



# Fiber Tracking

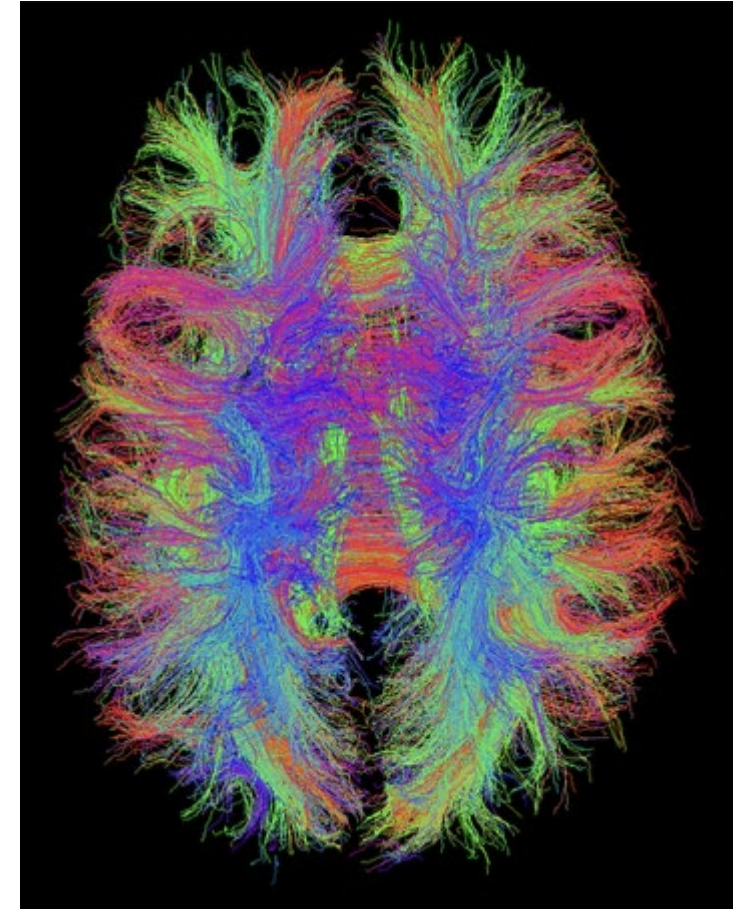
Scientific Visualization  
Professor Eric Shaffer

# Tractography: Fiber Tracking

“Recently, attention has been given to the visualization of 2D and 3D diffusion tensor fields from DT-MRI data. Although these methods provide nice visual cues, they do not attempt to recover the underlying anatomical structures, which are the white matter fiber tracts (bundles of axons) found within the brain.”

*Oriented tensor reconstruction: tracing neural pathways from diffusion tensor MRI*

