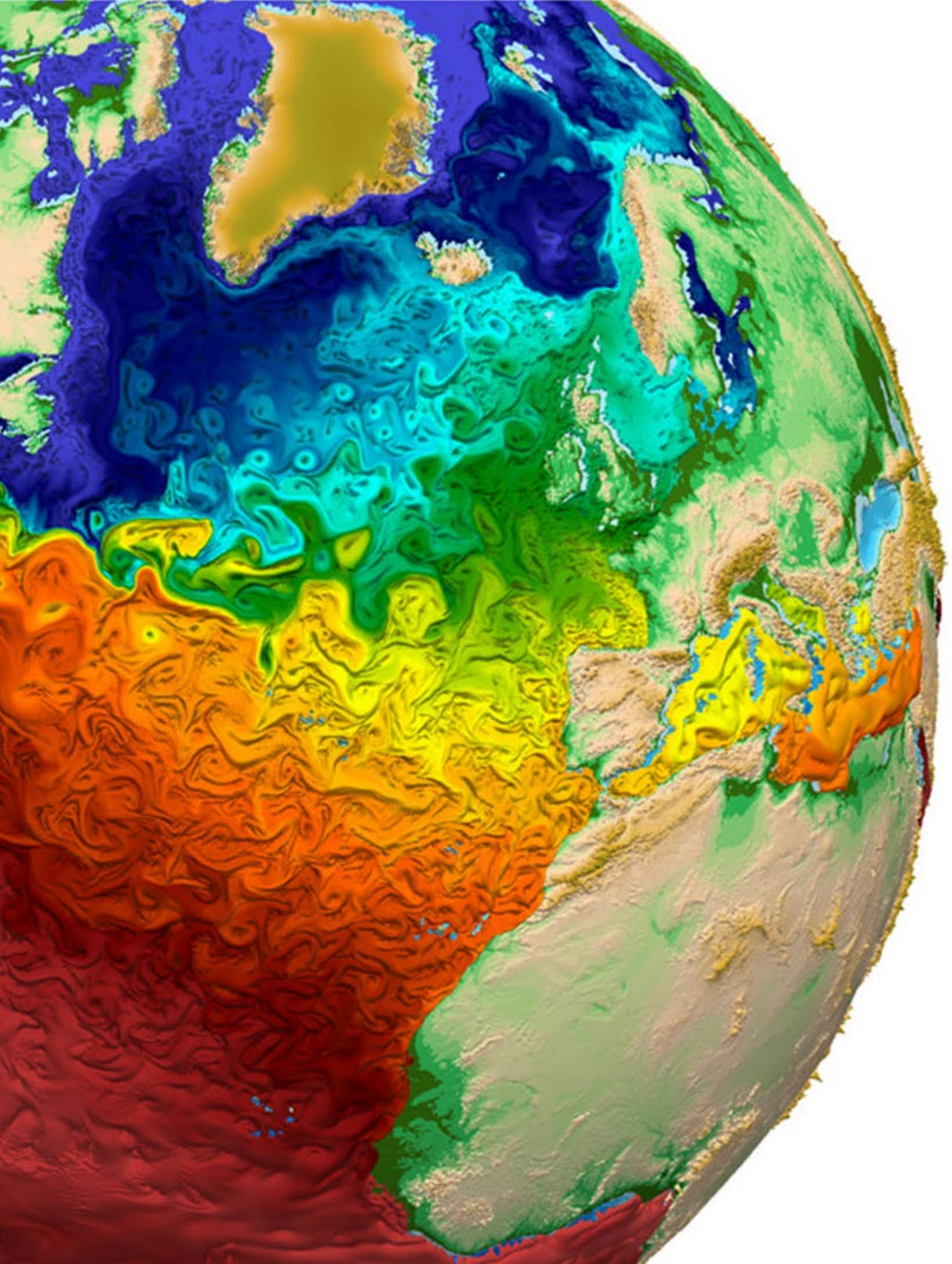


Contouring

Marching Tetrahedra

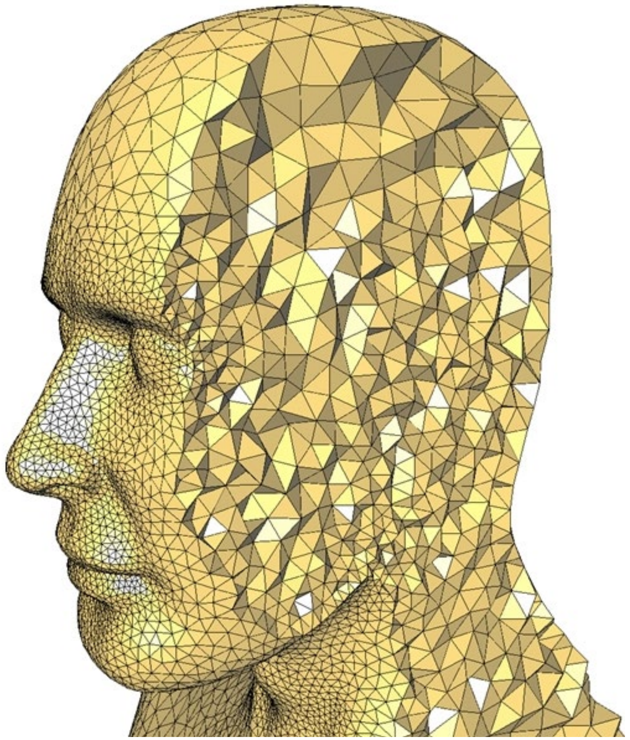
Scientific Visualization

Professor Eric Shaffer

ILLINOIS

Motivation

- Marching Cubes was patented
- MC applies to structured grids…not unstructured tetrahedral meshes
