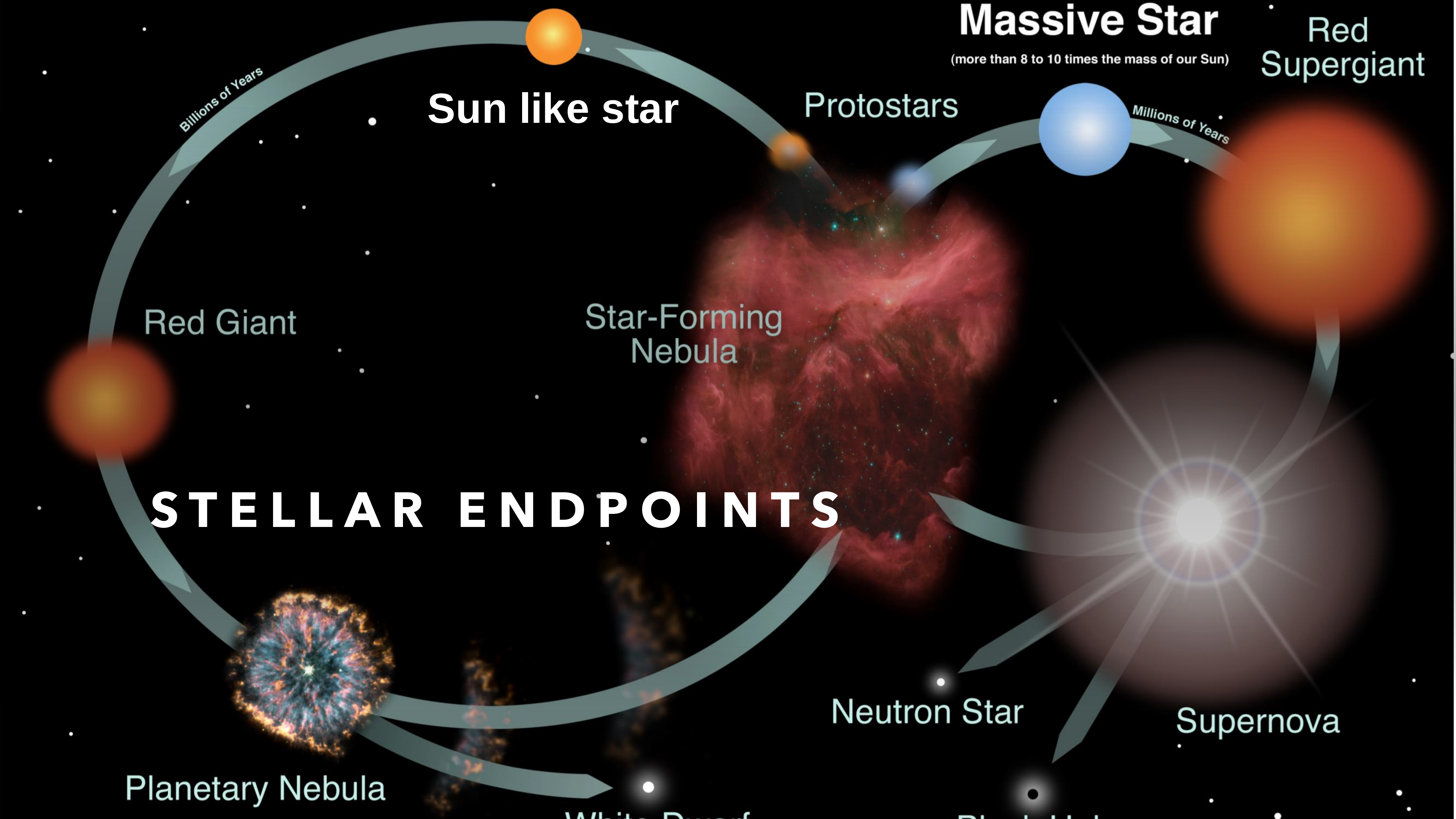
ASTR 1040
MIDTERM 2 REVIEW

11/12/2023

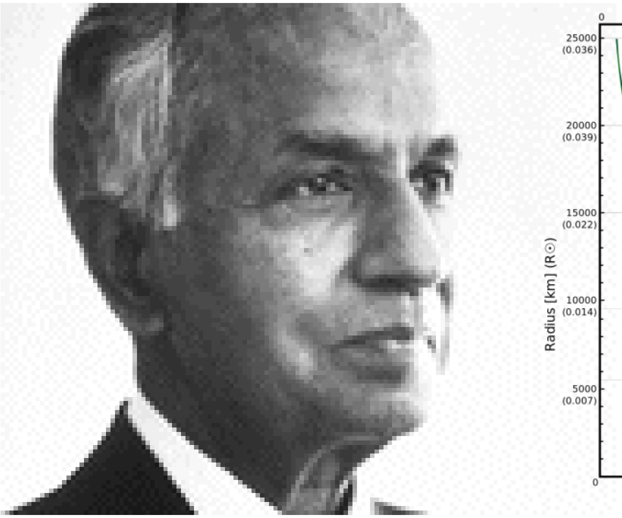
KIRK LONG CU BOULDER

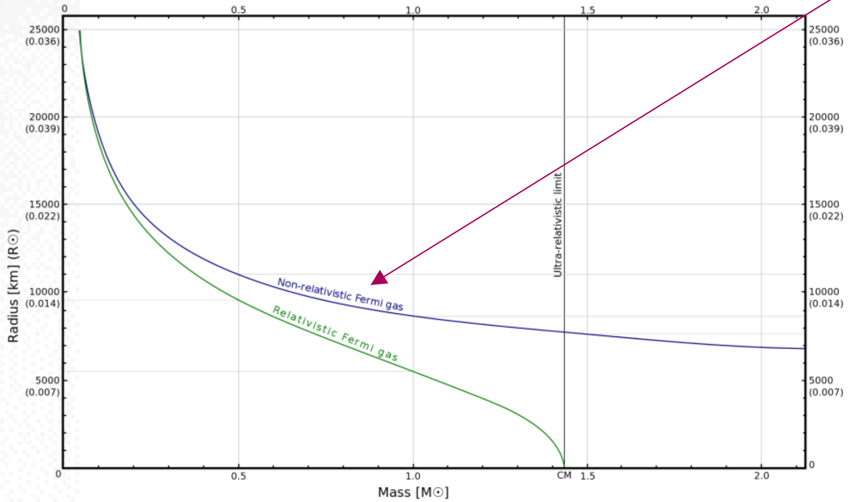
STELLAR ENDPOINTS

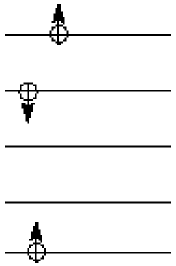


SUBRAHMANYAN CHANDRASEKHAR

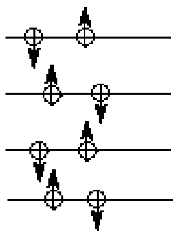
Chandrasekhar was studying degenerate “Fermi” gases – what particles do you think this situation applies to?







Regular gas: many unfilled energy levels. Particles free to move about and change energy levels.



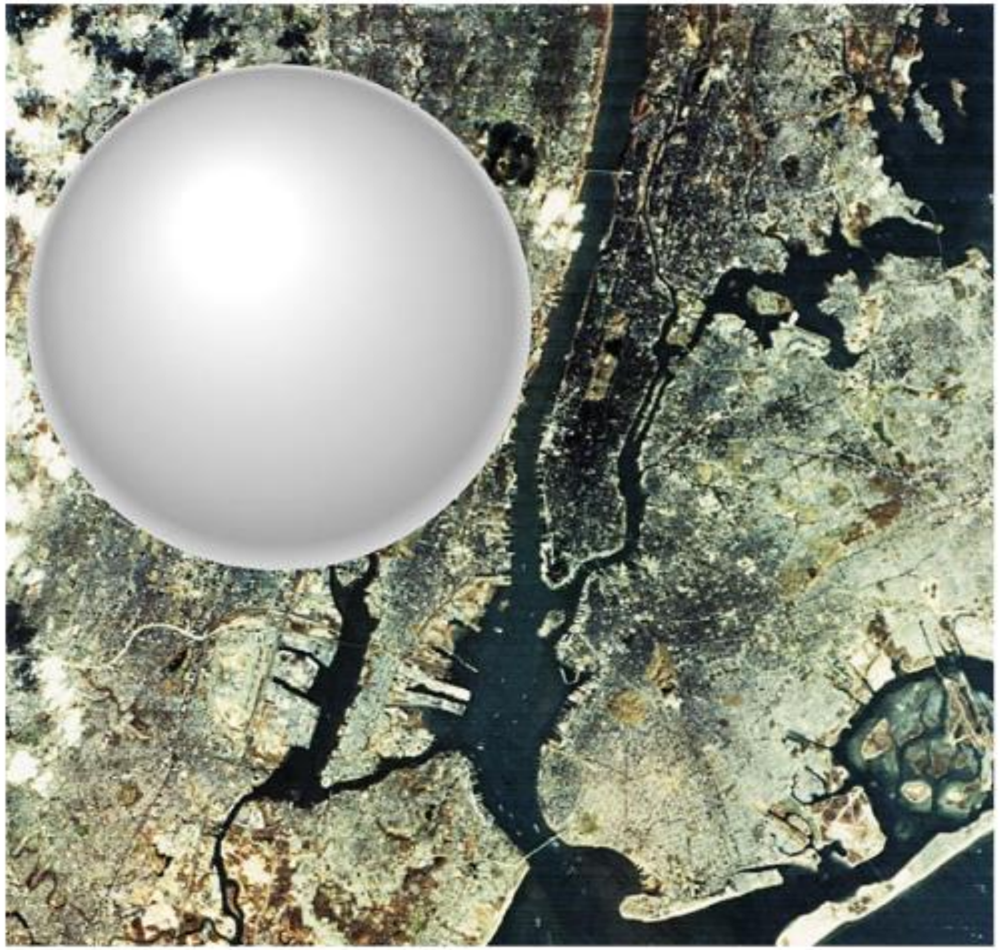
Degenerate gas: all lower energy levels filled with two particles each (opposite spins). Particles **locked** in place.

