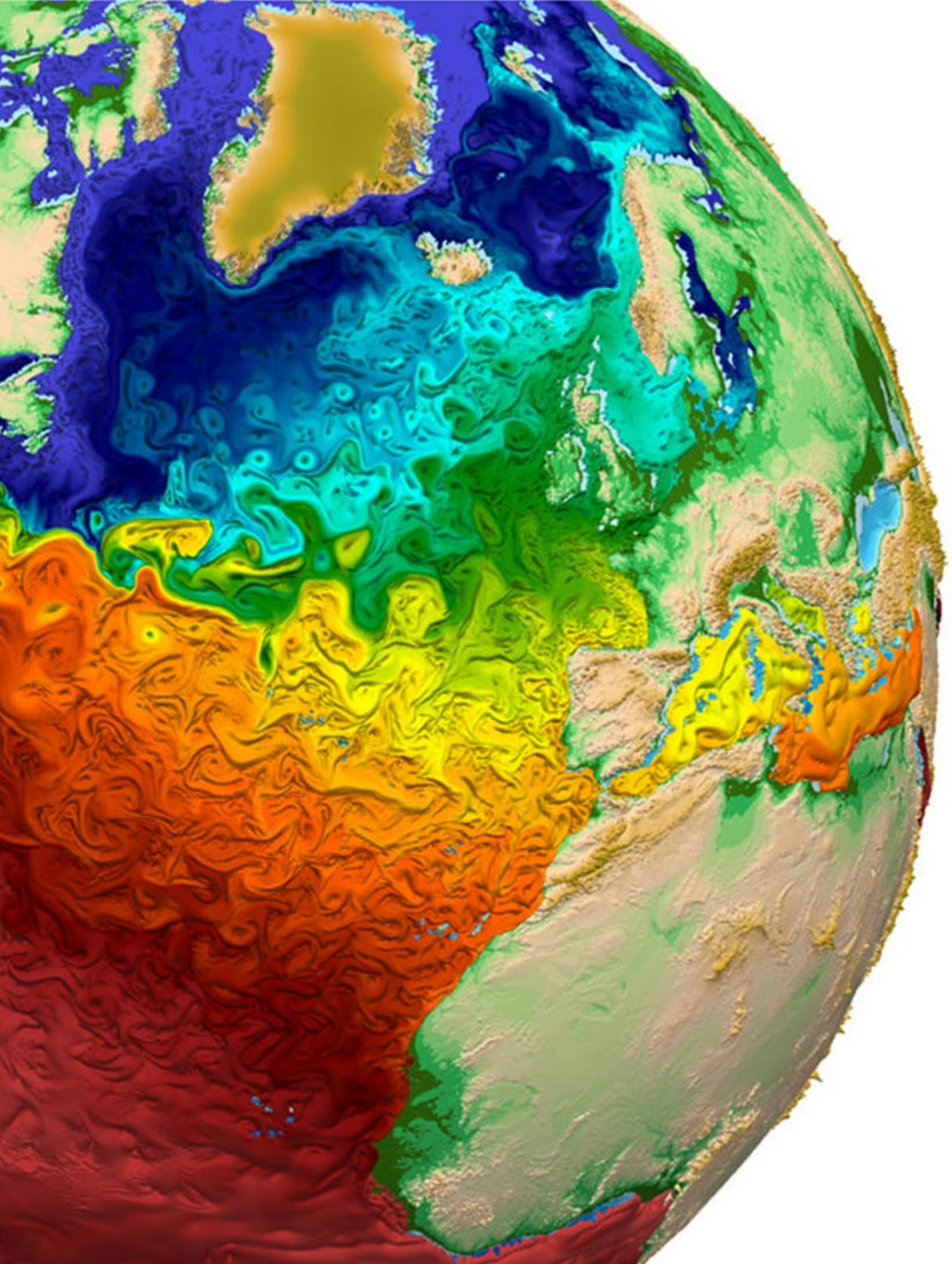


Domain Modeling

Triangulated Surface Meshes

Scientific Visualization

Professor Eric Shaffer

ILLINOIS

Polygonal Meshes

In rendering, we typically will generate an image by simulating the reflection of light off surfaces

