```

## 3.4 Import Data Structure

importTree, recurImportTree, and impTreeVMap contain the data necessary for the directed acyclic graph representing imports.

More is explained in ...

#### 3.5 Modules

The modules cell contains all modules that were checked and inferred already. Multiplicity means that there can be multiple module cells.

```
1
                <modules> //static information about a module
                    <module multiplicity="*">
2
3
                        <moduleName> .K </moduleName>
                        <moduleAlphaStar> .K </moduleAlphaStar>
4
                        <moduleBetaStar> .K </moduleBetaStar>
5
6
                        <moduleImpAlphas> .List </moduleImpAlphas>
                        <moduleImpBetas> .List </moduleImpBetas>
 7
8
                        <moduleCompCode> .K </moduleCompCode>
9
                        <moduleTempCode> .K </moduleTempCode>
10
                        <imports> .Set </imports>
                        <classes> //static information about a
11
                            module
12
                            <class multiplicity="*">
13
                                 <className> .K </className>
14
                            </class>
15
                        </classes>
16
                    </module>
17
                </modules>
```

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18		
19		
20	endmodule	

A sentence letting me know that the components here are the module specific versions of the ones just discussed.

# Chapter 4

## Context Sensitive Checks

I also placed the user defined types into data structures in order to perform several checks to make sure that the user did not have errors when creating types. Then the data structures will be transformed into a form that will be used for type inferencing. Section 4 of the Haskell 2010 report specifies the haskell type system. In the TopDecl sort, there are three type constructors that are used to create user defined datatypes. These are data, type, and newtype.

## 4.1 Datatype Constructors

```
syntax TopDecl ::= Decl [klabel('topdecldecl)]

'type" SimpleType "=" Type [klabel('type)]

"data" OptContext SimpleType OptConstrs
OptDeriving [klabel('data)]

"newtype" OptContext SimpleType "="
NewConstr OptDeriving [klabel('newtype)]
```

The end goal is to put the user defined types into a data structure which I can use to perform type inferencing. These three typecons are used to create user defined types.

## 4.1.1 Type

The first one is type,

```
1 type simpletype = type
```

This is used in a haskell program to declare a new type as a single type. In effect, it renames the type where both names now can be used to refer to the type.

1 type Username = String

Is one such example usage of type, it creates a new type Username, which is defined as just a string. Now the programmer can refer to Username or String to make a string.

#### 4.1.2 Data

The second one is data,

```
1 data [context =>] simpletype [= constrs] [deriving]
```

This allows a user to declare a new type that may include many fields and polymorphic types.

For instance:

```
1 data Date = Date Int Int Int
```

This is a new type that includes the type constructor *Date* followed by three integers.

```
1 data Poly a = Number a
```

This is a new polymorphic type with polymorphic parameter a, that has the type constructor Number.

### 4.1.3 Newtype

The third one is newtype,

```
1 newtype [context =>] simpletype = newconstr [deriving]
```

This is very similar to data except it only parses when the newtype has only one typecon and one field.

#### 4.1.4 Context

Another thing to note is that the

```
1 \quad [context =>]
```

part of the syntax for the types is deprecated. https://stackoverflow.com/questions/9345589/guardsvs-if-then-else-vs-cases-in-haskell

#### 4.2 Initial Data Structures

I implemented a map, called alpha, of new type names as the keys and their declared types as the entries. I then collected all appearances of the typecon type in the program, and put simpletype -> type in the alpha map. However, one of the things I needed to check for in the program was whether a user declared multiple definitions with type, so I could not use a map in K because they only allow unique keys with unique entries. So I initially used a set of tuples, and then changed it to a map after checking for multiple type declarations.

The second data structure I made is called T. T holds the user defined types created using data and newtype.

```
1 syntax KItem ::= TList(K) //list of T objects for every new type
    introduced by data and newtype
2 syntax KItem ::= TObject(K,K,K,K) //(module name, type name,
    entire list of poly type vars, list of inner T pieces)
3 syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type constructor,
    poly type vars, argument sorts, entire constr block, type
    name)
```

T is a list of TObjects, each TObject represents a single user defined datatype. It holds the name, the list of polymorphic parameters, and a list of inner T pieces. An inner T piece represents an option of what a type could be. It consists of a type constructor, a list of polymorphic parameters required for this option, the fields for this option, the entire subtree of the AST for this option unedited, and the type name again. I then used these data structures to perform these checks, and afterwards will transform them into a new data structure to perform type inferencing.

## 4.2.1 Example T

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
1 data CusBool = True2 | False2
```

Then the corresponding tempT should look like this.

#### 4.2.2 K Code

The following parses the tree and searches for the user defined types to place into the data structures.

```
//get alpha and beta
1
2
       syntax KItem ::= Module(K, K)
3
       syntax KItem ::= preModule(K,K) //(alpha, T)
4
       // STEP 1 CONSTRUCT T AND ALPHA
5
6
       // alpha = type
7
       // T = newtype and data, temporary data structure
8
9
       syntax KItem ::= initPreModule(K) [function]
10
       syntax KItem ::= getPreModule(K, K) [function] //(Current
           term, premodule)
11
       syntax KItem ::= makeT (K,K,K,K)
12
13
       syntax KItem ::= fetchTypes (K,K,K,K)
14
15
       syntax List ::= makeInnerT (K,K,K) [function] //LIST
16
       syntax List ::= getTypeVars(K) [function] //LIST
17
18
       syntax KItem ::= getCon(K) [function]
19
       syntax List ::= getArgSorts(K) [function] //LIST
20
21
       syntax KItem ::= AList(K)
22
       syntax KItem ::= AObject(K,K) //(1st -> 2nd) map without
           idempotency
23
       syntax KItem ::= ModPlusType(K,K)
24
       syntax KItem ::= TList(K) //list of T objects for every new
25
           type introduced by data and newtype
```

```
26
       syntax KItem ::= TObject(K,K,K,K) //(module name, type name,
            entire list of poly type vars, list of inner T pieces)
27
       syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type constructor,
            poly type vars, argument sorts, entire constr block,
           type name)
28
29
       rule initPreModule(J:K) => getPreModule(J,preModule(AList(.
           List), TList(.List)))
30
31
       rule getPreModule('bodytopdecls(I:K), J:K) => getPreModule(I
32
       rule getPreModule('topdeclslist('type(A:K,, B:K),, Rest:K),J
           :K) => fetchTypes(A,B,Rest,J) //constructalpha
33
34
35
       rule getPreModule ('topdeclslist ('data (A:K,, B:K,, C:K,, D:K)
           ,, Rest:K), J:K) => makeT(B, C, Rest, J)
36
       rule getPreModule ('topdeclslist ('newtype (A:K,, B:K,, C:K,, D
           :K),, Rest:K),J:K) => makeT(B,C,Rest,J)
37
38
39
       rule getPreModule('topdeclslist('topdecldecl(A:K),, Rest:K),
           J:K) => getPreModule(Rest, J)
40
       rule getPreModule ('topdeclslist ('class (A:K,, B:K,, C:K,, D:K
           ),, Rest:K),J:K) => getPreModule(Rest,J)
41
       rule getPreModule('topdeclslist('instance(A:K,, B:K,, C:K,,
           D:K),, Rest:K),J:K) => getPreModule(Rest,J)
42
       rule getPreModule ('topdeclslist ('default (A:K,, B:K,, C:K,, D
           :K),, Rest:K),J:K) => getPreModule(Rest,J)
43
       rule getPreModule('topdeclslist('foreign(A:K,, B:K,, C:K,, D
           :K),, Rest:K),J:K) => getPreModule(Rest,J)
44
       rule getPreModule(.TopDecls, J:K) => J
45
46
       rule <k> fetchTypes('simpleTypeCon(I:TyCon,, H:TyVars), '
           atypeGTyCon(C:K), Rest:K, preModule(AList(M:List), L:K))
           => getPreModule(Rest, preModule(AList(ListItem(AObject(
          ModPlusType(ModName, I), C)) M), L)) ...</k>
47
             <tempModule> ModName:KItem </tempModule>
48
49
       rule <k> makeT('simpleTypeCon(I:TyCon,, H:TyVars), D:K, Rest
           :K, preModule(AList(M:List), TList(ListInside:List))) =>
           getPreModule(Rest, preModule(AList(M), TList(ListItem(
           TObject(ModName, I, H, makeInnerT(I, H, D))) ListInside)))
```

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```
...</k>
50
            <tempModule> ModName: KItem </tempModule>
51
52
       rule makeInnerT(A:K,B:K,'nonemptyConstrs(C:K)) => makeInnerT
          (A,B,C)
53
       rule makeInnerT(A:K,B:K,'singleConstr(C:K)) => ListItem(
          InnerTPiece(getCon(C), getTypeVars(C), getArgSorts(C), C, A))
       rule makeInnerT(A:K,B:K,'multConstr(C:K,, D:K)) => ListItem(
54
          InnerTPiece(getCon(C), getTypeVars(C), getArgSorts(C), C, A))
           makeInnerT(A,B,D)
55
56
       rule getTypeVars('constrCon(A:K,, B:K)) => getTypeVars(B)
57
       rule getTypeVars('optBangATypes(A:K,, Rest:K)) =>
          getTypeVars(A) getTypeVars(Rest)
58
       rule getTypeVars('optBangAType('emptyBang(.KList),, Rest:K))
           => getTypeVars(Rest)
59
       rule getTypeVars('atypeGTyCon(A:K)) => .List
60
       rule getTypeVars('atypeTyVar(A:K)) => ListItem(A)
       rule getTypeVars(.OptBangATypes) => .List
61
62
63
       rule getCon('constrCon(A:K,, B:K)) => A
64
65
       rule getArgSorts('constrCon(A:K,, B:K)) => getArgSorts(B)
       rule getArgSorts('optBangATypes(A:K,, Rest:K)) =>
66
          getArgSorts(A) getArgSorts(Rest)
67
       rule getArgSorts('optBangAType('emptyBang(.KList),, Rest:K))
           => getArgSorts(Rest)
68
       rule getArgSorts('atypeGTyCon(A:K)) => ListItem(A)
       rule getArgSorts('atypeTyVar(A:K)) => .List
69
70
       rule getArgSorts(.OptBangATypes) => .List
71
73
74
       rule <k> preModule(A:K,T:K) => startTTransform ...</k>
75
            <tempAlpha> OldAlpha:K => A </tempAlpha>
76
            <tempT> OldT:K => T </tempT>
```

#### 4.3 Context Sensitive Checks

For my context sensitive checks, I perform several checks that are not inclusive to all context sensitive checks that need to be made in a complete Haskell semantics, but enough for the sanity of the type inference function.

#### K Code

The following code introduces all checks that need to be made.

```
// STEP 2 PERFORM CHECKS
1
 2
 3
       syntax KItem ::= "error"
4
       syntax KItem ::= "startChecks"
5
6
       syntax KItem ::= "checkNoSameKey"
 7
           //Keys of alpha and keys of T should be unique
8
       syntax KItem ::= "checkTypeConsDontCollide"
9
           //Make sure typeconstructors do not collide in T
10
       syntax KItem ::= "makeAlphaMap"
11
           //make map for alpha
12
       syntax KItem ::= "checkAlphaNoLoops"
13
            //alpha check for no loops
14
           //check alpha to make sure that everything points to a T
15
       syntax KItem ::= "checkArgSortsAreTargets"
16
              //Make sure argument sorts [U] [W,V] are in the set
                  of keys of alpha and targets of T, (keys of T)
17
       syntax KItem ::= "checkParUsed"
18 //NEED TO CHECK all the polymorphic parameters from right appear
       on left. RIGHT SIDE ONLY
   //NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY
19
20
21
       rule <k> startChecks
22
                 => checkNoSameKey
23
                    ~> (checkTypeConsDontCollide
24
                    ~> (makeAlphaMap
25
                    ~> (checkAlphaNoLoops
26
                    ~> (checkArgSortsAreTargets
27
                    ~> (checkParUsed))))) ...</k>
```

#### 4.3.1 Name Collision

The programmer should not be able to make two user defined datatypes with the same name, even if one is created using the constructor *data* and another is created using the constructor *type* for instance.

The following example should not be allowed.

```
1 data Date = Date Int
2 ;type Date = Contwo Int
```

#### K Code

The following is the code for the check.

```
1
       rule <k> checkTypeConsDontCollide
2
                 => tyConCollCheck(T,.List,.Set) ...</k>
             <tempT> T:K </tempT>
3
 4
       syntax KItem ::= tyConCollCheck(K,K,K) [function] //(TList,
5
           List of Tycons, Set of Tycons)
       syntax KItem ::= lengthCheck(K,K) [function]
6
 7
       rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:
           K, ListItem(InnerTPiece(Ty:K, E:K, F:K, H:K, G:K)) Inners:List
           )) Rest:List),J:List,D:Set) =>
9
                        tyConCollCheck(TList(ListItem(TObject(
                            ModName, A, B, Inners)) Rest), ListItem (Ty) J
                            , SetItem(Ty) D)
10
       rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:
           K,.List)) Rest:List), J:List, D:Set) =>
11
                        tyConCollCheck(TList(Rest),J,D)
12
       rule tyConCollCheck(TList(.List), J:List, D:Set) =>
13
                        lengthCheck(size(J), size(D))
14
15
       rule lengthCheck(A:Int, B:Int) => .K
16
                        requires A ==Int B
17
18
       rule lengthCheck(A:Int, B:Int) => error
19
                        requires A =/=Int B
```

#### 4.3.2 Type Constructor Collision

The programmer should not be able to use the same type constructors when making different options for their types or use the same type constructors for different types.

The following example should not be allowed.

```
1 data Date = Date Int | Date Bool
```

The following example should also not be allowed.

```
1 data Date = Date Int
2 ;data Datetwo = Date Int
```

#### K Code

The following is the code for the check.

```
1
       syntax KItem ::= keyCheck(K,K,K,K) [function] //(Alpha, T,
           List of names, Set of names)
2
3
       rule <k> checkNoSameKey
4
                 => keyCheck(A, T, .Set, .List) ...</k>
5
             <tempAlpha> A:K </tempAlpha>
6
            <tempT> T:K </tempT>
 7
8
       rule keyCheck(AList(ListItem(AObject(A:K,B:K)) C:List), T:K,
            D:Set, G:List) => keyCheck(AList(C), T, SetItem(A) D,
           ListItem(A) G)
9
       rule keyCheck(AList(.List), TList(ListItem(TObject(ModName:K
           , A:K,B:K,C:K)) Rest:List), D:Set, G:List) => keyCheck(
           AList(.List), TList(Rest), SetItem(A) D, ListItem(A) G)
10
       rule keyCheck(AList(.List), TList(.List), D:Set, G:List) =>
           lengthCheck(size(G), size(D))
11
12
13
       syntax KItem ::= makeAlphaM(K,K) [function] //(Alpha,
           AlphaMap)
14
       syntax KItem ::= tAlphaMap(K) //(AlphaMap) temp alphamap
15
16
       rule <k> makeAlphaMap
17
                 => makeAlphaM(A, .Map) ...</k>
18
            <tempAlpha> A:K </tempAlpha>
```

#### 4.3.3 Alpha Cycle Check

There should be no cycles in type renaming using type, and the type renaming chains using type should terminate with a type defined with data or newtype.

The following example should not be allowed.

```
1 type Birthday = Date
2 ;type Date = Birthday
```

The following example should also not be allowed if Date is not defined anywhere.

```
1 type Birthday = Date
```

#### K Code

The following is the code for the check.

```
syntax KItem ::= aloopCheck(K,K,K,K,K,K,K) [function] //(
1
           Alpha, List of Alpha, Set of Alpha, CurrNode, lengthcheck, T,
           BigSet)
2
       rule <k> checkAlphaNoLoops
3
4
                 => aloopCheck(A,.List,.Set,.K,T,.Set) ...</k>
5
            <tempAlphaMap> A:K </tempAlphaMap>
6
            <tempT> T:K </tempT>
 7
8
       //aloopCheck set and list to check cycles
9
       rule aloopCheck(Alpha:Map (A:KItem | -> B:KItem), D:List, G:
           Set, .K, .K, T:K, S:Set) => aloopCheck(Alpha, ListItem(B)
           ListItem(A) D, SetItem(B) SetItem(A) G, B, .K, T, S)
10
       rule aloopCheck(Alpha:Map (H |-> B:KItem), D:List, G:Set, H:
           KItem, .K,T:K,S:Set) => aloopCheck(Alpha, ListItem(B) D,
           SetItem(B) G, B, K, T, S)
```

```
11
12
       rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
           Set) => aloopCheck(Alpha, .List, .Set, .K, lengthCheck(
           size(G), size(D)), T, G S) //type rename loop ERROR
13
            requires (notBool H in keys(Alpha)) andBool (H in
                typeSet(T, .Set) orBool H in S)
14
       rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
15
           Set) => error //terminal alpha rename is not in T ERROR
16
            requires (notBool H in keys(Alpha)) andBool (notBool (H
                 in typeSet(T, .Set) orBool H in S))
17
18
19
       syntax Set ::= typeSet(K,K) [function] //(K, KSet)
20
       rule typeSet(TList(ListItem(TObject(ModName:K, A:K,B:K,C:K))
            Rest:List), D:Set) => typeSet(TList(Rest), SetItem(A) D)
21
       rule typeSet(TList(.List), D:Set) => D
22
23
       rule aloopCheck(.Map, .List, .Set, .K, .K,T:K, S:Set) => .K
```

## 4.3.4 Argument Sort Check

The argument sorts for types defined using the *data* keyword or the *newtype* keyword should be types that exist.

The following example should not be allowed if *Date* is not defined anywhere.

```
1 Data Birthday = Birthday Date
```

#### K Code

The following is the code for the check.

```
6
                 => argSortCheck(T,A,typeSet(T,.Set)) ...</k>
7
             <tempAlphaMap> A:K </tempAlphaMap>
8
             <tempT> T:K </tempT>
9
10
       rule argSortCheck(TList(ListItem(TObject(ModName:K, A:K,B:K,
           ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest
           :List, E:K, F:K)) InnerRest:List)) TListRest:List), AlphaMap
           :Map, Tset:Set) => argSortCheck(TList(ListItem(TObject(
           ModName, A, B, ListItem(InnerTPiece(C, D, ArgsRest, E, F))
           InnerRest)) TListRest), AlphaMap, Tset)
11
             requires ((Arg in keys(AlphaMap)) orBool (Arg in Tset))
12
13
       rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem (InnerTPiece (C:K, D:K, ListItem (Arg:KItem) ArgsRest
           :List, E:K, F:K)) InnerRest:List)) TListRest:List), AlphaMap
           :Map, Tset:Set) => error
14
             requires (notBool ((Arg in keys(AlphaMap)) orBool (Arg
                in Tset)))
15
16
       rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(C:K,D:K,.List,E:K,F:K)) InnerRest:
           List)) TListRest:List), AlphaMap:Map, Tset:Set) =>
           argSortCheck (TList(ListItem(TObject(ModName, A, B, InnerRest
           )) TListRest), AlphaMap, Tset)
17
       rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.
18
           List)) TListRest:List), AlphaMap:Map, Tset:Set) =>
           argSortCheck(TList(TListRest), AlphaMap, Tset)
19
20
       rule argSortCheck(TList(.List),AlphaMap:Map,Tset:Set) => .K
```

## 4.3.5 Polymorphic Parameter Check

The polymorphic parameters that appear on the right hand side of a *data* declaration need to appear on the left hand side as well.

The following example should not be allowed.

```
1 Data Newtype = New a b
```

Also, The polymorphic parameters that appear on the left hand side of a data declaration need to be unique. However, the parameters that appear

on the right hand side do not need to be unique.

The following example should not be allowed.

```
1 Data Newtype a a = New a
```

However, the following example should be allowed.

```
1 Data Newtype a = New a a
```

#### K Code

The following is the code for the check.

```
1 //NEED TO CHECK all the polymorphic parameters from right appear
        on left. RIGHT SIDE ONLY
2 //NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY
3
4
       syntax KItem ::= parCheck(K,K) [function] //(T,AlphaMap)
       syntax KItem ::= makeTyVarList(K,K,K) [function] //(TyVars,
5
           NewList)
6
       syntax KItem ::= lengthRet(K,K,K) [function]
7
8
       rule <k> checkParUsed
9
                 => parCheck(T,.K) ...</k>
10
             <tempT> T:K </tempT>
11
12
       rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,C:K))
            Rest:List),.K) => parCheck(TList(ListItem(TObject(
           ModName, A:K, B:K, C:K)) Rest:List), makeTyVarList(B, .List, .
           Set))
13
14
       rule parCheck (TList (ListItem (TObject (ModName: K, A: K, B: K,
           ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List
           ,D:K,E:K,F:K)) InnerRest:List)) Rest:List), NewSet:Set) =>
15
            parCheck (TList (ListItem (TObject (ModName, A, B, ListItem (
                InnerTPiece(C,ParRest,D,E,F)) InnerRest)) Rest),
                NewSet)
16
                requires Par in NewSet
17
18
       rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List
           ,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
            error
```

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```
19
                requires notBool (Par in NewSet)
20
21
       rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(C:K,.List,D:K,E:K,F:K)) InnerRest:
           List)) Rest:List), NewSet:Set) =>
22
            parCheck(TList(ListItem(TObject(ModName, A, B, InnerRest))
                 Rest), NewSet)
23
24
       rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.List
           )) Rest:List), NewSet:Set) =>
25
            parCheck(TList(Rest), NewSet)
26
27
       rule parCheck(TList(.List), NewSet:Set) => .K
28
29
       rule makeTyVarList('typeVars(A:K,,Rest:K),NewList:List,
           NewSet:Set) => makeTyVarList(Rest, ListItem(A) NewList,
           SetItem(A) NewSet)
30
       rule makeTyVarList(.TyVars, NewList:List, NewSet:Set) =>
31
           lengthRet(size(NewList), size(NewSet), NewSet)
32
33
       rule lengthRet(A:Int, B:Int, C:K) => C
34
                        requires A ==Int B
35
36
       rule lengthRet(A:Int, B:Int, C:K) => error
37
                        requires A =/=Int B
```

# Chapter 5

# Inferencing

Haskell's type system is a Hindley-Milner polymorphic type system that has been extended with type classes to account for overloaded functions.