  - ..tet meshes are popular
- Handling ambiguities in MC is not easy



ILLINOIS

Marching Tetrahedra: Algorithm

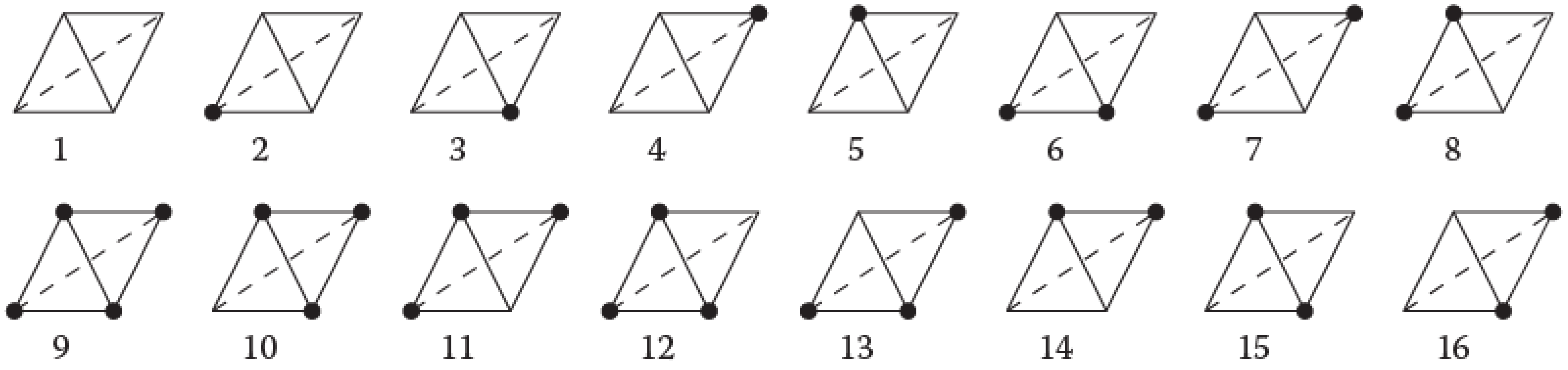
Read isosurface lookup table for tetrahedra

↓

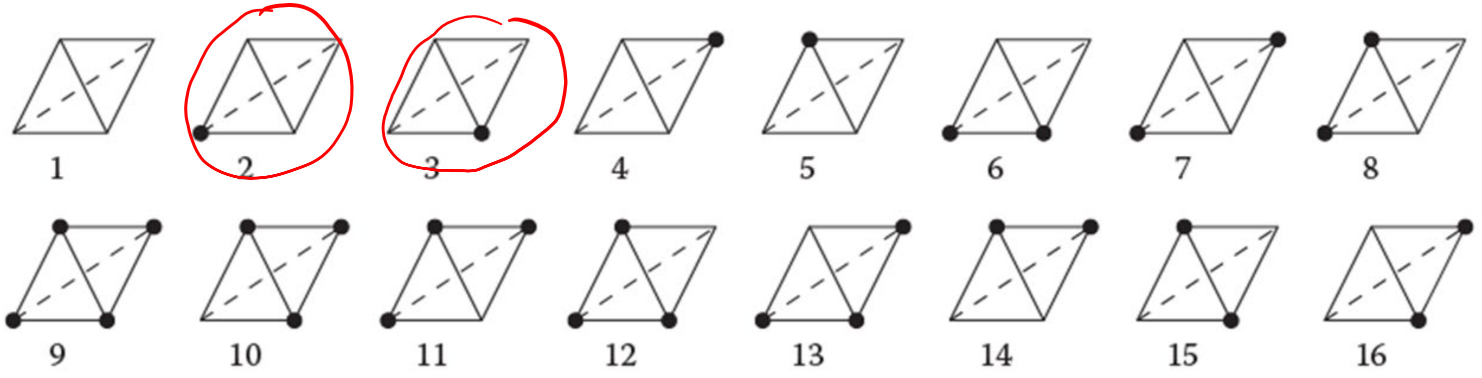
For each mesh tetrahedron, retrieve isosurface triangles