Combined ~quantum physics~ with relativity to show that there was a maximum mass a star could have before collapsing further (potentially into a black hole!). First completed calculation on boat ride to Oxford, was then unfortunately publicly ridiculed by Eddington (likely racism involved…) who did not want to believe in things like black holes. We now know the Chandrasekhar limit (named in his honor) to be ~ 1.4 solar masses, and this has been very well documented observationally! One of NASA’s flagship observatories, the Chandra x-ray telescope, is named in his honor.

Image credits: **L** – Wikipedia / Starchild NASA; **M** – Wikipedia / AllenMcC.; **R** – Nick Strobel

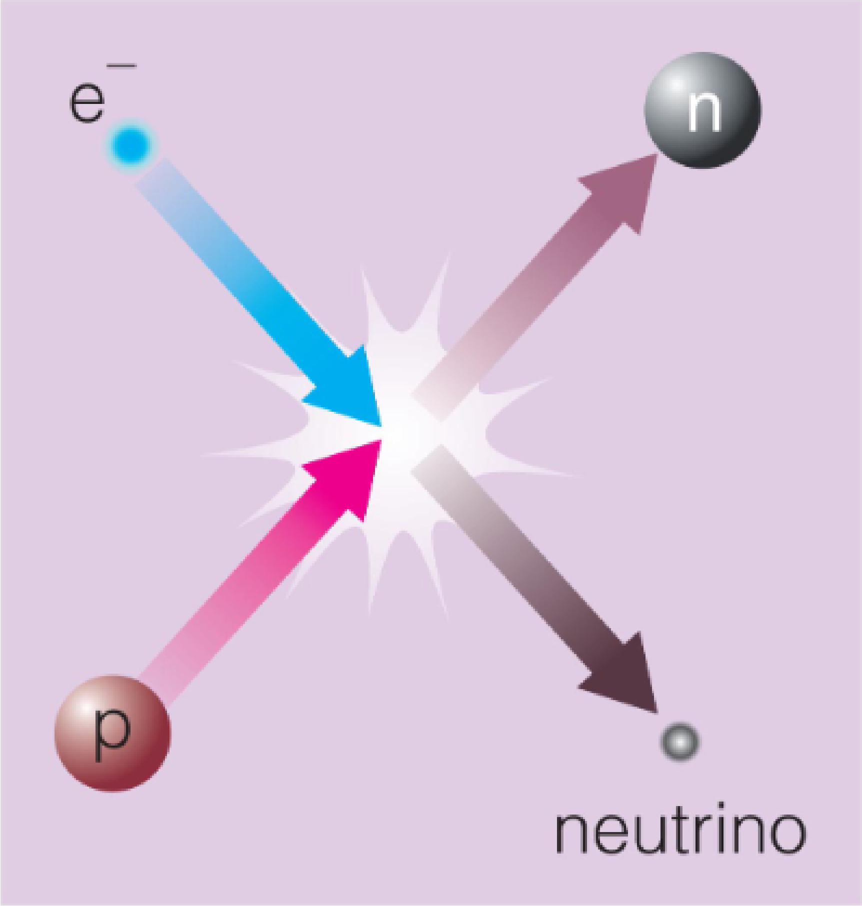
Neutron Stars

- A neutron star is the ball of neutrons left behind by a massive-star supernova.
- Degeneracy pressure of neutrons supports a neutron star against gravity.
- Neutron stars are about the size of a small city. Size ~ 10 km Mass ~ 1 - 3M_{Sun}
- Made of degenerate neutrons. More massive = smaller



Neutron Stars

- Electron degeneracy pressure goes away because electrons combine with protons, making neutrons and neutrinos.
- Neutrons collapse to the center, forming a **neutron star**.



Question: why are neutron stars so much denser than white dwarves?

DO PULSARS HAVE TO BE NEUTRON STARS?

Assuming a neutron star's surface can't spin faster than the speed of light, calculate a maximum possible radius of a pulsar given that they have periods measured as small as 0.001s. Using this fact, calculate **a characteristic density for a pulsar and compare to the density of a white dwarf ($\sim \mathbf{10^{10} kg/m^3}$).**

Hint: Set the centrifugal and gravitational forces at the equator equal to each other.

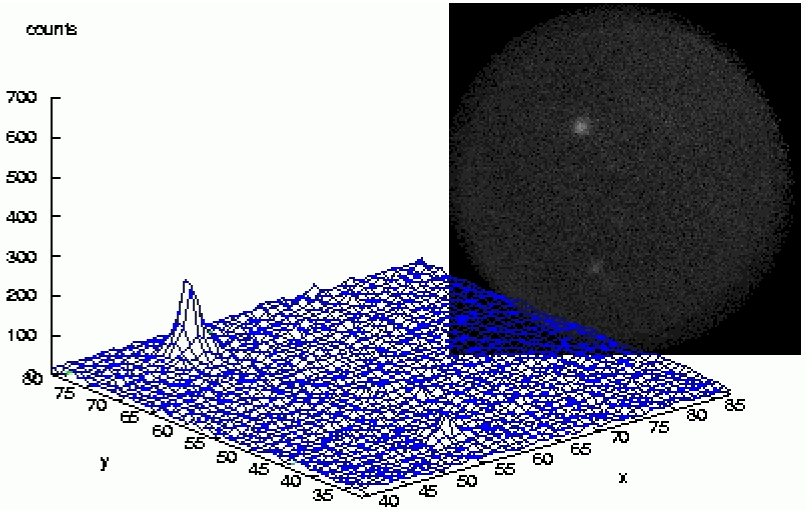
Formulae you may need:

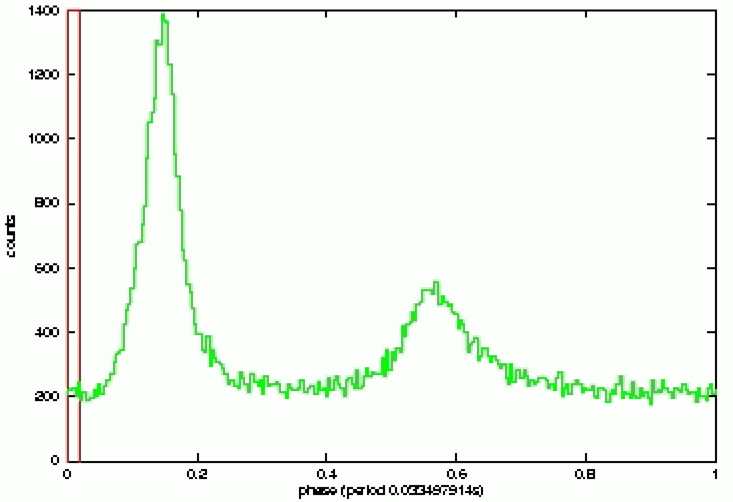
$$v = \omega r$$

$$F_G = G\left(\frac{Mm}{r^2}\right)$$

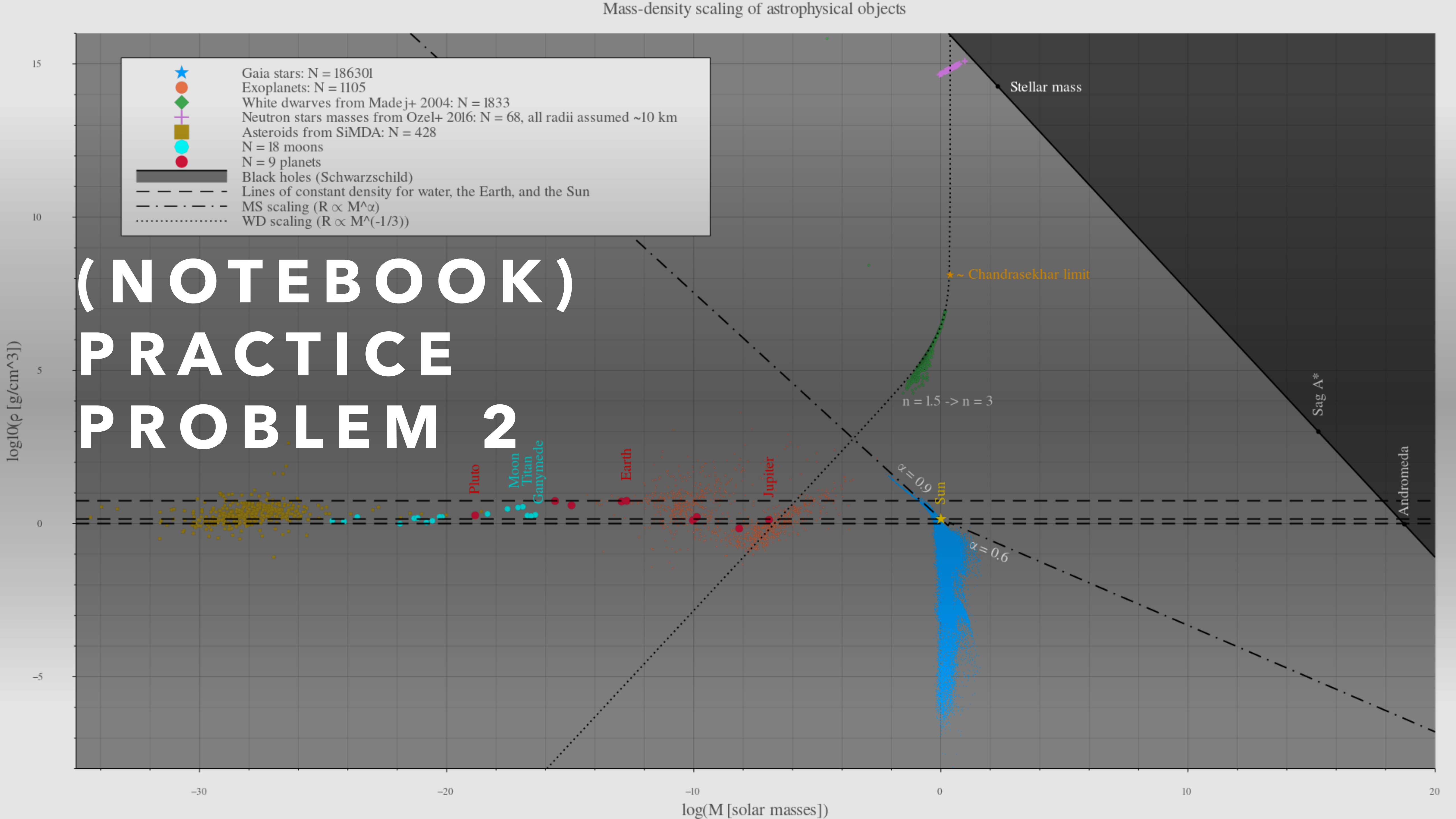
$$F_c = m\omega^2 r$$

$$\omega = 2\pi f = \frac{2\pi}{P}$$



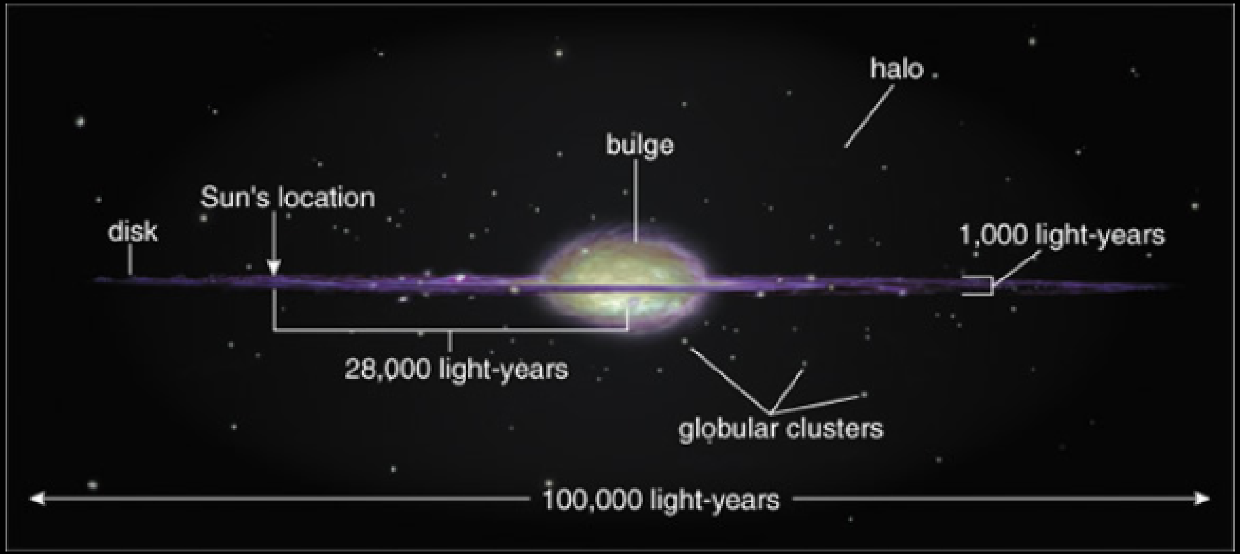


(NOTEBOOK) PRACTICE PROBLEM 2



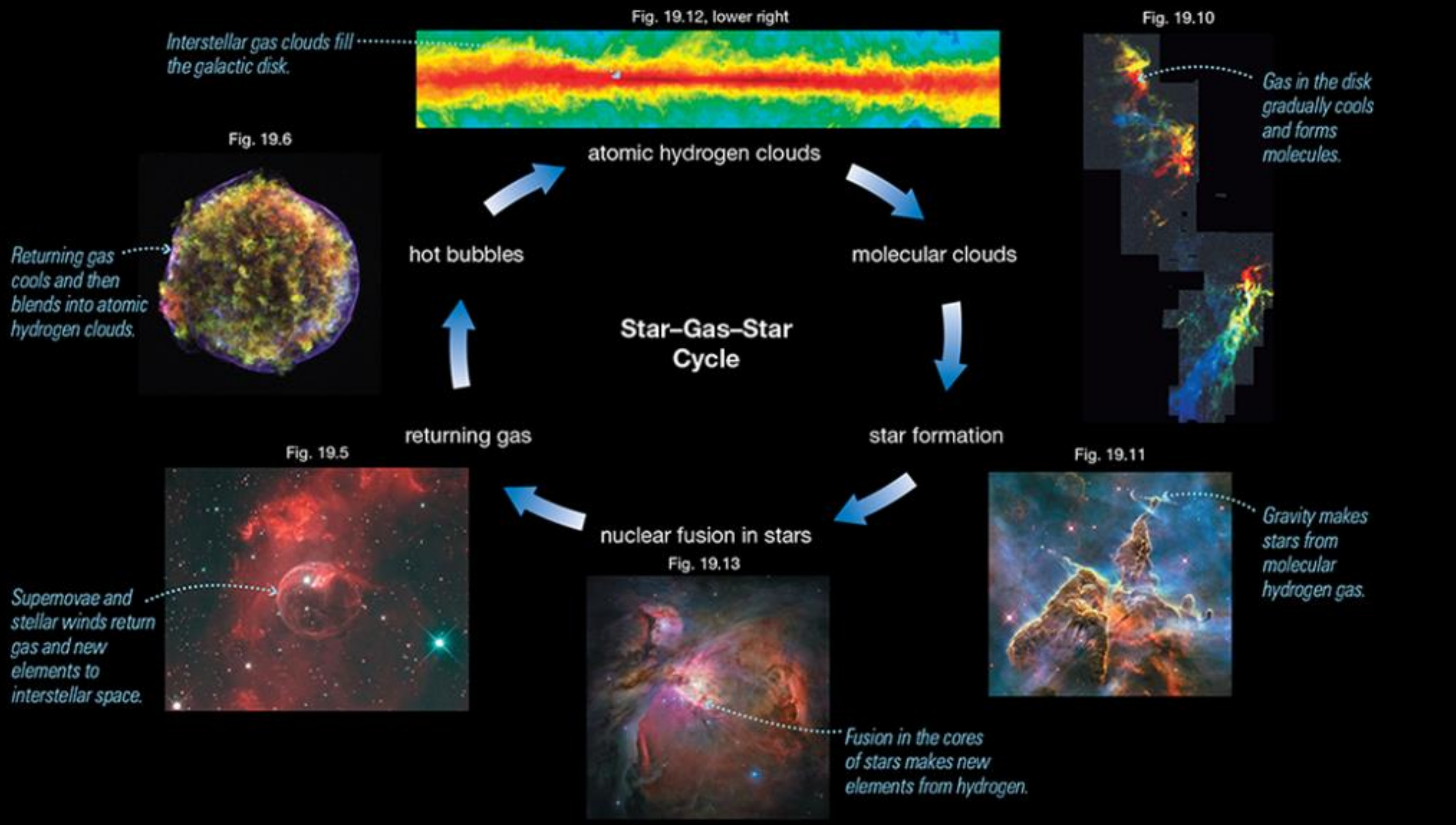
Our home: the Milky Way Galaxy

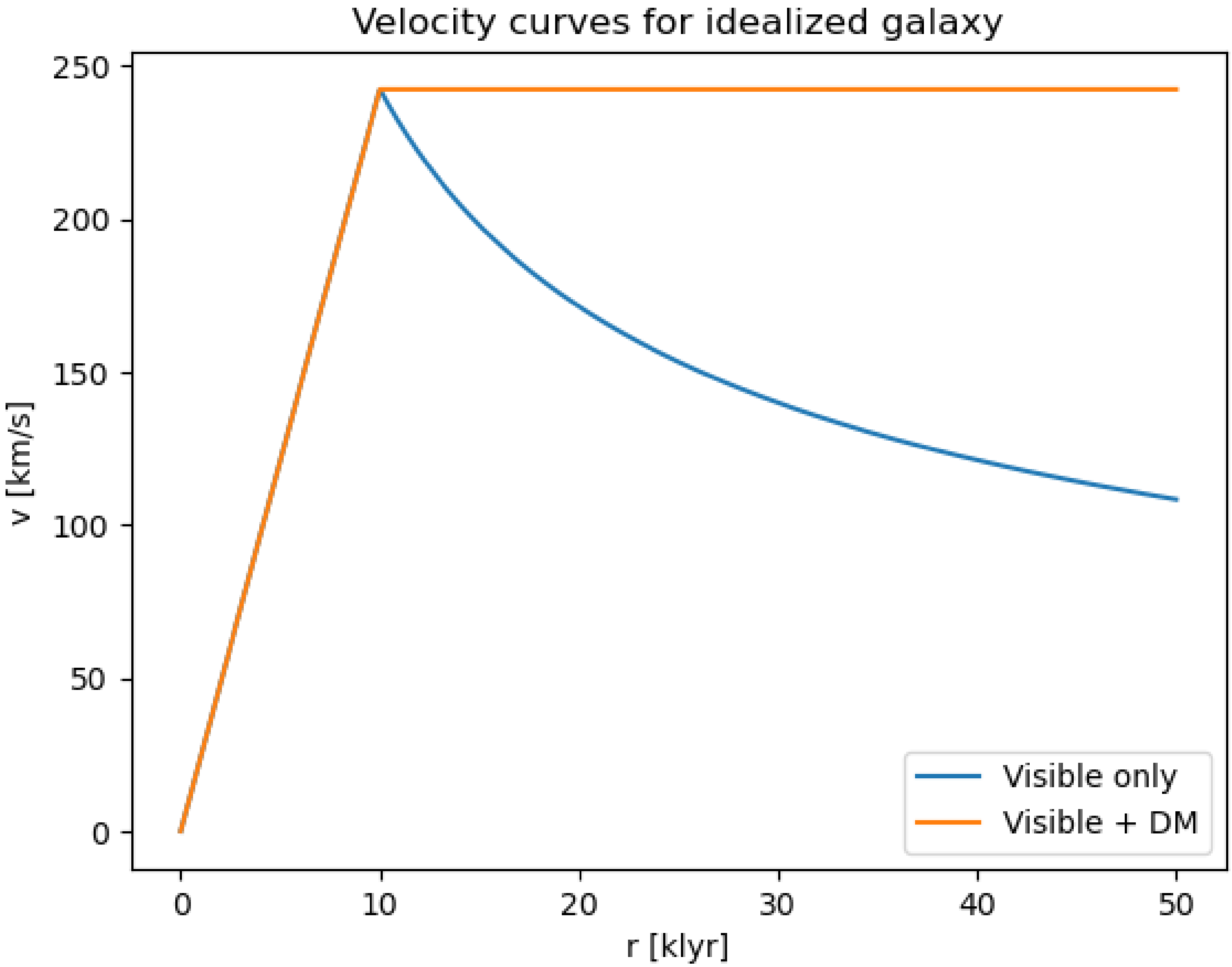
How would you describe its structure?



- Bulge: Spheroidal* central part of the galaxy.
- Disk: Flat plane of the galaxy.
- Spiral arms: Bright regions with active star formation.
- Halo: Spherical region surrounding the galaxy.
- Interstellar medium: The region between stars. It has gas and plasma!

Gas Recycling or “the cosmic baryon cycle”





WHAT MAKES UP A GALAXY?

- Gas
- Dust
- Stars
- Dark matter

Using the plot at left, estimate the ratio of dark matter to visible matter enclosed within 50 klyr?

HOW DO WE KNOW THERE IS A SUPERMASSIVE BLACK HOLE AT THE CENTER?

e.) Supermassive water blobs: [10 points]

r_p places an upper limit on the extent the mass interior to S2 *could* have. Suppose all of the mass driving the orbit was in a sphere with radius r_p — what would its density be? Compare your answer to the density of a black hole of the same mass. What if all of the mass interior to S2's orbit was a giant sphere of water (density $\sim 1000\ \mathrm{kg/m^3}$) — would we be able to tell the difference based on the orbit alone?

Google around and comment on a few other reasons we think the center of the galaxy is host to a supermassive black hole and not a supermassive blob of water.