[haskell 2010 report]

A type system is a set of rules that assign a property to various constructs in a programming language called type. A type is a property that allows the programmer to add constraints to programs.

https://en.wikipedia.org/wiki/Type\_system

The type inference function infers the type of the expression that is fed into it. A purpose of it is to ensure that all functions and function applications are allowed with regards to the types of the inputs and outputs.

## 5.1 Type theory

Type theory was created by Bertrand Russell to prevent Russell's Paradox for set theory, introduced by Georg Cantor. The issue was that not specifying a certain property for sets allowed sets to contain themselves in Naive Set Theory. So Bertrand Russell prevented this problem by specifying a property called type for objects, and objects cannot contain their own type.

https://plato.stanford.edu/entries/type-theory/

## 5.2 Data Structures

I needed to create a syntax for polymorphic types that may contain monomorphic type variables and polymorphic type variables.

Then I made a map from type constructor names to arities, called Delta.

Then I made a map from data constructors and term identifiers to their most general polymorphic types, called beta. The first part was converting the T data structure into beta, which is more suited for type inference.

#### 5.2.1 Transform T into Beta

```
1
       // STEP 3 Transform T into beta
 2
 3
       syntax KItem ::= "startTTransform"
4
       syntax KItem ::= "constructDelta"
5
       syntax KItem ::= "constructBeta"
7
       rule <k> startTTransform
8
                 => constructDelta
9
                    ~> (constructBeta) ...</k>
10
11
       rule <k> constructDelta
12
                 => makeDelta(T,.Map) ...</k>
13
             <tempT> T:K </tempT>
14
15
       syntax KItem ::= makeDelta(K, Map) [function] //(T, Delta)
16
       syntax KItem ::= newDelta(Map) //Delta
17
       syntax KItem ::= newBeta(Map) //beta
18
       syntax List ::= retPolyList(K,List) [function] //(T,Delta)
```

#### 5.2.2 Construct Delta

Delta contains the arity of the user defined dataTypes. So if a user defined datatype takes in two parameters, tempDelta will contain the number 2.

#### Example

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
1 data CusBool a b = True2 a | False2 b
```

Then the corresponding *tempDelta* should look like this.

```
1
       rule makeDelta(TList(ListItem(TObject(ModName:K,A:K,Polys:K,
           C:K)) Rest:List),M:Map) =>
2
             makeDelta(TList(Rest), M[ModPlusType(ModName, A) <- size(</pre>
                retPolyList(Polys,.List))])
3
       rule makeDelta(TList(.List), M:Map) => newDelta(M)
4
5
       rule retPolyList('typeVars(A:K,,Rest:K),NewList:List) =>
           retPolyList(Rest, ListItem(A) NewList)
       rule retPolyList(.TyVars,L:List) => L
6
7
8
       rule <k> newDelta(M:Map)
9
                 => .K ...</k>
10
             <tempDelta> OldDelta:K => M </tempDelta>
```

#### 5.2.3 Construct Beta

Beta is a map from the type constructor to its corresponding type.

A monomorphic data type looks like  $(a \to b) \to c$  In my K semantics, the equivalent data type looks like

```
1 forAll ( .Set , funtype ( funtype ( a , b ) , c ) )
```

A polymorphic data type looks like  $\forall abc . (a \rightarrow b) \rightarrow c$  In my K semantics, the equivalent data type looks like

```
1 forAll ( a b c , funtype ( funtype ( a , b ) , c ) )
```

#### Example

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
1 data CusBool = True2 | False2
```

Then the corresponding tempBeta should look like this. Note how the monomorphic datatype just has an empty forAll.

```
4 </tempBeta>
```

If the user makes the data type CusBool in module Simp5, and declares it with this example...

```
1 data CusBool a b = True2 a | False2 b
```

Then the corresponding tempBeta should look like this.

#### K Code

The following is the K Code.

```
1
       rule <k> constructBeta
2
                 => makeBeta(T,.Map) ...</k>
3
             <tempT> T:K </tempT>
4
 5
       syntax KItem ::= makeBeta(K,Map) [function] //(T,Beta,Delta)
6
       rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(Con:K,H:K,D:K,E:K,F:K)) InnerRest:
           List)) Rest:List), Beta:Map) =>
8
             makeBeta(TList(ListItem(TObject(ModName, A, B, InnerRest))
                 Rest), Beta[ModPlusType(ModName, Con) <- betaParser(E</pre>
                ,B,A)])
9
       rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,.List
           )) Rest:List),Beta:Map) =>
10
             makeBeta(TList(Rest), Beta)
11
       rule makeBeta(TList(.List), Beta:Map) =>
12
             newBeta(Beta)
13
14
       syntax KItem ::= betaParser(K,K,K) [function] //(Tree Piece,
           NewSyntax, Parameters, Constr)
       syntax Set ::= getTyVarsRHS(K,List) [function]
15
16
17
       syntax KItem ::= forAll(Set,K)
18
       syntax KItem ::= funtype(K,K)
```

```
19
20
       syntax Set ::= listToSet(List, Set) [function]
21
22
       rule listToSet(ListItem(A:KItem) L:List, S:Set) => listToSet
           (L, SetItem(A) S)
       rule listToSet(.List, S:Set) => S
23
24
25
26 //if optbangAtypes, need to see if first variable is a typecon
27 //if its a typecon then need to go into Delta and see the amount
        of parameters it has
28 //then count the number of optbangAtypes after the typecon
29
       rule betaParser('constrCon(A:K,, B:K), Par:K, Con:K) =>
           forAll(getTyVarsRHS(B,.List), betaParser(B, Par, Con))
30
       rule betaParser('optBangATypes('optBangAType('emptyBang(.
           KList),, 'atypeTyVar(Tyv:K)),, Rest:K), Par:K, Con:K) =>
           funtype(Tyv, betaParser(Rest, Par, Con))
31
       rule betaParser('optBangATypes('optBangAType('emptyBang(.
           KList),, 'baTypeCon(A:K,, B:K)),, Rest:K), Par:K, Con:K)
           => funtype('baTypeCon(A:K,, B:K), betaParser(Rest, Par,
32
       rule betaParser ('optBangATypes ('optBangAType ('emptyBang (...
           KList),, 'atypeGTyCon(Tyc:K)),, Rest:K), Par:K, Con:K) =>
            funtype(Tyc, betaParser(Rest, Par, Con))
33
       rule betaParser(.OptBangATypes, Par:K, Con:K) => '
           simpleTypeCon(Con,, Par)
34
35
       rule getTyVarsRHS(.OptBangATypes,Tylist:List) => listToSet(
           Tylist, .Set)
36
37
       rule <k> newBeta(M:Map)
38
                => .K ...</k>
39
            <tempBeta> OldBeta:K => M </tempBeta>
```

## 5.3 Definition of Substitution

A substitution is a set of variables and their replacements. Applying a substitution to an expression means to simultaneously replace each variable in the expression with the replacement term.

http://www.mathcs.duq.edu/simon/Fall04/notes-7-4/node3.html

## 5.4 Inference Algorithm

Much of the inference algorithm is the same as the one introduced in cs 421. [cs 421 gather exp ty substitution mp]

```
1 requires "haskell-syntax.k"
2 requires "haskell-configuration.k"
3 requires "haskell-preprocessing.k"
5 module HASKELL-TYPE-INFERENCING
6
       imports HASKELL-SYNTAX
7
       imports HASKELL-CONFIGURATION
8
       imports HASKELL-PREPROCESSING
9
10
       syntax KItem ::= "Bool" //Boolean
11
12
       // STEP 4 Type Inferencing
13
       syntax KItem ::= inferenceShell(K) [function]//Input,
           AlphaMap, Beta, Delta, Gamma
14
       //syntax KItem ::= typeInferenceFun(K, Map, Map, Map, Map, K, K) [
           function]//Input, Alpha, Beta, Delta, Gamma
15
       //syntax KItem ::= typeInferenceFun(Map,K,K) //Gamma,
           Expression, Guessed Type
16
       syntax Map ::= genGamma(K, Map, K) [function] //Apatlist,
           Gamma Type
17
       syntax KItem ::= genLambda(K,K) [function]
18
       syntax KItem ::= guessType(Int)
19
       syntax KItem ::= freshInstance(K, Int) [function]
20
       syntax Int ::= paramSize(K) [function]
21
22
23
       syntax KItem ::= mapBag(Map)
24
       syntax KResult ::= mapBagResult(Map)
25
26
       syntax Map ::= gammaSub(Map, Map, Map) [function]//
           substitution, gamma
27
28
       rule <k> performIndividualInferencing => inferenceShell(Code
           ) ...</k>
```

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```
29
            <tempModule> Mod:KItem </tempModule>
30
31
            <moduleName> 'moduleName(Mod) </moduleName>
            <moduleTempCode> Code:KItem </moduleTempCode>
32
33
34
       rule inferenceShell('topdeclslist('type(A:K,, B:K),, Rest:K)
           ) =>
35
            inferenceShell(Rest) //constructalpha
       rule inferenceShell('topdeclslist('data(A:K,, B:K,, C:K,, D:
36
           K),, Rest:K)) =>
37
            inferenceShell(Rest)
38
       rule inferenceShell('topdeclslist('newtype(A:K,, B:K,, C:K,,
            D:K),, Rest:K)) =>
39
            inferenceShell(Rest)
40
       rule inferenceShell('topdeclslist('class(A:K,, B:K,, C:K,, D
           :K),, Rest:K)) =>
41
            inferenceShell(Rest)
42
       rule inferenceShell('topdeclslist('instance(A:K,, B:K,, C:K
           ,, D:K),, Rest:K)) =>
43
            inferenceShell(Rest)
44
       rule inferenceShell('topdeclslist('default(A:K,, B:K,, C:K,,
            D:K),, Rest:K)) =>
45
            inferenceShell(Rest)
       rule inferenceShell('topdeclslist('foreign(A:K,, B:K,, C:K,,
46
            D:K),, Rest:K)) =>
47
            inferenceShell(Rest)
48
49
       rule inferenceShell('topdeclslist('topdecldecl(A:K),, Rest:K
            typeInferenceFun(.ElemList, .Map,A,guessType(0)) ~>
50
                inferenceShell(Rest)
51
52
53
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, '
           declFunLhsRhs(Fn:K,, Lhsrhs:K), Guess:K) =>
54
            typeInferenceFun(.ElemList, Gamma, Lhsrhs, Guess) ...</
                k>
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, '
55
           eqExpOptDecls(Ex:K,, Optdecls:K), Guess:K) =>
56
            typeInferenceFun(.ElemList, Gamma, Ex, Guess) ...</k>
```

#### 5.4.1 Variable Rule

$$\Gamma \vdash x : \tau \mid \text{unify}\{(\tau, \text{freshInstance}(\Gamma(x)))\}$$
 Variable

#### 5.4.2 Constant Rule

$$\frac{1}{\Gamma \vdash c : \tau \mid \text{unify}\{(\tau, \text{freshInstance}(\tau))\}} \text{ Constant}$$

### 5.4.3 Lambda Rule

$$\frac{[x:\tau_1] + \Gamma \vdash e:\tau_2 \mid \sigma}{\Gamma \vdash \backslash x \rightarrow e:\tau \mid \text{unify}\{(\sigma(\tau),\sigma(\tau_1 \rightarrow \tau_2))\} \circ \sigma} \text{ Lambda}$$

```
1
2     syntax KItem ::= typeInferenceFun(ElemList, Map, K, K) [
          strict(1)]
3     syntax KItem ::= typeInferenceFunLambda(ElemList, K, K, K) [
          strict(1)]
4     /* automatically generated by the strict(1) in typeInferenceFun
          or typeInferenceFunAux
```

```
5
      rule typeInferenceFunAux(Es:ElemList, C:K, A:K, B:K) => Es ~>
           typeInferenceFun(HOLE, C, A, B)
6
           requires notBool isKResult(Es)
7
      rule Es:KResult ~> typeInferenceFunAux(HOLE, C:K,A:K, B:K) =>
           typeInferenceFun(Es, C, A, B)
8
   */
9
10
       //lambda rule
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'lambdaFun(
11
           Apatlist:K,, Ex:K), Guess:KItem)
12
              => typeInferenceFunLambda(val(typeInferenceFun(.
                 ElemList, genGamma(Apatlist, Gamma, guessType(TypeIt)
                 ), genLambda(Apatlist,Ex), guessType(TypeIt +Int 1)
                 )), .ElemList, Guess, guessType(TypeIt), guessType(
                 TypeIt +Int 1)) \dots </k>
13
             <typeIterator> TypeIt:Int => TypeIt +Int 2 </
                typeIterator>
14
15
       rule <k> typeInferenceFunLambda (valValue (mapBagResult (Sigma:
           Map)), .ElemList, Tau:K, Tauone:K, Tautwo:K)
16
             => mapBagResult (compose (uniFun (ListItem (uniPair (typeSub
                (Sigma, Tau), typeSub(Sigma, funtype(Tauone, Tautwo)))))
                ,Sigma)) \dots </k>
```

## 5.4.4 Application Rule

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau \mid \sigma_1 \qquad \sigma_1(\Gamma) \vdash e_2 : \sigma_1(\tau_1) \mid \sigma_2}{\Gamma \vdash e_1 e_2 : \tau \mid \sigma_2 \circ \sigma_1} \text{ Application}$$

```
1
2
      syntax KItem ::= typeInferenceFunAppli(ElemList, Map, K, K,
          Map) [strict(1)]
3
4
      //application rule
5
      rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'funApp(Eone
          :K,, Etwo:K), Guess:KItem)
6
            => typeInferenceFunAppli(val(typeInferenceFun(.
                ElemList, Gamma, Eone, funtype(quessType(TypeIt),
                Guess))), .ElemList, Gamma, Etwo, guessType(TypeIt)
                , .Map) \dots </k>
7
           <typeIterator> TypeIt:Int => TypeIt +Int 1 </
               typeIterator>
```

```
8
9
       rule <k> typeInferenceFunAppli(valValue(mapBagResult(
           Sigmaone: Map)), .ElemList, Gamma: Map, Etwo: KItem,
           guessType(TypeIt:Int), .Map)
10
             => typeInferenceFunAppli(val(typeInferenceFun(.
                 ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo,
                 typeSub(Sigmaone, guessType(TypeIt)))), .ElemList,
                 .Map, .K, .K, Sigmaone) ...</k>
11
12
       rule <k> typeInferenceFunAppli(valValue(mapBagResult(
           Sigmatwo:Map)), .ElemList, .Map, .K, .K, Sigmaone:Map)
13
             => mapBagResult(compose(Sigmatwo, Sigmaone)) ...</k>
```

### 5.4.5 IfThenElse Rule

For functions, function guards, cases, and if-then-else are all equivalent.

```
\frac{\Gamma \vdash e_1 : \text{bool} \mid \sigma_1 \qquad \sigma_1(\Gamma) \vdash e_2 : \sigma_1(\tau) \mid \sigma_2 \qquad \sigma_2 \circ \sigma_1(\Gamma) \vdash e_3 : \sigma_2 \circ \sigma_1(\tau) \mid \sigma_3}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \tau \mid \sigma_3 \circ \sigma_2 \circ \sigma_1} \text{If ThenElse}
```

```
syntax KItem ::= typeInferenceFunIfThen(ElemList, Map, K, K,
           K, Map, Map) [strict(1)]
^{2}
3
      //if_then_else rule
4
      rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'ifThenElse(
          Eone:K,, Optsem:K,, Etwo:K,, Optsemtwo:K,, Ethree:K),
          Guess: KItem)
5
            => typeInferenceFunIfThen(val(typeInferenceFun(.
                ElemList, Gamma, Eone, Bool)), .ElemList, Gamma,
                Etwo, Ethree, Guess, .Map, .Map) ...</k>
6
7
      rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
          Sigmaone: Map)), .ElemList, Gamma: Map, Etwo: KItem, Ethree:
          KItem, Guess:KItem, .Map, .Map)
8
             => typeInferenceFunIfThen(val(typeInferenceFun(.
                ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo,
                typeSub(Sigmaone, Guess))), .ElemList, Gamma, .K,
                Ethree, Guess, Sigmaone, .Map) ...</k>
9
```

```
10
       rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
           Sigmatwo: Map)), .ElemList, Gamma: Map, .K, Ethree: KItem,
           Guess:KItem, Sigmaone:Map, .Map)
11
             => typeInferenceFunIfThen(val(typeInferenceFun(.
                 ElemList, gammaSub(compose(Sigmatwo, Sigmaone),
                 Gamma, .Map), Ethree, typeSub(compose(Sigmatwo,
                 Sigmaone), Guess))), .ElemList, .Map, .K, .K, .K,
                 Sigmaone, Sigmatwo) ...</k>
12
13
       rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
           Sigmathree: Map)), .ElemList, .Map, .K, .K, .K, Sigmaone:
          Map, Sigmatwo:Map)
14
             => mapBagResult(compose(compose(Sigmathree, Sigmatwo),
                  Sigmaone)) ...</k>
```

## 5.5 Mutual Recursion

Something to note is that mutually recursive functions are allowed in Haskell. For example:

$$fx = yx$$

$$yx = fx$$

This set of functions is allowed to compile. When run, the function just simply runs forever. This is unlike the OCaml semantics, which does not allow for mutually recursive functions.

The LetIn Rule must allow for mutual recursion. This means that an example program that must be inferred is

```
1 let \{f = \x -> y \x; y = \x -> f \x\} in (f \2)
```

In this example, the f is an expression that refers to y and y is an expression that refers to f.

#### 5.5.1 LetIn Rule

This let rule, unlike the let rule introduced in cs 421, allows for mutual recursion.

$$\frac{\Gamma \vdash \{\, \mathrm{decls}\,\} : \Delta \,|\, \sigma \qquad \sigma(\Delta + \Gamma) \vdash e : \tau \,|\, \sigma'}{\Gamma \vdash \mathrm{let}\, \{\, \mathrm{decls}\,\} \,\mathrm{in}\, \mathrm{exp} : \tau \,|\, \sigma' \circ \sigma}\, \mathrm{LetIn}$$

$$\frac{\Gamma \vdash \{\, \mathrm{decls}\,\} : \Delta \,|\, \sigma}{\Gamma \vdash \mathrm{module}\, \{\, \mathrm{decls}\,\} : \Delta \,|\, \sigma} \, \mathrm{Module}$$

$$\frac{\sigma_0' = \{\} \quad \operatorname{dom}(\Psi) = \{x_1, \dots, x_n\} \quad \sigma_{i+1}(\Psi + \Gamma) \vdash e_i : \Psi(x_i) \mid \sigma_j \quad \sigma_i' = \sigma_i \circ \sigma_{j+1}'}{\Gamma \vdash \{x_1 = e_1; \dots x_i = e_i; \dots x_n = e_n\} \text{ in exp} : \bigcup_{i=1}^n \{x_i \mapsto \operatorname{GEN}(\sigma_n'(\Gamma), \sigma_n'(\psi_i))\} \mid \sigma_n'} \operatorname{LetIn}$$

#### K Code

```
1
2
       syntax KItem ::= typeInferenceFunLetIn(ElemList, Map, Map, K
           , K, K, Int, Int, Map, Map) [strict(1)]
3
       syntax KItem ::= grabLetDeclName(K, Int) [function]
4
       syntax KItem ::= grabLetDeclExp(K, Int) [function]
5
       syntax KItem ::= mapLookup(Map, K) [function]
6
       syntax Map ::= makeDeclMap(K, Int, Map) [function]
7
       syntax Map ::= applyGEN(Map, Map, Map, Map) [function]
8
9
       //Haskell let in rule (let rec in exp + let in rule combined
10
       //qamma |- let rec f1 = e1 and f2 = e2 and f3 = e3 .... in e
11
       //beta, [f1 -> tau1, f2 -> tau2, f3 -> tau3,....] + gamma |-
            e1 : tau1 | sigma1, [f1 -> simga1(tau1), f2 -> sigma1(
           tau2), f3 \rightarrow sigma1(tau3),....] + sigma1(gamma) \mid - e2 :
           sigma1(tau2) | sigma2 [f1 -> sigma2 o sigma1(tau1), f2
           -> sigma2 o sigma1(tau2), f3 -> sigma2 o sigma1(tau3)
           ,....] + sigma2 o sigma1(gamma) |- e3 : sigma2 o sigma1(
           tau3) ..... [f1 -> gen(sigma_n o sigma2 o sigma1(tau1),
           sigma_n o sigma2 o sigma1(Gamma)), f2 -> gen(tau2), f3 ->
            gen(tau3),....] + gamma |- e : something
12
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'letIn(D:K,,
            E:K), Guess:KItem)
13
             => typeInferenceFunLetIn(.ElemList, Gamma, makeDeclMap
                 (D, TypeIt, .Map), D, E, Guess, O, TypeIt, .Map,
                 Beta) ...</k>
14
            <typeIterator> TypeIt:Int => TypeIt +Int size(
                makeDeclMap(D, TypeIt, .Map)) </typeIterator>
15
            <tempBeta> Beta:Map </tempBeta>
16
```