↓

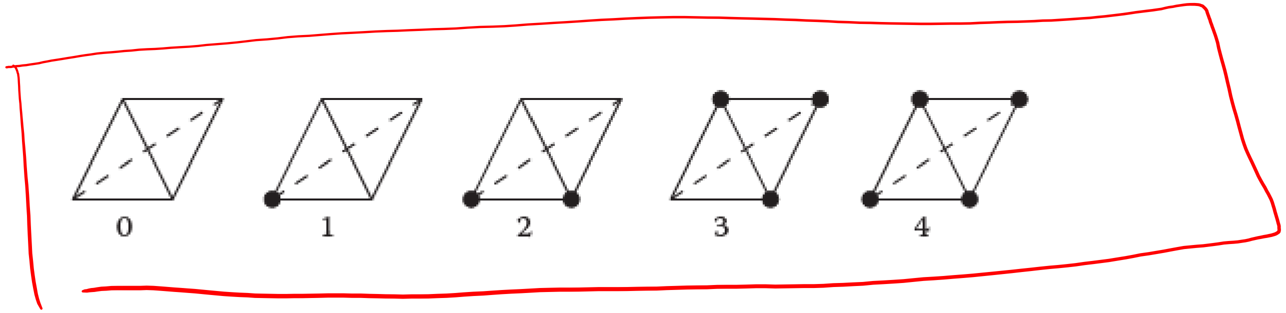
Compute isosurface vertex coordinates using linear interpolation



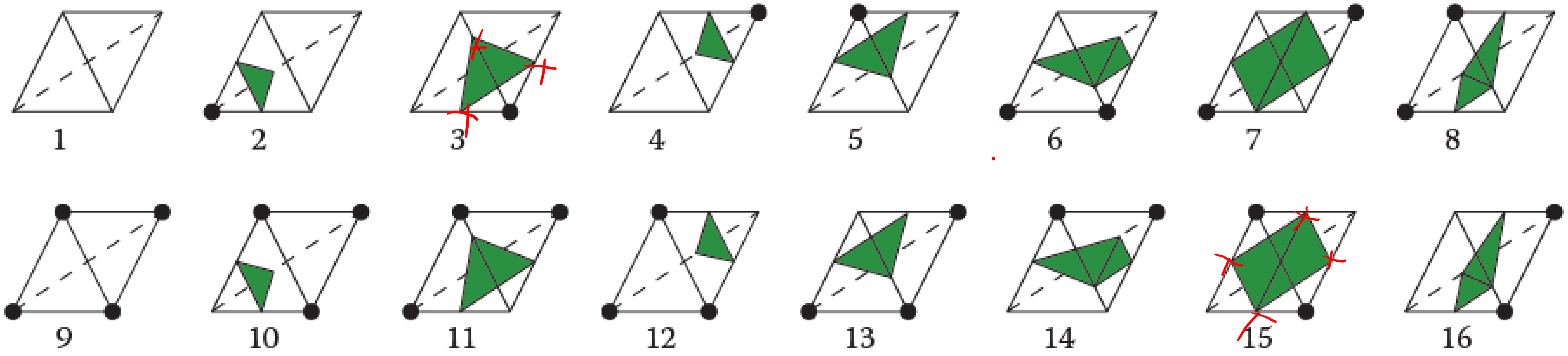
Cell Configurations



Only 5 equivalence classes of configurations if we allow rotation



Generating the Isosurface



Generate a vertex along each bipolar edge

For mesh edge $[p, q]$ with $f(p) = s_p$ and $f(q) = s_q$ and isovalue f_0

Place vertex at $(1 - t)p + tq$ where $t = \frac{(f_0 - s_p)}{(s_q - s_p)}$ $\in [0,1]$

s_p f_0 s_q

Using these vertices, generate one or two triangles inside the tetrahedron for the isosurface patch

Properties of the Isosurface

Assume:

- The isovalue does not equal the scalar value of any mesh vertex.
- The tetrahedral mesh is a partition of a 3-manifold with boundary.

