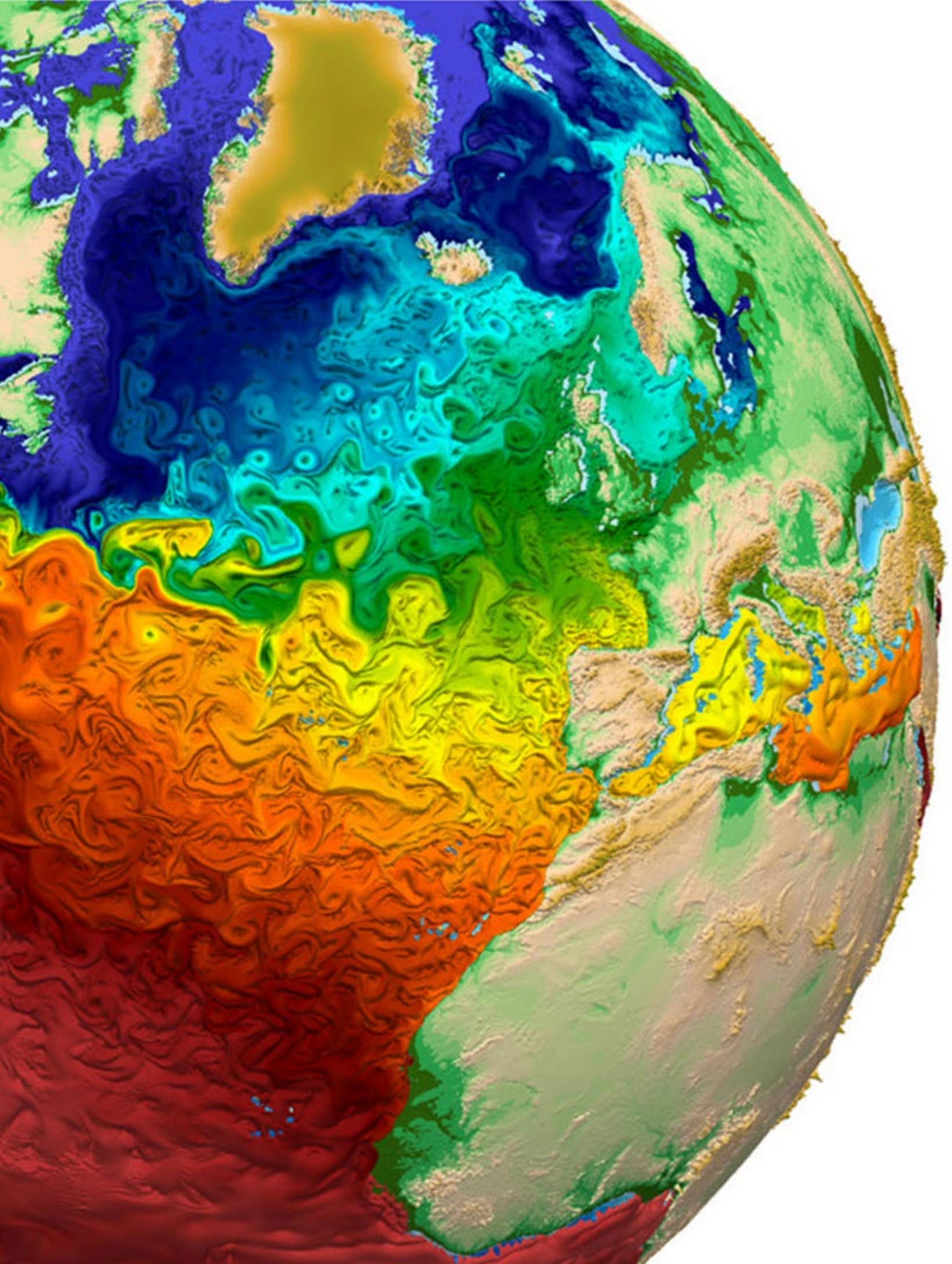

Tensor Visualization

What are Tensors?

Scientific Visualization

Professor Eric Shaffer

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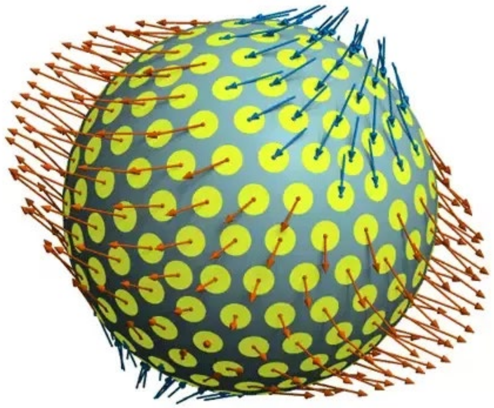
Tensors

“A tensor is just a machine that takes in some number of vectors and spits out some other vectors in a linear fashion. For example, the dot product can be viewed as a tensor that takes two vectors in and spits out a number. ”