```
17
       rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap
           :Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int
           , OldSigma:Map, Beta:Map)
18
              => typeInferenceFunLetIn(val(typeInferenceFun(.
                  ElemList, Gamma DeclMap, grabLetDeclExp(D, Iter),
                  mapLookup(DeclMap, grabLetDeclName(D, Iter)))), .
                  ElemList, Gamma, DeclMap, D, E, Guess, Iter,
                  TypeIt, OldSigma, Beta) ...</k>
19
             //=> typeInferenceFunLetIn(val(typeInferenceFun(
                 DeclMap, grabLetDeclExp(D, Iter +Int TypeIt), Guess
                 )), .ElemList, Gamma, DeclMap, D, E, Guess, Iter,
                 TypeIt, OldSigma) ...</k>
20
             requires Iter <Int (size(DeclMap))</pre>
21
22
       rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:
           Map)), .ElemList, Gamma: Map, DeclMap: Map, D: KItem, E:
          KItem, Guess: KItem, Iter: Int, TypeIt: Int, OldSigma: Map,
           Beta:Map)
23
             => typeInferenceFunLetIn(.ElemList, gammaSub(Sigma,
                 Gamma, .Map), gammaSub(Sigma, DeclMap, .Map), D, E,
                 typeSub(Sigma, Guess), Iter +Int 1, TypeIt, compose
                 (Sigma, OldSigma), Beta) ...</k>
24
             requires Iter <Int (size(DeclMap))</pre>
25
26
       rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap
           :Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int
           , OldSigma:Map, Beta:Map)
27
             => typeInferenceFunLetIn(val(typeInferenceFun(.
                 ElemList, Gamma applyGEN(Gamma, DeclMap, .Map, Beta
                 ), E, Guess)), .ElemList, Gamma, DeclMap, D, E,
                 Guess, Iter, TypeIt, OldSigma, Beta) ...</k>
28
             requires Iter >=Int (size(DeclMap))
29
30
       rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:
           Map)), .ElemList, Gamma: Map, DeclMap: Map, D: KItem, E:
          KItem, Guess: KItem, Iter: Int, TypeIt: Int, OldSigma: Map,
           Beta:Map)
31
             => mapBagResult(compose(Sigma, OldSigma))...</k>
32
             requires Iter >=Int (size(DeclMap))
1
2
       rule mapLookup((Name |-> Type:KItem) DeclMap:Map, Name:KItem
           ) => Type
```

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```
3
       rule mapLookup(DeclMap:Map, Name:KItem) => Name
4
            requires notBool(Name in keys(DeclMap))
5
6
       rule makeDeclMap('decls(Dec:K), TypeIt:Int, NewMap:Map) =>
           makeDeclMap(Dec, TypeIt, NewMap)
7
       rule makeDeclMap('declsList('declPatRhs('apatVar(Var:K),,
           Righthand:K),, Rest:K), TypeIt:Int, NewMap:Map) =>
           makeDeclMap('decls(Rest), TypeIt +Int 1, NewMap[Var <-</pre>
           guessType(TypeIt)])
       rule makeDeclMap(.DeclsList, TypeIt:Int, NewMap:Map) =>
8
           NewMap
9
10
       rule grabLetDeclName('decls(Dec:K), Iter:Int) =>
           grabLetDeclName(Dec, Iter)
11
       rule grabLetDeclName('declsList(Dec:K,, Rest:K), Iter:Int)
           => grabLetDeclName(Rest, Iter -Int 1)
12
            requires Iter >Int 0
13
       rule grabLetDeclName ('declsList ('declPatRhs ('apatVar (Var:K)
           ,, Righthand:K),, Rest:K), Iter:Int) => Var
14
            requires Iter <=Int 0
15
16
17
       rule grabLetDeclExp('decls(Dec:K), Iter:Int) =>
           grabLetDeclExp(Dec, Iter)
18
       rule grabLetDeclExp('declsList(Dec:K,, Rest:K), Iter:Int) =>
            grabLetDeclExp(Rest, Iter -Int 1)
19
            requires Iter >Int 0
20
       rule grabLetDeclExp('declsList('declPatRhs('apatVar(Var:K),,
            Righthand:K),, Rest:K), Iter:Int) => grabLetDeclExp(
           Righthand, Iter)
21
            requires Iter <=Int 0
22
       rule grabLetDeclExp('eqExpOptDecls(Righthand:K,, Opt:K),
           Iter:Int) => 'eqExpOptDecls(Righthand,, Opt)
23
24
       rule genGamma('apatVar(Vari:K), Gamma:Map, Guess:K) => Gamma
           [Vari <- Guess]</pre>
25
       rule genGamma('apatCon(Vari:K,, Pattwo:K), Gamma:Map, Guess:
           K) => Gamma[Vari <- Guess]</pre>
26
27
       rule genLambda('apatVar(Vari:K), Ex:K) => Ex
28
       rule genLambda('apatCon(Vari:K,, Pattwo:K), Ex:K) => '
           lambdaFun(Pattwo,, Ex)
29
```

## 5.6 Fresh Instance

```
rule freshInstance(guessType(TypeIt:Int), Iter:Int) =>
           guessType(TypeIt)
       rule freshInstance(forAll(.Set, B:K), Iter:Int) => B
3
       rule freshInstance(forAll(SetItem(C:KItem) A:Set, B:K), Iter
           :Int) => freshInstance(forAll(A, freshInstanceInner(C, B,
            Iter)), Iter +Int 1)
4
5
       syntax KItem ::= freshInstanceInner(K,K,Int) [function]
       rule freshInstanceInner(Repl:KItem, funtype(A:K, B:K), Iter:
           Int) => funtype(freshInstanceInner(Repl,A,Iter),
           freshInstanceInner(Repl, B, Iter))
8
       rule freshInstanceInner(Repl:KItem, Repl, Iter:Int) =>
           guessType(Iter)
9
       rule freshInstanceInner(Repl:KItem, Target:KItem, Iter:Int)
           => Target [owise]
10
11
       rule paramSize(forAll(A:Set, B:K)) => size(A)
12
       rule paramSize(A:K) => 0 [owise]
```

## 5.7 GEN

$$GEN(\Gamma, \tau) = \forall \alpha_1, \cdots, \alpha_n \cdot \tau$$

where

$$\{\alpha_1, \dots, \alpha_n\} = \text{freevarsty}(\tau) - \text{freevarsenv}(\Gamma)$$

$$\text{freevarsty}('\alpha) = \{'\alpha\}$$

$$\text{freevarsty}(c) = \{\}$$

where c is a type such as Int

```
freevarsty(c(\tau_1, \dots, \tau_n)) = \bigcup_{i=1}^n freevarsty(\tau_i)
            freevarsty(\forall \alpha_1, \dots, \alpha_n \cdot \tau) = freevarsty(\tau) - { \alpha_1, \dots, \alpha_n }
                     \operatorname{freevarsenv}(\Gamma) = \bigcup_{x \in \operatorname{dom}(\Gamma)} \operatorname{freevarsty}(\Gamma(x))
         rule applyGEN(Gamma:Map, (Key:KItem |-> Type:KItem) DeclMap
 1
             :Map, NewMap:Map, Beta:Map)
 2
            => applyGEN(Gamma, DeclMap, NewMap[Key <- gen(Gamma, Type
                , Beta)], Beta)
 3
 4
         rule applyGEN(Gamma:Map, .Map, NewMap:Map, Beta:Map)
 5
            => NewMap
 6
 7
        //GEN
 8
        //GEN(Gamma, Tau) => Forall alpha
9
10
        syntax KItem ::= gen(Map, K, Map) [function]
11
        syntax Set ::= freeVarsTy(K, Map) [function]
12
        syntax Set ::= freeVarsEnv(Map, Map) [function]
13
14
15
        rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map)
            => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set
            freeVarsEnv(Gamma, Beta), Tau)
16
        rule gen(Gamma:Map, Tau:KItem, Beta:Map) => forAll(
            freeVarsTy(Tau, Beta) -Set freeVarsEnv(Gamma, Beta), Tau)
             [owise]
17
        //rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map)
18
             => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set
            freeVarsEnv(Gamma, Beta), Tau)
19
20
        rule freeVarsTy(quessType(TypeIt:Int), Beta:Map) => SetItem(
            guessType(TypeIt:Int))
21
        rule freeVarsTy(funtype(Tauone:KItem, Tautwo:KItem), Beta:
            Map) => freeVarsTy(Tauone, Beta) freeVarsTy(Tautwo, Beta)
22
        rule freeVarsTy(Tau:KItem, Beta:Map) => .Set
23
              requires (forAll(.Set, Tau)) in values(Beta)
```

## 5.8 Unification Algorithm

1

2 3

4

5

6

7 8

9

```
Let S = \{(s_1, t_1), (s_2, t_2), \dots, (s_n, t_n)\} be a unification problem.
  Case S = \{\}: Unif(S) = Identity function (ie no substitution)
  Case S = (s, t) \cup S': Four main steps
  Delete: if s = t (they are the same term) then Unif(S) = Unif(S')
  Decompose: if s = f(q_1, \dots, q_m) and t = f(r_1, \dots, r_m) (same f, same m!),
then \text{Unif}(S) = \text{Unif}(\{(q_1, r_1), ..., (q_m, r_m)\} \cup S')
  Orient: if t = x is a variable, and s is not a variable, Unif(S) = Unif(\{(x, s)\} \cup \{(x, s)\})
S'
  Eliminate: if s = x is a variable, and x does not occur in t (the occurs
check), then
  Let \phi = x \mapsto t
  Let \psi = \text{Unif}(\phi(S'))
  \mathrm{Unif}(S) = \{x \mapsto \psi(t)\} \circ \psi
  Note:
          \{x \mapsto a\} \circ \{y \mapsto b\} = \{y \mapsto (\{x \mapsto a\}(b)\} \circ \{x \mapsto a\}
if y not in a
  [cs 421 class notes]
     //Unification
     syntax Map ::= uniFun(List) [function]
     syntax Bool ::= isVarType(K) [function]
     syntax Bool ::= notChildVar(K,K) [function]
     syntax KItem ::= uniPair(K,K)
     syntax List ::= uniSub(Map,K) [function] //apply
          substitution to unification
```

```
10
       syntax KItem ::= typeSub(Map,K) [function] //apply
           substitution to type
       syntax Map ::= compose(Map, Map) [function]
11
12
13
       rule uniFun(.List) => .Map //substi(.K,.K) is id
           substitution
14
15
       rule uniFun(ListItem(uniPair(S:K,S)) Rest:List) => uniFun(
           Rest) //delete rule
16
17
       rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List) => uniFun(
           ListItem(uniPair(T,S)) Rest) //orient rule
18
            requires isVarType(T) andBool (notBool isVarType(S))
19
20
       rule uniFun(ListItem(uniPair(funtype(A:K, B:K), funtype(C:K,
            D:K))) Rest:List) => uniFun(ListItem(uniPair(A, C))
           ListItem(uniPair(B, D)) Rest:List) //decompose rule
           function type
21
22
       rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List)
23
         => compose((S |-> typeSub(uniFun(uniSub((S |-> T), Rest)), T
             )),uniFun(uniSub((S |-> T),Rest))) //eliminate rule
24
       // => compose(uniFun(uniSub((S |-> T),Rest)),(S |-> typeSub
           (uniFun(uniSub((S |-> T), Rest)), T))) //eliminate rule
25
            requires isVarType(S) andBool notChildVar(S,T)
26
27
       rule isVarType(S:K) => true
28
            requires getKLabel(S) ==KLabel 'guessType
29
       rule isVarType(S:K) => false [owise]
30
31
       rule notChildVar(S:K,T:K) => true
32
33
       rule uniSub(Sigma:Map,.List) => .List
34
       rule uniSub(.Map,L:List) => L
35
       rule uniSub(Sigma:Map, Rest:List ListItem(uniPair(A:K, B:K))
           ) => uniSub(Sigma, Rest) ListItem(uniPair(typeSub(Sigma,
          A), typeSub(Sigma, B)))
36
37
       rule typeSub(Sigma:Map (Tau |-> Newtau:KItem), Tau:KItem) =>
           typeSub(Sigma (Tau |-> Newtau), Newtau)
38
       rule typeSub(Sigma:Map,funtype(Tauone:KItem,Tautwo:KItem))
           => funtype(typeSub(Sigma, Tauone), typeSub(Sigma, Tautwo))
       rule typeSub(Sigma:Map,Tau:KItem) => Tau [owise]
39
```

## 5.9 Composition of Substitutions

Composition of substitutions means that if the composition was applied to a type, it would first apply to the rightmost substitution and then afterwards apply to the substitution to the left of it, and so on.

```
syntax Map ::= composeIn(Map, Map, Map, K, K) [function]
1
2
3
       rule compose(Sigmaone:Map, Sigmatwo:Map) => composeIn(
           Sigmaone, Sigmatwo, .Map, .K, .K)
4
5
       rule composeIn(Sigmaone:Map, (Key:KItem |-> Type:KItem)
           Sigmatwo:Map, NewMap:Map, .K, .K) => composeIn(Sigmaone,
           Sigmatwo, NewMap, Key, Type)
6
7
       rule composeIn((Keyone |-> Typetwo:KItem) Sigmaone:Map,
           Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
            composeIn(Sigmaone, Sigmatwo, NewMap, Keyone, Typeone)
8
9
       rule composeIn((Typeone |-> Typetwo:KItem) Sigmaone:Map,
           Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
            composeIn((Typeone |-> Typetwo) Sigmaone, Sigmatwo,
           NewMap[Keyone <- Typetwo], .K, .K)</pre>
10
            requires notBool(Keyone in keys(Sigmaone))
11
12
       rule composeIn(Sigmaone:Map, Sigmatwo:Map, NewMap:Map,
           Keyone:KItem, Typeone:KItem) => composeIn(Sigmaone,
           Sigmatwo, NewMap[Keyone <- Typeone], .K, .K) [owise]</pre>
13
14
       rule composeIn(Sigmaone:Map, .Map, NewMap:Map, .K, .K) =>
           Sigmaone NewMap
15 endmodule
```

## 5.10 Examples

# Chapter 6

# Multiple Module Support

The next step is to implement multiple modules into the Haskell semantics. Similar to including files or objects in other programming languages, Haskell modules can include other modules and use functions, types, and typeclasses declared in the module. There are a several considerations and additional checks that need to be made. Modules need to include other modules There cannot be inclusion cycles Modules need to be able to access user defined types from the other modules that are included When referencing types from other modules, there is a scope where if the user makes another type of the same name in the current module then when the user references the type, it refers to the type from the current module. For instance:

```
1 File 1:
2 module File1 where
3 {data A = B
4 }
5 File 2:
6 module File2 where
7 {data A = B
8 ;data C = B A
9 }
```

This will compile. The A used in data C is File2.A However, If there are multiple types with the same name declared outside of the current module. If you try to refer to the type without the parent module, there will be a compiler error because there is ambiguity. For instance:

```
1 File 1:
2 module File1 where
3 {data A = B
4 }
5 File 2:
6 module File2 where
7 {data A = B
```

```
8 }
9 File 3:
10 module File3 where
11 {import File1
12 ;import File2
13 ;data C = B A
14 }
```

This will not compile because in File 3, type A is ambiguous and can mean File1.A or File2.A Type synonyms need to include polymorphism For instance: The user can write type A = B a

Since we need to check for module inclusion cycles and also build the set of user defined types for each module and included modules, I decided to use a tree. The plan for the algorithm is as follows 1. Construct tree for module inclusion 2. Check tree for cycles 3. Go to each leaf and recursively go up the tree and build alpha\* and beta\* for the types of the module and the children and desugar the scope so that each type specifies the scope.

Where alpha is the map of type synonyms declared in the current module and alpha\* is the map of type synonyms declared in the current module and all the included modules. Beta is the set of user defined types from using data and newtype declared in the current module. Beta\* is the set of user defined types from using data and newtype declared in the current module and all the included modules. Desugar the scope means that when the user references a type, desugar the reference to also include the parent module at all times. The syntax also needed to be changed to allow for multiple modules. The new syntax added is

This is because K cannot read mutiple files. So instead all the included modules for a program are dumped into one file and are seperated by the keyword ¡NEXTMODULE; This creates a list of modules called ModuleList.

```
1 //
2 requires "haskell-syntax.k"
3 requires "haskell-configuration.k"
4
5 module HASKELL-PREPROCESSING
```

```
6
       imports HASKELL-SYNTAX
 7
       imports HASKELL-CONFIGURATION
 8
9
       //USER DEFINED LIST
10
       //definition of ElemList
11
12
       //syntax KItem ::= ElemList
       syntax ElemList ::= List{Element,","} [strict]
13
         syntax Int ::= lengthOfList(ElemList) [function]
14 //
15
16 //
        rule lengthOfList(.ElemList) => 0
17 //
        rule lengthOfList(val(K:K),L:ElemList) => 1 +Int
      lengthOfList(L)
18 //
         rule lengthOfList(valValue(K:K),L:ElemList) => 1 +Int
      lengthOfList(L)
19
20
       syntax Element ::= val(K) [strict]
21
       syntax ElementResult ::= valValue(K)
22
       syntax Element ::= ElementResult
23
       syntax KResult ::= ElementResult
24
       rule val(K:KResult) => valValue(K) [structural]
25
26
       //form ElemList
27 //
        syntax ElemList ::= formElemList(K) [function]
28
29
       //CONVERT ~> TO List
30
       //list convert
        syntax List ::= convertToList(K) [function]
31 //
32 //
        rule convertToList(.K) => .List
33 //
      rule convertToList(A:KItem ~> B:K) => ListItem(A)
      convertToList(B)
34
35
36
       syntax KItem ::= dealWithImports(K,K)
37
38
       rule <k> 'modListSingle('module(A:K,, B:K)) =>
          dealWithImports(A,B) ...</k>
39
40
       (.Bag =>
41
             <module>...
                          //DOT DOT DOT MEANS OVERWRITE ONLY SOME
                OF THE DEFAULTS
42
               <moduleName> A </moduleName>
          ...</module>
43
```

```
44
       )
45
46
       rule <k> 'modList('module(A:K,, B:K),, C:K) =>
           dealWithImports(A,B) ~> C ...</k>
47
48
       (.Bag =>
49
             <module>...
                           //DOT DOT DOT MEANS OVERWRITE ONLY SOME
                 OF THE DEFAULTS
50
               <moduleName> A </moduleName>
51
          ...</module>
52
       )
53
54 //
        rule dealWithImports(Mod:K, A:K) => callInit(A)
55
56 //
         rule <k> dealWithImports(Mod:K, A:K) => callInit(A) ...</k</pre>
      >
57
58
       rule <k> dealWithImports(Mod:K, 'bodyimpandtop(A:K,, B:K))
           => .K ...</k>
           <importTree> L:List => L importListConvert(Mod, A) 
59
               importTree>
60
           <recurImportTree> L:List => L importListConvert(Mod, A)
               </recurimportTree>
61
62
           <moduleName> Mod </moduleName>
63
           <imports> S:Set (.Set => SetItem(A)) </imports>
64
           <moduleTempCode> OldTemp:K => B </moduleTempCode>
65
66
       rule <k> dealWithImports(Mod:K, 'bodyimpdecls(A:K)) => .K
           ...</k>
67
           <importTree> L:List => L importListConvert(Mod, A) 
               importTree>
           <recurImportTree> L:List => L importListConvert(Mod, A)
68
               </recurImportTree>
69
70
           <moduleName> Mod </moduleName>
71
           <imports> S:Set (.Set => SetItem(A)) </imports>
72
73 //
         rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) =>
      callInit(A) ...</k>
       rule <k> dealWithImports(Mod:K, 'bodytopdecls(B:K)) => .K
74
           ...</k>
75
```

```
76
           <moduleName> Mod </moduleName>
77
           <moduleTempCode> OldTemp:K => B </moduleTempCode>
78
79
       //importlist convert
80
       syntax List ::= importListConvert(K,K) [function]
       syntax KItem ::= impObject(K,K)
81
82
83
       rule importListConvert(Name:K, 'impDecls(A:K,, Rest:K)) =>
           importListConvert(Name, A) importListConvert(Name, Rest)
84
       rule importListConvert('moduleName(Name:K), 'impDecl(A:K,,
          Modid:K,, C:K,, D:K)) => ListItem(impObject(Name, Modid))
85
       rule importListConvert(Name:K, .ImpDecls) => .List
```

