

Understanding, Generating, and Evaluating Prediction Intervals

(In regression contexts with a frequentist orientation)

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Data Science @NetApp

posit
conf (2024)

github.com/brshallo/posit-2024

[in](#) [github](#)  bryanshalloway.com

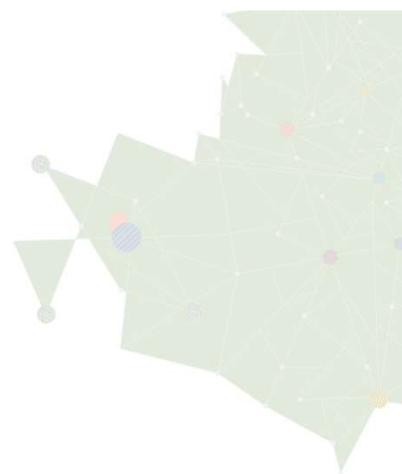


Rooftop solar's dark side

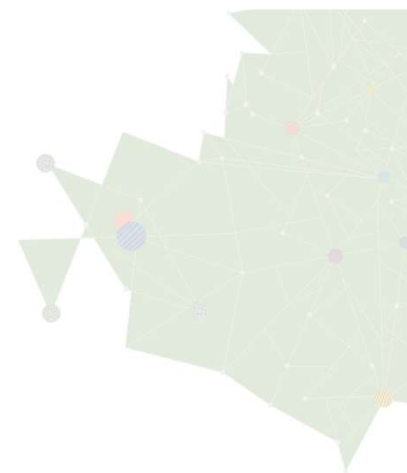
JULY 12, 2024 · 6:28 PM ET

"They had used about half as much electricity from their power company as the previous year..."

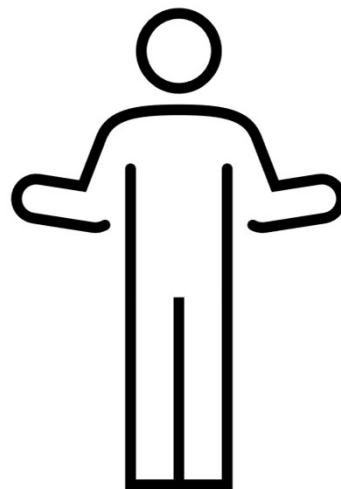
"But {the solar company} had promised these new panels would replace all of their power."



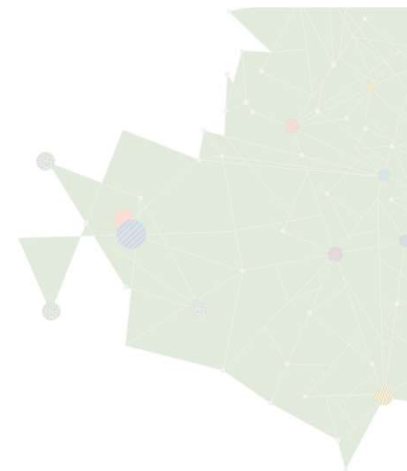
$\hat{f}(x_i)$



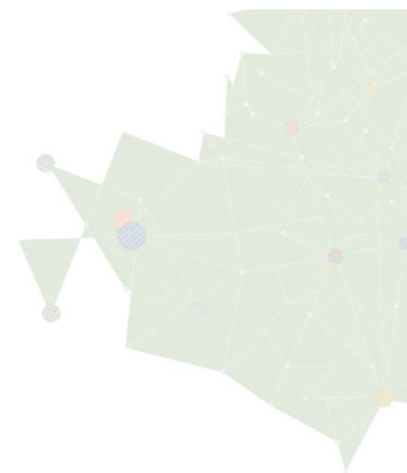
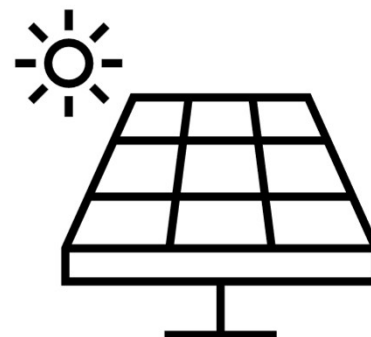
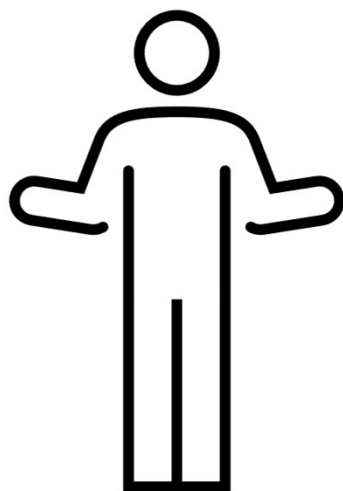
$\hat{f}(x_i)$