Rasterization engines most often use polygonal meshes to represent surfaces

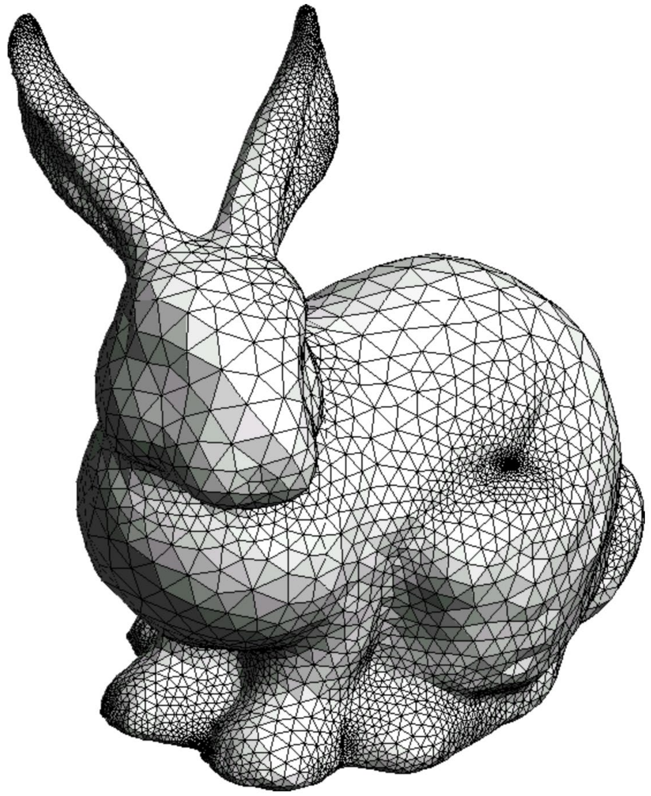
Modern GPUs are designed specifically to rasterize triangles

Many advantages to using triangles:

- Simplest 2D primitive…
- Any 2D polygon can be triangulated
- Can easily represent sharp surface features

Why do we say 2D when we are rendering in 3D?

Any disadvantages you can think of?