## 6.1 Module Inclusion Tree

```
/*NEW TODO ALGORITHM
2 1. Construct tree for module inclusion
3 2. Check tree for cycles
4 3. Go to each leaf and recursively go up the tree and build
      alpha* and beta* for the types of the module and the children
5 (and specify scoping) (desugar the scope so that each type
      specifies the scope) */
6
7
       syntax KItem ::= "checkImportCycle"
       syntax KItem ::= "recurseImportTree"
9
10 /*
         rule <k> performNextChecks
11
                => checkUseVars
                   ~> (checkLabelUses
12
                   ~> (checkBlockAddress(.K)
13
                   ~> (checkNoNormalBlocksHavingLandingpad(.K, TNS
14
                       -Set TES)
                   ~> (checkAllExpBlocksHavingLandingpad(.K, TES)
15
                   ~> (checkAllExpInFromInvoke(.K, TES)
16
17
                   ~> (checkLandingpad
18
                   ~> checkLandingDomResumes))))) ...</k> */
19
20
       rule <k> startImportRecursion => checkImportCycle
21
                                         ~> (recurseImportTree) ...
                                            k>
22
```

```
23
       syntax KItem ::= cycleCheck(K, Map, List, List) [function] //
           current node, map of all nodes to visited or not, stack,
           graph
24
       syntax Map ::= createVisitMap(List, Map) [function] //graph,
           visitmap
25
       syntax KItem ::= getUnvisitedNode(K,K, Map) [function] //
           visitmap
26
       syntax List ::= getNodeNeighbors(K, List) [function] //
           visitmap
27
28
       rule <k> checkImportCycle
29
                => cycleCheck(.K,createVisitMap(I, .Map),.List,I)
                    ...</k>
30
            <importTree> I:List </importTree>
            <impTreeVMap> .Map => createVisitMap(I, .Map) 
31
                impTreeVMap>
32
33
       syntax KItem ::= "visited"
34
       syntax KItem ::= "unvisited"
35
       syntax KItem ::= "none"
36
37
       rule createVisitMap(ListItem(impObject(A:K,B:K)) Rest:List,
          M:Map)
38
                => createVisitMap(Rest, M[A <- unvisited][B <-
                    unvisited])
39
       rule createVisitMap(.List, M:Map) => M
40
       rule getUnvisitedNode(.K, .K, .Map) => none
41
42
       rule getUnvisitedNode(.K, .K, (A:K |-> B:K) M:Map)
43
              => getUnvisitedNode(A, B, M)
       rule getUnvisitedNode(A:KItem, unvisited, M:Map) => A
44
       rule getUnvisitedNode(A:KItem, visited, M:Map)
45
46
               => getUnvisitedNode(.K, .K, M)
47
48
49
50
       rule getNodeNeighbors(Node:K,.List) => .List
51
       rule getNodeNeighbors(.K, Rest:List) => .List
52
53
       rule getNodeNeighbors(Node:KItem,ListItem(impObject(Node,B:
           KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
          ListItem(B)
```

```
54
       rule getNodeNeighbors (Node:KItem, ListItem(impObject(A:KItem,
           B:KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
55
            requires Node =/=K A
56
57
58
       rule cycleCheck(none, M:Map, .List, L:List) => .K
59
       rule cycleCheck(.K, M:Map, .List, I:List) => cycleCheck(
           getUnvisitedNode(.K, .K, M), M, .List, I)
60
       rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:List)
           => cycleCheck(Node, M, S, I)
       rule cycleCheck(Node:K, M:Map, S:List, I:List)
61
62
                     => cycleCheck(.K, M[Node <- visited],
                        getNodeNeighbors(Node, I) S, I)
63
            requires Node =/=K .K andBool Node =/=K none
64
       rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) =>
           cycleCheck (Node, M, S, I)
65
            requires S = /=K .List
66
67 /*
       rule cycleCheck(A:K, .K, .K, I:K) => cycleCheck(A,
68
           createVisitMap(I,.Map),.List,I)
69
70
71
72
       rule cycleCheck(Node:K, M:Map, S:List, I:K) => cycleCheck(.K
           , M[Node <- visited], getNodeNeighbors(Node,I) S, I)</pre>
73
       rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) =>
74
           cycleCheck(Node, M, S, I)
75
       //rule cycleCheck(.K, M:Map, .K, ListItem(impObject(A:K,B:K)
           ) Rest:List) => cycleCheck(ListItem(impObject(A:K,B:K))
           Rest:List)
76 */
```

## 6.2 Leaf DFS

```
6 //perform inferencing
7 //insert alpha* and beta* into importing modules
8 //remove all edges pointing to leaf
9
10
       syntax KItem ::= "leafDFS"
11
       syntax KItem ::= "getAlphaAndBeta"
12
       syntax KItem ::= "getAlphaBetaStar"
13
       syntax KItem ::= "performIndividualChecks"
14
       syntax KItem ::= "performIndividualInferencing"
15
      syntax KItem ::= "insertAlphaBetaStar"
       syntax KItem ::= "removeAllEdges"
16
17
       syntax KItem ::= "seeIfFinished"
18
19
       rule <k> recurseImportTree => leafDFS
20
                                   ~> (getAlphaAndBeta
21
                                   //~> (getAlphaBetaStar
22
                                      performIndividualInferencing
                                      ))...</k>
23
24 //rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) =>
      callInit(A) ...</k>
25
26 //
       rule <k> leaf
27 //
                 => cycleCheck(.K,createVisitMap(I, .Map),.List,I)
       ...</k>
             <importTree> I:List </importTree>
28 //
29
31
32
       syntax KItem ::= returnLeafDFS(K,List,Map) [function] //
          current node, map of all nodes to visited or not, stack,
          graph
33
       syntax KItem ::= innerLeafDFS(K, List) [function]
34
       syntax KItem ::= loadModule(K)
35
36
       rule <k> leafDFS
37
               => returnLeafDFS(.K,I,M) ...</k>
           <recurImportTree> I:List </recurImportTree>
38
           <impTreeVMap> M:Map </impTreeVMap>
39
40
```

```
41
       rule returnLeafDFS(.K,ListItem(impObject(Node:KItem,B:KItem)
          ) I:List,M:Map) => returnLeafDFS(B,I,M)
42
       rule returnLeafDFS(Node:KItem,I:List,M:Map) => returnLeafDFS
          (innerLeafDFS(Node, I), I, M)
43
            requires innerLeafDFS(Node, I) = /= K none
44
       rule returnLeafDFS(Node:KItem, I:List, M:Map) => loadModule(
          Node)
45
            requires innerLeafDFS(Node, I) == K none
46
47
       rule innerLeafDFS (Node:KItem, ListItem (impObject (Node, B:KItem
          )) I:List) => B
48
       rule innerLeafDFS (Node: KItem, ListItem (impObject (A: KItem, B:
          KItem)) I:List) => innerLeafDFS(Node, I)
49
            requires Node =/=K A
50
       rule innerLeafDFS(Node:KItem,.List) => none
        returnLeafDFS(Node:KItem,ListItem(impObject(Node,B:KItem))
       I:List,M:Map) => returnLeafDFS(B,I,M)
52
53
55
       //call before Checker Code
56
        rule <k> callInit(S:K) => initPreModule(S) ...</k>
58 //
              <tempModule> A:K => S </tempModule>
59
60
       rule <k> loadModule(S:KItem) => .K ...</k>
61
            <tempModule> A:K => S </tempModule>
62
63
       rule <k> getAlphaAndBeta => initPreModule(Code) ...</k>
64
            <tempModule> Mod:KItem </tempModule>
65
66
            <moduleName> 'moduleName (Mod) </moduleName>
67
            <moduleTempCode> Code:KItem </moduleTempCode>
```

## 6.3 Insert AlphaBetaStar

```
1 // syntax KItem ::= "insertAlphaBetaStar"
2
3     syntax KItem ::= insertABRec(K,List)
4     syntax KItem ::= insertAB(K)
5
```

### Draft of November 27, 2018 at 05:10

```
6
       rule <k> insertAlphaBetaStar => insertABRec(Mod, Imp) ...</k</pre>
             <tempModule> Mod:KItem </tempModule>
8
             <importTree> Imp:List </importTree>
9
10
       rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem,
           Node)) I:List) => insertAB(B) ~> insertABRec(Node, I)
           ...</k>
11
12
       rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem,
           C:KItem)) I:List) => insertABRec(Node, I) ...</k>
13
                 requires Node =/=K C
14
15
       rule \langle k \rangle insertAB(B) => .K ...\langle k \rangle
16
17
             <tempAlphaStar> Alph:KItem </tempAlphaStar>
18
             <tempBetaStar> Bet:KItem </tempBetaStar>
19
20
             <moduleName> 'moduleName(B) </moduleName>
21
             <moduleImpAlphas> ImpAlphas:List => ListItem(Alph)
                ImpAlphas </moduleImpAlphas>
22
             <moduleImpBetas> ImpBetas:List => ListItem(Bet)
                ImpBetas </moduleImpBetas>
23
24
25 endmodule
```

# Chapter 7

## Conclusion

This project helped me with my K knowledge. I also learned a lot about how compilers work and how programming languages are defined. Also, defining yet another real word language in K allows the K-framework to become more popular as another community can learn about it and utilize it. The more that real world languages are defined in K, the more potential K has to become a popular language in industry.

# Appendix A

# haskell-syntax.k

```
1 // Syntax from haskell 2010 Report
2 // https://www.haskell.org/onlinereport/haskell2010/haskellch10.
      html#x17-17500010
3
4 module HASKELL-SYNTAX
5
6
       syntax Integer ::= Token\{([0-9]+)\}
7
                   |(([0][0]|[0][0])[0-7]+)
8
                   |(([0][x] | [0][X])[0-9a-fA-F]+)| [onlyLabel]
9
10
       syntax CusFloat ::= Token\{([0-9]+[\.][0-9]+([e\ E
          ][ + -]?[0-9]+)?)
11
                                    |([0-9]+[e E][+-]?[0-9]+)|
                                       onlyLabel]
12
       syntax CusChar ::= Token{['](~[''\k])[']} [onlyLabel]
13
       14
       syntax VarId ::= Token{[a-z \setminus ][a-z A-z \setminus 0-9 \setminus ']*} [onlyLabel]
           | "size" [onlyLabel]
15
       syntax ConId ::= Token{[A-Z][a-zA-Z \setminus 0-9 \land ']*} [onlyLabel]
16
       syntax VarSym ::= Token{
17
      ([\! \# \$ \& \* \+ \/ \> \? \^][\! \# \$ \& \* \+ \.
         \/ \< \= \> \? \@ \\ \^ \| \- \~ \:]*)
18
     |[\-] | [\.]
19
     |([\.][\! \# \$ \% \& \* \+ \/ \< \= \> \? \@ \\ \^ \| \- \~
        \:][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~\:]*)
20
     | ([\-][\! \# \$ \% \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~
        \:][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~
        \:]*)
21
     | ([\@][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
22
     | ([\~][\! \# \$ \% \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~\:]+)
```

```
| ([\\][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
23
        \~ \:]+)
24
     | ([\|][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~\:]+)
     | ([\:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
25
        \~][\!\#\$\%\&\*\+\.\/\<\=\>\?\@\\\^\|\-
        \~\:]*)
     | ([\<][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \~
26
        \:][\! \# \$ \% \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~ \:]*)
     | ([\=][\! \# \$ \% \& \* \+ \. \/ \< \= \? \@ \\ \^ \| \~
27
        \:][\! \# \$ \& \* \+ \. \/ \< \= \> \? \@ \\ \^ \| \-
        \~\:]*)} [onlyLabel]
28
       syntax ConSym ::= Token{[\:][\! \# \$ \% \& \* \+ \. \/ \<
          \= \> \? \@ \\ \^ \| \- \~][\! \# \$ \& \* \+ \. \/ \<
           \= \> \? \@ \\ \^ \| \- \~ \:]*} [onlyLabel]
29
       syntax IntFloat ::= "(" Integer ")" [bracket] //NOT
30
          OFFICIAL SYNTAX
31
                         | "(" CusFloat ")" [bracket]
32
       syntax Literal ::= IntFloat | CusChar | CusString
33
       syntax TyCon ::= ConId
34
       syntax ModId ::= ConId | ConId "." ModId [klabel('conModId)
       syntax QTyCon ::= TyCon | ModId "." TyCon [klabel('conTyCon)
35
          1
36
       syntax QVarId ::= VarId | ModId "." VarId [klabel('qVarIdCon
          ) ]
37
       syntax QVarSym ::= VarSym | ModId "." VarSym [klabel('
          qVarSymCon)]
38
       syntax QConSym ::= ConSym | ModId "." ConSym [klabel('
          qConSymCon)]
39
       syntax TyVars ::= List{TyVar, ""} [klabel('typeVars)] //used
           in SimpleType syntax
40
       syntax TyVar ::= VarId
41
       syntax TyVarTuple ::= TyVar "," TyVar
                                                 [klabel('
          twoTypeVarTuple)]
42
                           | TyVar "," TyVarTuple [klabel('
                              typeVarTupleCon)]
43
       syntax Con ::= ConId | "(" ConSym ")" [klabel('
44
          conSymBracket)]
```

```
45
       syntax Var ::= VarId | "(" VarSym ")" [klabel('
          varSymBracket)]
46
       syntax QVar ::= QVarId | "(" QConSym ")" [klabel('
          qVarBracket)]
47
       syntax QCon ::= QTyCon | "(" GConSym ")" [klabel('
           gConBracket)]
48
       syntax QConOp ::= GConSym | "'" QTyCon "'"
49
                                                      [klabel('
          qTyConQuote)]
50
       syntax QVarOp ::= QVarSym | "'" QVarId "'"
                                                      [klabel('
          qVarIdQuote)]
51
       syntax VarOp ::= VarSym | "'" VarId "'"
                                                      [klabel('
          varIdQuote)]
52
       syntax ConOp ::= ConSym | "'" ConId "'"
                                                      [klabel('
          conIdQuote)]
53
54
       syntax GConSym ::= ":" | QConSym
55
       syntax Vars ::= Var
                     | Var "," Vars [klabel('varCon)]
56
57
       syntax VarsType ::= Vars "::" Type [klabel('varAssign)]
58
       syntax Ops ::= Op
59
                    | Op "," Ops [klabel('opCon)]
60
       syntax Fixity ::= "infixl" | "infixr" | "infix"
61
       syntax Op ::= VarOp | ConOp
62
       syntax CQName ::= Var | Con | QVar
63
64
       syntax QOp ::= QVarOp | QConOp
65
66
       syntax ModuleName ::= "module" ModId [klabel('moduleName)]
67
68
       syntax Module ::= ModuleName "where" Body
                                                          [klabel('
          module)]
69
                        | ModuleName Exports "where" Body [klabel('
                           moduleExp)]
70
                                                            [klabel('
                        | Body
                           moduleBody) ]
71
72
       syntax Body ::= "{" ImpDecls ";" TopDecls "}" [klabel('
          bodyimpandtop)]
                     | "{" ImpDecls "}" [klabel('bodyimpdecls)]
73
74
                     | "{" TopDecls "}" [klabel('bodytopdecls)]
75
76
       syntax ImpDecls ::= List(ImpDecl, ";") [klabel('impDecls)]
```

```
77
        syntax Exports ::= "(" ExportList OptComma ")"
78
        syntax ExportList ::= List{Export, ","}
79
80
        syntax Export ::= QVar
81
                        | QTyCon OptCQList
82
                         | ModuleName
83
        //optional cname list
84
85
        syntax OptCQList ::= "(..)"
86
                            | "(" CQList ")" [klabel('cqListBracket
87
                               //Liyi: a check needs to place in
                                  preprocessing to check
88
                               //if the CQList is a cname list or a
                                  qvar list.
                            "" [onlyLabel, klabel('
89
                               emptyOptCNameList)]
90
        syntax CQList ::= List{CQName, ","}
91
92
        syntax ImpDecl ::= "import" OptQualified ModId OptAsModId
            OptImpSpec [klabel('impDecl)]
93
                          "" [onlyLabel, klabel('emptyImpDecl)]
94
        syntax OptQualified ::= "qualified"
                               | "" [onlyLabel, klabel('
95
                                  emptyQualified)]
96
        syntax OptAsModId ::= "as" ModId
                             | ""
      [onlyLabel, klabel('
97
                                emptyOptAsModId)]
98
99
        syntax OptImpSpec ::= ImpSpec
100
                             " "
                                    [onlyLabel, klabel('
                                emptyOptImpSpec) ]
101
102
        syntax ImpSpecKey ::= "(" ImportList OptComma ")"
103
        syntax ImpSpec ::= ImpSpecKey
104
                         | "hiding" ImpSpecKey
105
106
        syntax ImportList ::= List{Import, ","}
107
108
        syntax Import ::= Var
109
                        | TyCon CQList
        syntax TopDecls ::= List{TopDecl, ";"} [klabel('topdeclslist
110
           ) ]
```

```
111
112
        syntax TopDecl ::= Decl [klabel('topdecldecl)]
113
                          > "type" SimpleType "=" Type [klabel('type)
114
                          | "data" OptContext SimpleType OptConstrs
                             OptDeriving [klabel('data)]
115
                          | "newtype" OptContext SimpleType "="
                             NewConstr OptDeriving [klabel('newtype)]
116
                          | "class" OptContext ConId TyVar OptCDecls
                             [klabel('class)]
117
                          | "instance" OptContext QTyCon Inst
                             OptIDecls [klabel('instance)]
118
                          | "default" Types [klabel('default)]
119
                          | "foreign" FDecl [klabel('foreign)]
120
121
        syntax FDecl ::= "import" CallConv CusString Var "::" FType
122
                        | "import" CallConv Safety CusString Var "::"
                            FType
123
                        | "export" CallConv Safety CusString Var "::"
                            FType
124
        //Liyi: fdecl needs to use special function in preprocessing
125
        // to get the actually elements from the impent and expent
            from the CusString
126
        //did string analysis
127
128
        syntax Safety ::= "unsafe" | "safe"
129
130
        syntax CallConv ::= "ccall" | "stdcall" | "cplusplus" | "jvm
            " | "dotnet"
131
        syntax FType ::= FrType
132
                        | FaType "->" FType // unsure about this one
                           syntax is ambiguous UNFINISHED
133
134
        syntax FrType ::= FaType
135
                        | "()"
136
137
        syntax FaType ::= QTyCon ATypeList
138
139
        //define declaration.
140
        syntax OptDecls ::= "where" Decls | "" [onlyLabel, klabel('
            emptyOptDecls)]
141
        syntax Decls ::= "{" DeclsList "}" [klabel('decls)]
142
        syntax DeclsList ::= List{Decl, ";"} [klabel('declsList)]
```

```
143
144
        syntax Decl ::= GenDecl
145
                       | FunLhs Rhs [klabel('declFunLhsRhs)]
146
                       | Pat Rhs [klabel('declPatRhs)]
147
148
        syntax OptCDecls ::= "where" CDecls | "" [onlyLabel, klabel
            ('emptyOptCDecls)]
149
        syntax CDecls ::= "{" CDeclsList "}"
150
        syntax CDeclsList ::= List{CDecl, ";"}
151
152
        syntax CDecl ::= GenDecl
153
                        | FunLhs Rhs
154
                        | Var Rhs
155
156
        syntax OptIDecls ::= "where" IDecls | "" [onlyLabel, klabel
            ('emptyOptIDecls)]
157
        syntax IDecls ::= "{" IDeclsList "}"
158
        syntax IDeclsList ::= List{IDecl, ";"} [klabel('ideclslist)]
159
160
        syntax IDecl ::= FunLhs Rhs [klabel('cdeclFunLhsRhs)]
161
                        | Var Rhs [klabel('cdeclVarRhs)]
162
                        "" [onlyLabel, klabel('emptyIDecl)]
163
164
        syntax GenDecl ::= VarsType
165
                          | Vars "::" Context "=>" Type [klabel('
                             genAssignContext)]
166
                          | Fixity Ops
167
                          | Fixity Integer Ops
168
                          "" [onlyLabel, klabel('emptyGenDecl)]
169
170
        //three optional data type for the TopDecl data operator.
171
        //deriving data type
172
        syntax OptDeriving ::= Deriving | "" [onlyLabel, klabel('
           emptyDeriving)]
173
        syntax Deriving ::= "deriving" DClass
174
                         | "deriving" "(" DClassList ")"
175
        syntax DClassList ::= List{DClass, ","}
176
        syntax DClass ::= QTyCon
177
178
        syntax FunLhs ::= Var APatList [klabel('varApatList)]
179
                        | Pat VarOp Pat [klabel('patVarOpPat)]
180
                         | "(" FunLhs ")" APatList [klabel('
                            funlhsApatList)]
```

```
181
182
        syntax Rhs ::= "=" Exp OptDecls [klabel('eqExpOptDecls)]
183
                      | GdRhs OptDecls [klabel('gdRhsOptDecls)]
184
185
        syntax GdRhs ::= Guards "=" Exp
186
                        | Guards "=" Exp GdRhs
187
        syntax Guards ::= "|" GuardList
188
        syntax GuardList ::= Guard | Guard "," GuardList [klabel('
            guardListCon) ]
189
        syntax Guard ::= Pat "<-" InfixExp</pre>
190
                        | "let" Decls
191
                        | InfixExp
192
193
        //definition of exp
194
        syntax Exp ::= InfixExp
195
                      > InfixExp "::" Type [klabel('expAssign)]
196
                      | InfixExp "::" Context "=>" Type [klabel('
                         expAssignContext)]
197
198
        syntax InfixExp ::= LExp
199
                           > "-" InfixExp [klabel('minusInfix)]
200
                           > LExp QOp InfixExp
201
202
        syntax LExp ::= AExp
203
                       > "\\" APatList "->" Exp [klabel('lambdaFun)]
204
                       | "let" Decls "in" Exp [klabel('letIn)]
205
                       | "if" Exp OptSemicolon "then" Exp
                          OptSemicolon "else" Exp [klabel('ifThenElse
                          ) ]
206
                       | "case" Exp "of" "{" Alts "}" [klabel('caseOf
                          ) ]
207
                       | "do" "{" Stmts "}" [klabel('doBlock)]
```