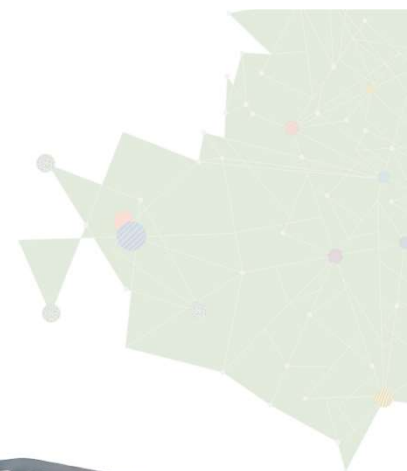
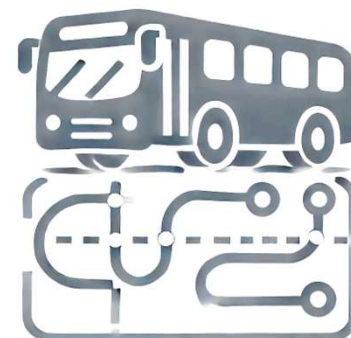
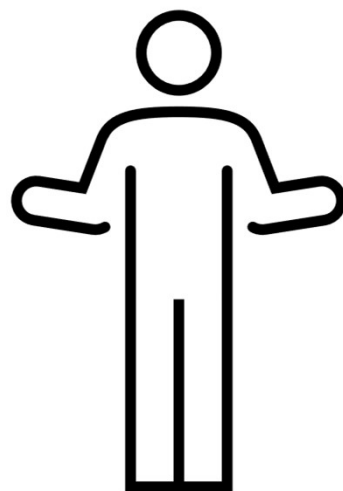
...



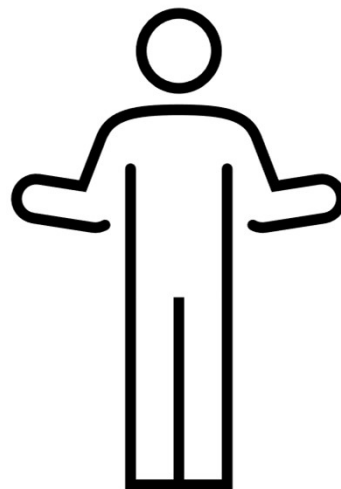
$$\hat{f}(x_i)$$

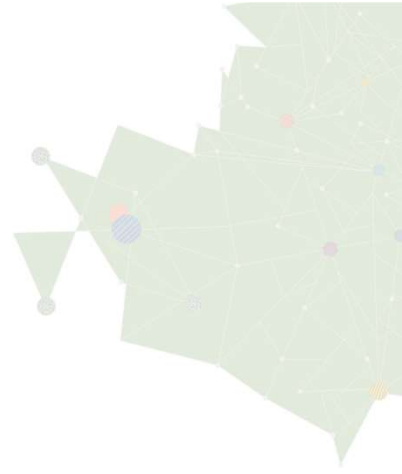
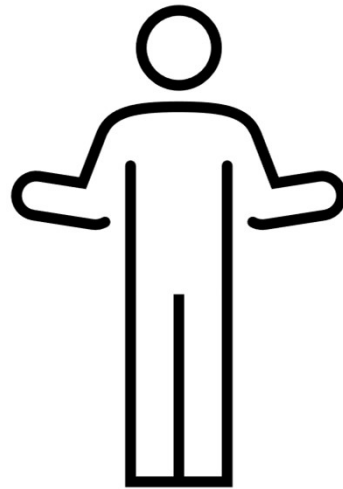


$$\hat{f}(x_i)$$

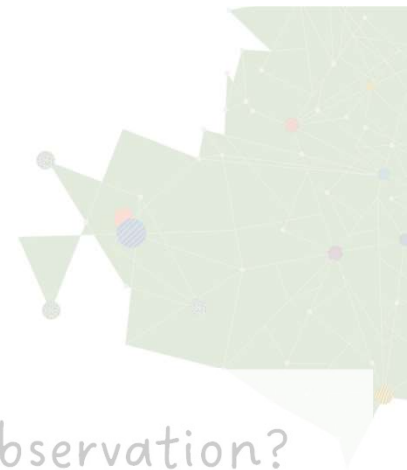


$$\hat{f}(x_i)$$



$\hat{f}(x_i) \dots$ 

$$\hat{f}(x_i) \dots$$



- Applicability → Is the model appropriate to use for this observation?
- Explainability → What attributes are driving the predicted value?
- Uncertainty → What's a reasonable range for this outcome?

...

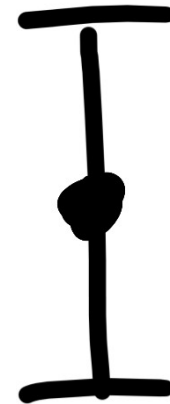


Point Estimate

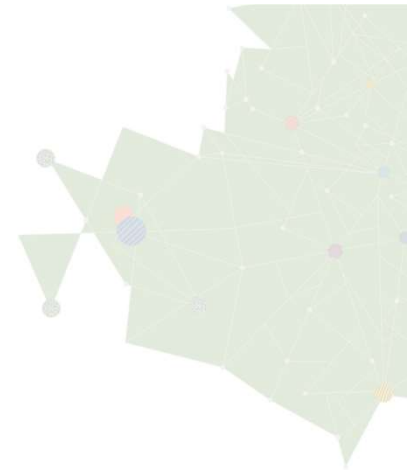


"This car will sell for \$12k"

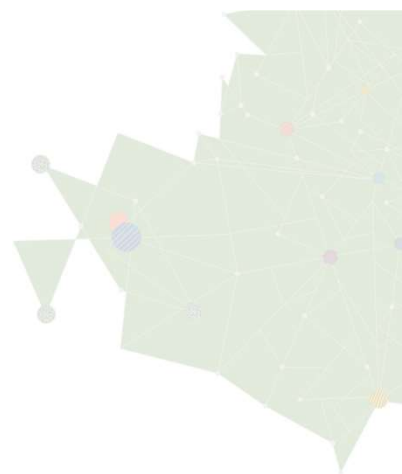
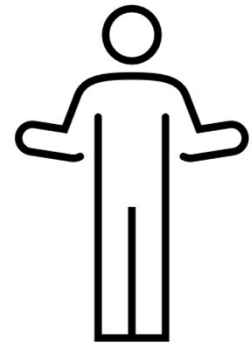
Prediction Interval