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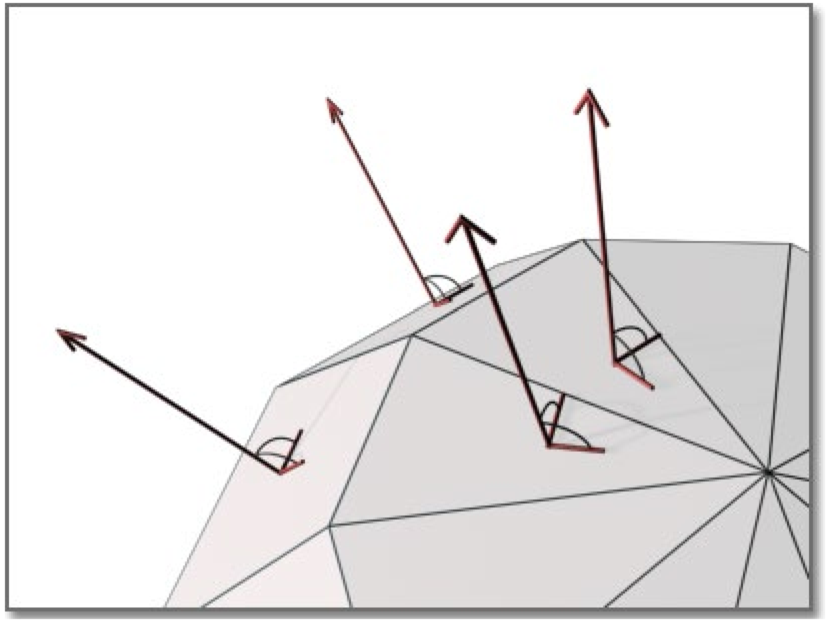
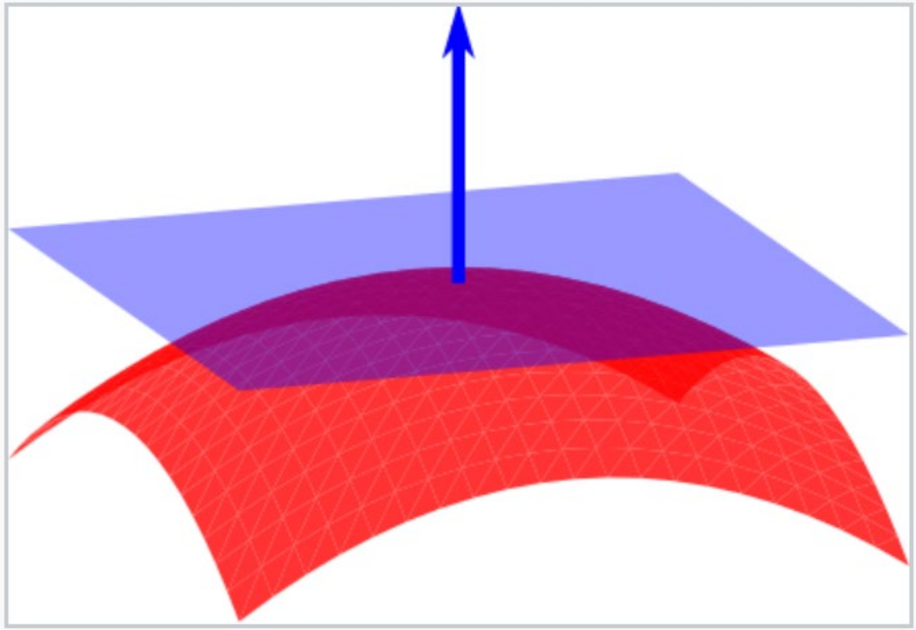
Vocabulary: Surface Normals

A normal is a vector that is perpendicular to object

Each triangle in a surface mesh has outward facing normal

The normal is just the vector perpendicular to the triangle





A normal to a surface at a point is the same as a normal to the tangent plane to the surface at the same point.

Courtesy Wikipedia

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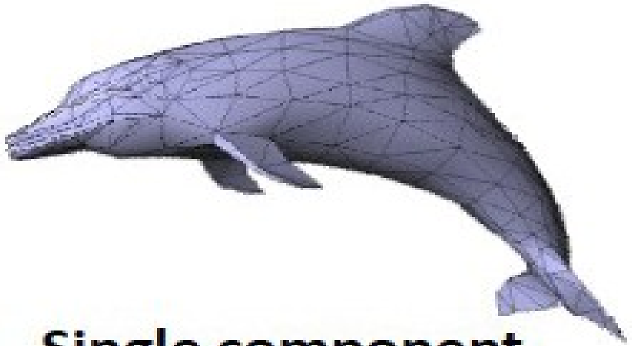
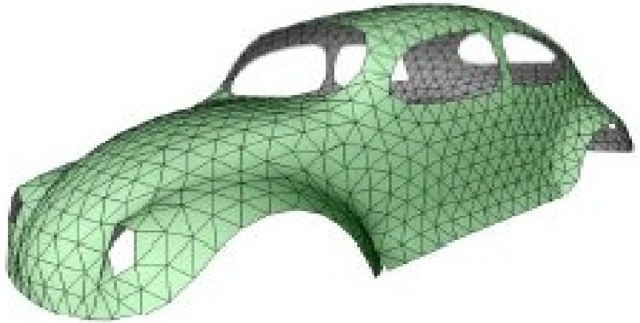
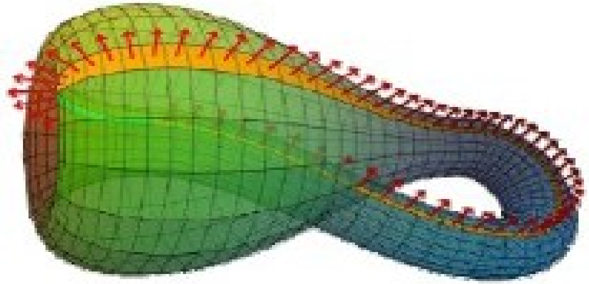
Vocabulary: Surface Mesh Properties



