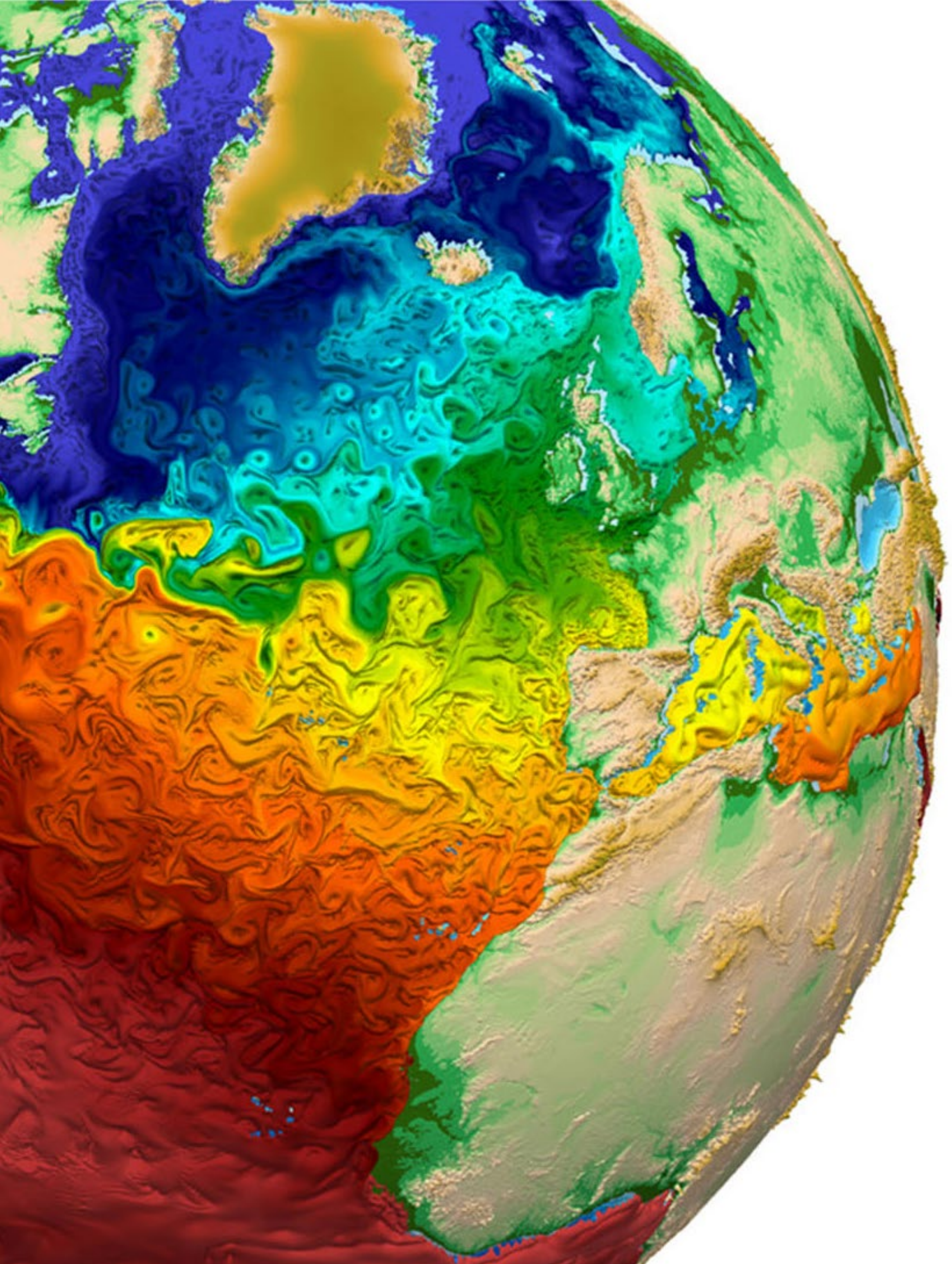

Contouring

Marching Cubes: Consistency and Correctness

Scientific Visualization

Professor Eric Shaffer

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Correctness and Consistency

The utility of the visualization is impacted by

Correctness

An isosurface is correct if it matches the behavior of a known function that describes the samples in the data set.