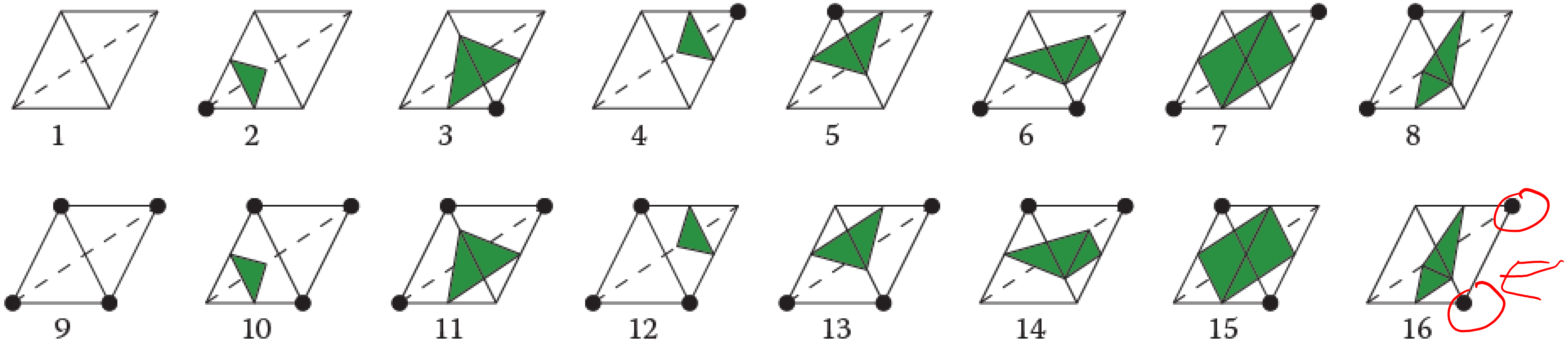
Resulting Properties

- The isosurface is a piecewise linear, orientable 2-manifold with boundary.
- The boundary of the isosurface lies on the boundary of the grid.
- The isosurface does not contain any zero-area triangles or duplicate triangles

Ambiguity

Marching Tetrahedra has no ambiguous configurations.

- The four vertices of each tetrahedron are connected by tetrahedron edges
- Each such edge is intersected at most once by the isosurface
- There is no possible ambiguity in the isosurface construction



Disadvantages

- Generates a large number of triangles
  - …more than Marching Cubes
- Unstructured nature of the tetrahedral mesh impacts scalability
  - Harder to manage storage
  - Partitioning for parallel processing