LExp is an important sort for the inference function. This is because LExp defines the different expression types which the inference function has specific rules for.

```
5
       syntax AExp ::= QVar [klabel('aexpQVar)]
6
                     | GCon [klabel('aexpGCon)]
7
                     | Literal [klabel('aexpLiteral)]
8
                     > AExp AExp [left, klabel('funApp)]
                     > QCon "{" FBindList "}"
9
10
                     | AExp "{" FBindList "}" //aexp cannot be gcon
                          UNFINISHED
                             //Liyi: first, does not understand the
11
                                  syntax, it is the Qcon {FBindlist}
12
                             //or QCon? Second, place a check in
                                 preprosssing.
13
                             //and also check the Fbindlist here
                                 must be at least one argument
                     > "(" Exp ")"
14
                                               [bracket]
                     | "(" ExpTuple ")"
15
                     | "[" ExpList "]"
16
17
                     | "[" Exp OptExpComma ".." OptExp "]"
                     | "[" Exp "|" Quals "]"
18
19
                     | "(" InfixExp QOp ")"
                     | "(" QOp InfixExp ")" //qop cannot be - (
20
                        minus) UNFINISHED
21
                              //Liyi: place a check here to check
                                  if QOp is a minus
22
23
24
       syntax OptExpComma ::= "," Exp | "" [onlyLabel, klabel('
          emptyExpComma) ]
25
       syntax OptExp ::= Exp | "" [onlyLabel, klabel('emptyExp)]
26
       syntax ExpList ::= Exp | Exp "," ExpList [right]
27
       syntax ExpTuple ::= Exp "," Exp
28
                                                 [right, klabel('
          twoExpTuple) ]
29
                         | Exp "," ExpTuple
                                                 [right, klabel('
                             expTupleCon) ]
30
31
       //constr datatypes
32
       syntax OptConstrs ::= "=" Constrs [klabel('nonemptyConstrs)
          ] | "" [onlyLabel, klabel('emptyConstrs)]
33
       syntax Constrs ::= Constr [klabel('singleConstr)] |
          Constr "|" Constrs [klabel('multConstr)]
34
       syntax Constr ::= Con OptBangATypes [klabel('constrCon)
          ] // (arity con = k, k 0) UNFINISHED
35
                            | SubConstr ConOp SubConstr
```

```
| Con "{" FieldDeclList "}"
36
37
38
       syntax NewConstr ::= Con AType [klabel('newConstrCon)]
39
                            | Con "{" Var "::" Type "}"
40
41
       syntax SubConstr ::= BType | "!" AType
42
       syntax FieldDeclList ::= List{FieldDecl, ","}
       syntax FieldDecl ::= VarsType
43
44
                           | Vars "::" "!" AType
45
46
47
       syntax OptBangATypes ::= List{OptBangAType, " "} [klabel('
          optBangATypes) ]
48
       syntax OptBangAType ::= OptBang AType [klabel('optBangAType)
49
       syntax OptBang ::= "!" | "" [onlyLabel, klabel('emptyBang)]
50
51
       syntax OptContext ::= Context "=>" | "" [onlyLabel, klabel('
           emptyContext)]
52
       syntax Context ::= Class
53
                        | "(" Classes ")"
54
55
       syntax Classes ::= List{Class, ","}
56
57
       syntax SimpleClass ::= QTyCon TyVar [klabel('classCon)]
58
59
       syntax Class
                          ::= SimpleClass
60
                             | QTyCon "(" TyVar ATypeList ")"
61
                                  //Liyi: a check in preprossing to
                                     check if the Atype list is
                                     empty
62
                                  //it must have at least one item
63
64
       //define type and simple type
       syntax SimpleType ::= TyCon TyVars [klabel('simpleTypeCon)
65
          ]
66
       syntax Type ::= BType
67
                     | BType "->" Type [klabel('typeArrow)]
68
       syntax BType ::= AType
69
                      | BType AType [klabel('baTypeCon)]
70
71
       syntax ATypeList ::= List{AType, ""} [klabel('atypeList)]
72
```

```
73
        syntax AType ::= GTyCon
                                                     [klabel('
           atypeGTyCon) ]
74
                       | TyVar
                                                     [klabel('
                          atypeTyVar)]
75
                        | "(" TypeTuple ")"
                                                    [klabel('
                          atypeTuple)]
76
                       | "[" Type "]"
                                                    [klabel('tyList)
77
                       | "(" Type ")"
                                                    [bracket]
78
        syntax TypeTuple ::= Type "," Type
                                                    [right,klabel('
           twoTypeTuple)]
79
                           | Type "," TypeTuple
                                                    [klabel('
                               typeTupleCon) ]
80
        syntax Types ::= List{Type, ","}
81
82
        syntax GConCommas ::= "," | "," GConCommas
83
        syntax GConCommon ::= "()" | "[]" | "(" GConCommas ")" //was
            incorrect syntax
        syntax GTyCon ::= QTyCon
84
85
                        | GConCommon
                        | "(->)"
86
87
88
        syntax GCon ::= GConCommon
89
                      | QCon
90
91
        //inst definition
        syntax Inst ::= GTyCon
92
                       | "(" GTyCon TyVars ")" //TyVars must be
93
                           distinct UNFINISHED
94
                        | "(" TyVarTuple ")" //TyVars must be
                          distinct
95
                       | "[" TyVar "]" [klabel('tyVarList)]
96
                        | "(" TyVar "->" TyVar ")" //TyVars must be
                          distinct
97
        //pat definition
98
        syntax Pat ::= LPat QConOp Pat
99
                     | LPat
100
101
        syntax LPat ::= APat
102
                      | "-" IntFloat [klabel('minusPat)]
103
                      | GCon APatList [klabel('lpatCon)]//arity
                          gcon = k UNFINISHED
104
```

```
105
        syntax APatList ::= APat | APat APatList [klabel('apatCon)]
106
107
        syntax APat ::= Var [klabel('apatVar)]
108
                     | Var "@" APat
109
                      l GCon
110
                      | QCon "{" FPats "}"
111
                      | Literal [klabel('apatLiteral)]
112
113
                      114
                      | "(" PatTuple ")"
115
                      | "[" PatList "]"
116
                      | "~" APat
117
118
        syntax PatTuple ::= Pat "," Pat [klabel('twoPatTuple
           ) ]
119
                          | Pat "," PatTuple [klabel('patTupleCon
                             ) ]
120
        syntax PatList ::= Pat
121
                          | Pat "," PatList [klabel('patListCon)
                             ]
122
123
        syntax FPats ::= List{FPat, ","}
124
        syntax FPat ::= QVar "=" Pat
125
126
        //definition of quals
127
        syntax Quals ::= Qual | Qual "," Quals [klabel('qualCon)]
128
129
        syntax Qual ::= Pat "<-" Exp
130
                      | "let" Decls
131
                      | Exp
132
133
        //definition of alts
134
        syntax Alts ::= Alt | Alt ";" Alts
135
136
        syntax Alt ::= Pat "->" Exp [klabel('altArrow)]
137
                    | Pat "->" Exp "where" Decls
138
                     "" [onlyLabel, klabel('emptyAlt)]
139
140
        //definition of stmts
141
        syntax Stmts ::= StmtList Exp OptSemicolon
142
        syntax StmtList ::= List{Stmt, ""}
143
        syntax Stmt ::= Exp ";"
144
                     | Pat "<-" Exp ";"
```

# Appendix B

# haskell-configuration.k

```
1 requires "haskell-syntax.k"
3 module HASKELL-CONFIGURATION
4
       imports HASKELL-SYNTAX
5
6
       syntax KItem ::= "startImportRecursion"
       syntax KItem ::= callInit(K)
8
       //syntax KItem ::= initPreModule(K) [function]
9
       //syntax KItem ::= tChecker(K) [function]
10
11
       configuration
12
           <T>
13
                <k> $PGM:ModuleList ~> startImportRecursion </k>
14
                <tempModule> .K </tempModule>
15
               <tempCode> .K </tempCode>
16
                <typeIterator> 1 </typeIterator>
17
               <tempAlpha> .K </tempAlpha>
18
                <tempAlphaMap> .Map </tempAlphaMap>
19
                <tempBeta> .Map </tempBeta>
20
                <tempT> .K </tempT>
21
                <tempDelta> .Map </tempDelta>
22
                <tempAlphaStar> .K </tempAlphaStar>
23
                <tempBetaStar> .K </tempBetaStar>
24
                <importTree> .List </importTree>
25
                <recurImportTree> .List </recurImportTree>
26
                <impTreeVMap> .Map </impTreeVMap>
27
                <modules> //static information about a module
28
                    <module multiplicity="*">
29
                        <moduleName> .K </moduleName>
30
                        <moduleAlphaStar> .K </moduleAlphaStar>
31
                        <moduleBetaStar> .K </moduleBetaStar>
32
                        <moduleImpAlphas> .List </moduleImpAlphas>
33
                        <moduleImpBetas> .List </moduleImpBetas>
```

```
34
                        <moduleCompCode> .K </moduleCompCode>
35
                        <moduleTempCode> .K </moduleTempCode>
36
                        <imports> .Set </imports>
37
                        <classes> //static information about a
38
                            <class multiplicity="*">
39
                                <className> .K </className>
40
                            </class>
41
                        </classes>
42
                    </module>
43
               </modules>
44
           </T>
45
46 endmodule
```