Consistency

If each component of an isosurface is continuous then it is topologically consistent.

- Also topologically consistent if the isosurface’s only holes are on the data set’s boundary.

Standard MC provides neither of these…leading to many extension

CFD data in particular is often generated by non-linear functions and so will not be strictly correctly represented by a standard MC surface

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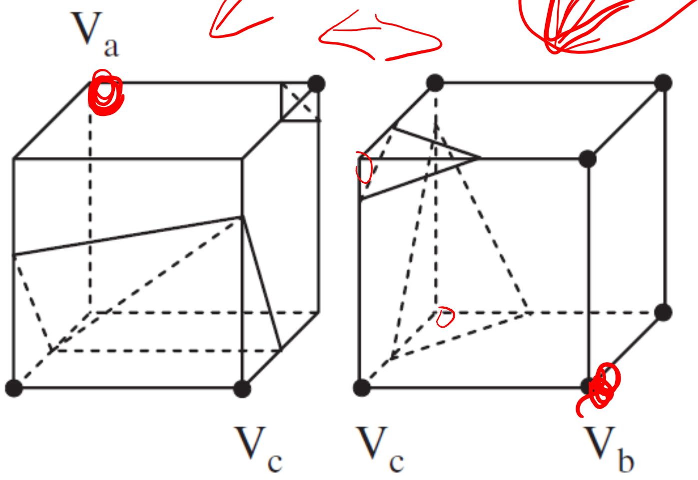
Face Ambiguity: Example

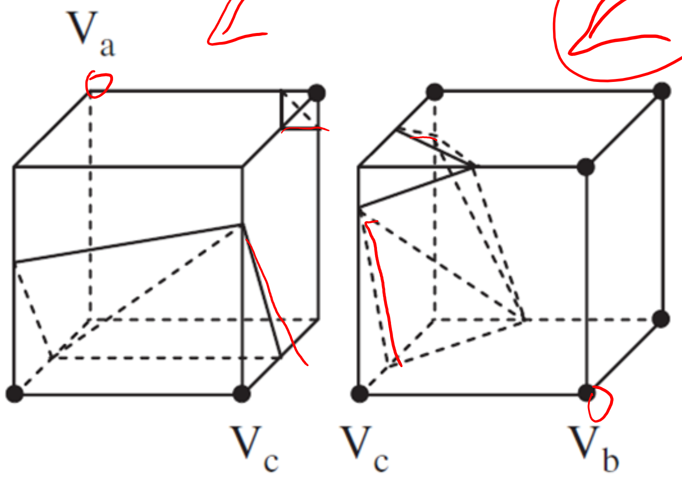
Several cell cases can be facetized in multiple ways

- Cases 3, 6, 7, 10, 12, and 13

Example (cases 6 and 3):