"I'm 80% sure this car will sell for
between \$10k and \$14k"



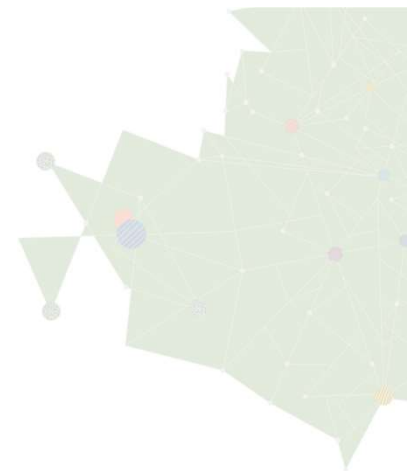
$\hat{f} \longrightarrow \$\$$

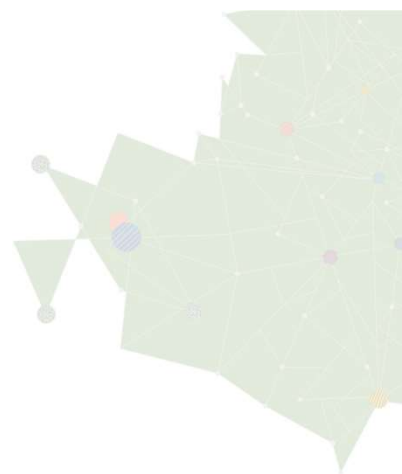
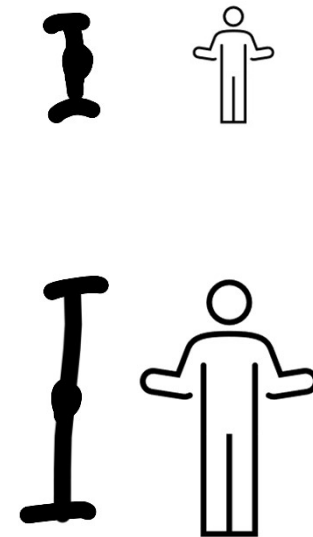
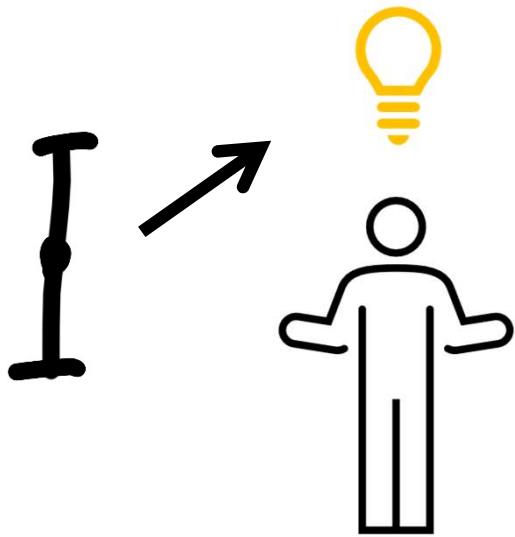


