Comparing dominance of tennis' big three

via multiple-output Bayesian quantile regression models

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Introduction

Big Three

- Roger Federer
- Rafael Nadal
- Novak Djokovic

- Won 63 out of 77 Grand Slam tournaments, between Wimbledon in 2003 until 2022.

Dominance

- List of Grand Slam Winners:

Table 1: January-2022

Player	Titles
1. Roger Federer	20
1. Rafael Nadal	20
1. Novak Djokovic	20
4. Pete Sampras	14
5. Roy Emerson	12

Table 2: Currently, August-2022

Player	Titles
1. Rafael Nadal	22
2. Novak Djokovic	21
3. Roger Federer	20
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Question: Who is more dominant between the Big Three?

How to measure dominance in a tennis match

Important notes:

- A tennis match is divided into sets and games.
- A player with most sets wins the match.
- A player can win more games, but still lose the match.
    - Example: 7-6, 0-6, 7-6.

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- Solution:
  - **Relative points:** ratio points won/lost in a match.
  - **Duration** of the match.

Data

- Data organised by Jeff Sackmann in the repository:
  - https://github.com/JeffSackmann/tennis_atp
- All matches from the Big Three, between 1998 and the US Open in 2021.
  - Excluding Davis Cup and Olympic Games matches.
  - Also matches played on carpet.

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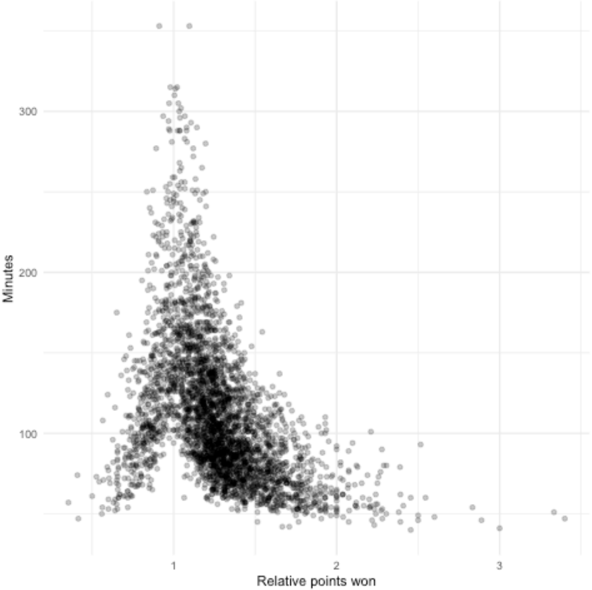
  - Excluding Davis Cup and Olympic Games matches.
  - Also matches played on carpet.
- We should condition on some variables:
  - type of tournament (Grand Slam, Masters 1000, ...);
  - surface (clay, grass and hard courts);
  - wins and losses;
  - rank of opponent.

Data distribution



Bayesian quantile regression for multiple output response variables

Directional quantile regression model

- Response variable is defined as $Y \in \mathbb{R}^k$.
- Directional index can be defined by $\tau \in \mathcal{B}^k := \{v \in \mathbb{R}^k : 0 < ||v|| < 1.\}$.
  - $\tau = \tau u, \tau \in (0, 1)$.
  - Direction: $u \in \mathcal{S}^{k-1} := \{z \in \mathbb{R}^k : ||z|| = 1\}$;
- Define Γ_u, an arbitrary $k \times (k-1)$ matrix of unit vectors.
  - $(u \vdots \Gamma_u)$ is an orthonormal basis of $\mathbb{R}^k$.

DEFINITION:

The τth quantile of Y is the τth quantile hyperplane obtained from the regression:

- $Y_u := u' Y$ on the marginals of $Y^{\perp} := \Gamma_u{}' Y$ with an intercept term.

Estimation setup

The τth quantile of Y is any element of the collection Λ_τ of hyperplanes

$$\lambda_\tau := \{y \in \mathbb{R}^k : u^{'} y = \hat{b}_\tau \Gamma^{'}_u y + \hat{a}_\tau\},$$

such that $(\hat{a}_\tau, \hat{b}_\tau)$ are the solutions of the minimization problem

$$\min_{(a_\tau, b_\tau) \in \mathbb{R}^k} E[\rho_\tau(u^{'} y - b_\tau \Gamma^{'}_u y - a_\tau)].$$

where $\rho_\tau(u)$ is a known loss function in the quantile regression literature defined as

$$\rho_\tau(u) = u(\tau - \mathbb{I}(u < 0)), \quad 0 < \tau < 1.$$

Upper and lower halfspaces

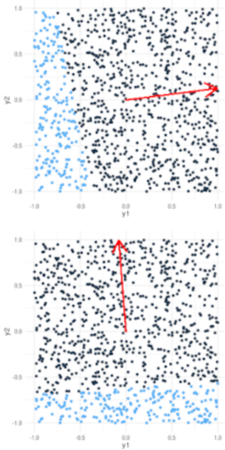
With predictor variables, we have

$$\lambda_\tau(X) = \{u^{'}y = \hat{b}_\tau \Gamma_u^{'} y + x^{'}\hat{\beta}_\tau + \hat{a}_\tau\},$$