Inconsistent



Consistent

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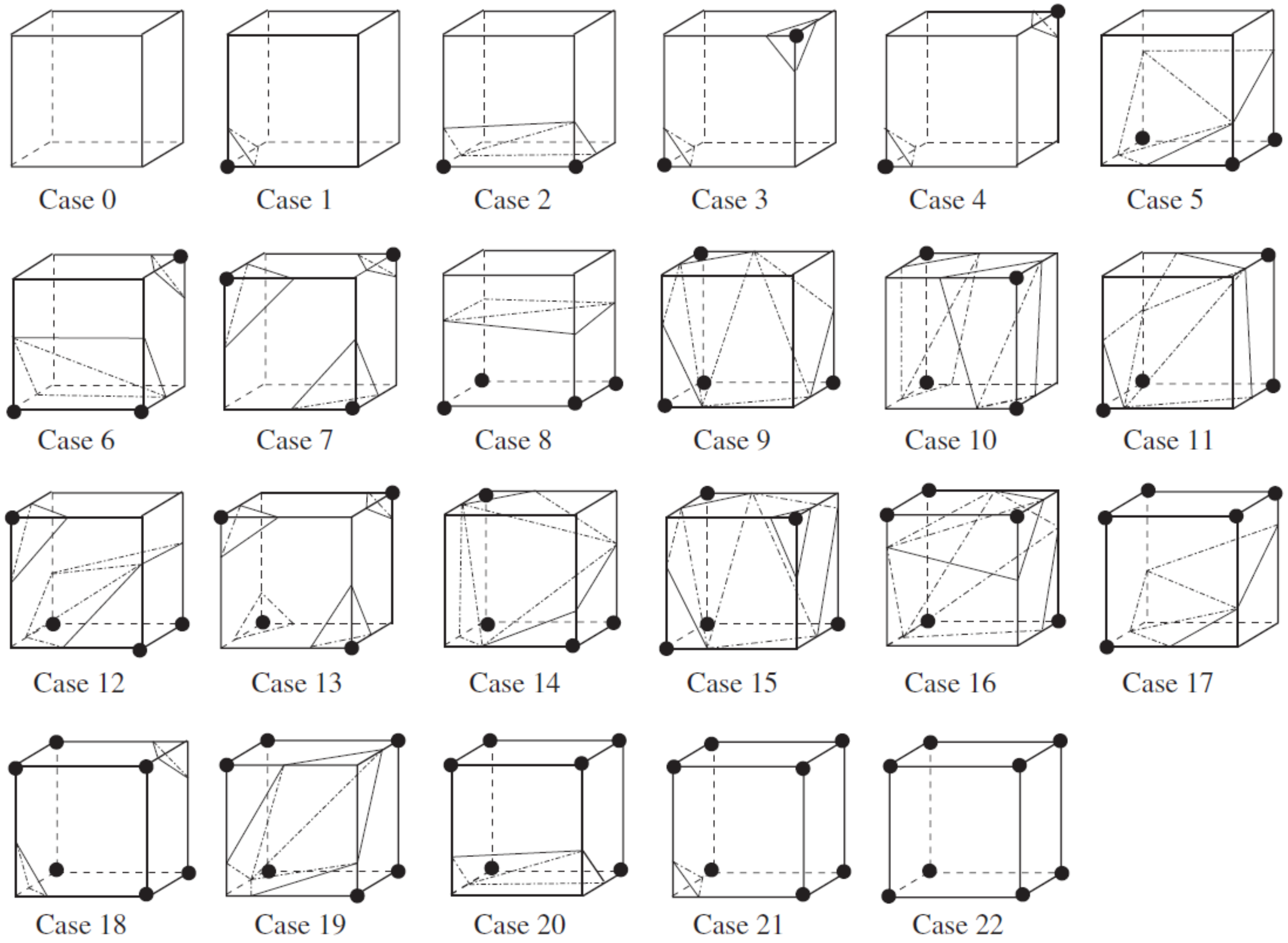
Face Ambiguity: One Solution

Build set of cases only considering rotational symmetry



Will produce a topologically consistent but not necessarily correct isosurface

G. M. Nielson, "On marching cubes," in *IEEE Transactions on Visualization and Computer Graphics*, July-Sept. 2003

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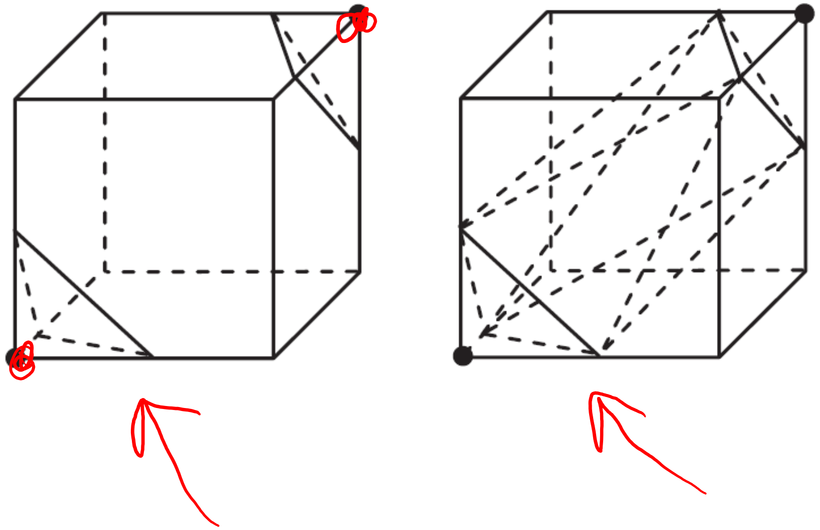
Internal Ambiguity