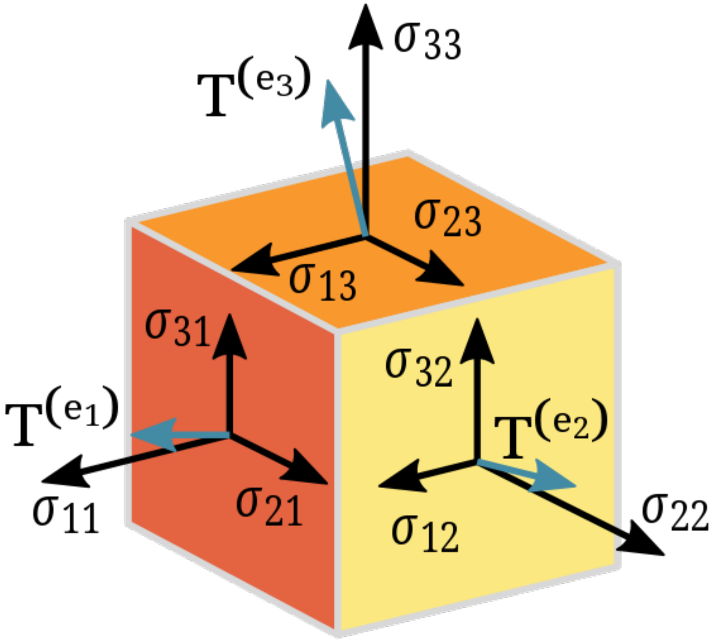


Stress forces acting on a particle in a homogeneous continuous medium under a uniform linear stress, as a function of the orientation of a surface element on the particle's boundary.

What is a Tensor

In mathematics, a **tensor** is an algebraic object that describes a (multilinear) relationship between sets of algebraic objects related to a vector space. Objects that tensors may map between include vectors and scalars, and even other tensors.

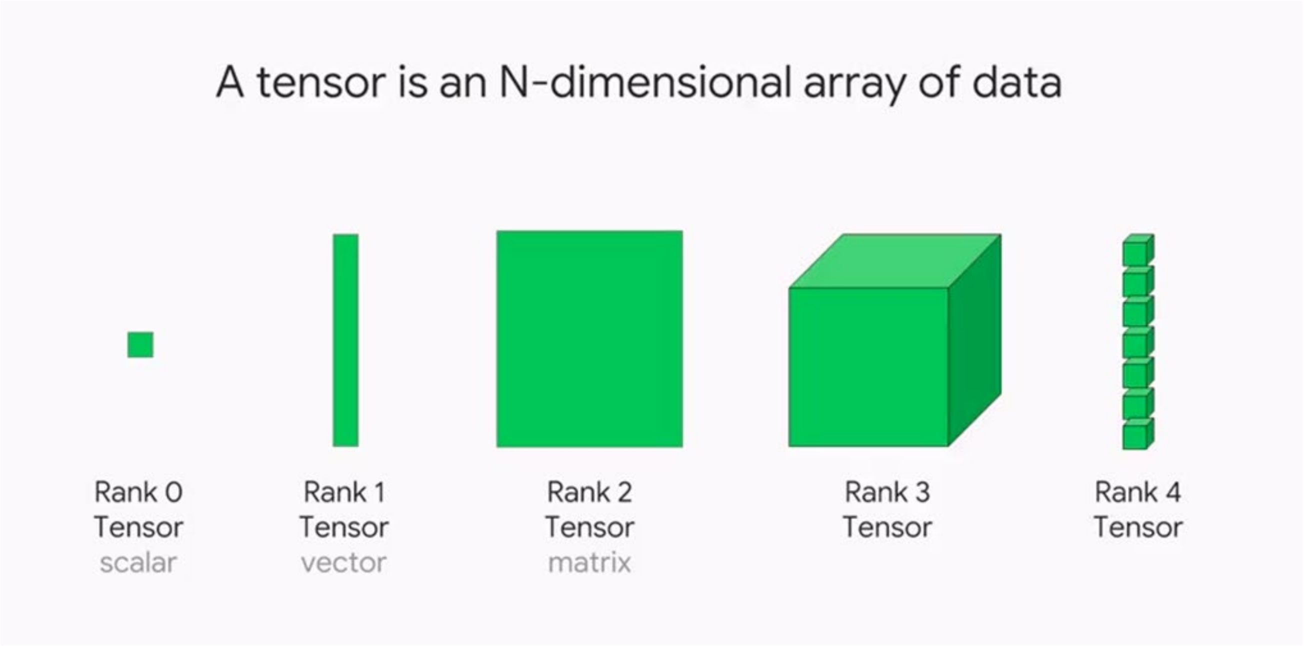
-wikipedia



Is Tensor a Matrix?

A tensor is often thought of as a generalized matrix.

- A multi-dimensional array of numbers
- A rank 2 tensor is a matrix
- Not all matrices are tensors



The TensorFlow machine learning platform name derives from the operations that such neural networks perform on multidimensional data arrays, which are referred to as tensors. ...