L. Zhukov, A. H. Barr

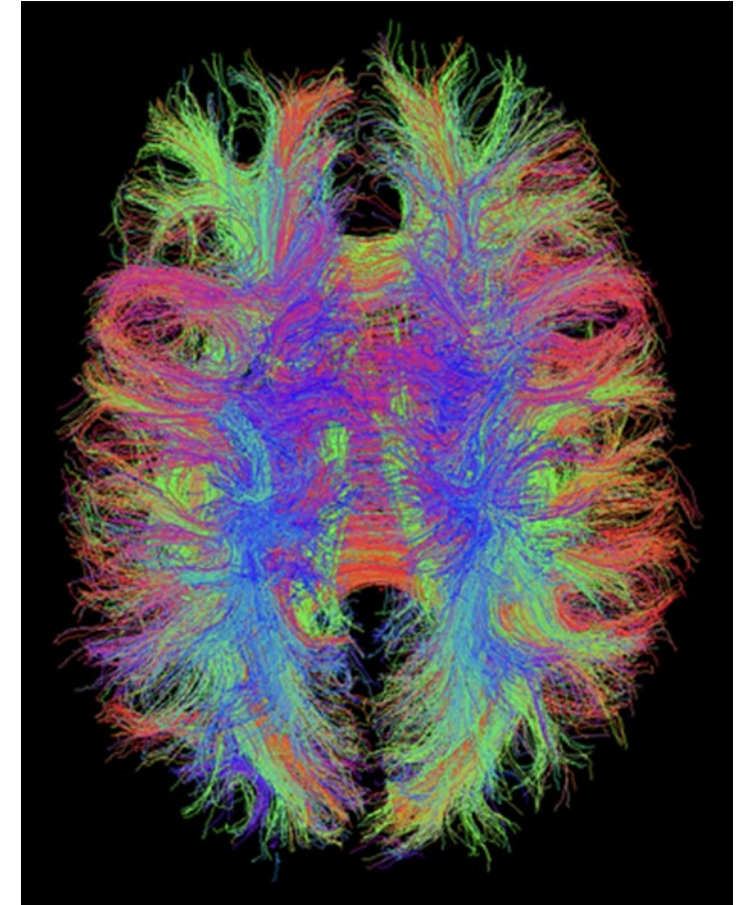




# Applications of Fiber Tracking

“To illustrate the potential for tractography to provide new information in clinical neuroscience we review recent studies in three broad areas: First, use of tractography for quantitative comparisons of specific white matter pathways in disease; second, evidence from tractography for the presence of qualitatively different pathways in congenital disorders or following recovery; third, use of tractography to gain insights into normal brain anatomy that can aid our understanding of the consequences of localised pathology, or guide interventions.”

*Just pretty pictures? What diffusion tractography can add in clinical neuroscience.* Johansen-Berg, H., & Behrens, T. E. (2006).



# DT-MRI Basics

Neural fibers are comprised mostly of bundles of long cylindrical cells  
Filled with fluid and are bounded by less-water-permeable cell membranes

The diffusion properties can be represented with a symmetric second order tensor

$$\mathbf{T} = \begin{pmatrix} T^{xx} & T^{xy} & T^{xz} \\ T^{yx} & T^{yy} & T^{yz} \\ T^{zx} & T^{zy} & T^{zz} \end{pmatrix}$$

The 6 independent values (the tensor is symmetric)  
The tensor elements vary continuously with spatial location

# Principal Component Analysis

$$\mathbf{T} = \begin{pmatrix} T^{xx} & T^{xy} & T^{xz} \\ T^{yx} & T^{yy} & T^{yz} \\ T^{zx} & T^{zy} & T^{zz} \end{pmatrix}$$

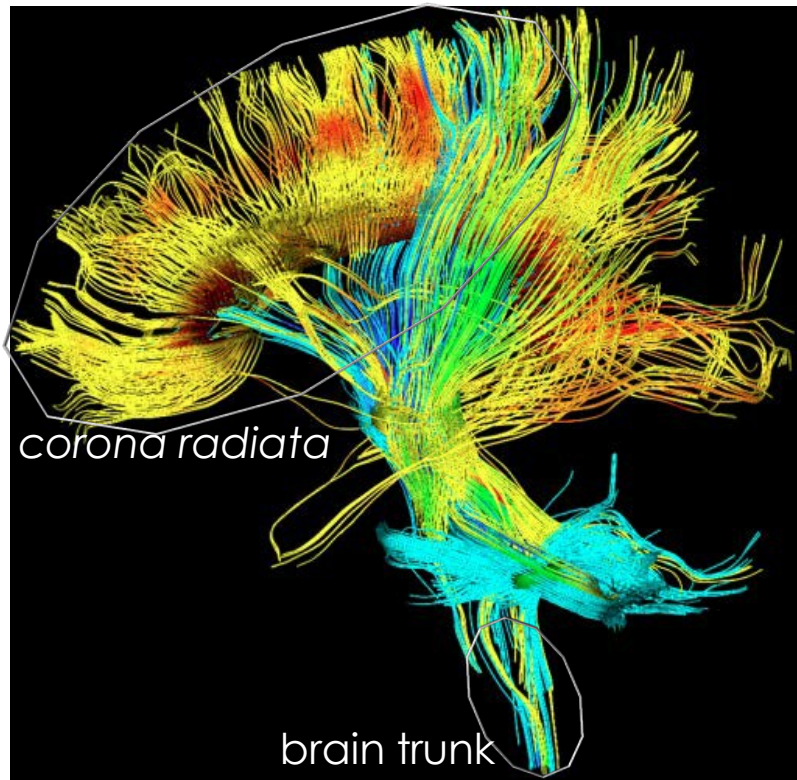
$$\underline{\mathbf{T}\mathbf{e}_i = \lambda_i \mathbf{e}_i}$$