Solution: density is just mass divided by volume, so for all the material inside a sphere with radius r_p the density would be:

$$\rho = \frac{M}{4/3\pi r_p^3}$$

We can calculate this in the code cell below:

In [7]:
```
r_p = a*(1-e)
ρ = M/(4/3*np.pi*r_p**3)
ρ.si
```

Out[7]: $0.00032053375\ \frac{\mathrm{kg}}{\mathrm{m^3}}$

This is quite small (~3e-4 kg/m^3). To get the density of the black hole we can repeat the calculation done above but use R_s instead of r_p:

In [8]:
```
R_s = 2*G*M/c**2
ρBH = M/(4/3*np.pi*R_s**3)
ρBH.si
```

Out[8]: $1207713\ \frac{\mathrm{kg}}{\mathrm{m^3}}$

This is huge (roughly 1e6 kg/m^3) in comparison (roughly 4 billion times for dense)! Because the density of water is between these two values, gravitationally the effect of a giant blob of water with the same mass would have the same effect on the orbit of our star and we'd have no way to tell. It seems highly unlikely that we end up with a giant ball of water at the center of our galaxy when we don't see giant balls of water anywhere else, but based on the star's orbit and minimum distance technically we can't tell. There are many great answers for further proof as to why it probably isn't a blob of water (radio emission from synchotron radiation, high energy emission from inside the orbit, flare events, Occam's razor, etc.) but the best one is in the next question.

CEPHEIDS

What does slope of a log-log plot represent?

What are cepheid variable stars and why are they useful to us?

What does it mean for something to be a standard candle?