# Appendix C

# haskell-preprocessing.k

```
1 //
 2 requires "haskell-syntax.k"
3 requires "haskell-configuration.k"
 5 module HASKELL-PREPROCESSING
 6
       imports HASKELL-SYNTAX
       imports HASKELL-CONFIGURATION
 8
9
       //USER DEFINED LIST
10
       //definition of ElemList
11
12
       //syntax KItem ::= ElemList
13
       syntax ElemList ::= List{Element,","} [strict]
14 //
      syntax Int ::= lengthOfList(ElemList) [function]
15
16 //
        rule lengthOfList(.ElemList) => 0
17 //
         rule lengthOfList(val(K:K),L:ElemList) => 1 +Int
      lengthOfList(L)
18 //
         rule lengthOfList(valValue(K:K),L:ElemList) => 1 +Int
      lengthOfList(L)
19
20
       syntax Element ::= val(K) [strict]
       syntax ElementResult ::= valValue(K)
22
       syntax Element ::= ElementResult
23
       syntax KResult ::= ElementResult
24
       rule val(K:KResult) => valValue(K) [structural]
25
       //form ElemList
27 //
         syntax ElemList ::= formElemList(K) [function]
28
29
       //CONVERT ~> TO List
30
       //list convert
31 //
        syntax List ::= convertToList(K) [function]
```

```
32 //
         rule convertToList(.K) => .List
33 //
         rule convertToList(A:KItem ~> B:K) => ListItem(A)
      convertToList(B)
34
35
36
       syntax KItem ::= dealWithImports(K,K)
37
38
       rule <k> 'modListSingle('module(A:K,, B:K)) =>
           dealWithImports(A,B) ...</k>
39
40
       (.Bag =>
             <module>... //DOT DOT DOT MEANS OVERWRITE ONLY SOME
41
                 OF THE DEFAULTS
42
               <moduleName> A </moduleName>
          ...</module>
43
44
       )
45
46
       rule <k> 'modList('module(A:K,, B:K),, C:K) =>
          dealWithImports(A,B) ~> C ...</k>
47
48
       (.Bag =>
             <module>... //DOT DOT DOT MEANS OVERWRITE ONLY SOME
49
                 OF THE DEFAULTS
               <moduleName> A </moduleName>
50
51
          ...</module>
52
      )
53
         rule dealWithImports(Mod:K, A:K) => callInit(A)
54 //
55
         rule <k> dealWithImports(Mod:K, A:K) => callInit(A) ...</k</pre>
57
58
       rule <k> dealWithImports(Mod:K, 'bodyimpandtop(A:K,, B:K))
           => .K ...</k>
           <importTree> L:List => L importListConvert(Mod, A) 
59
               importTree>
60
           <recurImportTree> L:List => L importListConvert(Mod, A)
               </recurImportTree>
61
62
           <moduleName> Mod </moduleName>
63
           <imports> S:Set (.Set => SetItem(A)) </imports>
           <moduleTempCode> OldTemp:K => B </moduleTempCode>
64
65
```

```
66
       rule <k> dealWithImports(Mod:K, 'bodyimpdecls(A:K)) => .K
          ...</k>
67
          <importTree> L:List => L importListConvert(Mod, A) 
              importTree>
68
          <recurImportTree> L:List => L importListConvert(Mod, A)
              </recurImportTree>
69
70
          <moduleName> Mod </moduleName>
71
          <imports> S:Set (.Set => SetItem(A)) </imports>
72
73 //
        rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) =>
      callInit(A) ...</k>
74
       rule <k> dealWithImports(Mod:K, 'bodytopdecls(B:K)) => .K
          ...</k>
75
76
          <moduleName> Mod </moduleName>
77
          <moduleTempCode> OldTemp:K => B </moduleTempCode>
78
79
       //importlist convert
80
       syntax List ::= importListConvert(K,K) [function]
81
       syntax KItem ::= impObject(K,K)
82
83
       rule importListConvert(Name:K, 'impDecls(A:K,, Rest:K)) =>
          importListConvert(Name, A) importListConvert(Name, Rest)
84
       rule importListConvert('moduleName(Name:K), 'impDecl(A:K,,
          Modid:K,, C:K,, D:K)) => ListItem(impObject(Name, Modid))
85
       rule importListConvert(Name:K, .ImpDecls) => .List
86
88
89
       /*NEW TODO ALGORITHM
90 1. Construct tree for module inclusion
91 2. Check tree for cycles
92 3. Go to each leaf and recursively go up the tree and build
      alpha* and beta* for the types of the module and the children
93 (and specify scoping) (desugar the scope so that each type
      specifies the scope) */
94
95
       syntax KItem ::= "checkImportCycle"
96
       syntax KItem ::= "recurseImportTree"
97
98 /*
       rule <k> performNextChecks
```

```
99
                 => checkUseVars
100
                     ~> (checkLabelUses
101
                     ~> (checkBlockAddress(.K)
102
                     ~> (checkNoNormalBlocksHavingLandingpad(.K, TNS
103
                     ~> (checkAllExpBlocksHavingLandingpad(.K, TES)
104
                     ~> (checkAllExpInFromInvoke(.K, TES)
105
                     ~> (checkLandingpad
106
                     ~> checkLandingDomResumes)))))) ...</k> */
107
108
        rule <k> startImportRecursion => checkImportCycle
109
                                           ~> (recurseImportTree) ...
                                              k>
110
111
        syntax KItem ::= cycleCheck(K, Map, List, List) [function] //
            current node, map of all nodes to visited or not, stack,
            graph
112
        syntax Map ::= createVisitMap(List, Map) [function] //graph,
            visitmap
113
        syntax KItem ::= getUnvisitedNode(K, K, Map) [function] //
            visitmap
114
        syntax List ::= getNodeNeighbors(K, List) [function] //
            visitmap
115
116
        rule <k> checkImportCycle
117
                 => cycleCheck(.K,createVisitMap(I, .Map),.List,I)
                     ...</k>
             <importTree> I:List </importTree>
118
119
             <impTreeVMap> .Map => createVisitMap(I, .Map) 
                 impTreeVMap>
120
121
        syntax KItem ::= "visited"
122
        syntax KItem ::= "unvisited"
123
        syntax KItem ::= "none"
124
125
        rule createVisitMap(ListItem(impObject(A:K,B:K)) Rest:List,
           M:Map)
126
                 => createVisitMap(Rest, M[A <- unvisited][B <-
                     unvisited])
127
        rule createVisitMap(.List, M:Map) => M
128
129
        rule getUnvisitedNode(.K, .K, .Map) => none
130
        rule getUnvisitedNode(.K, .K, (A:K |-> B:K) M:Map)
```

```
131
               => getUnvisitedNode(A, B, M)
132
        rule getUnvisitedNode(A:KItem, unvisited, M:Map) => A
133
        rule getUnvisitedNode(A:KItem, visited, M:Map)
                => getUnvisitedNode(.K, .K, M)
134
135
136
137
138
        rule getNodeNeighbors(Node:K,.List) => .List
139
        rule getNodeNeighbors(.K, Rest:List) => .List
140
141
        rule getNodeNeighbors(Node:KItem,ListItem(impObject(Node,B:
            KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
            ListItem(B)
142
        rule getNodeNeighbors (Node:KItem, ListItem(impObject(A:KItem,
            B:KItem)) Rest:List) => getNodeNeighbors(Node, Rest)
143
             requires Node =/=K A
144
145
        rule cycleCheck(none, M:Map, .List, L:List) => .K
146
147
        rule cycleCheck(.K, M:Map, .List, I:List) => cycleCheck(
            getUnvisitedNode(.K, .K, M), M, .List, I)
148
        rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:List)
            => cycleCheck(Node, M, S, I)
        rule cycleCheck(Node:K, M:Map, S:List, I:List)
149
150
                      => cycleCheck(.K, M[Node <- visited],
                         getNodeNeighbors(Node, I) S, I)
151
             requires Node =/=K .K andBool Node =/=K none
152
        rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) =>
           cycleCheck (Node, M, S, I)
153
             requires S = /=K .List
154
155 /*
156
        rule cycleCheck(A:K, .K, .K, I:K) => cycleCheck(A,
            createVisitMap(I,.Map),.List,I)
157
158
159
160
        rule cycleCheck(Node:K, M:Map, S:List, I:K) => cycleCheck(.K
            , M[Node <- visited], getNodeNeighbors(Node,I) S, I)</pre>
161
162
        rule cycleCheck(.K, M:Map, ListItem(Node:K) S:List, I:K) =>
            cycleCheck (Node, M, S, I)
```

```
163
       //rule cycleCheck(.K, M:Map, .K, ListItem(impObject(A:K,B:K)
           ) Rest:List) => cycleCheck(ListItem(impObject(A:K,B:K))
           Rest:List)
164 */
165
167
168 //COPY IMPORT GRAPH, NEED SECOND GRAPH FOR RECURSING, ADDITIONAL
        GRAPH FOR SELECTING IMPORTS FOR ALPHA* AND BETA*
169 //DFS for leaf
170 //acquire alpha and beta for leaf
171 //merge alpha and beta with imports to produce alpha* and beta*
172 //perform checks
173 //perform inferencing
174 //insert alpha* and beta* into importing modules
175 //remove all edges pointing to leaf
176
177
       syntax KItem ::= "leafDFS"
178
       syntax KItem ::= "getAlphaAndBeta"
179
       syntax KItem ::= "getAlphaBetaStar"
180
       syntax KItem ::= "performIndividualChecks"
       syntax KItem ::= "performIndividualInferencing"
181
182
       syntax KItem ::= "insertAlphaBetaStar"
183
       syntax KItem ::= "removeAllEdges"
184
       syntax KItem ::= "seeIfFinished"
185
186
       rule <k> recurseImportTree => leafDFS
187
                                    ~> (getAlphaAndBeta
188
                                    //~> (getAlphaBetaStar
189
                                    ~> (
                                       performIndividualInferencing
                                       ))...</k>
190
191 //rule <k> dealWithImports(Mod:K, 'bodytopdecls(A:K)) =>
       callInit(A) ...</k>
192
193 //
        rule <k> leaf
194 //
                  => cycleCheck(.K,createVisitMap(I, .Map),.List,I)
        ...</k>
195 //
              <importTree> I:List </importTree>
196
```

```
198
199
       syntax KItem ::= returnLeafDFS(K,List,Map) [function] //
          current node, map of all nodes to visited or not, stack,
          graph
200
       syntax KItem ::= innerLeafDFS(K,List) [function]
201
       syntax KItem ::= loadModule(K)
202
203
       rule <k> leafDFS
204
               => returnLeafDFS(.K,I,M) ...</k>
205
            <recurImportTree> I:List </recurImportTree>
206
            <impTreeVMap> M:Map </impTreeVMap>
207
208
       rule returnLeafDFS(.K,ListItem(impObject(Node:KItem,B:KItem)
          ) I:List, M:Map) => returnLeafDFS(B, I, M)
209
       rule returnLeafDFS(Node:KItem,I:List,M:Map) => returnLeafDFS
          (innerLeafDFS(Node, I), I, M)
210
            requires innerLeafDFS(Node, I) =/=K none
211
       rule returnLeafDFS(Node:KItem, I:List, M:Map) => loadModule(
212
            requires innerLeafDFS(Node, I) == K none
213
214
       rule innerLeafDFS (Node:KItem, ListItem (impObject (Node, B:KItem
          )) I:List) => B
215
       rule innerLeafDFS(Node:KItem,ListItem(impObject(A:KItem,B:
          KItem)) I:List) => innerLeafDFS(Node, I)
216
            requires Node =/=K A
217
       rule innerLeafDFS(Node:KItem,.List) => none
218 //
         returnLeafDFS(Node:KItem,ListItem(impObject(Node,B:KItem))
        I:List,M:Map) => returnLeafDFS(B,I,M)
219
220
222
223
       //call before Checker Code
224 //
         rule <k> callInit(S:K) => initPreModule(S) ...</k>
225 //
              <tempModule> A:K => S </tempModule>
226
227
       rule <k> loadModule(S:KItem) => .K ...</k>
228
            <tempModule> A:K => S </tempModule>
229
```

```
230
       rule <k> getAlphaAndBeta => initPreModule(Code) ...</k>
231
            <tempModule> Mod:KItem </tempModule>
232
233
            <moduleName> 'moduleName (Mod) </moduleName>
234
            <moduleTempCode> Code:KItem </moduleTempCode>
235
236
238
       //get alpha and beta
239
       syntax KItem ::= Module(K, K)
240
       syntax KItem ::= preModule(K,K) //(alpha, T)
241
242
       // STEP 1 CONSTRUCT T AND ALPHA
243
       // alpha = type
244
       // T = newtype and data, temporary data structure
245
246
       syntax KItem ::= initPreModule(K) [function]
247
       syntax KItem ::= getPreModule(K, K) [function] //(Current
           term, premodule)
248
       syntax KItem ::= makeT (K,K,K,K)
249
250
       syntax KItem ::= fetchTypes (K,K,K,K)
251
252
       syntax List ::= makeInnerT (K,K,K) [function] //LIST
253
       syntax List ::= getTypeVars(K) [function] //LIST
254
255
       syntax KItem ::= getCon(K) [function]
256
       syntax List ::= getArgSorts(K) [function] //LIST
257
258
       syntax KItem ::= AList(K)
259
       syntax KItem ::= AObject(K,K) //(1st -> 2nd) map without
           idempotency
260
       syntax KItem ::= ModPlusType(K,K)
261
262
       syntax KItem ::= TList(K) //list of T objects for every new
           type introduced by data and newtype
263
       syntax KItem ::= TObject(K,K,K,K) //(module name, type name,
            entire list of poly type vars, list of inner T pieces)
264
       syntax KItem ::= InnerTPiece(K,K,K,K,K) //(type constructor,
            poly type vars, argument sorts, entire constr block,
           type name)
265
```

```
266 //
          rule initPreModule('module(I:ModuleName,, J:K)) =>
       getPreModule(J, preModule(AList(.List), TList(.List)))
          rule initPreModule('moduleExp(I:ModuleName,, L:K,, J:K))
267 //
       => getPreModule(J,preModule(AList(.List),TList(.List)))
268
          rule initPreModule('moduleBody(J:Body)) => getPreModule(J,
       preModule(AList(.List), TList(.List)))
269
270
        rule initPreModule(J:K) => getPreModule(J,preModule(AList(.
           List), TList(.List)))
271
272
        rule getPreModule('bodytopdecls(I:K), J:K) => getPreModule(I
273
        rule getPreModule('topdeclslist('type(A:K,, B:K),, Rest:K),J
            :K) => fetchTypes(A,B,Rest,J) //constructalpha
274
275
276
        rule getPreModule ('topdeclslist ('data(A:K,, B:K,, C:K,, D:K)
            ,, Rest:K), J:K) => makeT(B,C,Rest,J)
277
        rule getPreModule ('topdeclslist ('newtype (A:K,, B:K,, C:K,, D
            :K),, Rest:K), J:K) => makeT(B,C,Rest,J)
278
279
280
        rule getPreModule('topdeclslist('topdecldecl(A:K),, Rest:K),
            J:K) => getPreModule(Rest, J)
281
        rule getPreModule ('topdeclslist ('class (A:K,, B:K,, C:K,, D:K
            ),, Rest:K),J:K) => getPreModule(Rest,J)
282
        rule getPreModule('topdeclslist('instance(A:K,, B:K,, C:K,,
           D:K),, Rest:K),J:K) => getPreModule(Rest,J)
283
        rule getPreModule ('topdeclslist ('default (A:K,, B:K,, C:K,, D
            :K),, Rest:K),J:K) => getPreModule(Rest,J)
284
        rule getPreModule ('topdeclslist ('foreign (A:K,, B:K,, C:K,, D
            :K),, Rest:K),J:K) => getPreModule(Rest,J)
285
        rule getPreModule(.TopDecls, J:K) => J
286
287
        //rule getPreModule('module(I:ModuleName,L:K, J:K)) =>
           preModule(J)
288
289
        rule <k> fetchTypes('simpleTypeCon(I:TyCon,, H:TyVars), '
           atypeGTyCon(C:K), Rest:K, preModule(AList(M:List), L:K))
           => getPreModule(Rest, preModule(AList(ListItem(AObject(
           ModPlusType(ModName, I), C)) M), L)) ...</k>
290
             <tempModule> ModName: KItem </tempModule>
291
```

```
292
        rule <k> makeT('simpleTypeCon(I:TyCon,, H:TyVars), D:K, Rest
           :K, preModule(AList(M:List), TList(ListInside:List))) =>
           getPreModule(Rest, preModule(AList(M), TList(ListItem(
           TObject(ModName, I, H, makeInnerT(I, H, D))) ListInside)))
           ...</k>
293
             <tempModule> ModName: KItem </tempModule>
294
295
        rule makeInnerT(A:K,B:K,'nonemptyConstrs(C:K)) => makeInnerT
           (A,B,C)
296
        rule makeInnerT(A:K,B:K,'singleConstr(C:K)) => ListItem(
           InnerTPiece(getCon(C), getTypeVars(C), getArgSorts(C), C, A))
297
        rule makeInnerT(A:K,B:K,'multConstr(C:K,, D:K)) => ListItem(
           InnerTPiece(getCon(C), getTypeVars(C), getArgSorts(C), C, A))
            makeInnerT(A,B,D)
298
299
        rule getTypeVars('constrCon(A:K,, B:K)) => getTypeVars(B)
300
        rule getTypeVars('optBangATypes(A:K,, Rest:K)) =>
           getTypeVars(A) getTypeVars(Rest)
301
        rule getTypeVars('optBangAType('emptyBang(.KList),, Rest:K))
            => getTypeVars(Rest)
302
        rule getTypeVars('atypeGTyCon(A:K)) => .List
303
        rule getTypeVars('atypeTyVar(A:K)) => ListItem(A)
304
        rule getTypeVars(.OptBangATypes) => .List
305
306
        //rule getCon('emptyConstrs()) => .K
307
        //rule getCon('nonemptyConstrs(A:K)) => getCon(A)
308
        rule getCon('constrCon(A:K,, B:K)) => A
309
310
        //rule getArgSorts('constrCon(A:K,, B:K)) => B
311
        rule getArgSorts('constrCon(A:K,, B:K)) => getArgSorts(B)
312
        rule getArgSorts('optBangATypes(A:K,, Rest:K)) =>
           getArgSorts(A) getArgSorts(Rest)
313
        rule getArgSorts('optBangAType('emptyBang(.KList),, Rest:K))
            => getArgSorts(Rest)
314
        rule getArgSorts('atypeGTyCon(A:K)) => ListItem(A)
315
        rule getArgSorts('atypeTyVar(A:K)) => .List
316
        rule getArgSorts(.OptBangATypes) => .List
317
319
320
        rule <k> preModule(A:K,T:K) => startTTransform ...</k>
321
             <tempAlpha> OldAlpha:K => A </tempAlpha>
```

```
322
            <tempT> OldT:K => T </tempT>
323
325
326
       // STEP 2 PERFORM CHECKS
327
328
        syntax KItem ::= "error"
329
330
        syntax KItem ::= "startChecks"
331
        syntax KItem ::= "checkNoSameKey"
332
           //Keys of alpha and keys of T should be unique
        syntax KItem ::= "checkTypeConsDontCollide"
333
334
           //Make sure typeconstructors do not collide in T
        syntax KItem ::= "makeAlphaMap"
335
336
           //make map for alpha
337
        syntax KItem ::= "checkAlphaNoLoops"
338
           //alpha check for no loops
339
           //check alpha to make sure that everything points to a T
        syntax KItem ::= "checkArgSortsAreTargets"
340
341
              //Make sure argument sorts [U] [W,V] are in the set
                  of keys of alpha and targets of T, (keys of T)
342
        syntax KItem ::= "checkParUsed"
343 //NEED TO CHECK all the polymorphic parameters from right appear
        on left. RIGHT SIDE ONLY
344 //NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY
345
346 //
         rule <k> preModule(A:K,T:K) => startChecks ...</k>
347 //
              <tempAlpha> OldAlpha:K => A </tempAlpha>