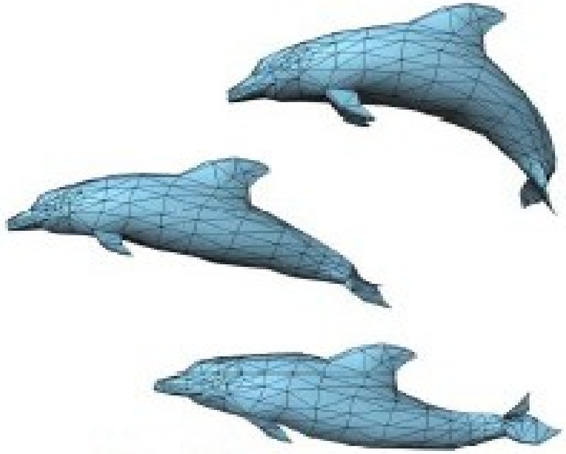
Single component, closed, triangular, orientable manifold



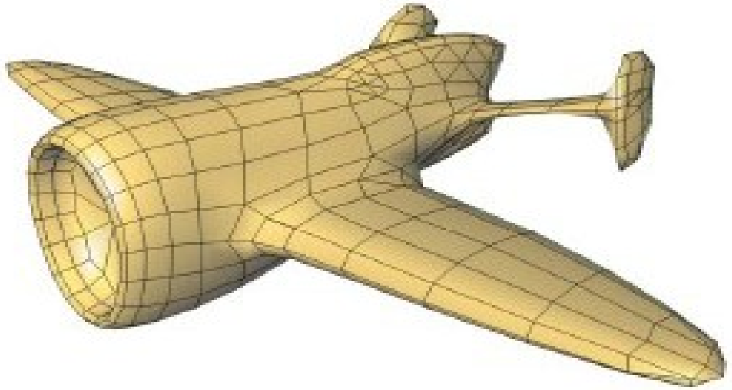
With boundaries



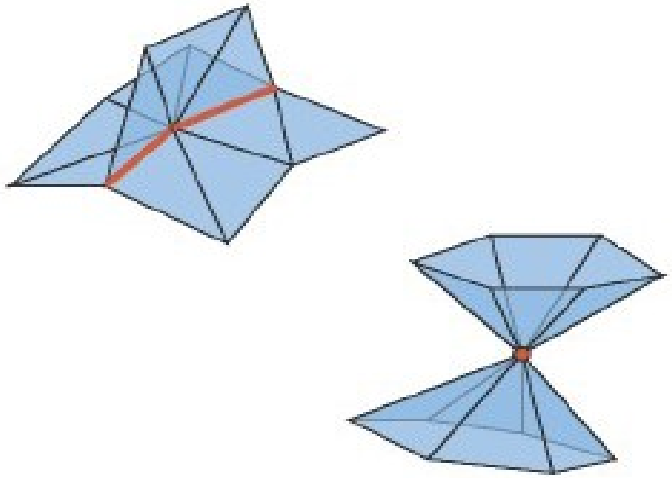
Not orientable



Multiple components

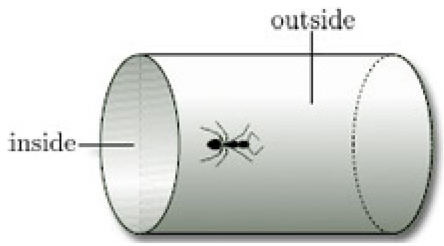


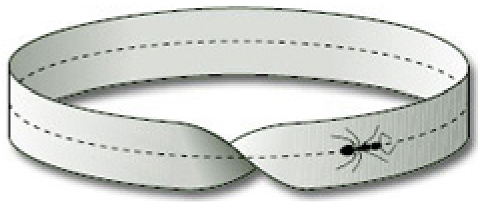
Not only triangles



Non manifold

Orientability

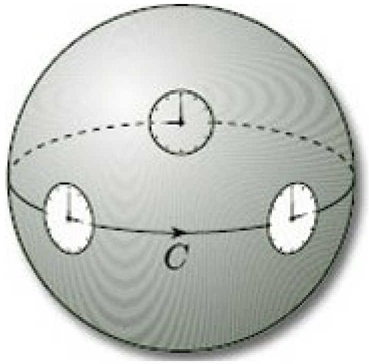