We can say that each element $(\hat{a}_\tau, \hat{b}_\tau, \hat{\beta}_\tau)$ define an upper closed quantile halfspace

$$\begin{aligned} H^+_{\tau u} &= H^+_{\tau u}(\hat{a}_\tau, \hat{b}_\tau, \hat{\beta}_\tau) \\ &= \{y \in \mathbb{R}^k : u^{'}y \geq \hat{b}_\tau \Gamma_u^{'} y + x^{'}\hat{\beta}_\tau + \hat{a}_\tau\} \end{aligned}$$

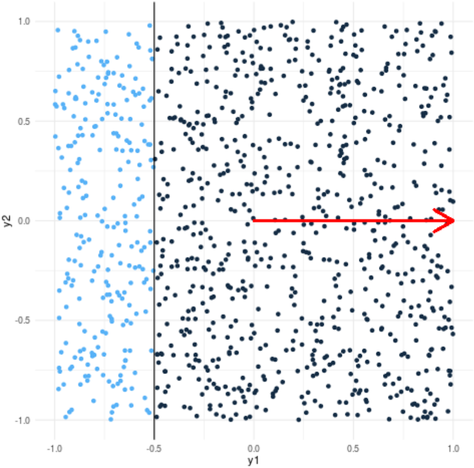
and an analogous lower open quantile halfspace switching $\geq$ for $<$.



Properties

- Probabilistic nature of quantiles:

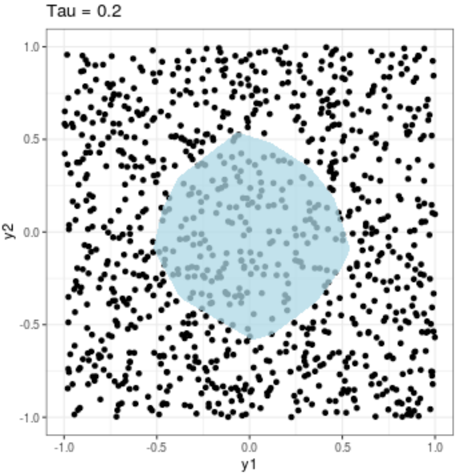
$$P(Y \in H^-_{\tau u}) = \tau,$$



Quantile region

Moreover, fixing τ we are able to define the τ quantile region $R(\tau)$ as

$$R(\tau) = \bigcap_{u \in \mathcal{S}^{k-1}} H^+_{\tau u}.$$



Bayesian directional quantile regression model

Consider the mixture representation of the asymmetric Laplace distribution

$$\begin{aligned} Y_i|w_i &\sim N(\mu + \theta w_i, \psi^2 \sigma w_i) \\ w_i &\sim \mathsf{Exp}(\sigma) \\ &\Updownarrow \\ Y &\sim AL(\mu, \sigma, \tau) \end{aligned}$$

Then one can consider that, for each direction u,

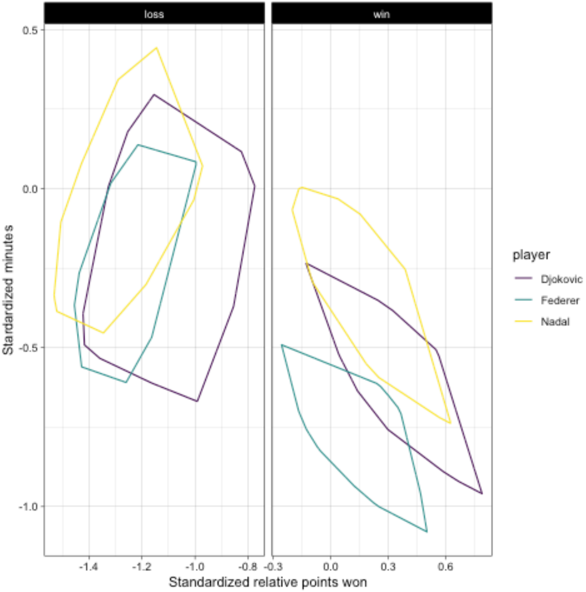
$$Y_u|b_\tau, \beta_\tau, \sigma, w \sim N(Y^\perp b_\tau + x^{'}\beta_\tau + \theta w_i, \psi^2 \sigma w_i),$$