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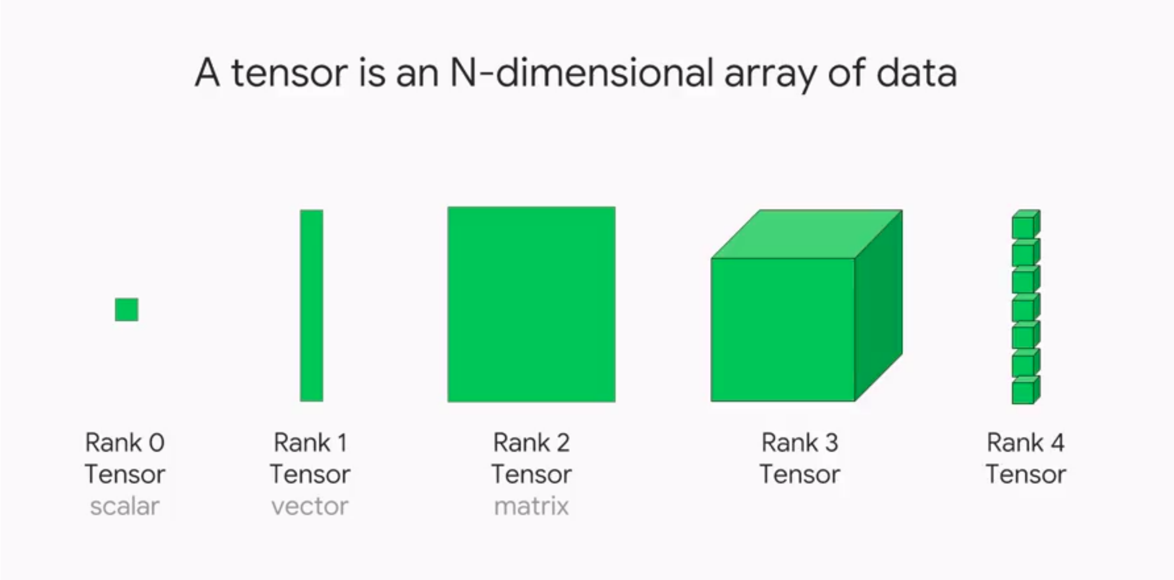
What is a Tensor?

Explanation 1: Dimensionality

- scalar: a 0D array of values e.g. 1 value
- vector: a 1D array of values e.g. 3 values
- tensor: a 2D matrix of values e.g. 3x3 = 9 values

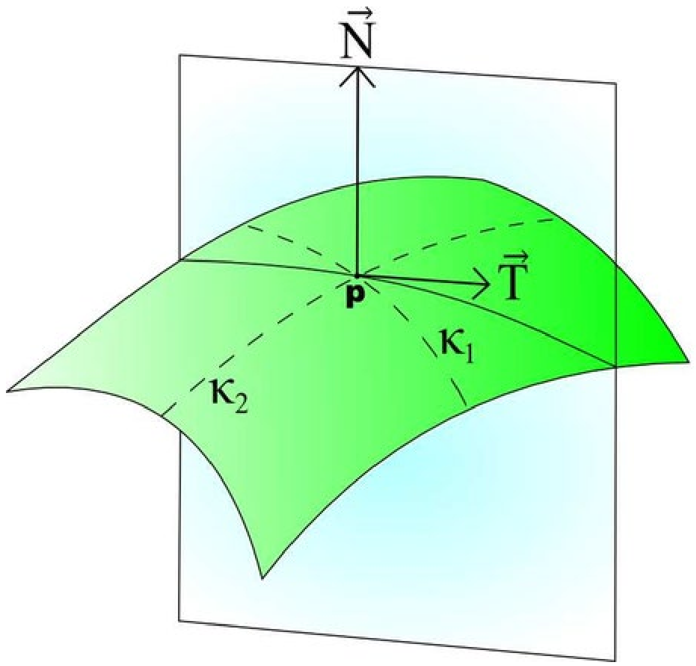


We will focus only on rank-2 tensors

What is a Tensor?

Explanation 2: Analysis

- scalar: **magnitude** (of some signal at a point in space)
- vector: **magnitude** and **direction** (of some signal at some point in space)
- tensor: **variation** of **magnitude** (of some signal at some point in space)



What is a Tensor?

Explanation 3: As a function

- **scalar:** at $\mathbf{x} \in \mathbf{R}^3$, measure some value $s \in \mathbf{R}$
- **vector:** at $\mathbf{x} \in \mathbf{R}^3$, measure some magnitude and direction $\mathbf{v} \in \mathbf{R}^3$
- **tensor:** at $\mathbf{x} \in \mathbf{R}^3$ *and* in a direction $\mathbf{v} \in \mathbf{R}^3$, measure some magnitude $s \in \mathbf{R}$

Fields

So we have different kinds of fields (i.e. **functions** of a variable $\mathbf{x} \in \mathbf{R}^3$):

Scalar fields $s : \mathbf{R}^3 \rightarrow \mathbf{R}$
Vector fields $\mathbf{v} : \mathbf{R}^3 \rightarrow \mathbf{R}^3$
Tensor fields $\mathbf{T} : \mathbf{R}^3 \times \mathbf{R}^3 \rightarrow \mathbf{R}$

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