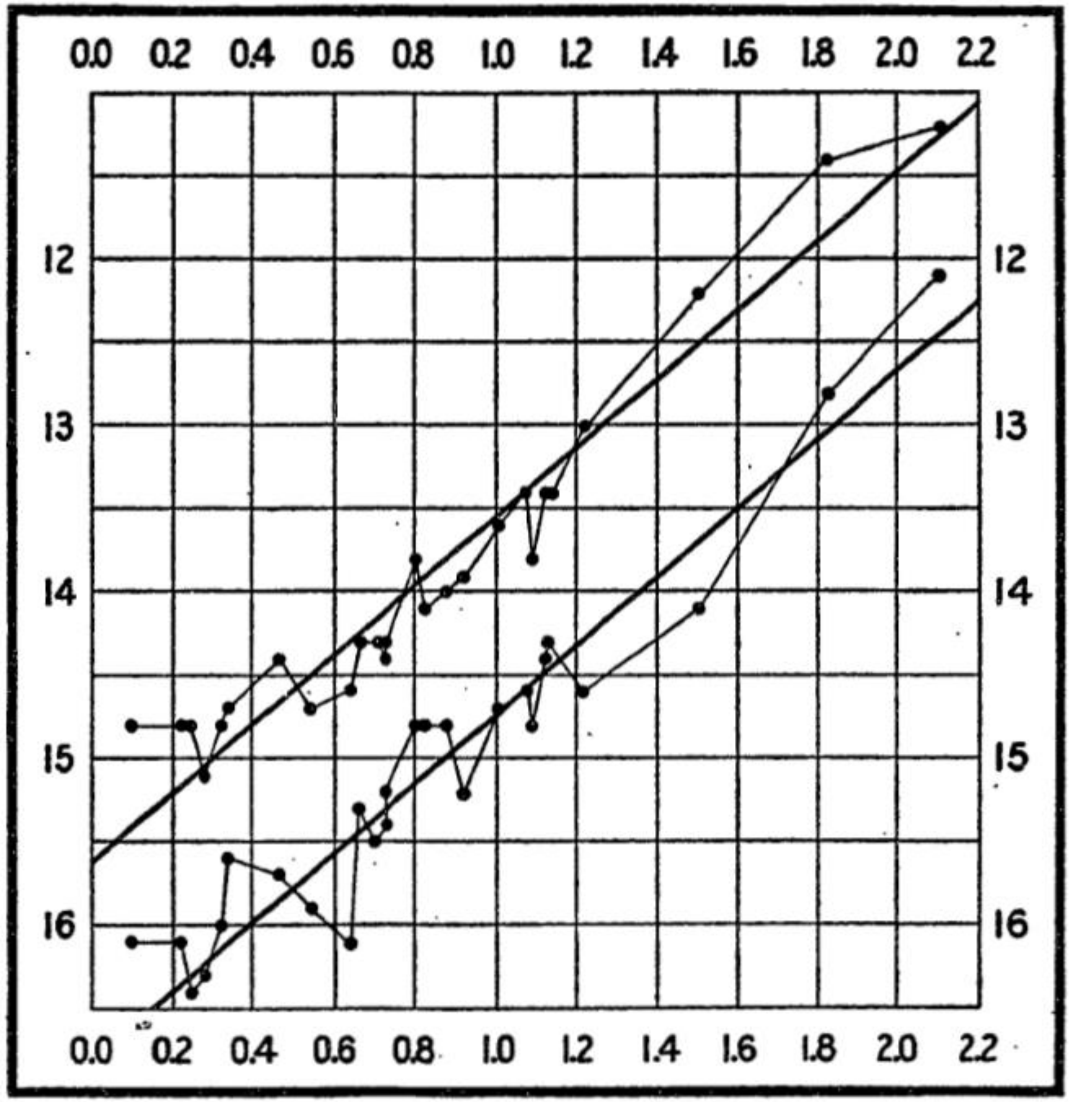


FIG. 2.

From Leavitt 1912, x-axis = log(period), y-axis = log(M)

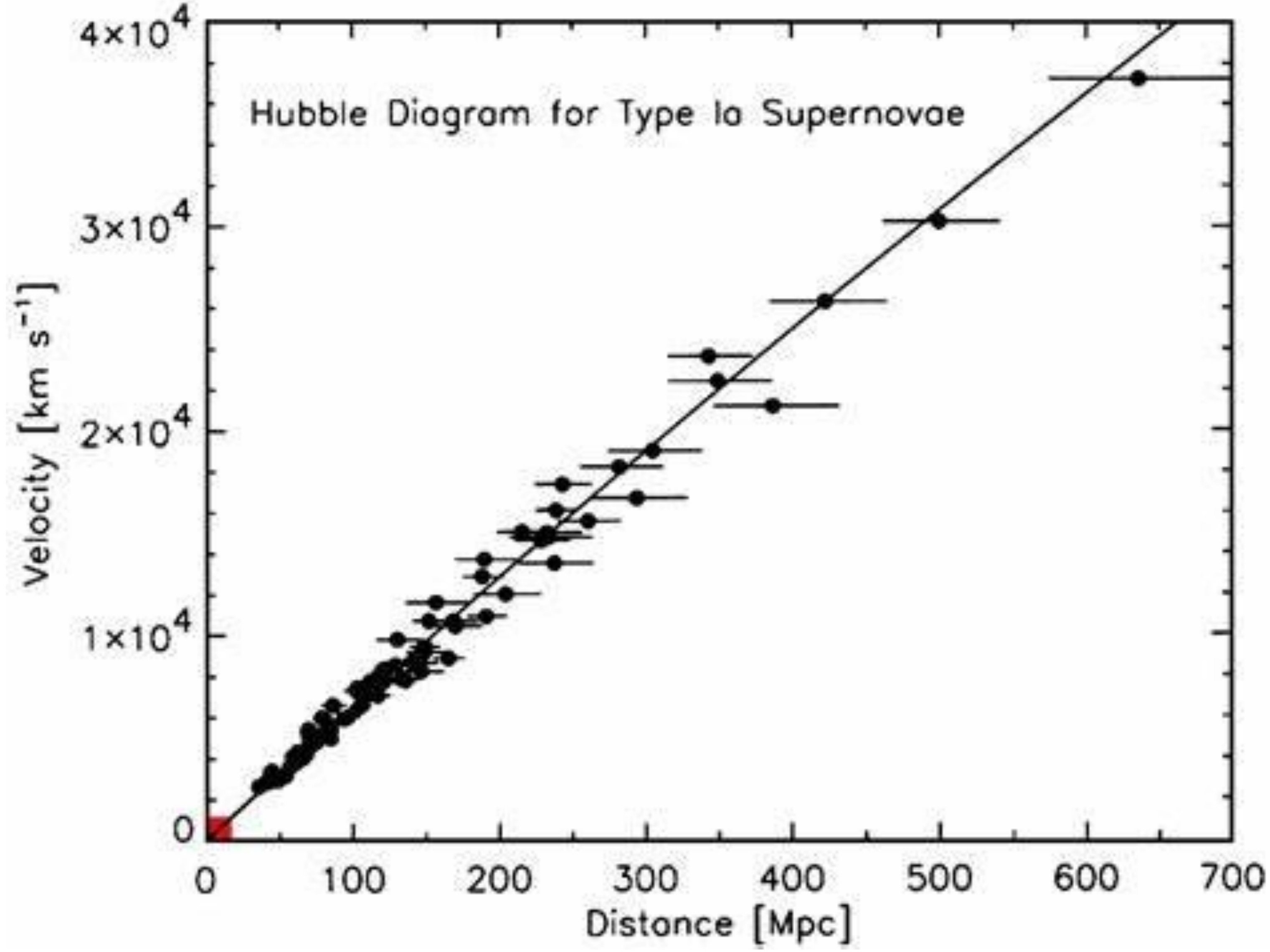
HUBBLE'S LAW

What is H_0 for type 1a supernovae?

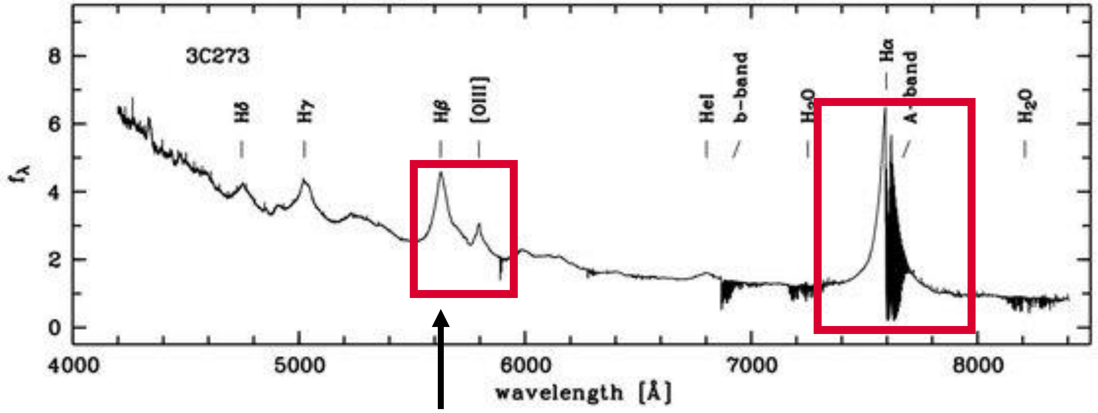
How do we estimate the age of the universe from the Hubble constant? What would higher or lower values of H_0 imply about our universe's age?