Application results

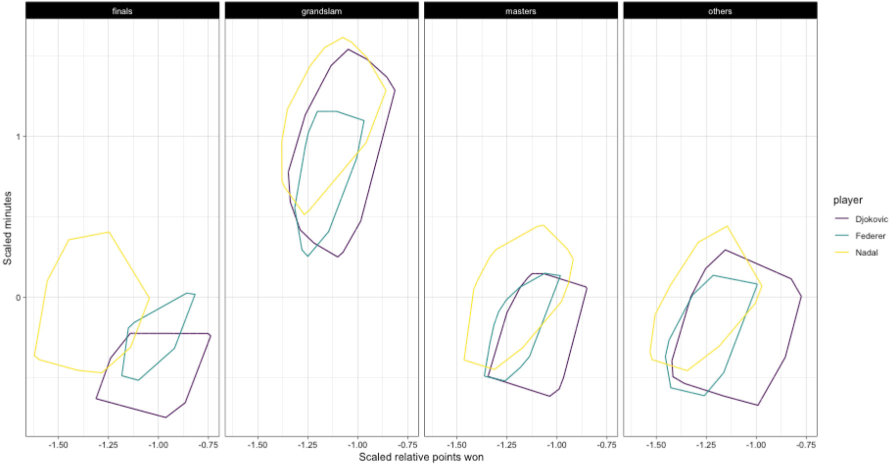
Model choices

- Y_1: Relative points won.
- Y_2: Minutes played.
- Covariates:
  - Player (Federer, Nadal, Djokovic);
  - Surface;
  - Win or loss;
  - Type of tournament;
  - Top 20 player opponent or not;
- For the model, we fix $\tau = 0.25$ and consider 180 directions in the unit circle.
- We consider interaction effects between player and the other covariates.

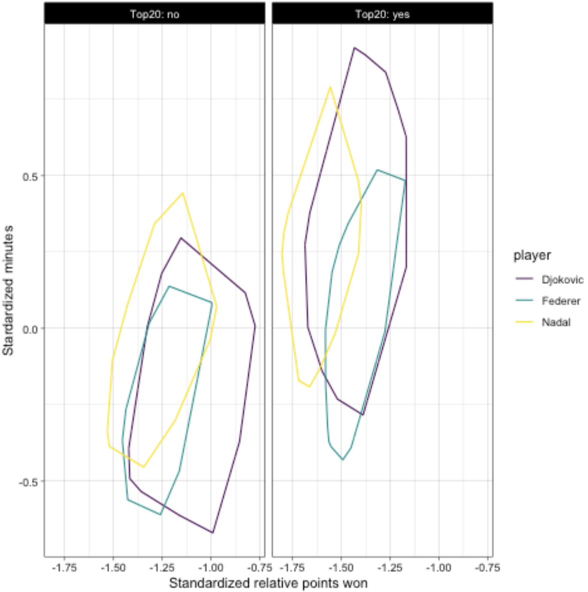
Effect of win and losses



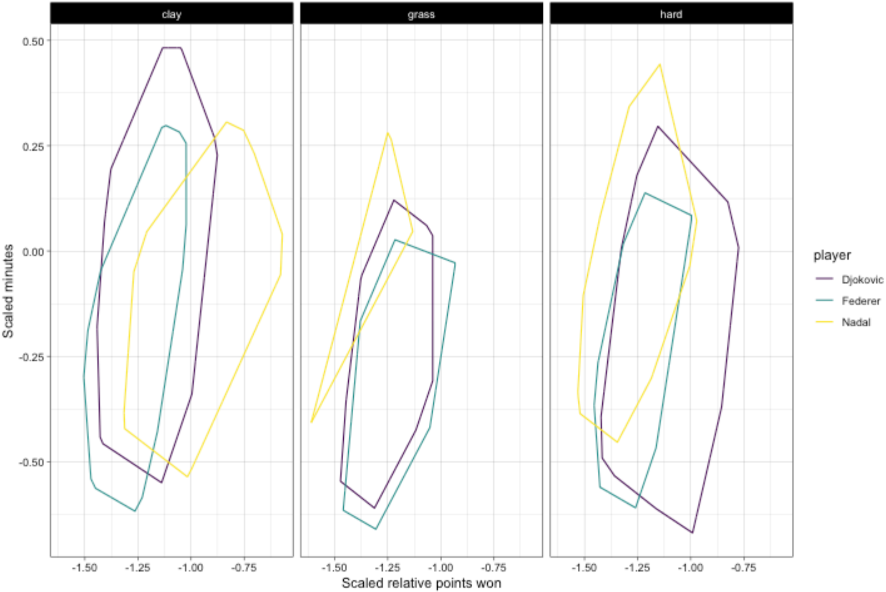
Effect of tournament



Effect of Top 20



Effect of surface



Final discussion

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- Nadal's dominance in clay courts is unmatched.
- Federer dominance in grass courts is also visible.
- The same way as Djokovic dominance in hard courts.
- In the time dimension, Federer shows an edge during wins.
- For most comparisons, Djokovic seems the most dominant player.

Thank you!

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