The 3-D local axis direction of the neuron fibers will be the dominant eigenvector of the tensor  
There should be one large eigenvalue, and two small eigenvalues

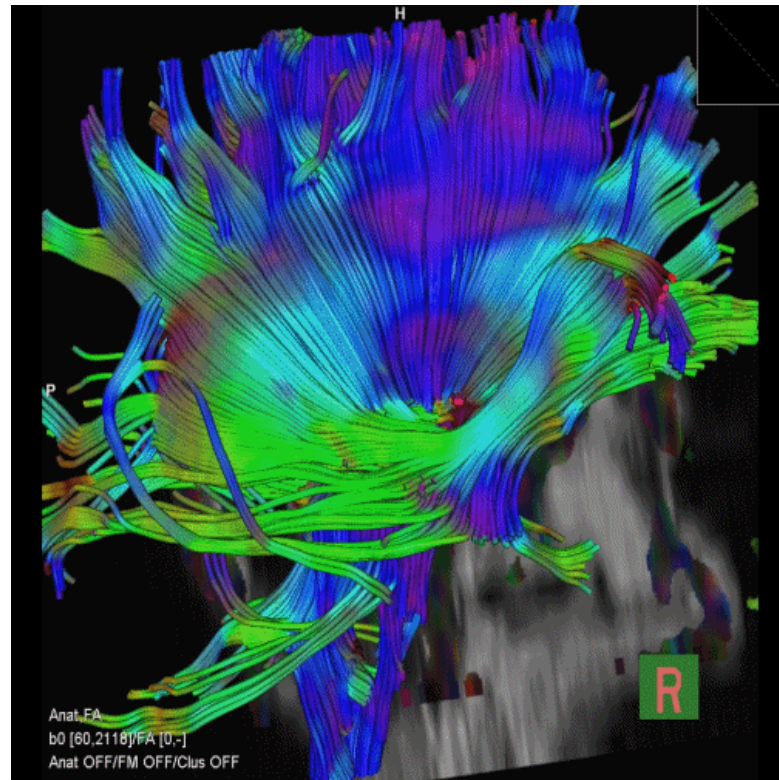
$$\mathbf{T} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}^T$$

# Fiber Tracking

- consider major eigenvector field
- trace streamlines
  - **seed**: in regions with high anisotropy (i.e. where fibers are)
  - **stop**: when anisotropy gets too low (i.e. when we leave fibers)



streamlines, brain overview



stream tubes, brain detail

# Seeding

Choose to seed points exhibiting strong anisotropy

These likely lie on fibers

Measure

$$c_\ell = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

Values close to 1 indicate strong linear diffusion  $\lambda_1 \gg \lambda_2 \approx \lambda_3$