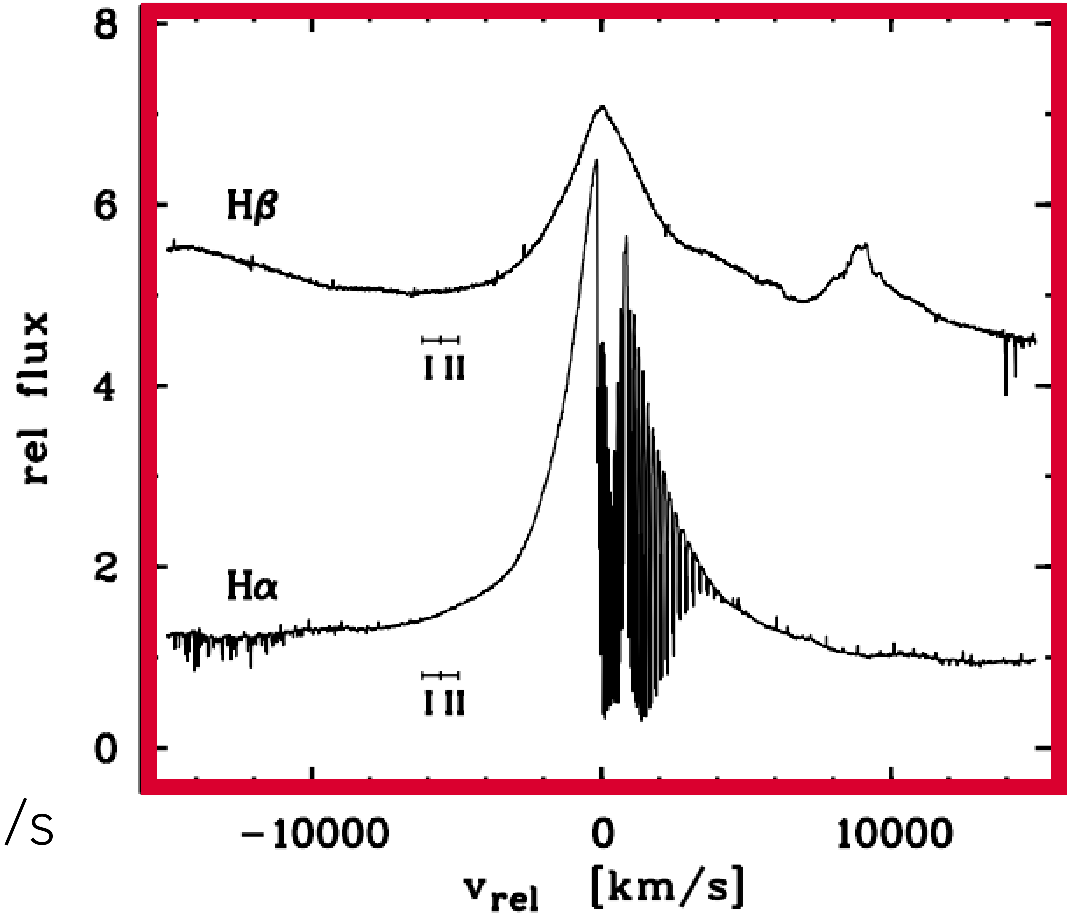
Is the Hubble constant constant in time?

Why do these observations imply the universe has no center and no edge?



REDSHIFT





$$\Delta v = c\frac{\Delta\lambda}{\lambda_c} \approx c\left(\frac{100}{5600}\right) \approx 0.02c \approx 5{,}000 \text{ km/s}$$

1. The $H\beta$ transition has a rest wavelength of 486 nm = 4860 Angstroms. Using the plot above, what is the redshift of quasar 3C 273? Using Hubble's law, what is its distance?

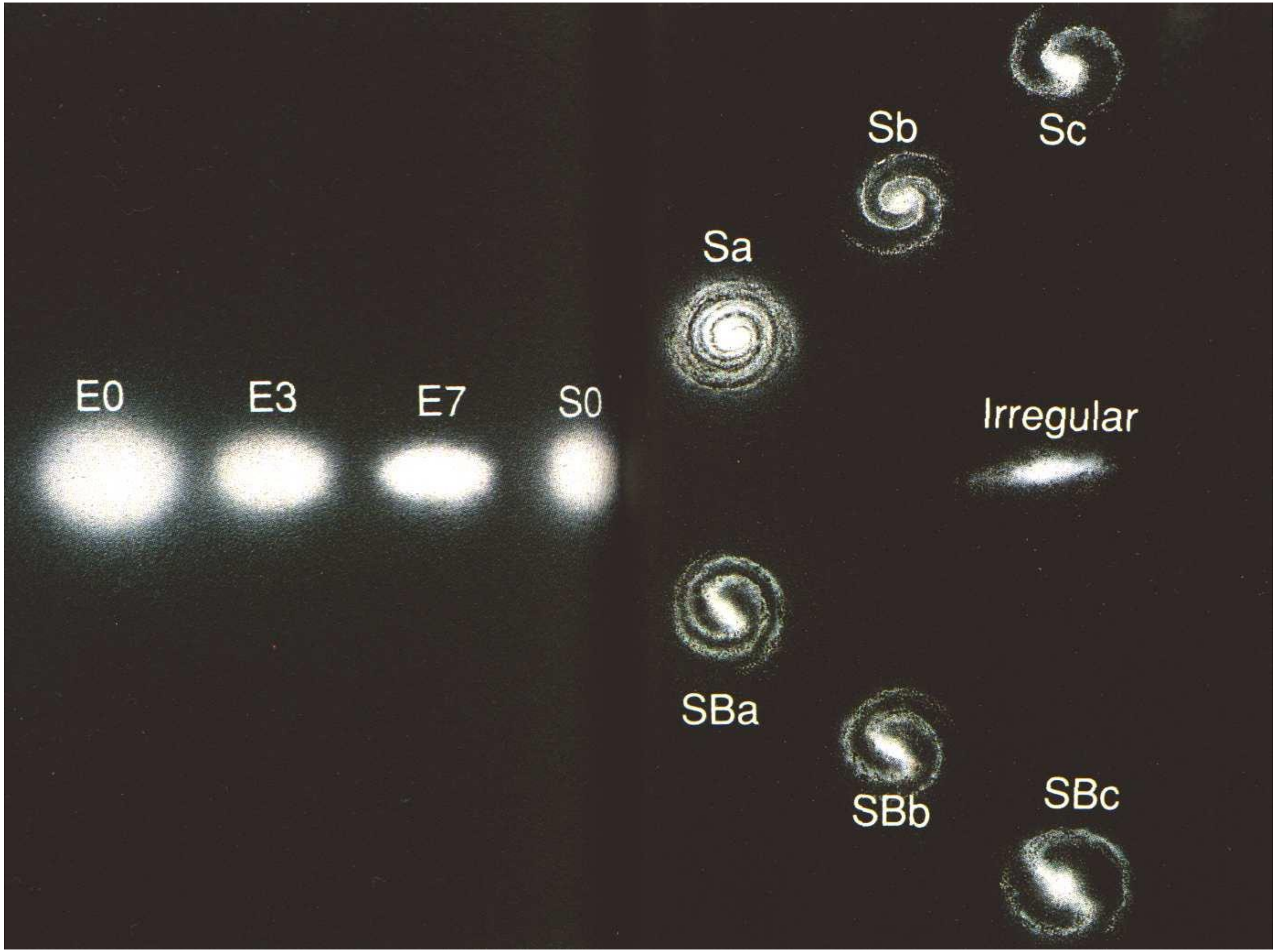
2. As shown in the plot above and in the calculation at left, the width of the emission line indicates an orbital speed of ~5,000 km/s. If the gas is orbiting at ~150 light days, what is the mass of the supermassive black hole at the heart of 3C 273?

HUBBLE'S TUNING FORK

Do galaxies evolve from right to left or left to right, and why? What did Hubble think?

What feature(s) really matters for determining whether a galaxy is older or younger?

What is more likely to collide: galaxies or stars? How do galaxy collisions influence their evolution?