348 //
              <tempT> OldT:K => T </tempT>
349
350
351 /*
        rule <k> performNextChecks
352
                => checkUseVars
353
                   ~> (checkLabelUses
354
                   ~> (checkBlockAddress(.K)
355
                   ~> (checkNoNormalBlocksHavingLandingpad(.K, TNS
                      -Set TES)
356
                   ~> (checkAllExpBlocksHavingLandingpad(.K, TES)
                   ~> (checkAllExpInFromInvoke(.K, TES)
357
358
                   ~> (checkLandingpad
                   ~> checkLandingDomResumes))))) ...</k> */
359
360
```

```
361
        rule <k> startChecks
362
                  => checkNoSameKey
363
                     ~> (checkTypeConsDontCollide
364
                     ~> (makeAlphaMap
365
                     ~> (checkAlphaNoLoops
                     ~> (checkArgSortsAreTargets
366
367
                     ~> (checkParUsed))))) ...</k>
368
369
        rule <k> checkTypeConsDontCollide
370
                  => tyConCollCheck(T,.List,.Set) ...</k>
371
             <tempT> T:K </tempT>
372
373
        //syntax KItem ::= tChecker(K) [function]
374
        syntax KItem ::= tyConCollCheck(K,K,K) [function] //(TList,
            List of Tycons, Set of Tycons)
375
        syntax KItem ::= lengthCheck(K,K) [function]
        //syntax KItem ::= tyConCollCheck(K,K,K) [function]
376
377
        //syntax K ::= innerCollCheck(K) [function]
        //syntax K ::= tyConCollCheckPasser(K, K) [function]
378
379
380
        //rule tChecker(preModule(Alpha:Map,T:K,Mod:K)) =>
            tyConCollCheck(innerCollCheck(T), preModule(Alpha, T, Mod))
381
382
        //rule tyConCollCheck(.K,preModule(Alpha:Map,H:K,Mod:K)) =>
            tyConCollCheck(innerCollCheck(H), preModule(Alpha, H, Mod))
383
384
        rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:
            K, ListItem(InnerTPiece(Ty:K, E:K, F:K, H:K, G:K)) Inners:List
            )) Rest:List),J:List,D:Set) =>
385
                         tyConCollCheck(TList(ListItem(TObject(
                            ModName, A, B, Inners)) Rest), ListItem(Ty) J
                             , SetItem(Ty) D)
386
        rule tyConCollCheck(TList(ListItem(TObject(ModName:K, A:K,B:
            K,.List)) Rest:List),J:List,D:Set) =>
387
                         tyConCollCheck(TList(Rest),J,D)
388
        rule tyConCollCheck(TList(.List), J:List, D:Set) =>
389
                         lengthCheck(size(J), size(D))
390
391
        rule lengthCheck(A:Int, B:Int) => .K
392
                         requires A ==Int B
393
394
        rule lengthCheck(A:Int, B:Int) => error
395
                         requires A =/=Int B
```

```
396
397
        //rule tyConCollCheck(TList(TObject(A:K,B:K,C:K) ~> Rest:K),
            J:K) => tyConCollCheckPasser(TList(innerCollCheck(TObject
            (A:K,B:K,C:K)) ~> Rest:K),J:K)
398
        syntax KItem ::= keyCheck(K,K,K,K) [function] //(Alpha, T,
            List of names, Set of names)
399
400
        rule <k> checkNoSameKev
401
                  => keyCheck(A, T, .Set, .List) ...</k>
402
              <tempAlpha> A:K </tempAlpha>
403
             <tempT> T:K </tempT>
404
        //rule <k> checkAlphaNoSameKey
405
        //
                    => akeyCheck(.K, .Set) ...</k>
406
407
        rule keyCheck(AList(ListItem(AObject(A:K,B:K)) C:List), T:K,
             D:Set, G:List) => keyCheck(AList(C), T, SetItem(A) D,
            ListItem(A) G)
408
        rule keyCheck(AList(.List), TList(ListItem(TObject(ModName:K
            , A:K,B:K,C:K)) Rest:List), D:Set, G:List) => keyCheck(
            AList(.List), TList(Rest), SetItem(A) D, ListItem(A) G)
409
        rule keyCheck(AList(.List), TList(.List), D:Set, G:List) =>
            lengthCheck(size(G), size(D))
410
411
412
        syntax KItem ::= makeAlphaM(K,K) [function] //(Alpha,
            AlphaMap)
413
        syntax KItem ::= tAlphaMap(K) //(AlphaMap) temp alphamap
414
415
        rule <k> makeAlphaMap
416
                  => makeAlphaM(A, .Map) ...</k>
417
              <tempAlpha> A:K </tempAlpha>
418
419
        rule makeAlphaM(AList(ListItem(AObject(A:K,B:K)) C:List), M:
            Map) => makeAlphaM(AList(C), M[A <- B])</pre>
420
        rule makeAlphaM(AList(.List), M:Map) => tAlphaMap(M)
421
422
        rule \langle k \rangle tAlphaMap(M:K) => .K ...\langle k \rangle
423
              <tempAlphaMap> OldAlphaMap:K => M </tempAlphaMap>
424
425 //
          syntax KItem ::= tkeyCheck(K,K,K) [function] //(T,List of
       T, Set of T)
426
427 //
         rule <k> checkTNoSameKey
```

```
428 //
                   => tkeyCheck(T, .Set, T) ...</k>
429 //
               <tempT> T:K </tempT>
430
431 //
          rule tkeyCheck(TList(ListItem(TObject(A:K,B:K,C:K)) Rest:
       List), D:Set, G:K) => tkeyCheck(TList(Rest), SetItem(A) D, G)
          rule tkeyCheck(TList(.List), D:Set, TList(G:List)) =>
432 //
       lengthCheck(size(G), size(D))
433
434
        syntax KItem ::= aloopCheck(K,K,K,K,K,K,K) [function] //(
           Alpha, List of Alpha, Set of Alpha, CurrNode, lengthcheck, T,
           BigSet)
435
436
        rule <k> checkAlphaNoLoops
437
                 => aloopCheck(A,.List,.Set,.K,.K,T,.Set) ...</k>
438
             <tempAlphaMap> A:K </tempAlphaMap>
439
             <tempT> T:K </tempT>
440
441
        //aloopCheck set and list to check cycles
442
        rule aloopCheck(Alpha:Map (A:KItem | -> B:KItem), D:List, G:
            Set, .K, .K, T:K, S:Set) => aloopCheck(Alpha, ListItem(B)
           ListItem(A) D, SetItem(B) SetItem(A) G, B, .K,T,S)
443
        rule aloopCheck(Alpha:Map (H |-> B:KItem), D:List, G:Set, H:
           KItem, .K,T:K,S:Set) => aloopCheck(Alpha, ListItem(B) D,
           SetItem(B) G, B, K, T, S)
444
445
        rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
           Set) => aloopCheck(Alpha, .List, .Set, .K, lengthCheck(
           size(G), size(D)), T, G S) //type rename loop ERROR
446
             requires (notBool H in keys(Alpha)) andBool (H in
                 typeSet(T, .Set) orBool H in S)
447
448
        rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K,T:K,S:
           Set) => error //terminal alpha rename is not in T ERROR
449
             requires (notBool H in keys(Alpha)) andBool (notBool (H
                  in typeSet(T, .Set) orBool H in S))
450
451
452
        syntax Set ::= typeSet(K,K) [function] //(K, KSet)
453
        rule typeSet(TList(ListItem(TObject(ModName:K, A:K,B:K,C:K))
             Rest:List), D:Set) => typeSet(TList(Rest), SetItem(A) D)
454
        rule typeSet(TList(.List), D:Set) => D
455
```

```
456 //
          rule aloopCheck(Alpha:Map, D:List, G:Set, H:KItem, .K) =>
       keys(Alpha) ~> H
457 //
               requires notBool H in keys (Alpha)
458
459
        rule aloopCheck(.Map, .List, .Set, .K, .K, T:K, S:Set) => .K
460 //
          rule aloopCheck(AList(Front:List ListItem(AObject(H,B:K))
       C:List), D:List, G:Set, H:ConId) => aloopCheck(AList(C:List),
        ListItem(B) D, SetItem(B) G, B)
461
462
463 //
          syntax KItem ::= TList(K) //list of T objects for every
       new type introduced by data and newtype
          syntax KItem ::= TObject(K,K,K) //(type name, entire list
464 //
       of poly type vars, list of inner T pieces)
465 //
          syntax KItem ::= InnerTPiece(K, K, K, K, K) //(type
       constructor, poly type vars, argument sorts, entire constr
       block, type name)
466
467 //Make sure argument sorts [U] [W,V] are in the set of keys of
       alpha and targets of T, (keys of T)
468
469
        syntax KItem ::= argSortCheck(K,K,K) [function] //(T,
           AlphaMap)
470
471
        rule <k> checkArgSortsAreTargets
472
                 => argSortCheck(T,A,typeSet(T,.Set)) ...</k>
473
             <tempAlphaMap> A:K </tempAlphaMap>
474
             <tempT> T:K </tempT>
475
476
        rule argSortCheck(TList(ListItem(TObject(ModName:K, A:K,B:K,
           ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest
            :List, E:K, F:K)) InnerRest:List)) TListRest:List), AlphaMap
            :Map, Tset:Set) => argSortCheck(TList(ListItem(TObject(
           ModName, A, B, ListItem(InnerTPiece(C, D, ArgsRest, E, F))
            InnerRest)) TListRest), AlphaMap, Tset)
477
             requires ((Arg in keys(AlphaMap)) orBool (Arg in Tset))
478
479
        rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(C:K,D:K,ListItem(Arg:KItem) ArgsRest
            :List, E:K, F:K)) InnerRest:List)) TListRest:List), AlphaMap
            :Map, Tset:Set) => error
480
             requires (notBool ((Arg in keys(AlphaMap)) orBool (Arg
                 in Tset)))
```

```
481
482
        rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(C:K,D:K,.List,E:K,F:K)) InnerRest:
           List)) TListRest:List), AlphaMap:Map, Tset:Set) =>
            argSortCheck (TList(ListItem(TObject(ModName, A, B, InnerRest
            )) TListRest), AlphaMap, Tset)
483
484
        rule argSortCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.
           List)) TListRest:List), AlphaMap:Map, Tset:Set) =>
            argSortCheck(TList(TListRest), AlphaMap, Tset)
485
486
        rule argSortCheck(TList(.List),AlphaMap:Map,Tset:Set) => .K
487
488 //NEED TO CHECK all the polymorphic parameters from right appear
        on left. RIGHT SIDE ONLY
489 //NEED TO CHECK UNIQUENESS FOR POLY PARAM ON LEFT SIDE ONLY
490
491
        syntax KItem ::= parCheck(K,K) [function] //(T,AlphaMap)
492
        syntax KItem ::= makeTyVarList(K,K,K) [function] //(TyVars,
           NewList)
493
        syntax KItem ::= lengthRet(K,K,K) [function]
494
495
        rule <k> checkParUsed
496
                 => parCheck(T,.K) ...</k>
497
             <tempT> T:K </tempT>
498
499
        //rule makeParLists(TList(ListItem(TObject(A:K, ListItem(Arg:
           KItem) PolyList:List,C:K)) Rest:List),Tlist:List,Tset:Set
            ) => makeParLists(TList(ListItem(TObject(A, PolyList, C))
            Rest),ListItem(Arg) Tlist,SetItem(Arg) Tset)
500
        rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,C:K))
             Rest:List),.K) => parCheck(TList(ListItem(TObject(
           ModName, A:K, B:K, C:K)) Rest:List), makeTyVarList(B, .List, .
            Set))
501
502
        rule parCheck (TList (ListItem (TObject (ModName: K, A: K, B: K,
           ListItem(InnerTPiece(C:K,ListItem(Par:KItem) ParRest:List
            ,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
503
             parCheck(TList(ListItem(TObject(ModName, A, B, ListItem(
                 InnerTPiece(C,ParRest,D,E,F)) InnerRest)) Rest),
                 NewSet)
504
                requires Par in NewSet
505
```

```
506
        rule parCheck (TList (ListItem (TObject (ModName: K, A: K, B: K,
           ListItem (InnerTPiece (C:K, ListItem (Par:KItem) ParRest:List
           ,D:K,E:K,F:K)) InnerRest:List)) Rest:List),NewSet:Set) =>
            error
507
               requires notBool (Par in NewSet)
508
509
        rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,
           ListItem(InnerTPiece(C:K,.List,D:K,E:K,F:K)) InnerRest:
           List)) Rest:List), NewSet:Set) =>
510
             parCheck(TList(ListItem(TObject(ModName, A, B, InnerRest))
                 Rest), NewSet)
511
512
        rule parCheck(TList(ListItem(TObject(ModName:K,A:K,B:K,.List
           )) Rest:List), NewSet:Set) =>
513
             parCheck(TList(Rest), NewSet)
514
515
        rule parCheck(TList(.List), NewSet:Set) => .K
516
517
        rule makeTyVarList('typeVars(A:K,,Rest:K),NewList:List,
           NewSet:Set) => makeTyVarList(Rest, ListItem(A) NewList,
           SetItem(A) NewSet)
518
519
        rule makeTyVarList(.TyVars, NewList:List, NewSet:Set) =>
           lengthRet(size(NewList), size(NewSet), NewSet)
520
521
        rule lengthRet(A:Int, B:Int, C:K) => C
522
                        requires A ==Int B
523
524
        rule lengthRet(A:Int, B:Int, C:K) => error
525
                        requires A =/=Int B
526
527
        //rule argSortCheck(TList(ListItem(TObject(A:K,B:K,C:K)
528
530
531
        // STEP 3 Transform T into beta
532
533
        syntax KItem ::= "startTTransform"
534
        syntax KItem ::= "constructDelta"
535
        syntax KItem ::= "constructBeta"
536
537
        rule <k> startTTransform
```

```
538
                  => constructDelta
539
                     ~> (constructBeta) ...</k>
540
541
        rule <k> constructDelta
542
                  => makeDelta(T,.Map) ...</k>
543
              <tempT> T:K </tempT>
544
545
        syntax KItem ::= makeDelta(K,Map) [function] //(T,Delta)
546
        syntax KItem ::= newDelta(Map) //Delta
547
        syntax KItem ::= newBeta(Map) //beta
548
        syntax List ::= retPolyList(K,List) [function] //(T,Delta)
549
550
        rule makeDelta(TList(ListItem(TObject(ModName:K,A:K,Polys:K,
            C:K)) Rest:List),M:Map) =>
              makeDelta(TList(Rest), M[ModPlusType(ModName, A) <- size(</pre>
551
                 retPolyList(Polys,.List))])
        rule makeDelta(TList(.List),M:Map) => newDelta(M)
552
553
        rule retPolyList('typeVars(A:K,,Rest:K),NewList:List) =>
554
            retPolyList(Rest, ListItem(A) NewList)
555
        rule retPolyList(.TyVars,L:List) => L
556
557
        rule <k> newDelta(M:Map)
                  => .K ...</k>
558
559
              <tempDelta> OldDelta:K => M </tempDelta>
560
561
        rule <k> constructBeta
562
                  => makeBeta(T,.Map) ...</k>
563
              <tempT> T:K </tempT>
564
565
        syntax KItem ::= makeBeta(K, Map) [function] //(T, Beta, Delta)
566
567
        rule makeBeta(TList(ListItem(TObject(ModName:K,A:K,B:K,
            ListItem(InnerTPiece(Con:K,H:K,D:K,E:K,F:K)) InnerRest:
            List)) Rest:List), Beta:Map) =>
568
              makeBeta(TList(ListItem(TObject(ModName, A, B, InnerRest))
                  Rest), Beta[ModPlusType (ModName, Con) <- betaParser (E</pre>
                 ,B,A)])
569
        rule makeBeta (TList (ListItem (TObject (ModName: K, A: K, B: K, . List
            )) Rest:List),Beta:Map) =>
570
             makeBeta(TList(Rest), Beta)
571
        rule makeBeta(TList(.List), Beta:Map) =>
572
              newBeta(Beta)
```

```
573 //
          rule makeBeta (TList (ListItem (TObject (ModName: K, A: K, B: K,
       ListItem(InnerTPiece(C:K,H:K,D:K,E:K,F:K)) InnerRest:List))
       Rest:List),Beta:Map) =>
574 //
               makeBeta (TList (ListItem (TObject (ModName, A, B, InnerRest
       )) Rest), Beta)
575
576
        syntax KItem ::= betaParser(K, K, K) [function] //(Tree Piece,
           NewSyntax, Parameters, Constr)
577
        syntax Set ::= getTyVarsRHS(K,List) [function]
578
579
        syntax KItem ::= forAll(Set,K)
580
        syntax KItem ::= funtype(K,K)
581
582
        syntax Set ::= listToSet(List, Set) [function]
583
584
        rule listToSet(ListItem(A:KItem) L:List, S:Set) => listToSet
            (L, SetItem(A) S)
585
        rule listToSet(.List, S:Set) => S
586
587
588 //if optbangAtypes, need to see if first variable is a typecon
589 //if its a typecon then need to go into Delta and see the amount
        of parameters it has
590 //then count the number of optbangAtypes after the typecon
591
        rule betaParser('constrCon(A:K,, B:K), Par:K, Con:K) =>
            forAll(getTyVarsRHS(B,.List), betaParser(B, Par, Con))
592
        rule betaParser('optBangATypes('optBangAType('emptyBang(.
           KList),, 'atypeTyVar(Tyv:K)),, Rest:K), Par:K, Con:K) =>
            funtype(Tyv, betaParser(Rest, Par, Con))
593
        rule betaParser('optBangATypes('optBangAType('emptyBang(.
           KList),, 'baTypeCon(A:K,, B:K)),, Rest:K), Par:K, Con:K)
           => funtype('baTypeCon(A:K,, B:K), betaParser(Rest, Par,
           Con))
594
        rule betaParser('optBangATypes('optBangAType('emptyBang(.
           KList),, 'atypeGTyCon(Tyc:K)),, Rest:K), Par:K, Con:K) =>
             funtype(Tyc, betaParser(Rest, Par, Con))
595
        rule betaParser(.OptBangATypes, Par:K, Con:K) => '
            simpleTypeCon(Con,, Par)
596 //
          rule betaParser('optBangATypes('optBangAType('emptyBang(.
       KList),, 'atypeGTyCon(Tyc:K)),, Rest:KItem)) => getTypeVars(A
       ) getTypeVars(Rest)
597 //
          rule getTypeVars('optBangAType('emptyBang(.KList),, Rest:K
       )) => getTypeVars(Rest)
```

```
598 //
          rule getTypeVars('atypeGTyCon(A:K)) => .List
599 //
          rule getTypeVars('atypeTyVar(A:K)) => ListItem(A)
600 //
          rule getTypeVars(.OptBangATypes) => .List
601
602
        rule getTyVarsRHS(.OptBangATypes,Tylist:List) => listToSet(
           Tylist, .Set)
603
604
        rule <k> newBeta(M:Map)
605
                => .K ...</k>
606
             <tempBeta> OldBeta:K => M </tempBeta>
607
609
610 //
          syntax KItem ::= "insertAlphaBetaStar"
611
612
        syntax KItem ::= insertABRec(K,List)
613
        syntax KItem ::= insertAB(K)
614
615
        rule <k> insertAlphaBetaStar => insertABRec(Mod, Imp) ...</k</pre>
616
             <tempModule> Mod:KItem </tempModule>
617
             <importTree> Imp:List </importTree>
618
619
        rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem,
           Node)) I:List) => insertAB(B) ~> insertABRec(Node, I)
           ...</k>
620
621
        rule <k> insertABRec(Node:KItem, ListItem(impObject(B:KItem,
           C:KItem)) I:List) => insertABRec(Node, I) ...</k>
622
                 requires Node =/=K C
623
624
        rule \langle k \rangle insertAB(B) => .K ...\langle k \rangle
625
626
             <tempAlphaStar> Alph:KItem </tempAlphaStar>
627
             <tempBetaStar> Bet:KItem </tempBetaStar>
628
629
             <moduleName> 'moduleName(B) </moduleName>
630
             <moduleImpAlphas> ImpAlphas:List => ListItem(Alph)
                ImpAlphas </moduleImpAlphas>
631
             <moduleImpBetas> ImpBetas:List => ListItem(Bet)
                ImpBetas </moduleImpBetas>
632
```