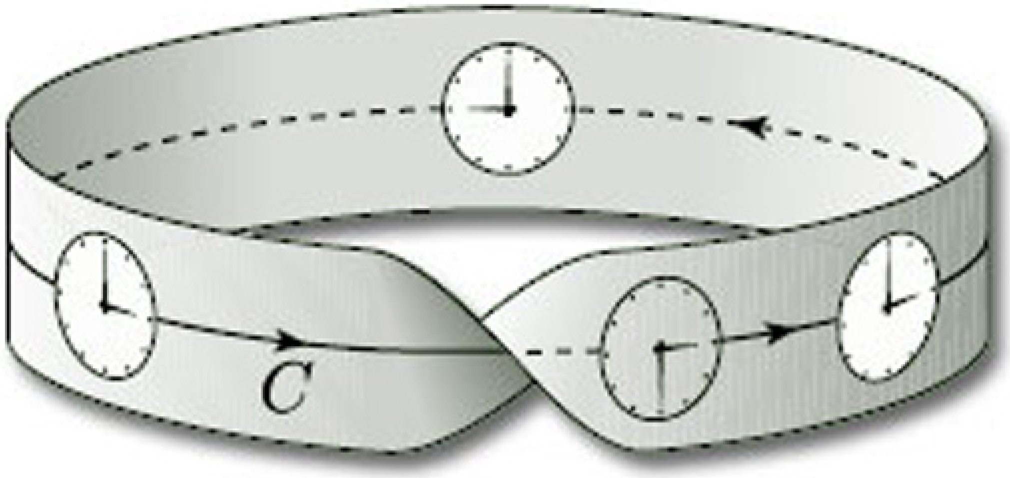


Not that relevant to this course….but it is interesting….

Imagine sliding a clock face around a surface.

On an orientable surface the clock face will return to starting point and appear the same. (Sphere)
On a non-orientable surface it will be a mirror image of the original clock face (Moebius Strip)
…there is no consistent definition of clockwise on a non-orientable surface.