# Tensor Interpolation

Reconstruct the continuous tensor field using linear interpolation

Value of a tensor inside a voxel is a linear combination of 8 corner values

Interpolation is tri-linear

Interpolation is component-wise

$$\begin{aligned} \mathbf{T}(x, y, z) = & \mathbf{T}_{ijk} (1-x)(1-y)(1-z) + \\ & + \mathbf{T}_{i+1,jk} x(1-y)(1-z) + \mathbf{T}_{i,j+1,k} (1-x)y(1-z) \\ & + \mathbf{T}_{ij,k+1} (1-x)(1-y)z + \mathbf{T}_{i+1,j,k+1} x(1-y)z \\ & + \mathbf{T}_{i,j+1,k+1} (1-x)yz + \mathbf{T}_{i+1,j+1,k} xy(1-z) \\ & + \mathbf{T}_{i+1,j+1,k+1} xyz \end{aligned}$$

For general position  
replace  $x$  with  $\frac{x-x_{min}}{x_{max}-x_{min}}$   
Etc.



# Tensor Interpolation

We can use trilinear component-wise interpolation because:  
Any linear combination of symmetric tensors remains a symmetric tensor

Component-wise interpolation of eigenvectors would not lead to correct results:  
A linear interpolation between two unit vectors is not a unit vector anymore

$$\begin{aligned}\mathbf{T}(x, y, z) = & \mathbf{T}_{ijk} (1 - x)(1 - y)(1 - z) + \\ & + \mathbf{T}_{i+1,jk} x(1 - y)(1 - z) + \mathbf{T}_{i,j+1,k} (1 - x)y(1 - z) \\ & + \mathbf{T}_{ij,k+1} (1 - x)(1 - y)z + \mathbf{T}_{i+1,j,k+1} x(1 - y)z \\ & + \mathbf{T}_{i,j+1,k+1} (1 - x)yz + \mathbf{T}_{i+1,j+1,k} xy(1 - z) \\ & + \mathbf{T}_{i+1,j+1,k+1} xyz\end{aligned}$$

# Moving Least Squares (MLS)

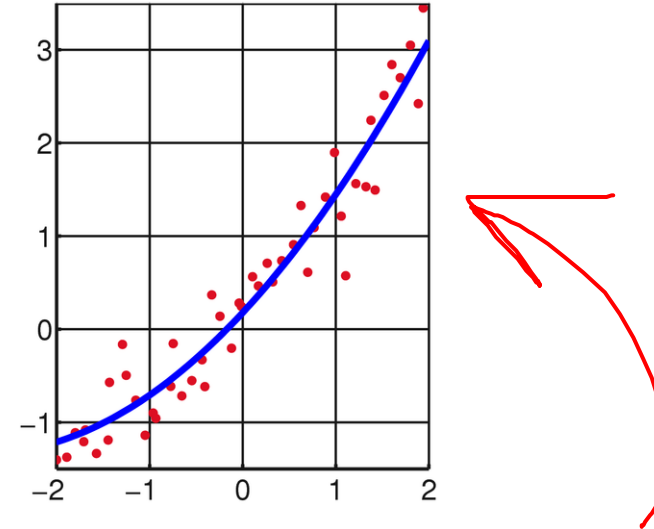
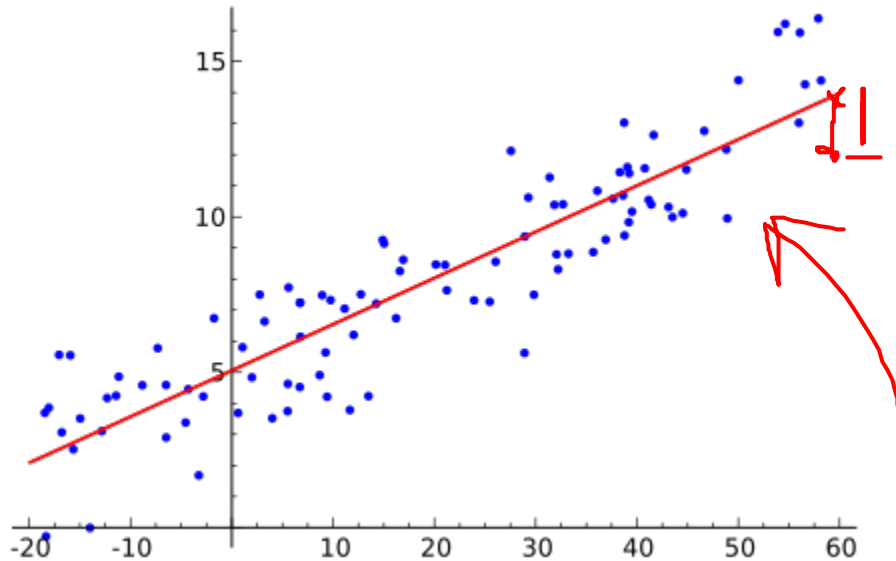
Experimental data is noisy