The facetization of a cube that has no ambiguous faces can still have internal ambiguity

Internal ambiguity does not cause any topological inconsistency but it can yield an incorrect isosurface

Internal ambiguity can arise in cases 4, 6, 7, 10, 12, and 13.



There are many, many ways people have addressed both kinds of ambiguity…we'll just discuss one

Trilinear Interpolant

Often don't know the underlying function of the field

How can we determine correctness?

One approach: assume trilinear interpolant of the samples is correct function

Within a unit cube the interpolant is

$$
\begin{aligned}
T(x,y,z) = {} & (1-x)(1-y)(1-z)T(0,0,0) \\
& + (1-x)(1-z)zT(0,0,1) \\
& + (1-x)y(1-z)T(0,1,0) + (1-x)yzT(0,1,1) \\
& + x(1-y)(1-z)T(1,0,0) + x(1-y)zT(1,0,1) \\
& + xy(1-z)T(1,1,0) + xyzT(1,1,1).
\end{aligned}
$$

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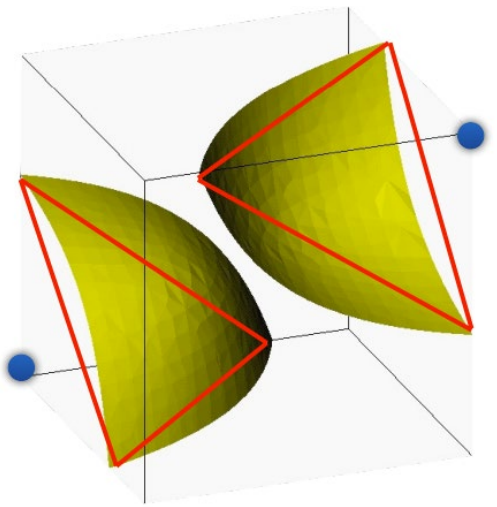
Trilinear Interpolant

$$
\begin{aligned}
T(x,y,z) &= (1-x)(1-y)(1-z)T(0,0,0) \\
&+ (1-x)(1-y)zT(0,0,1) \\
&+ (1-x)y(1-z)T(0,1,0) + (1-x)yzT(0,1,1) \\
&+ x(1-y)(1-z)T(1,0,0) + x(1-y)zT(1,0,1) \\
&+ xy(1-z)T(1,1,0) + xyz(1,1,1) \\
&= \left(\frac{x_1-x}{x_1-x_0}\right)\left(\frac{y_1-y}{y_1-y_0}\right)\left(\frac{z_1-z}{z_1-z_0}\right)T(x_0,y_0,z_0) \\
&+ \left(\frac{x_1-x}{x_1-x_0}\right)\left(\frac{y_1-y}{y_1-y_0}\right)\left(\frac{z-z_0}{z_1-z_0}\right)T(x_0,y_0,z_1) \\
&+ \left(\frac{x_1-x}{x_1-x_0}\right)\left(\frac{y-y_0}{y_1-y_0}\right)\left(\frac{z_1-z}{z_1-z_0}\right)T(x_0,y_1,z_0) \\
&\;\;\vdots \\
&+ \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{y-y_0}{y_1-y_0}\right)\left(\frac{z-z_0}{z_1-z_0}\right)T(x_1,y_1,z_1).
\end{aligned}
$$

True isosurface of a trilinear function is a cubic curved surface

Contours are hyperbola

MC approximates with triangles



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Trilinear Interpolant

Can make isosurface consistent with trilinear interpolant

Resolve ambiguities as trilinear interpolant would

G. M. Nielson, "On marching cubes," in IEEE Transactions on Visualization and Computer Graphics, July-Sept. 2003

…3 step process to assure correctness assuming trilinear interpolant is underlying function