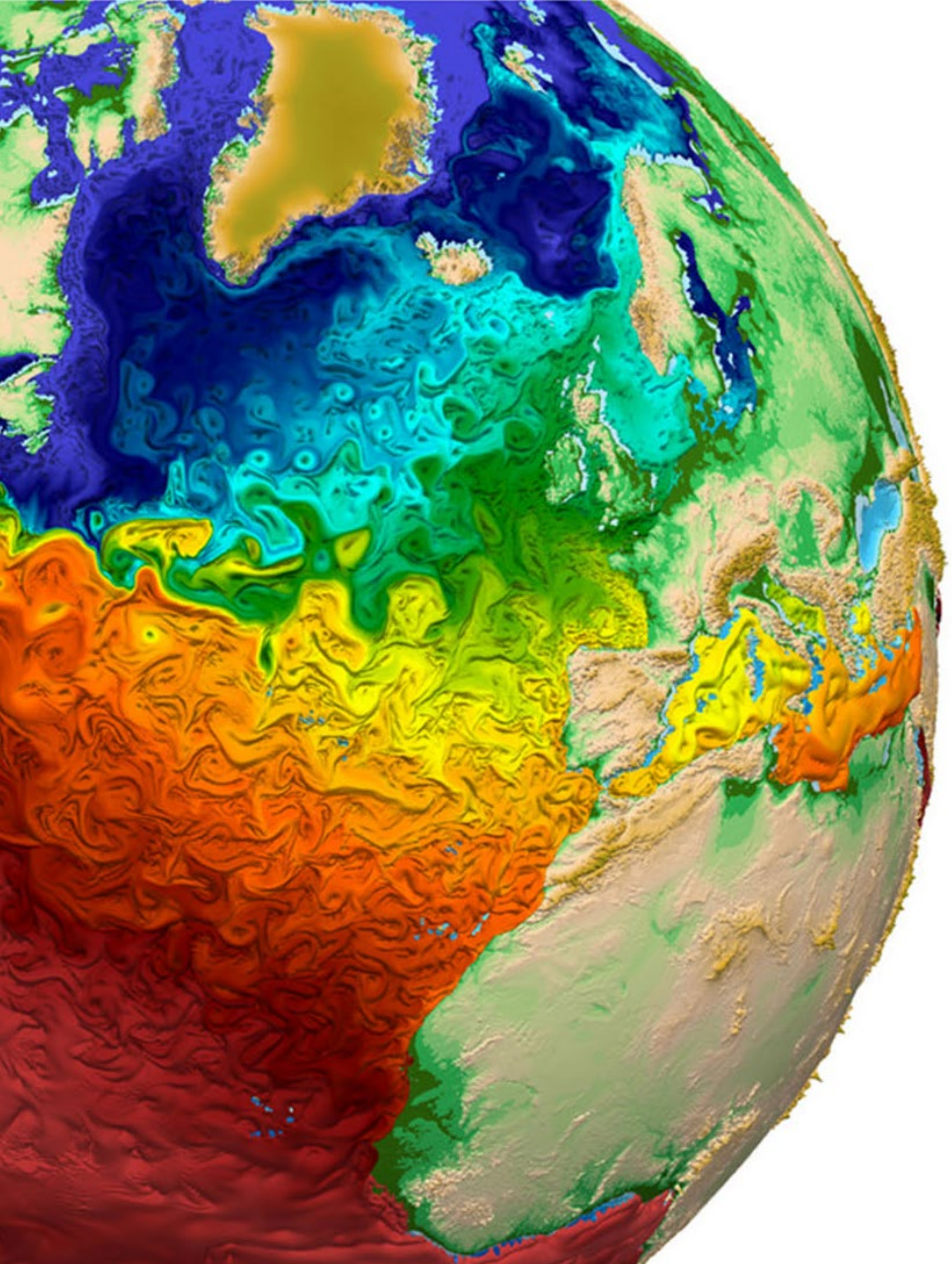

Vector Visualization

Derived Quantities

Scientific Visualization

Professor Eric Shaffer

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Visualizing Derived Scalar Quantities

Compute derived scalar quantities from vector fields $\mathrm{v}(x, y, z) = \langle v_x, v_y, v_z \rangle$