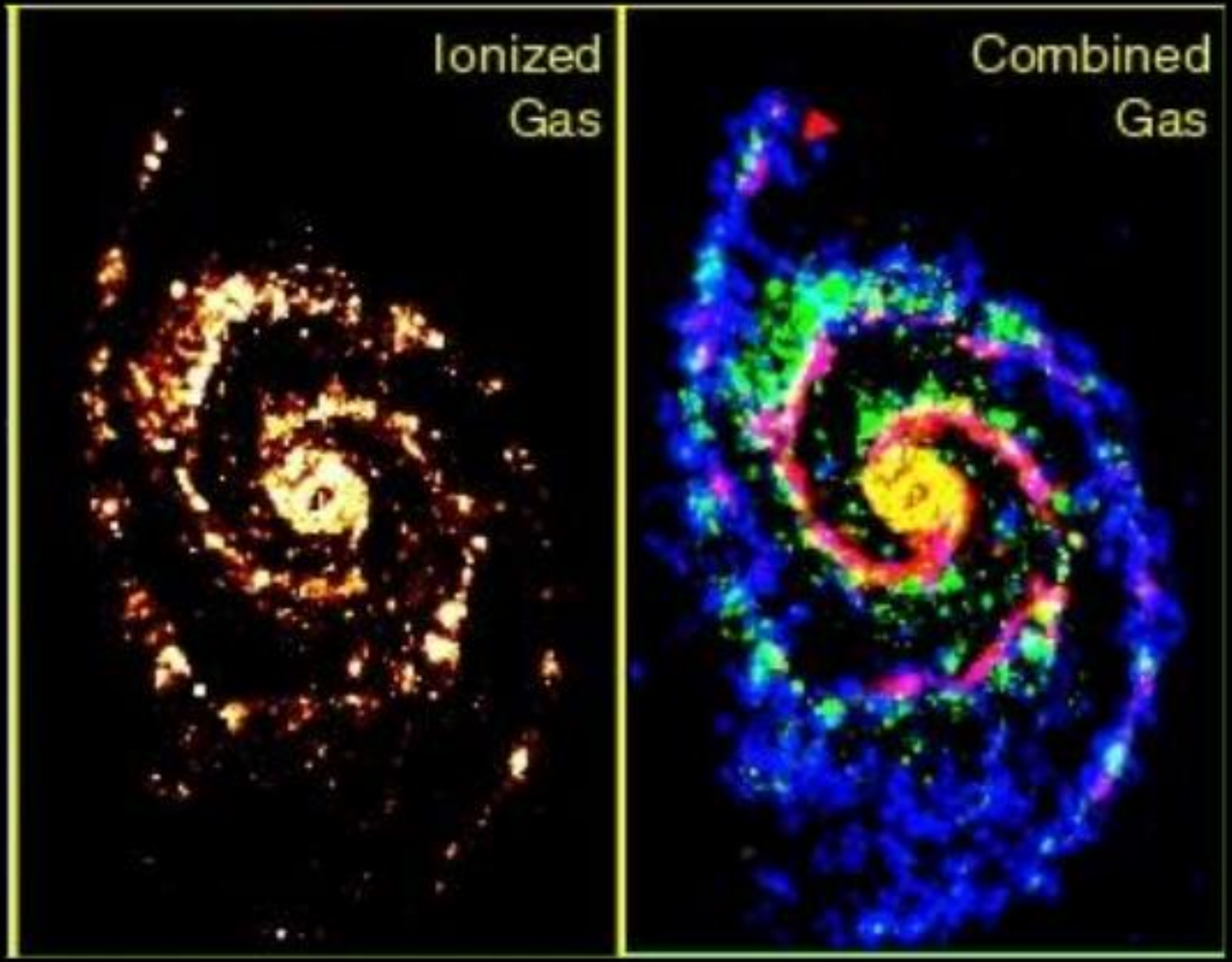


- A typical galaxy has a gas mass of ~10^8Msun, a star formation rate of 1Msun/year. How long will it take the galaxy to run out of gas?

A. A million years
B. $\boxed{\text{A hundred million years}}$
C. A billion years
D. A hundred billion years

1% of the age of the universe!

What's happening?



How might supermassive black holes influence galaxy properties? Why is this surprising?

SPECIAL AND GENERAL RELATIVITY

Special:

Applies for *constant* velocity (no acceleration) only

Speed of light + laws of physics = constant for all observers

Pretty much everything else -> *relative*

General:

Same as special, but considers accelerating reference frames (which break some special relativistic degeneracies)

MATH – THE LORENTZ FACTOR

To calculate the change in distance / time from one reference frame to another we use the **Lorentz factor**:

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time dilation: $\Delta t' = \gamma \Delta t$

Moving frame ($\Delta t'$) · Rest frame (Δt)

Length contraction: $\Delta x' = \frac{\Delta x}{\gamma}$

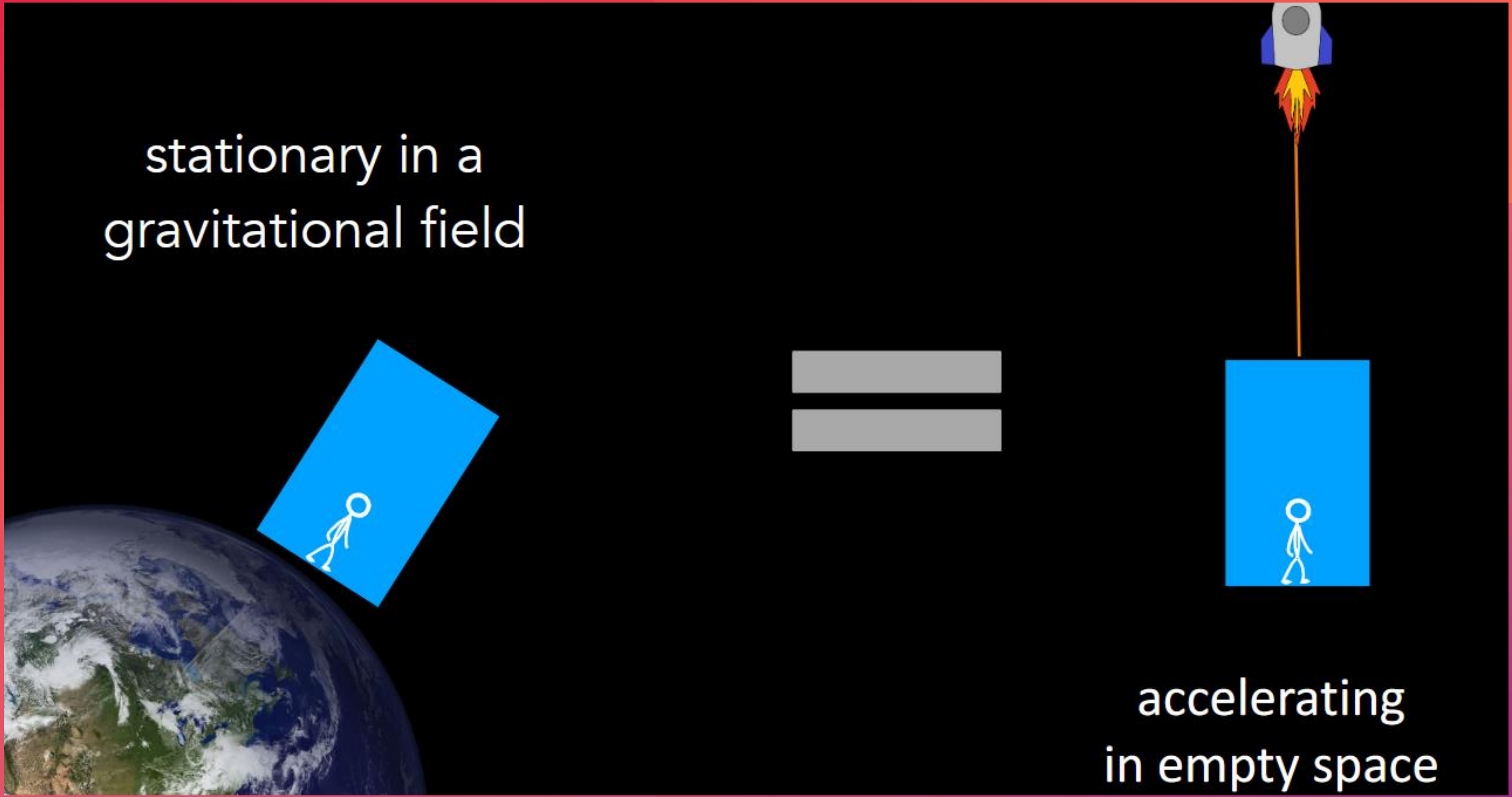
Rest frame (of object) (Δx) · Moving frame ($\Delta x'$)