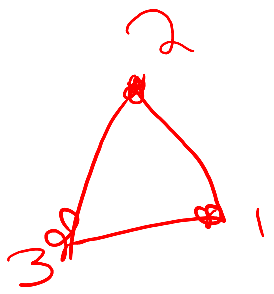


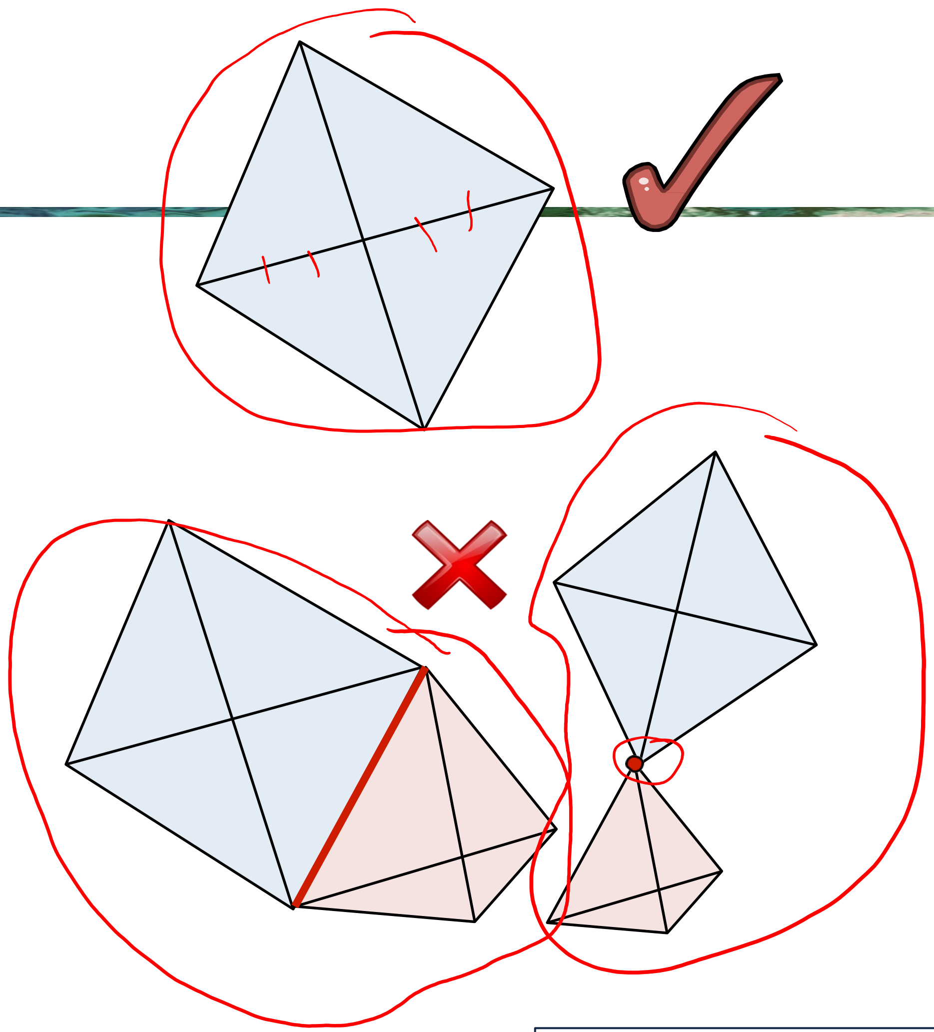
Surface Mesh Properties

Manifold:

1.Every edge connects exactly two faces

2. Vertex neighborhood is “disk-like”





The blue and pink meshes above are tetrahedra, like 4-sided pyramids

Orientable: Consistent normals

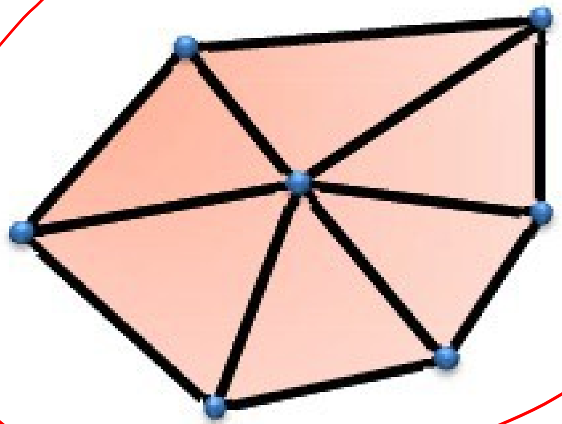
Watertight: Orientable + Manifold

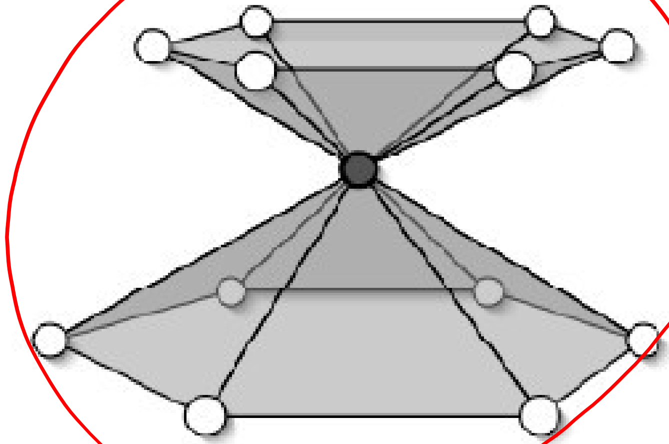
Boundary: Some edges bound only one face