Use known scalar visualization methods for these quantities

Divergence $\operatorname{div} \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ equivalent to $\operatorname{div} \mathbf{v} = \lim_{\Gamma \to 0} \frac{1}{|\Gamma|} \int_{\Gamma} (\mathbf{v} \cdot \mathbf{n}_{\Gamma}) ds$

Example:

$$\mathrm{v}(x, y, z) = \langle xy^2, xy^2, zy \rangle$$
$$\mathrm{v}(1,2,3) = \langle 4,4,6 \rangle$$

$$\operatorname{div} \mathrm{v}(x, y, z) = y^2 + 2xy + y$$
$$\operatorname{div} \mathrm{v}(1,2,3) = 4 + 4 + 2 = 10$$

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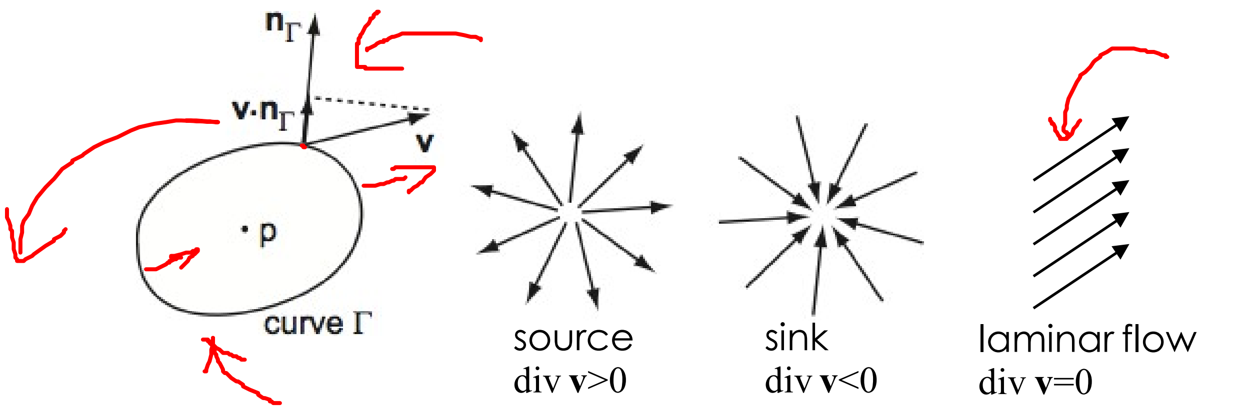
- think of vector field as encoding a fluid flow
- intuition: degree to which the field converges or diverges at a point
- given a field $\mathbf{v} : \mathbf{R}^3 \to \mathbf{R}^3$, $\operatorname{div} \mathbf{v} : \mathbf{R}^3 \to \mathbf{R}$ is



$\operatorname{div} \mathbf{v}$ is sometimes denoted as $\nabla \cdot v$

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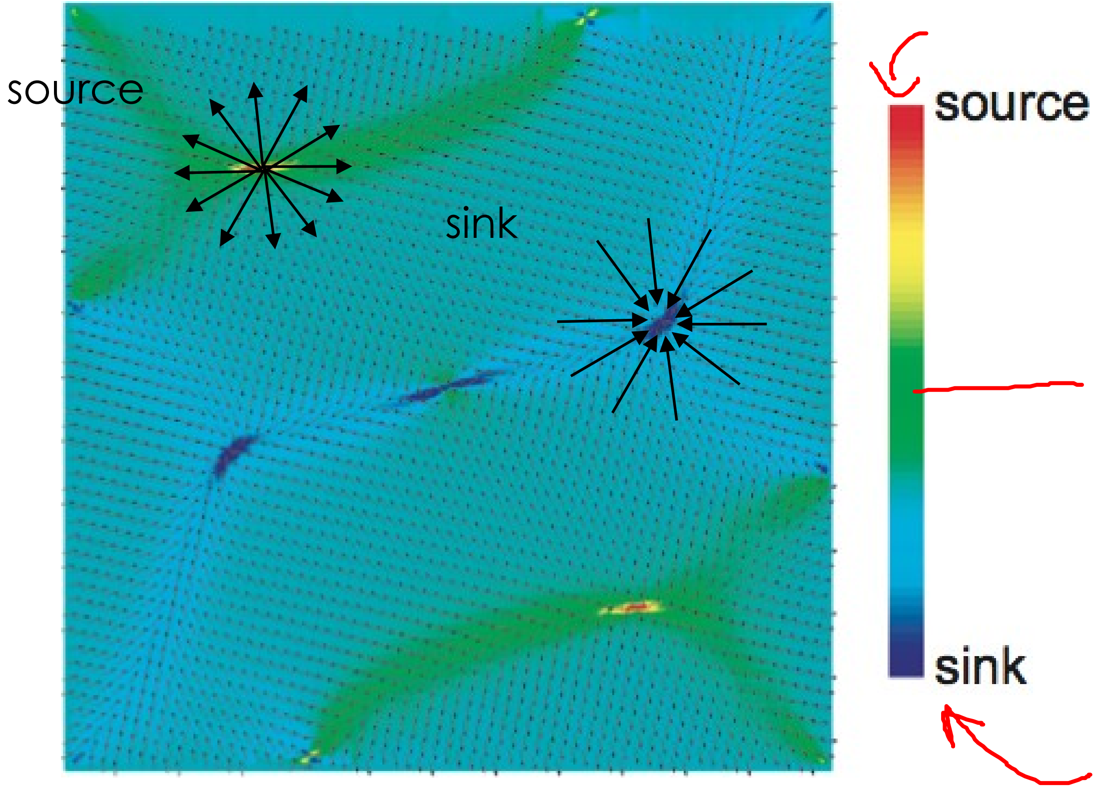
Example: Divergence