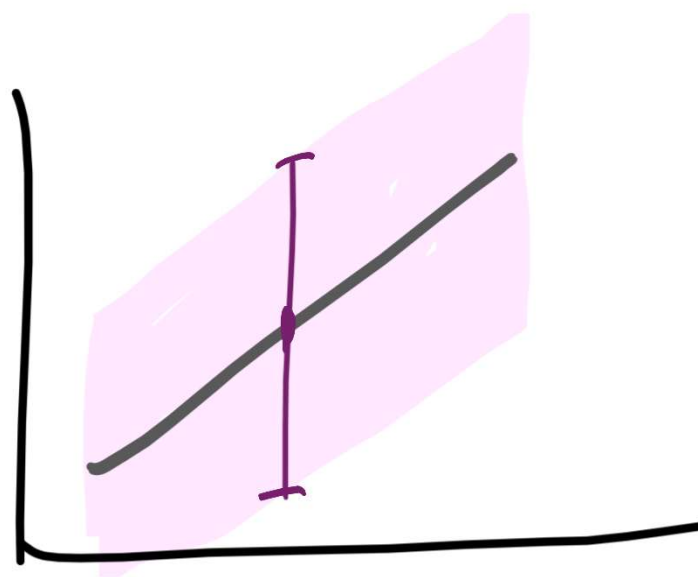
$\hat{f} \longrightarrow \$\$$



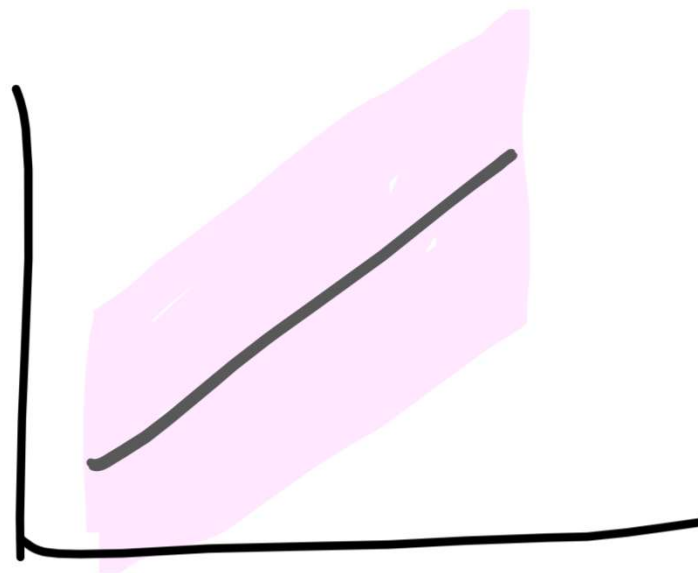
CARVANA



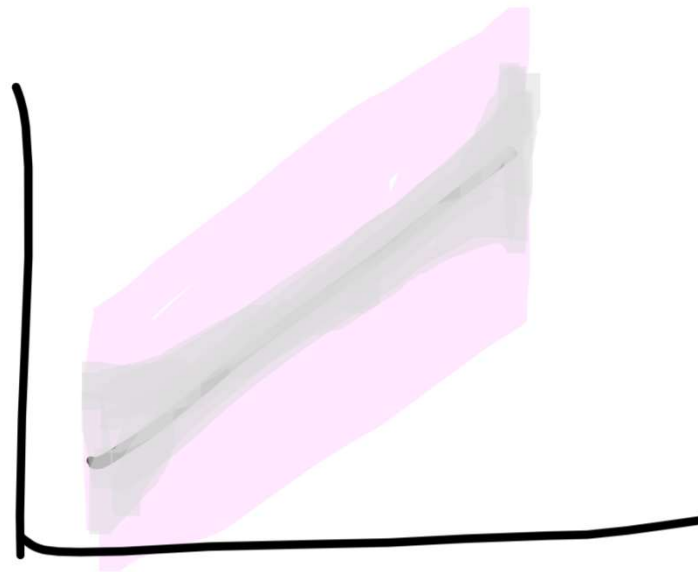




Confidence Intervals vs Prediction Intervals



Confidence Intervals vs Prediction Intervals

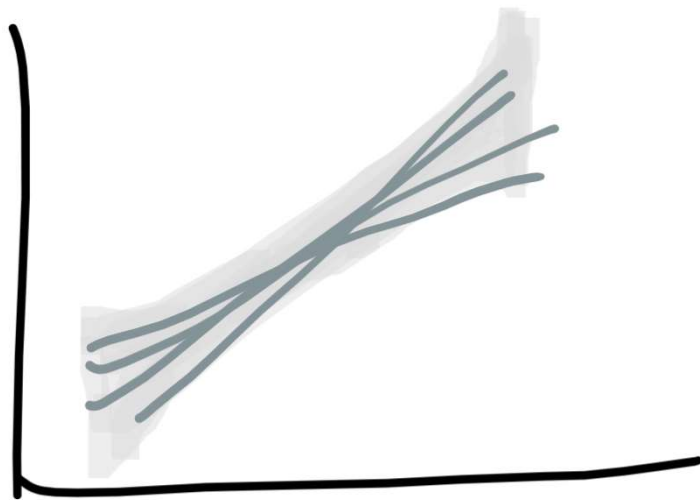


Confidence Intervals

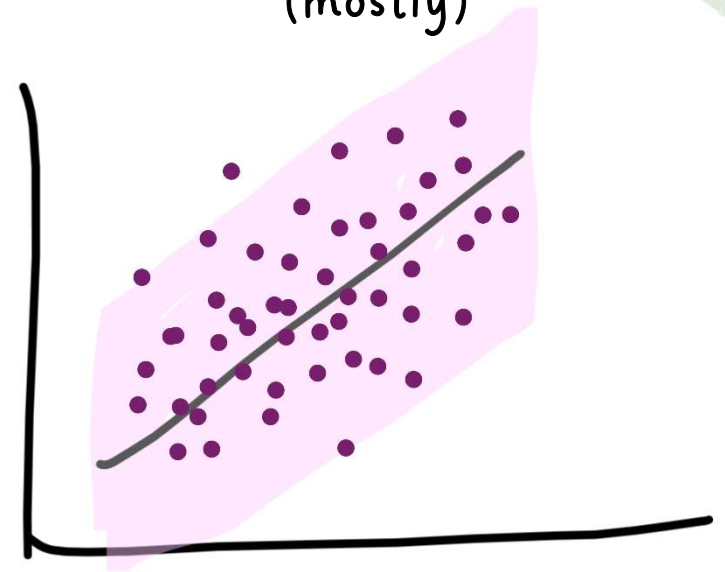
vs

Prediction Intervals

Uncertainty in Fitting Model
(mostly)



Uncertainty in Sample
(mostly)



Confidence Intervals

vs

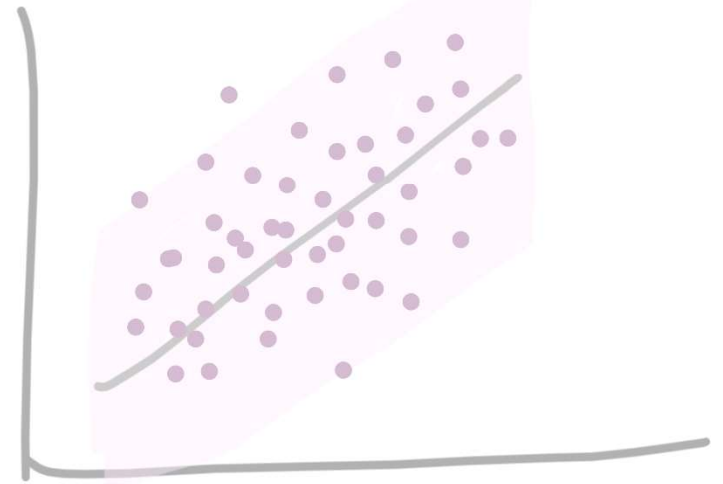
Prediction Intervals

Uncertainty in Fitting Model
(mostly)