endmodule

# Appendix D

# haskell-type-inferencing.k

```
1 requires "haskell-syntax.k"
2 requires "haskell-configuration.k"
3 requires "haskell-preprocessing.k"
5 module HASKELL-TYPE-INFERENCING
6
       imports HASKELL-SYNTAX
       imports HASKELL-CONFIGURATION
8
       imports HASKELL-PREPROCESSING
9
10
       syntax KItem ::= "Bool" //Boolean
11
12
       // STEP 4 Type Inferencing
13
       syntax KItem ::= inferenceShell(K) [function]//Input,
           AlphaMap, Beta, Delta, Gamma
14
       //syntax KItem ::= typeInferenceFun(K, Map, Map, Map, Map, K, K) [
           function]//Input, Alpha, Beta, Delta, Gamma
15
       //syntax KItem ::= typeInferenceFun(Map,K,K) //Gamma,
           Expression, Guessed Type
16
       syntax Map ::= genGamma(K, Map, K) [function] //Apatlist,
           Gamma Type
17
       syntax KItem ::= genLambda(K,K) [function]
18
       syntax KItem ::= guessType(Int)
        syntax KItem ::= lambdaReturn(K,K,K)
20
       syntax KItem ::= freshInstance(K, Int) [function]
21
       syntax Int ::= paramSize(K) [function]
22
23
       syntax KItem ::= mapBag(Map)
25
       syntax KResult ::= mapBagResult(Map)
26
27
       syntax Map ::= gammaSub(Map, Map, Map) [function]//
           substitution, gamma
28
```

```
29
       rule <k> performIndividualInferencing => inferenceShell(Code
           ) ...</k>
30
            <tempModule> Mod:KItem </tempModule>
31
32
            <moduleName> 'moduleName (Mod) </moduleName>
33
            <moduleTempCode> Code:KItem </moduleTempCode>
34
35
       rule inferenceShell('topdeclslist('type(A:K,, B:K),, Rest:K)
           ) =>
36
            inferenceShell(Rest) //constructalpha
37
       rule inferenceShell('topdeclslist('data(A:K,, B:K,, C:K,, D:
          K),, Rest:K)) =>
38
            inferenceShell(Rest)
39
       rule inferenceShell('topdeclslist('newtype(A:K,, B:K,, C:K,,
            D:K),, Rest:K)) =>
40
            inferenceShell(Rest)
41
       rule inferenceShell('topdeclslist('class(A:K,, B:K,, C:K,, D
           :K),, Rest:K)) =>
42
            inferenceShell(Rest)
       rule inferenceShell('topdeclslist('instance(A:K,, B:K,, C:K
43
           ,, D:K),, Rest:K)) =>
            inferenceShell(Rest)
44
45
       rule inferenceShell('topdeclslist('default(A:K,, B:K,, C:K,,
            D:K),, Rest:K)) =>
46
            inferenceShell(Rest)
       rule inferenceShell('topdeclslist('foreign(A:K,, B:K,, C:K,,
47
            D:K),, Rest:K)) =>
48
            inferenceShell(Rest)
49
50
       rule inferenceShell('topdeclslist('topdecldecl(A:K),, Rest:K
51
            typeInferenceFun(.ElemList, .Map,A,quessType(0)) ~>
                inferenceShell(Rest)
52
53
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, '
54
           declFunLhsRhs(Fn:K,, Lhsrhs:K), Guess:K) =>
            typeInferenceFun(.ElemList, Gamma, Lhsrhs, Guess) ...</
55
56
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, '
           eqExpOptDecls(Ex:K,, Optdecls:K), Guess:K) =>
57
            typeInferenceFun(.ElemList, Gamma, Ex, Guess) ...</k>
58
```

```
59
       //T-App
60
       //rule typeInferenceFun('aexpQVar(Var:K), Alpha:Map, Beta:
          Map, Delta:Map, (Var |-> Sigma:K) Gamma:Map,.K,.K) =>
           Sigma
61
       //Gamma Proves x:phi(tau) if Gamma(x) = \forall alpha_1,
           ..., alpha n . tau
62
       //where phi replaces all occurrences of alpha_1, ...,
           alpha_n by monotypes tau_1, ..., tau_n
63
64
       rule <k> typeInferenceFun(.ElemList, (Var |-> Type:K) Gamma:
          Map, 'aexpQVar(Var:K), Guess:KItem)
65
             => mapBagResult (uniFun (ListItem (uniPair (Guess,
                 freshInstance(Type, TypeIt))))) ...</k> //Variable
66
            <typeIterator> TypeIt:Int => TypeIt +Int paramSize(Type
                ) </typeIterator>
67
68
       //rule typeInferenceFun('aexpGCon(Gcon:K), Alpha:Map, (Gcon
           |-> Sigma:K) Beta:Map, Delta:Map, Gamma:Map,.K,.K) =>
           Sigma //T-App
69
       //rule typeInferenceFun('aexpGCon(Gcon:K), Alpha:Map, Lol:
          Map, Delta:Map, Gamma:Map, .K, .K) => Sigma //T-App
70
            <tempBeta> (Gcon |-> Sigma:K) Beta:Map </tempBeta>
       //
71
72
       rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'aexpGCon('
          conTyCon(Mid:K,, Gcon:K)), Guess:KItem)
73
             => mapBagResult (uniFun (ListItem (uniPair (Guess,
                 freshInstance(Type, TypeIt))))) ...</k> //Constant
74
            <tempBeta> (ModPlusType (Mid, Gcon) |-> Type:K) Beta:Map
                </tempBeta>
75
            <typeIterator> TypeIt:Int => TypeIt +Int paramSize(Type
                ) </typeIterator>
76
77
       //lambda rule
78 //
        rule <k> typeInferenceFun(Gamma:Map, 'lambdaFun(Apatlist:K
      ,, Ex:K), Guess:KItem)
79 //
               => typeInferenceFun(genGamma(Apatlist,Gamma,
      quessType(TypeIt)), qenLambda(Apatlist,Ex), quessType(TypeIt
      +Int 1))
80 //
               >> lambdaReturn(Guess, guessType(TypeIt), guessType(
      TypeIt +Int 1)) \dots </k>
```

```
81 //
               <typeIterator> TypeIt:Int => TypeIt +Int 2 </
       typeIterator>
82
83 //
         rule <k> Sigma:Map ~> lambdaReturn(Tau:K, Tauone:K, Tautwo
84 //
                => compose (uniFun (ListItem (uniPair (typeSub (Sigma, Tau
       ), typeSub(Sigma, funtype(Tauone, Tautwo))))), Sigma) ...</k>
85
86
       syntax KItem ::= typeInferenceFun(ElemList, Map, K, K) [
           strict(1)]
87
       syntax KItem ::= typeInferenceFunLambda(ElemList, K, K, K) [
           strict(1)]
88 /* automatically generated by the strict(1) in typeInferenceFun
       or typeInferenceFunAux
89
       rule typeInferenceFunAux(Es:ElemList, C:K, A:K, B:K) => Es ~>
           typeInferenceFun(HOLE, C, A, B)
90
            requires notBool isKResult(Es)
91
       rule Es:KResult ~> typeInferenceFunAux(HOLE, C:K,A:K, B:K) =>
           typeInferenceFun(Es, C, A, B)
92 */
93
94
        //lambda rule
        rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'lambdaFun(
95
           Apatlist:K,, Ex:K), Guess:KItem)
96
              => typeInferenceFunLambda(val(typeInferenceFun(.
                  ElemList, genGamma(Apatlist, Gamma, guessType(TypeIt)
                  ), genLambda(Apatlist,Ex), guessType(TypeIt +Int 1)
                  )), .ElemList, Guess, guessType(TypeIt),guessType(
                  TypeIt +Int 1)) \dots </k>
97
             <typeIterator> TypeIt:Int => TypeIt +Int 2 </
                 typeIterator>
98
        rule <k> typeInferenceFunLambda (valValue (mapBagResult (Sigma:
99
           Map)), .ElemList, Tau:K, Tauone:K, Tautwo:K)
100
             => mapBagResult (compose (uniFun (ListItem (uniPair (typeSub
                 (Sigma, Tau), typeSub(Sigma, funtype(Tauone, Tautwo)))))
                 ,Sigma)) \dots </k>
101
102
        //rule <k> substi(S:Map) ~> lambdaReturn(Tau:K, Tauone:K,
            Tautwo:K)
103
        //
               => S[Tauone] ...</k>
104
105
```

```
106
        //syntax KItem ::= appliReturn(Map, K, K, Map)
        //syntax KItem ::= typeChildSub(Map, K) [function]
107
108
109
        syntax KItem ::= typeInferenceFunAppli(ElemList, Map, K, K,
           Map) [strict(1)]
110
111
        //application rule
112
        rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'funApp(Eone
            :K,, Etwo:K), Guess:KItem)
113
              => typeInferenceFunAppli(val(typeInferenceFun(.
                  ElemList, Gamma, Eone, funtype(guessType(TypeIt),
                  Guess))), .ElemList, Gamma, Etwo, guessType(TypeIt)
                  , .Map) ...</k>
114
             <typeIterator> TypeIt:Int => TypeIt +Int 1 </
                 typeIterator>
115
116
        rule <k> typeInferenceFunAppli(valValue(mapBagResult(
            Sigmaone: Map)), . ElemList, Gamma: Map, Etwo: KItem,
            quessType(TypeIt:Int), .Map)
117
              => typeInferenceFunAppli(val(typeInferenceFun(.
                  ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo,
                  typeSub(Sigmaone, quessType(TypeIt)))), .ElemList,
                  .Map, .K, .K, Sigmaone) ...</k>
118
119
        rule <k> typeInferenceFunAppli(valValue(mapBagResult(
           Sigmatwo:Map)), .ElemList, .Map, .K, .K, Sigmaone:Map)
120
              => mapBagResult(compose(Sigmatwo, Sigmaone)) ...</k>
121
122 //
          rule <k> Sigmaone: Map ~> appliReturn (Gamma: Map, Etwo: KItem
       , guessType(TypeIt:Int), .Map)
123 //
                => typeInferenceFun(gammaSub(Sigmaone, Gamma, .Map),
        Etwo, typeSub(Sigmaone, guessType(TypeIt)))
                ~> appliReturn(.Map, .K, .K, Sigmaone) ...</k>
124 //
125
126 //
          rule <k> Sigmatwo:Map ~> appliReturn(.Map, .K, .K,
       Sigmaone:Map)
127 //
                => compose(Sigmatwo, Sigmaone) ...</k>
128
129
        syntax KItem ::= typeInferenceFunIfThen(ElemList, Map, K, K,
            K, Map, Map) [strict(1)]
130
131
        //if then else rule
```

```
132
        rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'ifThenElse(
           Eone:K,, Optsem:K,, Etwo:K,, Optsemtwo:K,, Ethree:K),
           Guess:KItem)
133
              => typeInferenceFunIfThen(val(typeInferenceFun(.
                  ElemList, Gamma, Eone, Bool)), .ElemList, Gamma,
                  Etwo, Ethree, Guess, .Map, .Map) ...</k>
134
135
        rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
           Sigmaone: Map)), . ElemList, Gamma: Map, Etwo: KItem, Ethree:
           KItem, Guess:KItem, .Map, .Map)
136
              => typeInferenceFunIfThen(val(typeInferenceFun(.
                  ElemList, gammaSub(Sigmaone, Gamma, .Map), Etwo,
                  typeSub(Sigmaone, Guess))), .ElemList, Gamma, .K,
                  Ethree, Guess, Sigmaone, .Map) ...</k>
137
138
        rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
           Sigmatwo:Map)), .ElemList, Gamma:Map, .K, Ethree:KItem,
           Guess:KItem, Sigmaone:Map, .Map)
139
              => typeInferenceFunIfThen(val(typeInferenceFun(.
                  ElemList, gammaSub(compose(Sigmatwo, Sigmaone),
                  Gamma, .Map), Ethree, typeSub(compose(Sigmatwo,
                  Sigmaone), Guess))), .ElemList, .Map, .K, .K, .K,
                  Sigmaone, Sigmatwo) ...</k>
140
141
        rule <k> typeInferenceFunIfThen(valValue(mapBagResult(
           Sigmathree: Map)), .ElemList, .Map, .K, .K, .K, Sigmaone:
           Map, Sigmatwo:Map)
142
              => mapBagResult(compose(compose(Sigmathree, Sigmatwo),
                   Sigmaone)) \dots < /k >
143
144
        syntax KItem ::= typeInferenceFunLetIn(ElemList, Map, Map, K
            , K, K, Int, Int, Map, Map) [strict(1)]
145
        syntax KItem ::= grabLetDeclName(K, Int) [function]
146
        syntax KItem ::= grabLetDeclExp(K, Int) [function]
147
        syntax KItem ::= mapLookup(Map, K) [function]
148
        syntax Map ::= makeDeclMap(K, Int, Map) [function]
149
        syntax Map ::= applyGEN(Map, Map, Map, Map) [function]
150
        //Haskell let in rule (let rec in exp + let in rule combined
151
            )
        //gamma |- let rec f1 = e1 and f2 = e2 and f3 = e3 .... in e
152
            =>
```

```
153
        //beta, [f1 -> tau1, f2 -> tau2, f3 -> tau3,...] + gamma |-
             e1 : tau1 | sigma1, [f1 \rightarrow simga1(tau1), f2 \rightarrow sigma1(
            tau2), f3 \rightarrow sigma1(tau3),....] + sigma1(gamma) \mid - e2:
            sigma1(tau2) | sigma2 [f1 -> sigma2 o sigma1(tau1), f2
            -> sigma2 o sigma1(tau2), f3 -> sigma2 o sigma1(tau3)
            ,....] + sigma2 o sigma1(gamma) |- e3 : sigma2 o sigma1(
            tau3) .... [f1 -> gen(sigma_n o sigma2 o sigma1(tau1),
            sigma_n o sigma2 o sigma1(Gamma)), f2 -> gen(tau2), f3 ->
             gen(tau3),....] + gamma |- e : something
154
        rule <k> typeInferenceFun(.ElemList, Gamma:Map, 'letIn(D:K,,
             E:K), Guess:KItem)
155
              => typeInferenceFunLetIn(.ElemList, Gamma, makeDeclMap
                  (D, TypeIt, .Map), D, E, Guess, O, TypeIt, .Map,
                  Beta) ...</k>
156
             <typeIterator> TypeIt:Int => TypeIt +Int size(
                 makeDeclMap(D, TypeIt, .Map)) </typeIterator>
157
             <tempBeta> Beta:Map </tempBeta>
158
159
        rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap
            :Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int
            , OldSigma:Map, Beta:Map)
160
               => typeInferenceFunLetIn(val(typeInferenceFun(.
                   ElemList, Gamma DeclMap, grabLetDeclExp(D, Iter),
                   mapLookup(DeclMap, grabLetDeclName(D, Iter)))), .
                   ElemList, Gamma, DeclMap, D, E, Guess, Iter,
                   TypeIt, OldSigma, Beta) ...</k>
161
              //=> typeInferenceFunLetIn(val(typeInferenceFun(
                  DeclMap, grabLetDeclExp(D, Iter +Int TypeIt), Guess
                  )), .ElemList, Gamma, DeclMap, D, E, Guess, Iter,
                  TypeIt, OldSigma) ...</k>
162
              requires Iter <Int (size(DeclMap))</pre>
163
164
        rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:
            Map)), .ElemList, Gamma: Map, DeclMap: Map, D: KItem, E:
            KItem, Guess: KItem, Iter: Int, TypeIt: Int, OldSigma: Map,
           Beta:Map)
165
              => typeInferenceFunLetIn(.ElemList, gammaSub(Sigma,
                  Gamma, .Map), gammaSub(Sigma, DeclMap, .Map), D, E,
                  typeSub(Sigma, Guess), Iter +Int 1, TypeIt, compose
                  (Sigma, OldSigma), Beta) ...</k>
166
              requires Iter <Int (size(DeclMap))</pre>
167
```

```
168
        rule <k> typeInferenceFunLetIn(.ElemList, Gamma:Map, DeclMap
            :Map, D:KItem, E:KItem, Guess:KItem, Iter:Int, TypeIt:Int
            , OldSigma:Map, Beta:Map)
169
              => typeInferenceFunLetIn(val(typeInferenceFun(.
                  ElemList, Gamma applyGEN (Gamma, DeclMap, .Map, Beta
                  ), E, Guess)), .ElemList, Gamma, DeclMap, D, E,
                  Guess, Iter, TypeIt, OldSigma, Beta) ...</k>
170
              requires Iter >=Int (size(DeclMap))
171
172
        rule <k> typeInferenceFunLetIn(valValue(mapBagResult(Sigma:
            Map)), .ElemList, Gamma: Map, DeclMap: Map, D: KItem, E:
           KItem, Guess: KItem, Iter: Int, TypeIt: Int, OldSigma: Map,
           Beta:Map)
173
              => mapBagResult(compose(Sigma, OldSigma))...</k>
174
              requires Iter >=Int (size(DeclMap))
175
176
        rule mapLookup((Name |-> Type:KItem) DeclMap:Map, Name:KItem
            ) => Type
177
        rule mapLookup(DeclMap:Map, Name:KItem) => Name
178
             requires notBool(Name in keys(DeclMap))
179
180
        //rule makeDeclMap('decls(A:K), TypeIt:Int, NewMap:Map) =>
            makeDeclMap(A, TypeIt +Int 1, NewMap)
181
        rule makeDeclMap('decls(Dec:K), TypeIt:Int, NewMap:Map) =>
            makeDeclMap(Dec, TypeIt, NewMap)
182
        rule makeDeclMap('declsList('declPatRhs('apatVar(Var:K),,
            Righthand:K),, Rest:K), TypeIt:Int, NewMap:Map) =>
            makeDeclMap('decls(Rest), TypeIt +Int 1, NewMap[Var <-</pre>
            quessType(TypeIt)])
183
        rule makeDeclMap(.DeclsList, TypeIt:Int, NewMap:Map) =>
            NewMap
184
185
        rule grabLetDeclName('decls(Dec:K), Iter:Int) =>
            grabLetDeclName(Dec, Iter)
186
        rule grabLetDeclName('declsList(Dec:K,, Rest:K), Iter:Int)
            => grabLetDeclName(Rest, Iter -Int 1)
187
             requires Iter >Int 0
188
        rule grabLetDeclName ('declsList ('declPatRhs ('apatVar (Var:K)
            ,, Righthand:K),, Rest:K), Iter:Int) => Var
189
             requires Iter <=Int 0
190
191
```

```
192
        rule grabLetDeclExp('decls(Dec:K), Iter:Int) =>
            grabLetDeclExp(Dec, Iter)
193
        rule grabLetDeclExp('declsList(Dec:K,, Rest:K), Iter:Int) =>
             grabLetDeclExp(Rest, Iter -Int 1)
194
             requires Iter >Int 0
195
        rule grabLetDeclExp('declsList('declPatRhs('apatVar(Var:K),,
             Righthand:K),, Rest:K), Iter:Int) => grabLetDeclExp(
            Righthand, Iter)
196
             requires Iter <=Int 0
197
        rule grabLetDeclExp('eqExpOptDecls(Righthand:K,, Opt:K),
            Iter:Int) => 'eqExpOptDecls(Righthand,, Opt)
198
199
        rule genGamma('apatVar(Vari:K), Gamma:Map, Guess:K) => Gamma
            [Vari <- Guess]</pre>
200
        rule genGamma('apatCon(Vari:K,, Pattwo:K), Gamma:Map, Guess:
            K) => Gamma[Vari <- Guess]</pre>
201
202
        rule genLambda('apatVar(Vari:K), Ex:K) => Ex
        rule genLambda('apatCon(Vari:K,, Pattwo:K), Ex:K) => '
203
            lambdaFun(Pattwo,, Ex)
204
205
206
        rule gammaSub(Sigma:Map, (Key:KItem |-> Type:KItem) Gamma:
           Map, Newgamma:Map)
207
            => gammaSub(Sigma, Gamma, Newgamma[Key <- typeSub(Sigma,
                 Type) ] )
208
        // => gammaSub(Sigma, Gamma, Newgamma[Key <- typeChildSub(</pre>
            Sigma, Type) ] )
209
210
        rule gammaSub(Sigma:Map, .Map, Newgamma:Map)
211
          => Newgamma
212
213
        //rule typeChildSub((guessType(TypeIt) |-> Type:KItem) Sigma
            :Map, guessType(TypeIt:Int)) => Type
214
215
        //rule typeChildSub(Sigma:Map, guessType(TypeIt:Int)) =>
            guessType(TypeIt)
216
               requires notBool (guessType(TypeIt) in keys(Sigma))
217
218
        rule freshInstance(guessType(TypeIt:Int), Iter:Int) =>
            guessType(TypeIt)
219
        rule freshInstance(forAll(.Set, B:K), Iter:Int) => B
```

```
220
        rule freshInstance(forAll(SetItem(C:KItem) A:Set, B:K), Iter
            :Int) => freshInstance(forAll(A, freshInstanceInner(C, B,
             Iter)), Iter +Int 1)
221
222
        syntax KItem ::= freshInstanceInner(K,K,Int) [function]
223
224
        rule freshInstanceInner(Repl:KItem, funtype(A:K, B:K), Iter:
            Int) => funtype(freshInstanceInner(Repl,A,Iter),
            freshInstanceInner(Repl,B,Iter))
225
        rule freshInstanceInner(Repl:KItem, Repl, Iter:Int) =>
           guessType(Iter)
226
        rule freshInstanceInner(Repl:KItem, Target:KItem, Iter:Int)
           => Target [owise]
227
228
        rule paramSize(forAll(A:Set, B:K)) => size(A)
229
        rule paramSize(A:K) => 0 [owise]
230
231
232
         rule applyGEN(Gamma:Map, (Key:KItem |-> Type:KItem) DeclMap
             :Map, NewMap:Map, Beta:Map)
233
           => applyGEN(Gamma, DeclMap, NewMap[Key <- gen(Gamma, Type
               , Beta)], Beta)
234
235
         rule applyGEN(Gamma:Map, .Map, NewMap:Map, Beta:Map)
236
           => NewMap
237
238
        //GEN
239
        //GEN(Gamma, Tau) => Forall alpha
240
241
        syntax KItem ::= gen(Map, K, Map) [function]
242
        syntax Set ::= freeVarsTy(K, Map) [function]
243
        syntax Set ::= freeVarsEnv(Map, Map) [function]
244
        //syntax KItem ::= setBag(Set)
245 //
          syntax Set ::= listToSet(List, Set) [function]
246
247
248
        rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map)
            => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set
            freeVarsEnv(Gamma, Beta), Tau)
249
        rule gen(Gamma:Map, Tau:KItem, Beta:Map) => forAll(
            freeVarsTy(Tau, Beta) -Set freeVarsEnv(Gamma, Beta), Tau)
             [owise]
250
```

```
251
        //rule gen(Gamma:Map, forAll(Para:Set, Tau:KItem), Beta:Map)
            => forAll(freeVarsTy(forAll(Para:Set, Tau), Beta) -Set
           freeVarsEnv(Gamma, Beta), Tau)
252
253
        rule freeVarsTy(guessType(TypeIt:Int), Beta:Map) => SetItem(
           quessType(TypeIt:Int))
254
        rule freeVarsTy(funtype(Tauone:KItem, Tautwo:KItem), Beta:
           Map) => freeVarsTy(Tauone, Beta) freeVarsTy(Tautwo, Beta)
255
        rule freeVarsTy(Tau:KItem, Beta:Map) => .Set
256
            requires (forAll(.Set, Tau)) in values(Beta)
257
        rule freeVarsTy(forAll(Para:Set, Tau:KItem), Beta:Map) =>
           freeVarsTy(Tau, Beta) -Set Para
258
        rule freeVarsEnv(Gamma:Map, Beta:Map) => listToSet(values(
           Beta), .Set)
259
260
261 //
         rule listToSet(ListItem(A:KItem) L:List, S:Set) =>
       listToSet(L, SetItem(A) S)
         rule listToSet(.List, S:Set) => S
262 //
263
265
266
        //Unification
267
268
        syntax Map ::= uniFun(List) [function]
269
        //syntax List ::= uniSub(K,K,K) [function]
270
        syntax Bool ::= isVarType(K) [function]
271
        syntax Bool ::= notChildVar(K,K) [function]
272
        syntax KItem ::= uniPair(K,K)
273
274
        syntax List ::= uniSub(Map,K) [function] //apply
           substitution to unification
275
276
        syntax KItem ::= typeSub(Map,K) [function] //apply
           substitution to type
277
        syntax Map ::= compose(Map, Map) [function]
278
279
        // syntax KItem ::= Map
280
281
        rule uniFun(.List) => .Map //substi(.K,.K) is id
           substitution
282
```

```
283
        rule uniFun(ListItem(uniPair(S:K,S)) Rest:List) => uniFun(
           Rest) //delete rule
284
285
        // rule uniFun(SetItem(I:K)) => .K //uniFun(Rest) //delete
286
287
        rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List) => uniFun(
           ListItem(uniPair(T,S)) Rest) //orient rule
288
             requires isVarType(T) andBool (notBool isVarType(S))
289
290
        //rule uniFun(ListItem(uniPair(forAll(Svars:List,S:K),forAll
            (.List, T:K))) Rest:List, Sigma:Map) => uniFun(ListItem(
           uniPair(forAll(.List,T),forAll(Svars,S))) Rest,Sigma) //
           orient rule
291
          // requires Svars =/=K .List
292
293
        //rule uniFun(ListItem(uniPair(quessType(S:Int), forAll(.List
            ,T:K))) Rest:List,Sigma:Map) => uniFun(ListItem(uniPair(
            forAll(.List,T:K), quessType(S))) Rest, Sigma) //orient
           rule
294
295
       // rule uniFun(ListItem(uniPair(forAll(.List,S:K),T:K)) Rest:
          List, Sigma: Map) => uniFun(uniSub('aexpQVar(Var), T, Rest),
          Sigma['aexpQVar(Var) <- T]) //eliminate rule</pre>
296
       //
               requires notChildVar('aexpQVar(Var:K),T)
297
298
        rule uniFun(ListItem(uniPair(funtype(A:K, B:K), funtype(C:K,
            D:K))) Rest:List) => uniFun(ListItem(uniPair(A, C))
           ListItem(uniPair(B, D)) Rest:List) //decompose rule
            function type
299
300
        rule uniFun(ListItem(uniPair(S:K,T:K)) Rest:List)
301
          => compose((S |-> typeSub(uniFun(uniSub((S |-> T), Rest)), T
              )),uniFun(uniSub((S |-> T),Rest))) //eliminate rule
302
        // => compose(uniFun(uniSub((S |-> T),Rest)),(S |-> typeSub
            (uniFun(uniSub((S |-> T), Rest)), T))) //eliminate rule
303
             requires isVarType(S) andBool notChildVar(S,T)
304
305
        rule isVarType(S:K) => true
306
             requires getKLabel(S) ==KLabel 'guessType
307
        rule isVarType(S:K) => false [owise]
308
309
        rule notChildVar(S:K,T:K) => true
```

```
310
311
        rule uniSub(Sigma:Map,.List) => .List
312
        rule uniSub(.Map,L:List) => L
313
        rule uniSub(Sigma:Map, Rest:List ListItem(uniPair(A:K, B:K))
           ) => uniSub(Sigma, Rest) ListItem(uniPair(typeSub(Sigma,
           A), typeSub(Sigma, B)))
314
315
        //rule typeSub(substi(.Map), Tau:KItem) => Tau
316
        rule typeSub(Sigma:Map (Tau |-> Newtau:KItem), Tau:KItem) =>
           typeSub(Sigma (Tau |-> Newtau), Newtau)
317
        rule typeSub(Sigma:Map,funtype(Tauone:KItem,Tautwo:KItem))
            => funtype(typeSub(Sigma, Tauone), typeSub(Sigma, Tautwo))
318
        rule typeSub(Sigma:Map,Tau:KItem) => Tau [owise]
319
320
        syntax Map ::= composeIn(Map, Map, Map, K, K) [function]
321
322
        rule compose(Sigmaone:Map, Sigmatwo:Map) => composeIn(
            Sigmaone, Sigmatwo, .Map, .K, .K)
323
324
        rule composeIn(Sigmaone:Map, (Key:KItem |-> Type:KItem)
            Sigmatwo:Map, NewMap:Map, .K, .K) => composeIn(Sigmaone,
            Sigmatwo, NewMap, Key, Type)
325
326
        rule composeIn((Keyone |-> Typetwo:KItem) Sigmaone:Map,
            Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
             composeIn(Sigmaone, Sigmatwo, NewMap, Keyone, Typeone)
327
328
        rule composeIn((Typeone |-> Typetwo:KItem) Sigmaone:Map,
            Sigmatwo:Map, NewMap:Map, Keyone:KItem, Typeone:KItem) =>
             composeIn((Typeone |-> Typetwo) Sigmaone, Sigmatwo,
           NewMap[Keyone <- Typetwo], .K, .K)</pre>
329
             requires notBool(Keyone in keys(Sigmaone))
330
331
        rule composeIn(Sigmaone:Map, Sigmatwo:Map, NewMap:Map,
            Keyone:KItem, Typeone:KItem) => composeIn(Sigmaone,
            Sigmatwo, NewMap[Keyone <- Typeone], .K, .K) [owise]</pre>
332
333
        rule composeIn(Sigmaone:Map, .Map, NewMap:Map, .K, .K) =>
            Sigmaone NewMap
334
335
        //rule composeIn(Sigmaone:Map, .Map, .K, .K) =>
            Sigmaone
336
```

```
337
        //rule composeIn((Key:KItem |-> Type:KItem) Sigmaone:Map, .
            Map, NewMap:Map) => composeIn(Sigmaone, .Map, NewMap[Key
            <- mapLookup(Sigmaone, Type)])
338
339
        //rule compose(Sigmaone:Map, Sigmatwo:Map) => updateMap(
            Sigmaone, Sigmatwo)
340
        //rule compose(Sigmaone:Map, (Keytwo:KItem |-> Valtwo:KItem)
             Sigmatwo:Map) => compose(Sigmaone[Keytwo <- Valtwo][</pre>
            Valtwo <- mapLookup(Sigmaone, Keytwo)], Sigmatwo)</pre>
341
        //rule compose(Sigmaone:Map, (Key:KItem |-> Type:KItem)
            Sigmatwo:Map, .K) => compose(Sigmaone[Type <- mapLookup(</pre>
            Sigmaone, Key)][Key <- Type], Sigmatwo, mapLookup(</pre>
            Sigmaone, Key))
342
        //rule compose(Sigmaone:Map, (Key:KItem |-> Type:KItem)
            Sigmatwo:Map) => composeIn(Sigmaone, Sigmatwo, mapLookup(
            Sigmaone, Key))
343
        //
                requires (notBool (Type in values(Sigmaone))) andBool
             (Type =/=K mapLookup(Sigmaone, Key))
344
        //rule compose(Sigmaone:Map, (Key:KItem |-> Type:KItem)
            Sigmatwo:Map) => compose(Sigmaone[Key <- Type][Type <-</pre>
            mapLookup(Sigmaone, Key)], Sigmatwo)
345
               requires (notBool (Type in values(Sigmaone))) andBool
             (Type =/=K mapLookup(Sigmaone, Key))
          rule compose(Sigmaone:Map, (Keytwo:KItem |-> Valtwo:KItem)
346 //
         Sigmatwo:Map) => compose(Sigmaone[Valtwo <- mapLookup(</pre>
        Sigmaone, Keytwo)], Sigmatwo)
347 //
                requires (Valtwo in values (Sigmaone)) and Bool (Valtwo
         =/=K mapLookup(Sigmaone, Keytwo))
348
        //rule compose(Sigmaone:Map, (Keytwo:KItem | -> Valtwo:KItem)
             Sigmatwo: Map) => compose ((Keytwo |-> Valtwo) Sigmaone,
            Sigmatwo)
        // requires notBool (Keytwo in keys(Sigmaone))
349
350
        //rule compose(Sigmaone:Map, .Map) => Sigmaone
351
        // rule compose(substi(Sone:K, Tone:K), substi(Stwo:K, Ttwo:K))
             => substi(typeSub(substi(Stwo, Ttwo), Sone), Tone)
352
353
354
       // rule notChildVar('aexpQVar(Var:K),T)
355
356
357
        //T-Var
358 //
         rule typeInferenceFun('funApp(Eone:K,, Etwo:K), Alpha:Map,
         Beta:Map, Delta:Map, Gamma:Map, .K, .K) =>
```

```
359 //
               typeInferenceFun('funApp(Eone,, Etwo), Alpha, Beta,
       Delta, Gamma, typeInferenceFun (Eone, Alpha, Beta, Delta, Gamma
       ,.K,.K),typeInferenceFun(Etwo,Alpha, Beta, Delta, Gamma,.K,.K
360 //
          rule typeInferenceFun('funApp(Eone:K,, Etwo:K), Alpha:Map,
        Beta: Map, Delta: Map, Gamma: Map, funtype (Tauone: K, Tautwo: K),
        Tauone) => Tautwo
361
362
        //T-Lam
363 //
         rule typeInferenceFun('lambdaFun(Apatlist:K,, Ex:K), Alpha
       :Map, Beta:Map, Delta:Map, Gamma:Map, .K, .K) =>
364 //
               typeInferenceFun('lambdaFun(Apatlist,, Ex), Alpha,
       Beta, Delta, Gamma, typeInferenceFun(Ex, Alpha, Beta, Delta,
       genGamma(Apatlist, Gamma), .K, .K), .K)
365
366 //
         rule typeInferenceFun('lambdaFun(Apatlist:K,, Ex:K), Alpha
       :Map, Beta:Map, Delta:Map, Gamma:Map, Tautwo:K, .K) => Tautwo
367
368 endmodule
```

# References