- compute using definition with partial derivatives
- visualize using color mapping



- gives a good impression of where the flow 'enters' and 'exits' some domain

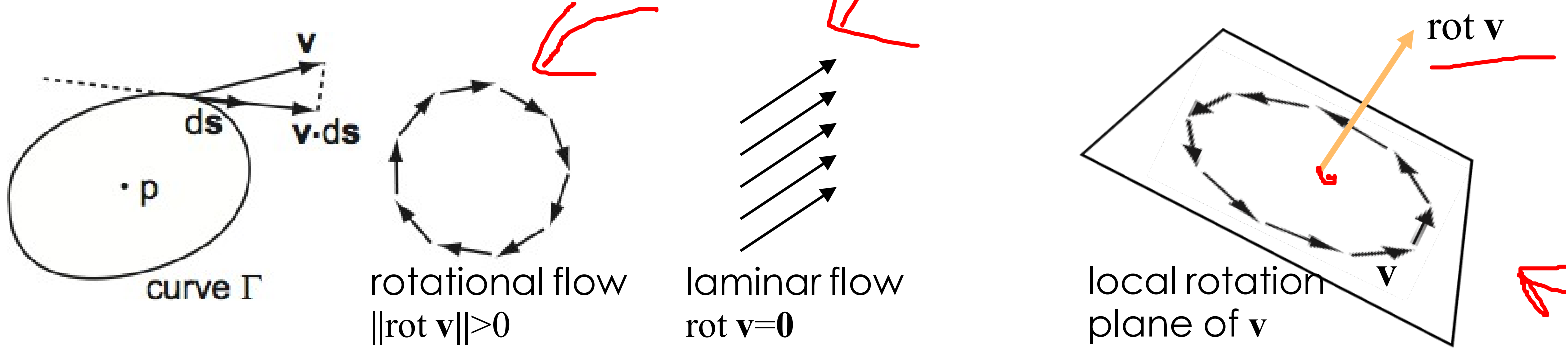
Vorticity

Vorticity or Rotor

- vector quantity...a field locally orthogonal to the plane of rotation
- consider again a vector field as encoding a fluid flow
- intuition: magnitude is how quickly the flow 'rotates' around each point?
- given a field $\mathbf{v} : \mathbf{R}^3 \rightarrow \mathbf{R}^3$, rot $\mathbf{v} : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ is