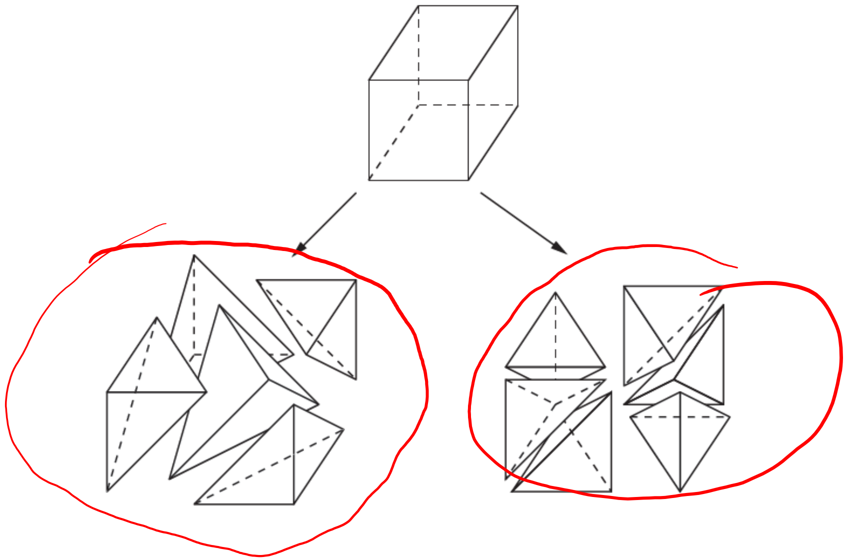
All these issues can be overcome…

Cube Tetrahedralization

Can apply marching tetrahedra to regular grids

Decompose cubes into tetrahedra

Can choose to use either 5 or 6 tetrahedra per cube

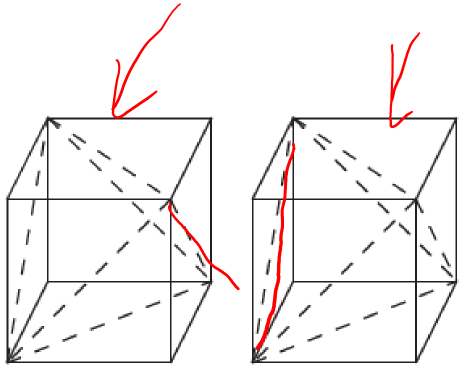


Cube Tetrahedralization

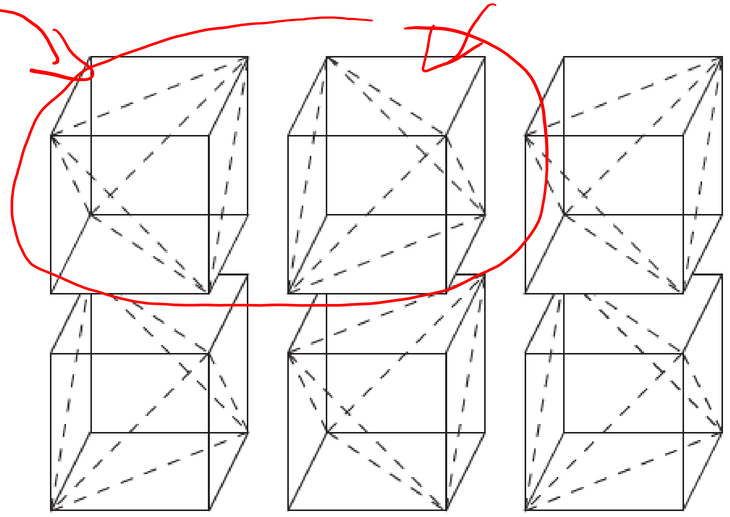
This creates diagonal facets on the square faces of the cube

Diagonals of adjacent cube faces must match

- Using 6 tetrahedra per cube results in matching diagonals
- If 5 tetrahedra are used, need to alternate the decomposition



Mismatched diagonals

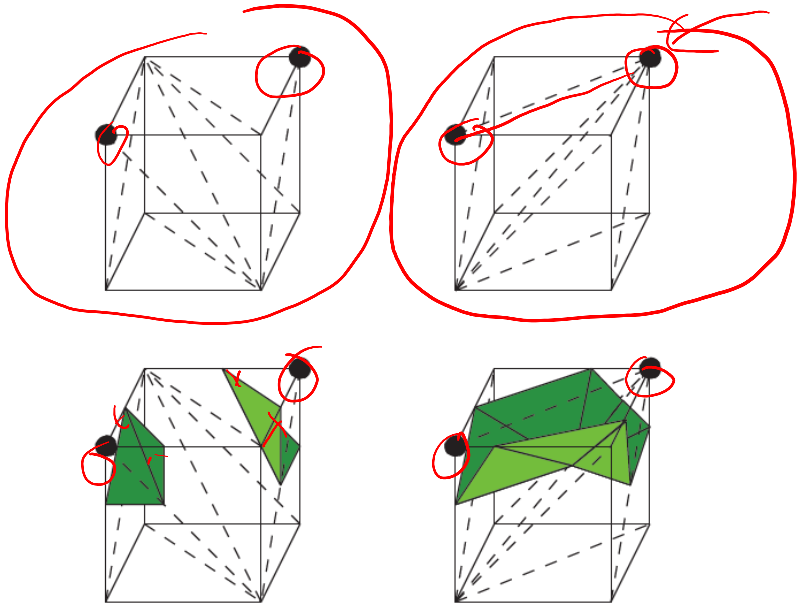


Alternating decomposition: matched diagonals

Where did the Ambiguity Go?

- MC has ambiguous cases
- MT does not

Choice of tetrahedral decomposition orientation resolves the ambiguity



Isocontouring a Triangle Mesh

Extremely simple algorithm

…no table

…each triangle has at most one contour edge

…place the vertices of contour edge using linear interpolation.