- Needs to be filtered for best results
- Gaussian filter will not work well, will blur the directional information

MLS - find a low degree polynomial:

- which best fits the data, in the least squares sense
- in the small region around the point of interest
- replace the data value at that point value of the polynomial at that point
  - Here data = tensor

# Least Squares Best Fit



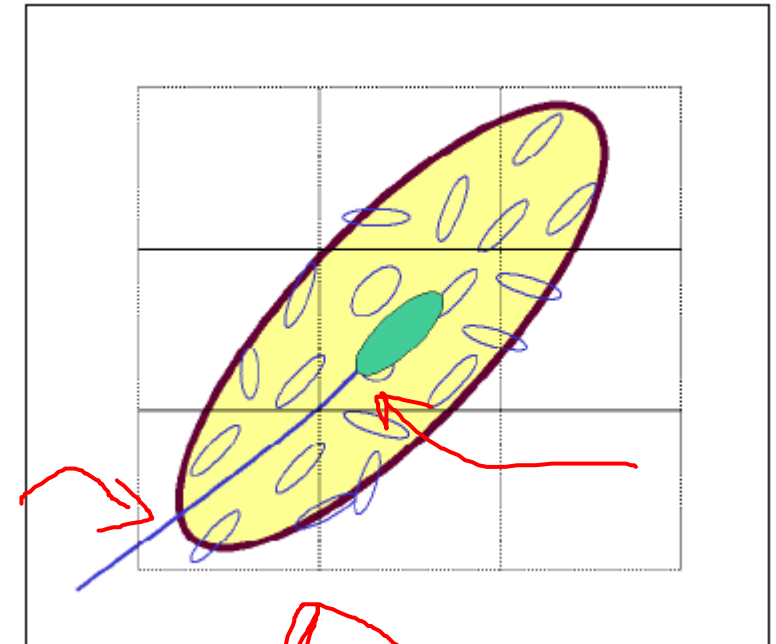
Least squares best fit for data  $(x_i, y_i)$

- Finds coefficients of a polynomial  $F(X)$
- Minimizes sum of the squared error between  $F(x_i)$  and  $y_i$
- Moving Least Squares considers a subset of the data in a sliding window

# Algorithm

```
1. User inputs starting region
2. For every starting point P in the region
{
    Tp = filter(T, P, sphere);
    cl = anisotropy(Tp);
    if (cl > eps){
        e1 = direction(Tp);
        trace1 = fibertrace(P, e1);
        trace2 = fibertrace(P, -e1);
        trace = | trace1 + trace2;
    }
}
```