produced by taking the curl of the flow field

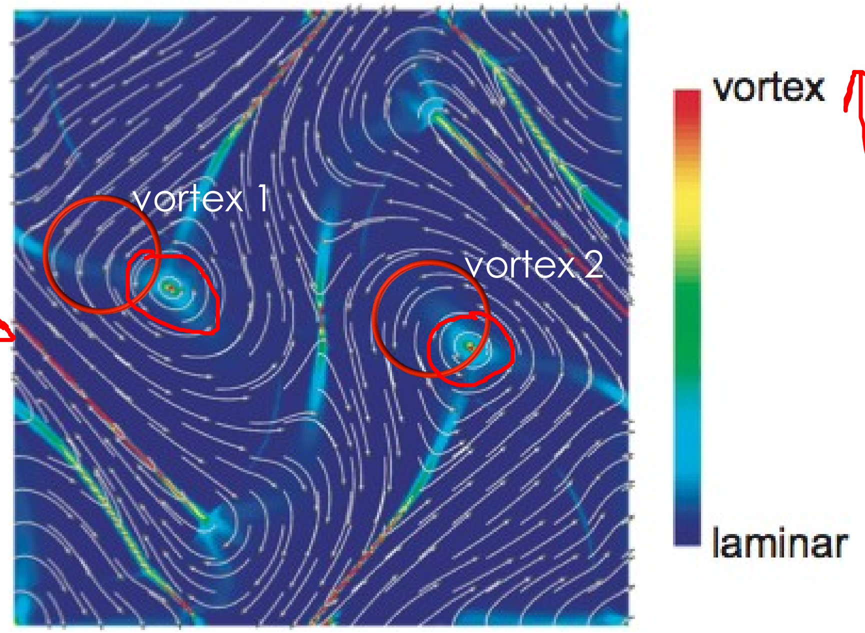
$$\text{rot } \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}, \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$



rot $\mathbf{v}$ is sometimes denoted as $\nabla \times v$

Visualizing Vorticity

- compute using definition with partial derivatives
- visualize magnitude $\|\mathbf{rot}\ \mathbf{v}\|$ using e.g. color mappin



- very useful in practice to find vortices = regions of high vorticity
- these are highly important in flow simulations (aerodynamics, hydrodynamics)

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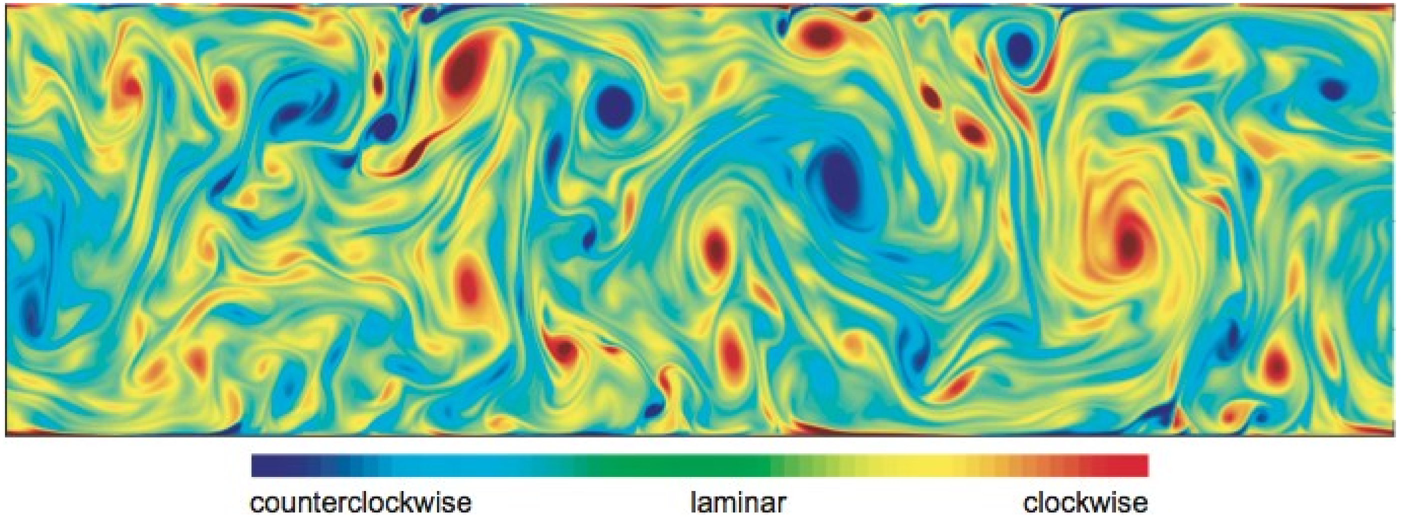
Visualizing Vorticity

Example of vorticity

- 2D fluid flow
- simulated by solving Navier-Stokes equations
- visualized using vorticity

Express $\mathrm{v}(x, y) = \langle v_x, v_y, 0 \rangle \rightarrow \mathrm{rot}\,\mathrm{v} = \left\langle 0,0, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right\rangle$



$-$ 0 $+$

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