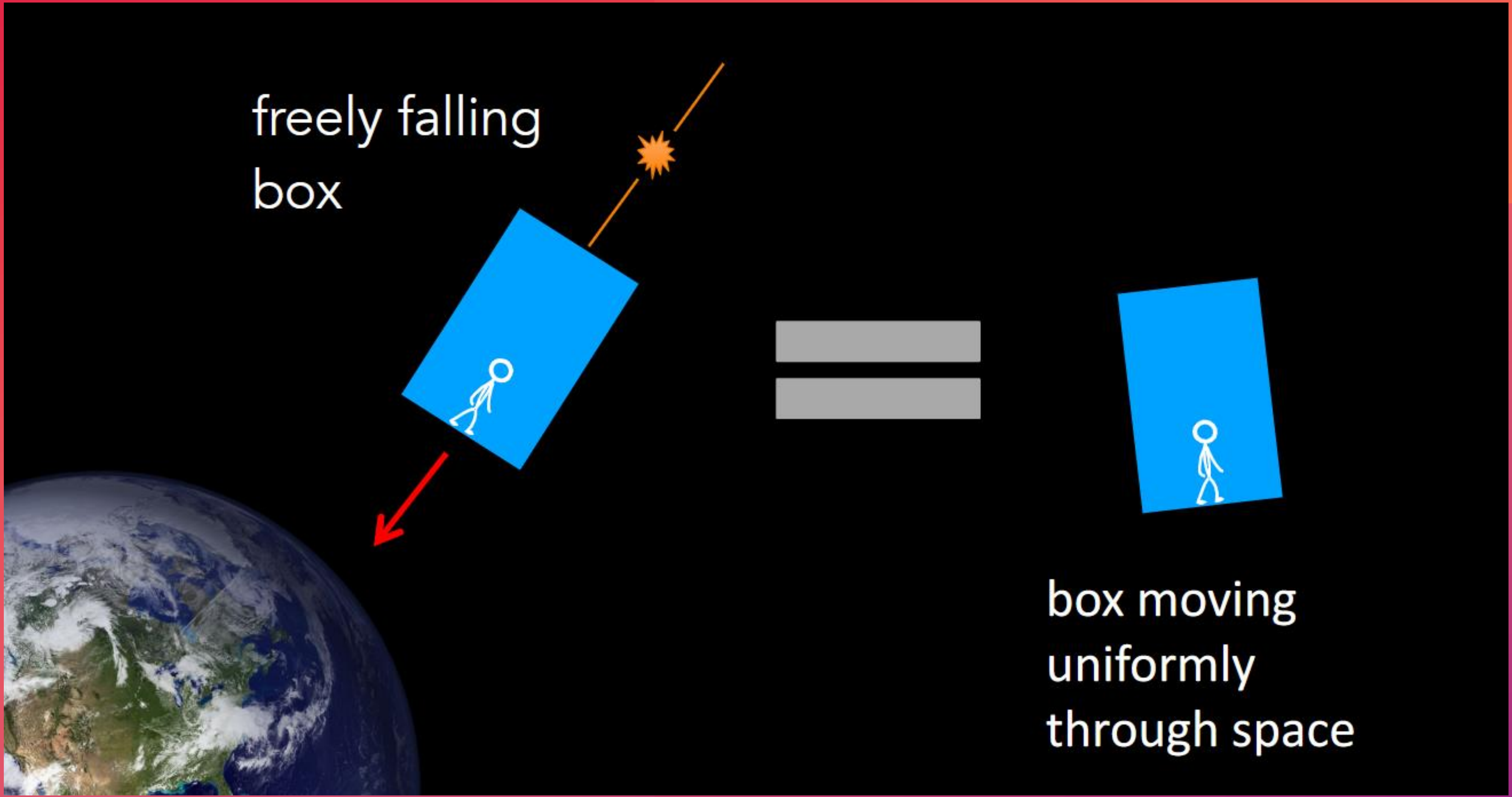
Practice problem 2:

a. If it takes me 10 years to cross the galaxy in my spaceship, but people on Earth think it took 100,000 years, what is γ? How big is the galaxy from my perspective?

b. Mark Kelly holds the record for longest consecutive time spent in space at 215 days. He lived on the ISS, which orbits the Earth at ~8,000 m/s. If you travelled in a spaceship at this speed for 215 days (from Earth's perspective) how much of a discrepancy between clocks on Earth and clocks on your spaceship would there be?

Twin paradox


stationary in a
gravitational field
accelerating
in empty space


freely falling
box
box moving
uniformly
through space

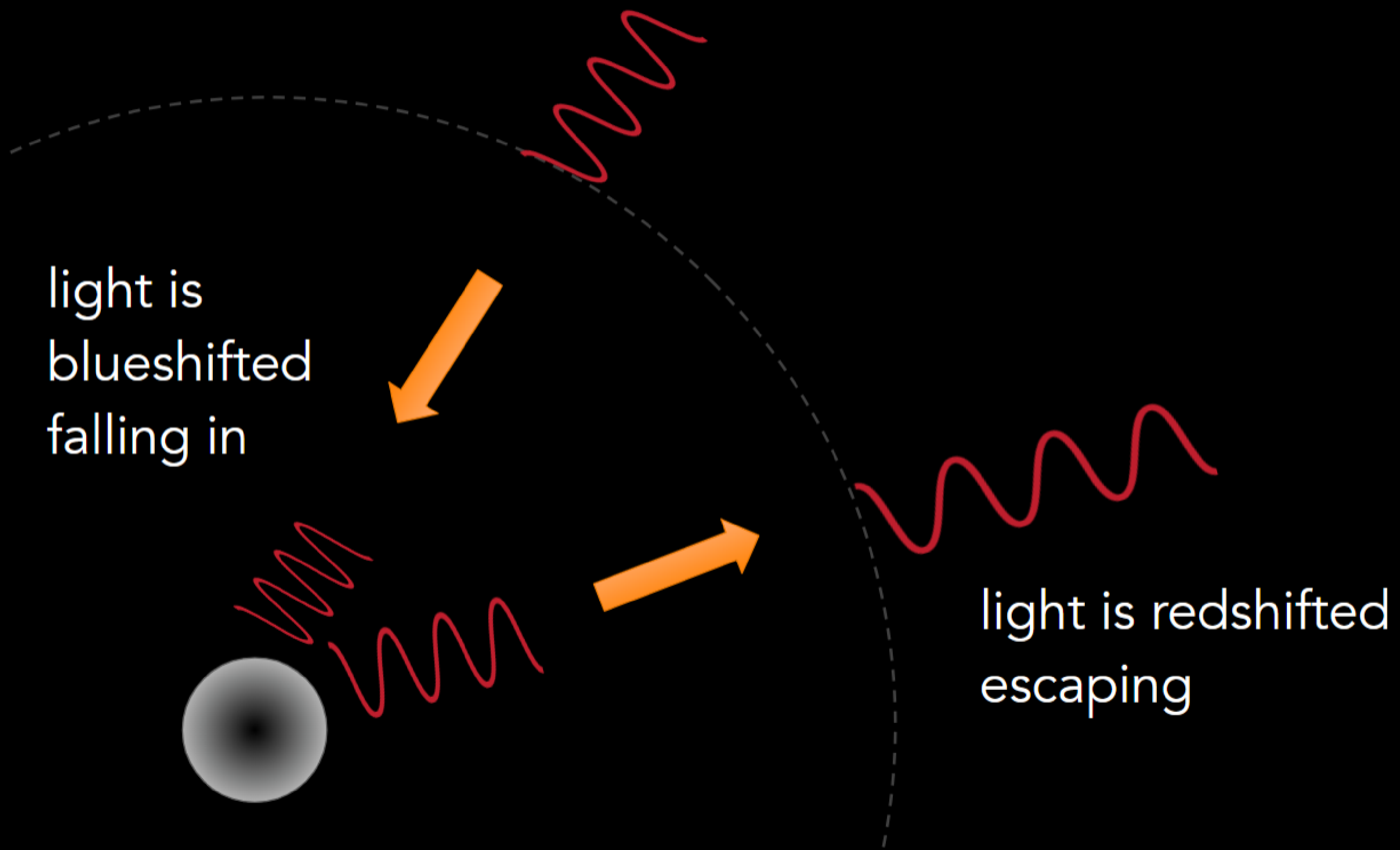
EQUIVALENCE PRINCIPLE

- Laws of physics in a freely falling frame are indistinguishable from those in a uniformly moving frame in a gravity free Universe
- Laws of physics in an accelerating frame are indistinguishable from those in a stationary frame in a gravitational field

How does this explain the gravitational bending of light? Why can't Newtonian gravity explain this?

What changes (conceptually) between Newtonian gravity and GR? What keeps the Earth in orbit around the Sun?

Gravitational time dilation



MIDTERM 2 REVIEW: QUALITATIVE

- What is the difference between a white dwarf (type I) supernova and a massive star supernova? What is left behind in both cases? Why can we use type I supernovae as standard candles and, conversely, why can't we use type II?
- What is a black hole? What is the event horizon? How do we infer the presence of black holes from observations?
- What is degeneracy pressure, and how is it different from "normal" pressures? What quantum effect causes it? Bonus: why do neutron stars have radii ~1000 times smaller than white dwarves?
- Considering special relativity alone, explain the concepts of length contraction and time dilation and how they are related. In the twin paradox, which twin ends up younger and why?
- What causes gravitational time dilation and gravitational redshift, and how are the two connected?
- Would you rather fall into a supermassive or stellar mass black hole, and why?
- What is the equivalence principle?
- Why are galaxies more likely to collide than stars?

MIDTERM 2 REVIEW: QUANTITATIVE

- Given the Hubble constant is measured at roughly $H_0 \approx 70 \frac{\frac{\mathrm{km}}{\mathrm{s}}}{\mathrm{Mpc}}$, estimate the age of the universe. Explain (conceptually) how this works. Suppose we had evolved much earlier in the universe (say 1/4th of the Universe's age today) – would our measurement of the Hubble constant change? If so, what would its value be?
- Calculate the density of a supermassive black hole ($10^7\ M_\odot$) and a stellar mass black hole ($10\ M_\odot$). Which is greater and why?
- Using conservation of energy, derive the equation for escape velocity. Setting the escape velocity equal to the speed of light, derive the Schwarzschild radius. The escape velocity of Earth from its surface is roughly 11 km/s, but rockets never go this fast – how then have we managed to explore the solar system?
- Using the circular velocity formula, derive the relationship in Kepler's third law: $P^2 \propto a^3$
- The last place you can stably orbit a black hole is at 1.5 times the Schwarzschild radius. Assuming the orbit was Keplerian, what speed is associated with such an orbit? Answer in terms of the speed of light. Does the mass of the object matter?