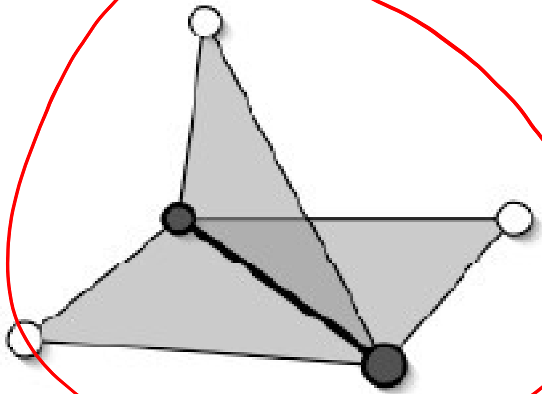
Ordered: Vertices in CCW order when viewed from normal

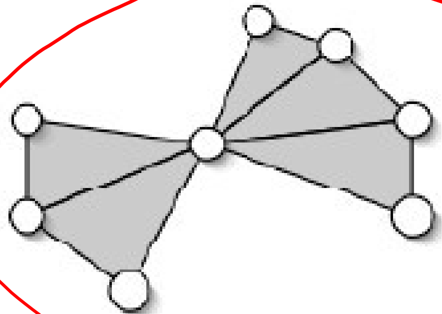
2-Manifold Mesh Examples

Disk-shaped neighborhoods





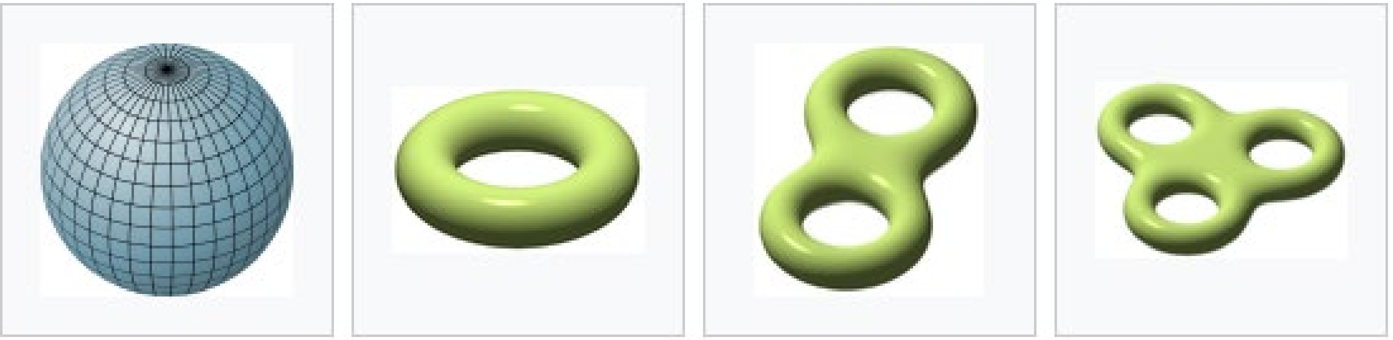




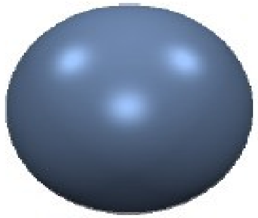
non-manifolds

Genus

Genus of orientable surfaces

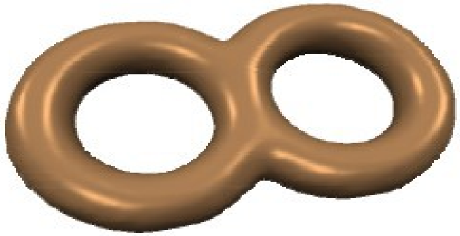


genus 0 genus 1 genus 2 genus 3



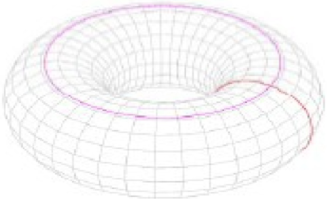
Genus 0

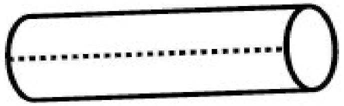
Genus 1



Genus 2

Genus ?