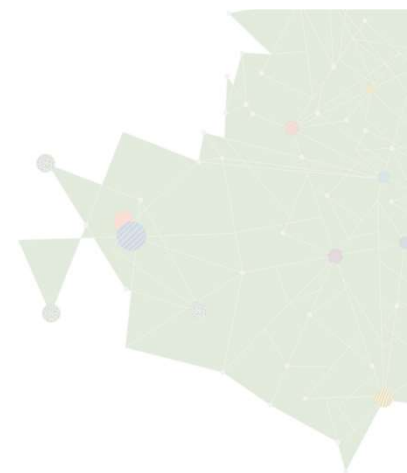
90% sure the average weight among
all 3 mo olds is 12 - 12.5 lbs

Uncertainty in Sample
(mostly)

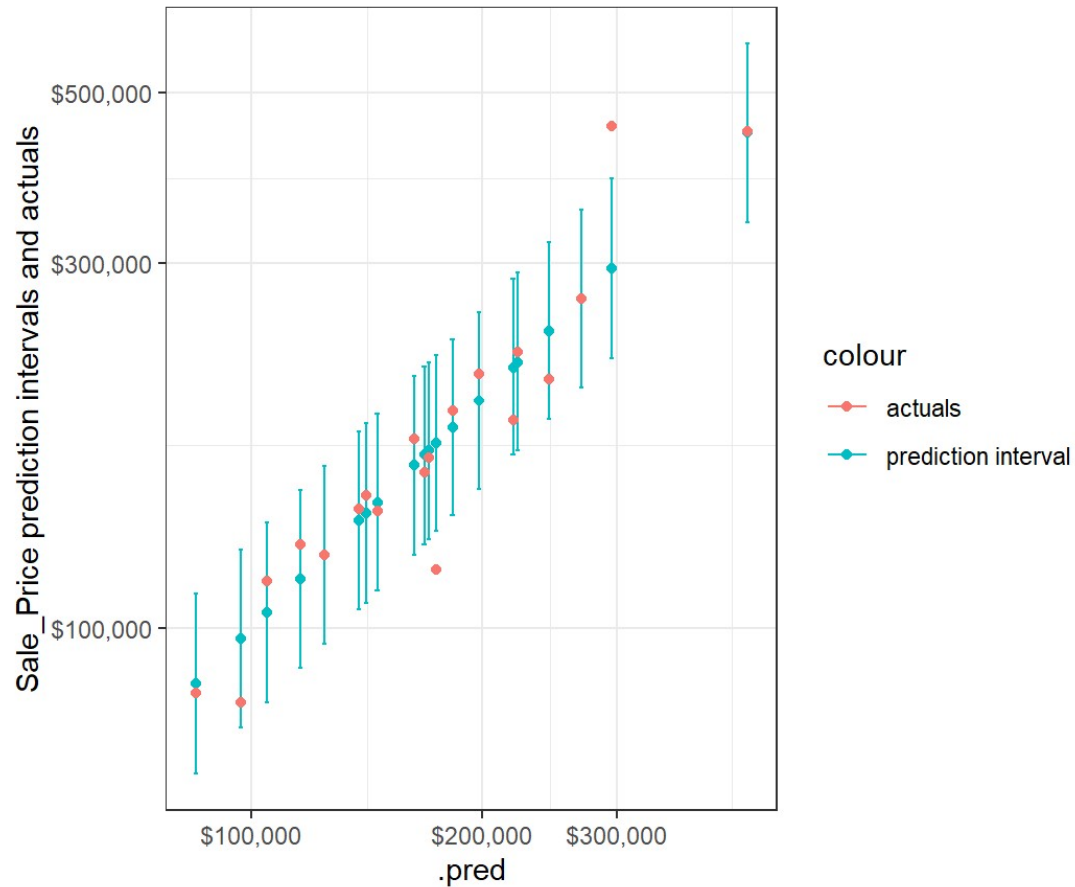


90% sure the weight of a random
3 mo old will be 10 - 14 lbs

```
predict(lm_fit, data_test,  
  type = "pred_int",  
  level = 0.90)  
#> # A tibble: 14 × 2  
#>   .pred_lower .pred_upper  
#>   <dbl>      <dbl>  
#> 1      17.6      24.5  
#> 2      18.5      25.4  
#> 3      18.6      25.5  
#> 4      18.8      25.7  
#> 5      18.1      25.1  
#> 6       4.19     11.2  
#> 7       2.72      9.67  
#> 8      17.0      23.9  
#> 9      18.4      25.3  
#> 10     18.4      25.3  
#> 11     18.4      25.3  
#> 12     17.7      24.6  
#> 13       5.27     12.2  
#> 14       4.26     11.2
```



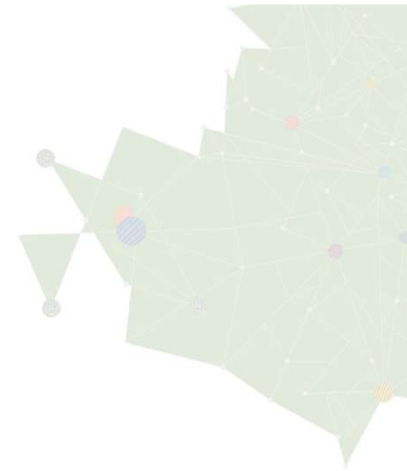
90% prediction intervals on a holdout dataset
Linear model (analytic method)