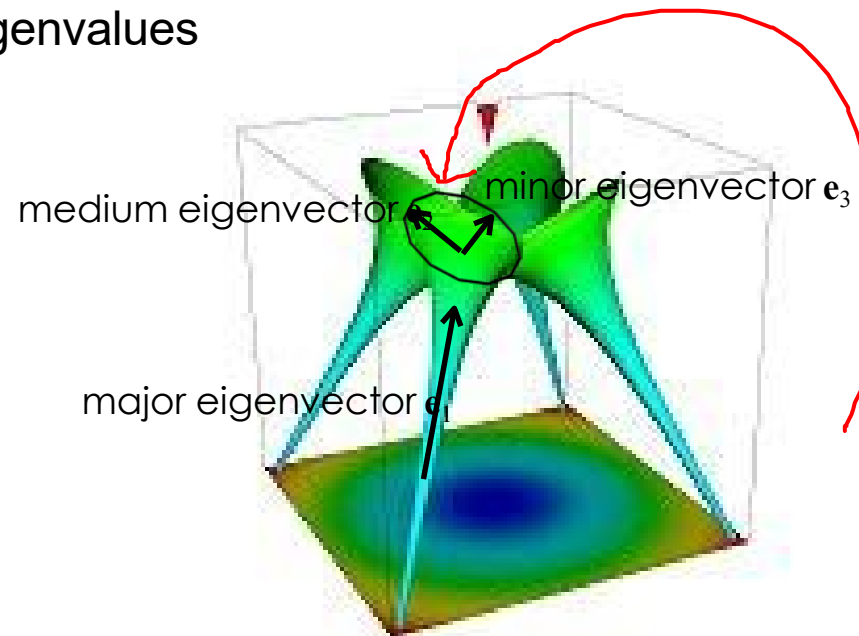
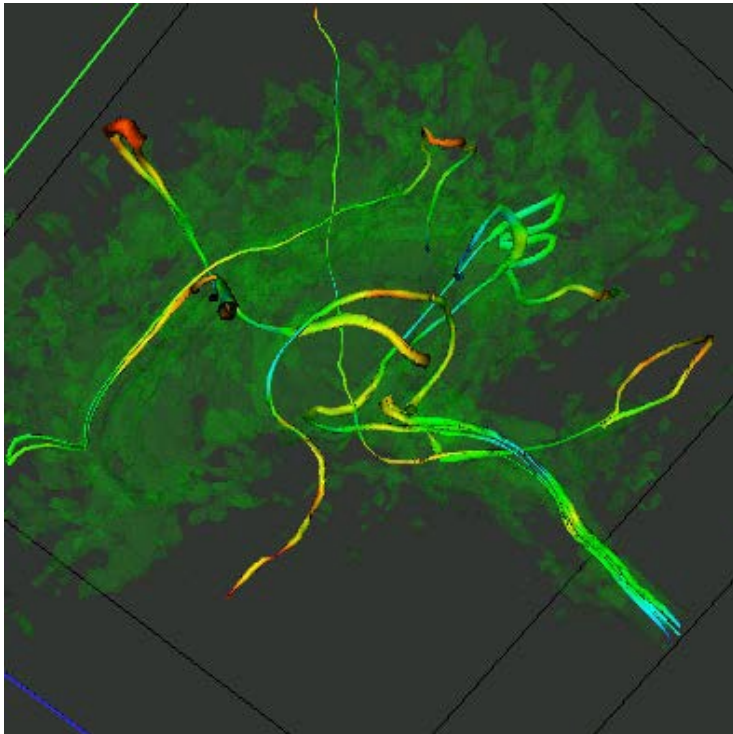
```
trace = fibertrace(P, e){
    trace->add(P);
    do {
        Pn = integrate_forward(P, e1, dt);
        Tp = filter(T, Pn, ellipsoid, e1);
        cl = anisotropy(Tp)
        if (cl > eps){
            trace->add(Pn);
            P = Pn;
            e1 = direction(Tp);
        }
    } while (cl > eps)
    return(trace);
}
```





# Hyperstreamlines

- trace stream tubes in major eigenvector field (like so far)
- use an **elliptic** cross-section
  - oriented along medium + minor eigenvectors
  - scaled with medium + minor eigenvalues



Tube color is often mapped to direction...giving viewers another cue to help understand the fiber path through 3D space.

Tube cross-section shows diffusion across fibers

- Thin, round tubes: we're in a fiber **bundle**
- Thick, flat tubes: we're in a fiber **sheet**
- Thick, round tubes: we're **exiting** a fiber