Euler Characteristic

For a closed (no boundary), manifold, connected surface mesh:

$$V-E+F=2(1-G)$$

$$V-E+F=2$$

V = number of vertices
E = number of edges
F = number of faces
G = genus (number of holes in the surface)

A **2-manifold** is a surface (locally like a plane)

Leonhard Euler
1707 - 1783

Can you think of anything else named for him?

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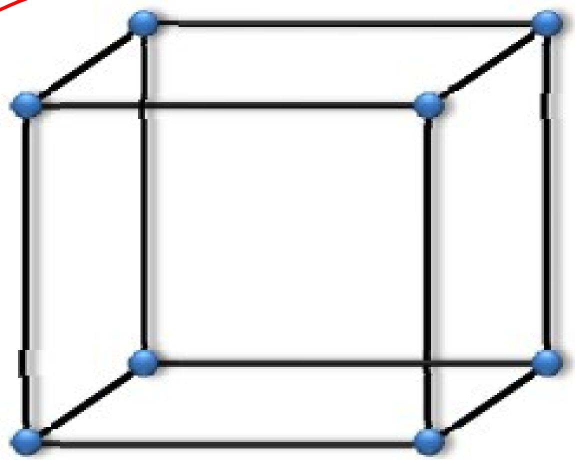
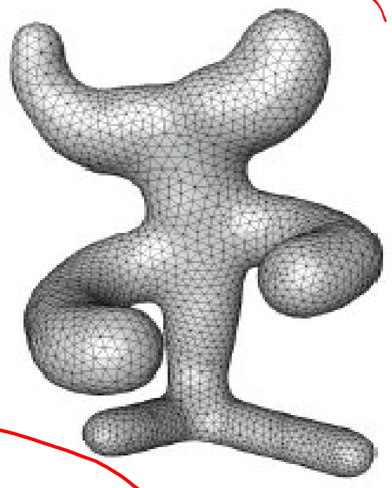
Euler Characteristic for Closed 2-Manifold Polygonal Meshes

$$V + F - E = \boxed{\chi}$$ Euler characteristic



$V = 8$
$E = 12$
$F = 6$
$\chi = 8 + 6 - 12 = 2$

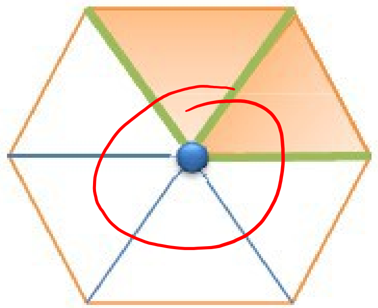


$V = 3890$
$E = 11664$
$F = 7776$
$\chi = 2$

...and if they are triangle meshes

- *Triangle* mesh statistics

$$E \approx 3V$$
$$F \approx 2V$$



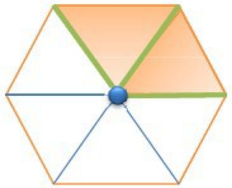
- Avg. valence ≈ 6
  Show using Euler Formula

Aside from being totally interesting on their own, these formulas are useful for computing memory usage

…and if they are triangle meshes

- *Triangle* mesh statistics

  $E \approx 3V$
  $F \approx 2V$



- Avg. valence ≈ 6
  Show using Euler Formula

$$V - E + F = 2$$

$$V - \frac{3F}{2} + F = 2$$

$$V - F/2 = 2$$

$$V = F/2 + 2$$

$$2V = F + 4$$

$$\boxed{2V \approx F}$$