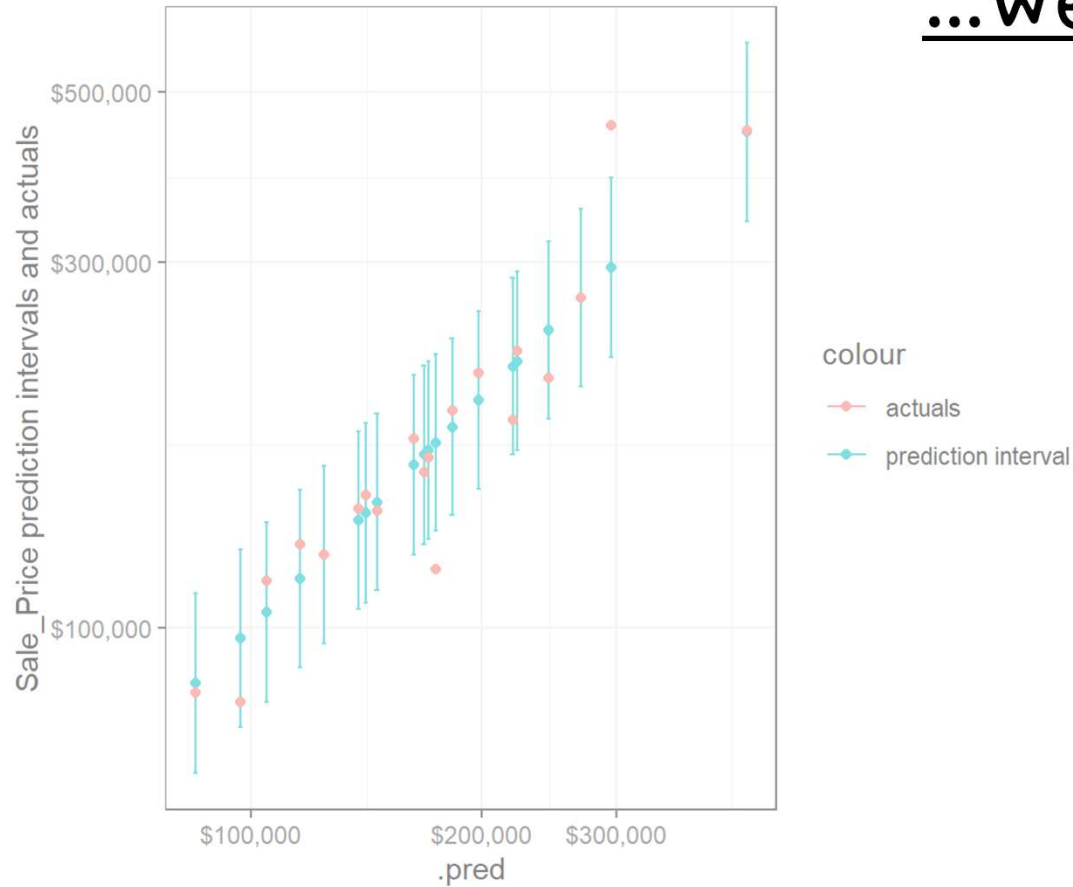
• coverage

• interval width

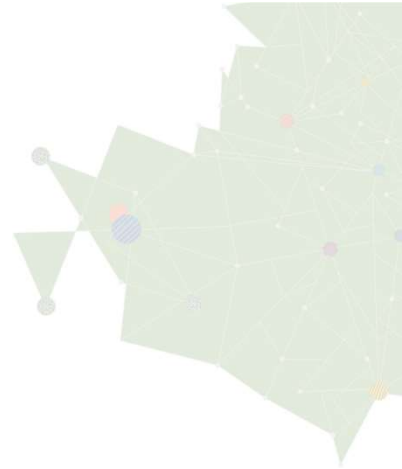
...



90% prediction intervals on a holdout dataset
Linear model (analytic method)



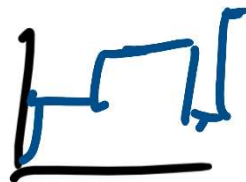
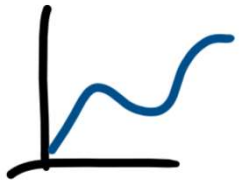
...weaknesses



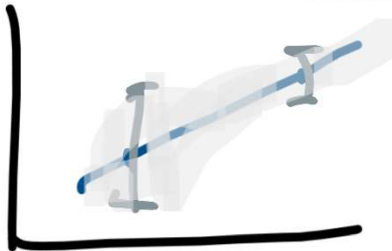


...weaknesses

- No guarantee of coverage



- Other model types



- Assumptions

...



desired...

- Coverage guaranteed

-
- Model Agnostic

-
- Assumption free
(and flexible)

...weaknesses

- No guarantee of coverage

- Other model types

- Assumptions

...

Conformal Prediction

(Mostly) assumption free uncertainty quantification

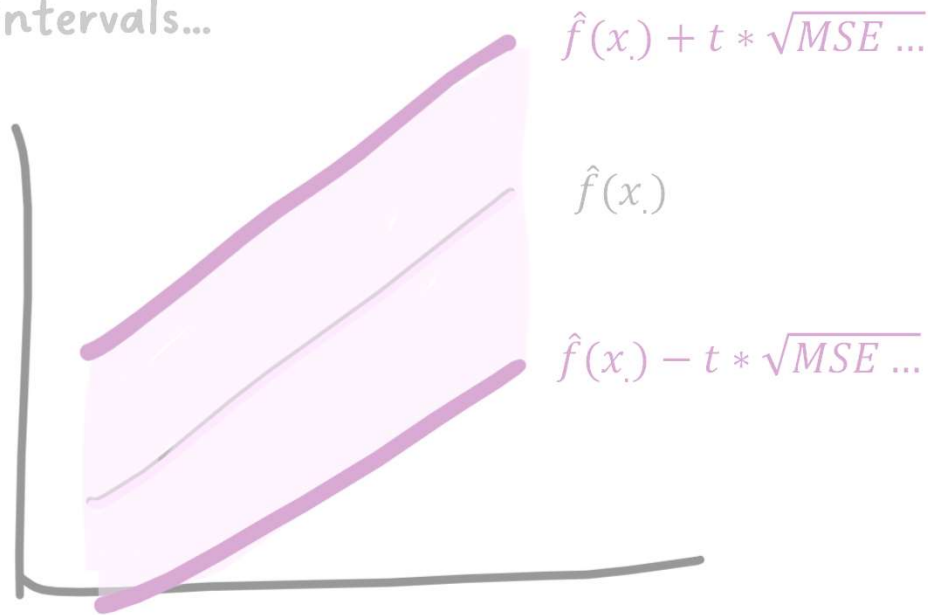


Conformalized quantile regression is...
"the best way to do conformal prediction for regression."
-Stephen Bates

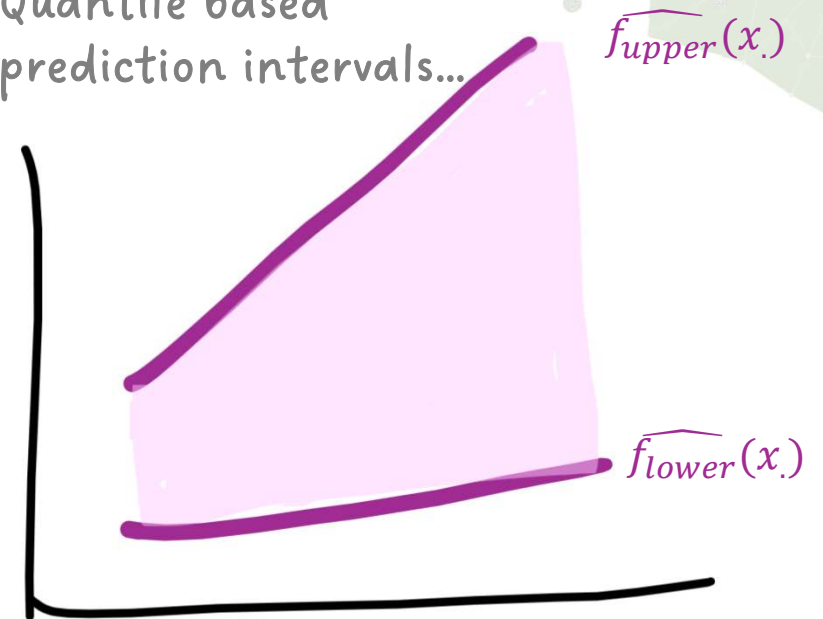
coauthor of A Gentle Introduction to Conformal Prediction and Distribution-Free Uncertainty Quantification

1. Don't fit expected values, but quantiles

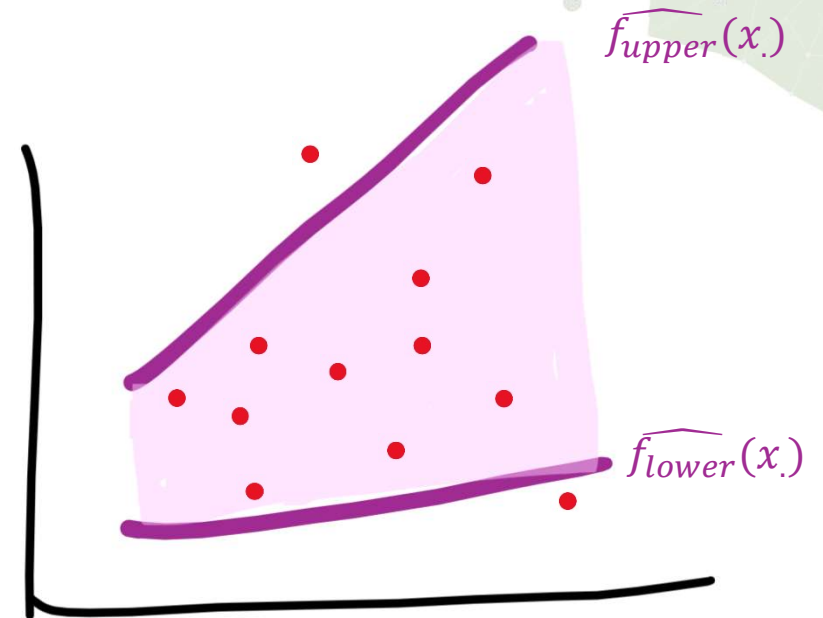
Classical prediction intervals...



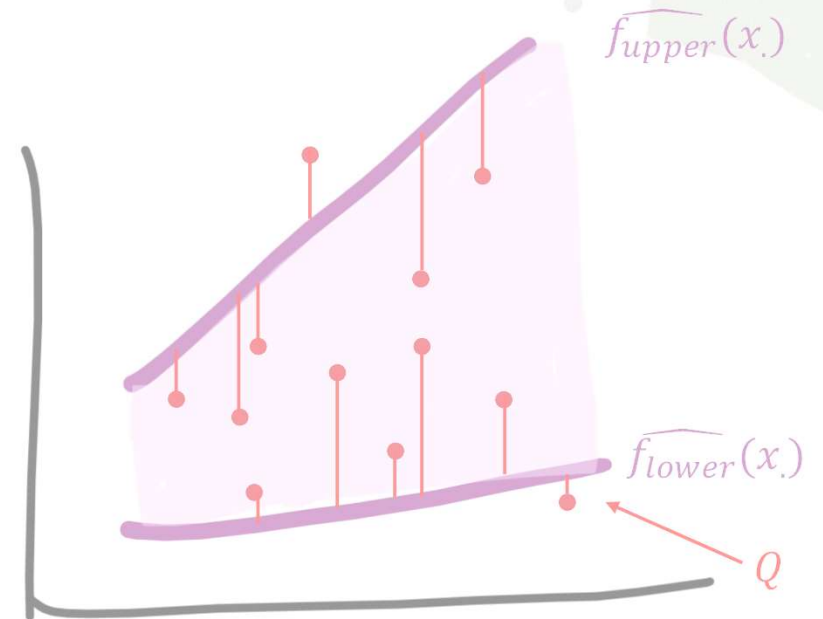
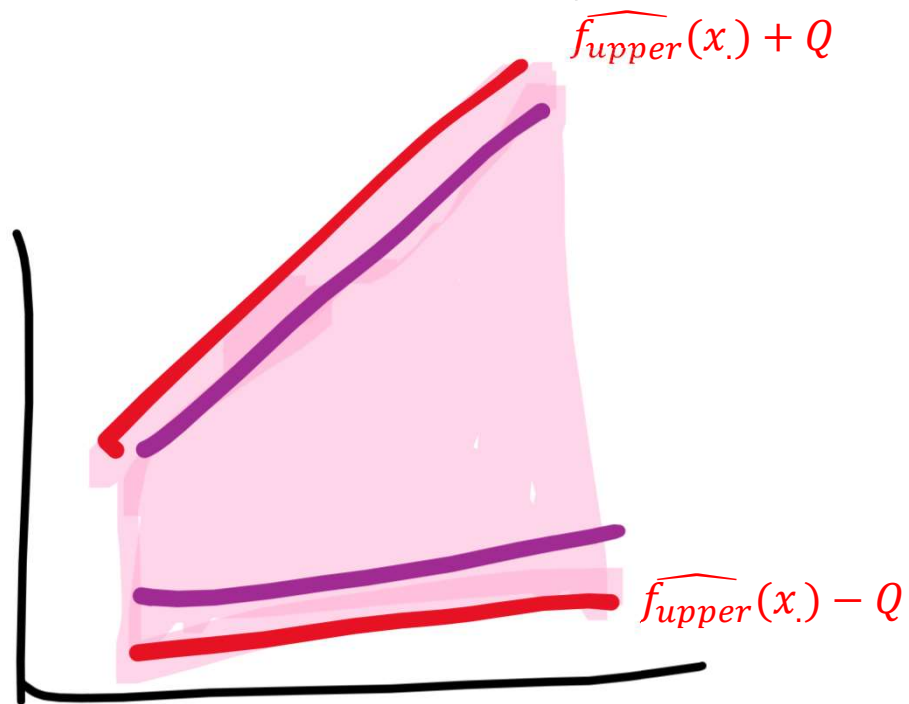
Quantile based prediction intervals...



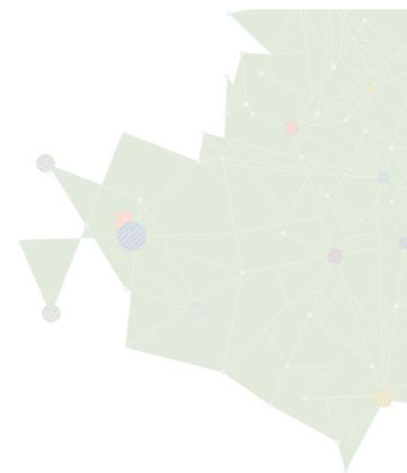
2. Calibrate, adjust quantiles



2. Calibrate, adjust quantiles



Conformalized Quantile Regression Resources

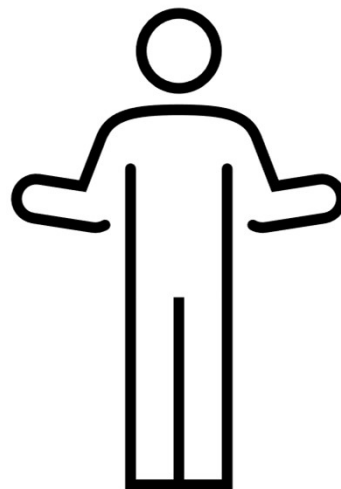


- `probably::int conformal_quantile()`
Conformal Inference with Tidymodels - posit::conf(2023); Kuhn
(<https://youtu.be/vJ4BYJSg734?si=cjpXabfmAad1FuBK>)
- A Gentle Introduction to Conformal Prediction and Distribution-Free Uncertainty Quantification; Angelopoulos, Bates
(<https://people.eecs.berkeley.edu/~angelopoulos/blog/posts/gentle-intro/>)
- Introduction To Conformal Prediction with Python; Molnar
- MAPIE: Model Agnostic Prediction Interval Estimator (python)
<https://mapie.readthedocs.io/en/latest/>
- Practical Guide to Applied Conformal Prediction...; Manokhin

Also:

- Understanding Prediction Intervals; Shalloway
(<https://www.bryanshalloway.com/2021/03/18/intuition-on-uncertainty-of-predictions-introduction-to-prediction-intervals/>)

$\hat{f}(x_i)$



...

