

ASTR 1040 RECITATION 8

RELATIVITY

10/24/2023

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HOUSEKEEPING

- a) Sorry for delay in grading homeworks – have talked to grader about it and hope they will be in soon
- b) Please submit your homeworks (on time) and make sure they are legible!
- c) Will only be in AHR 4-5 next two weeks

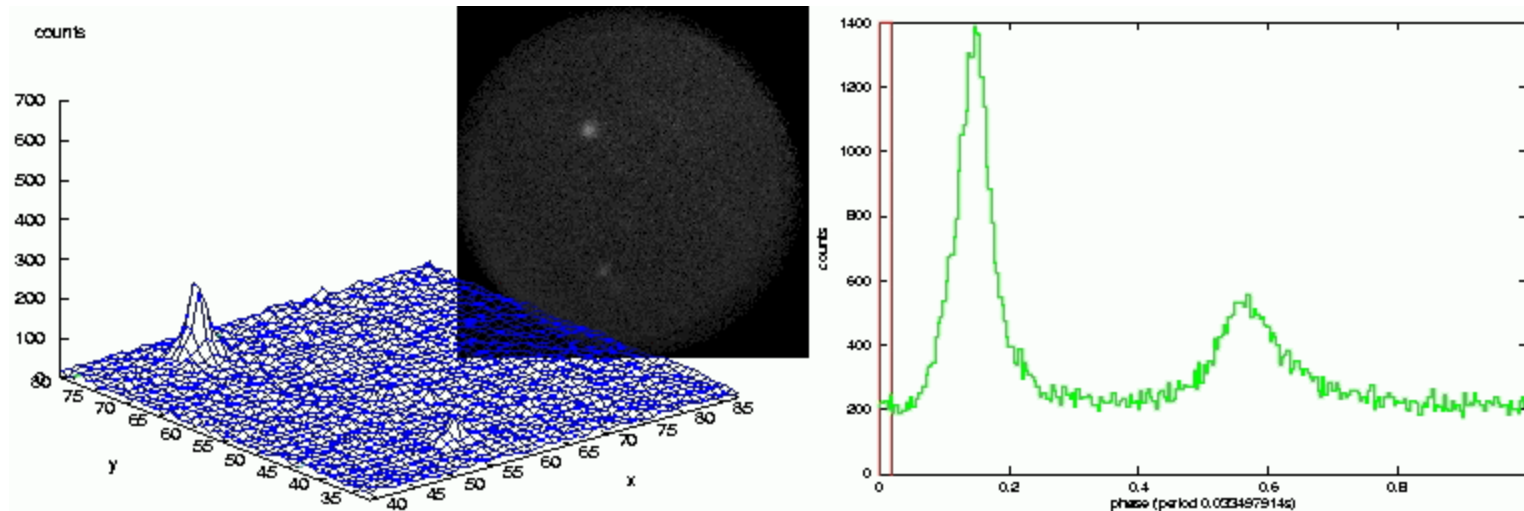
DO PULSARS HAVE TO BE NEUTRON STARS?

Assuming a neutron star's surface can't spin faster than the speed of light, calculate a maximum possible radius of a pulsar given that they have periods measured as small as 0.001s. Using this fact, calculate a characteristic density for a pulsar and compare to the density of a white dwarf ($\sim 10^{10}\text{kg/m}^3$).

Hint: Set the centrifugal and gravitational forces at the equator equal to each other.

Formulae you may need:

$$\begin{aligned}v &= \omega r \\ F_G &= G \left(\frac{Mm}{r^2} \right) \\ F_c &= m\omega^2 r \\ \omega &= 2\pi f = \frac{2\pi}{P}\end{aligned}$$



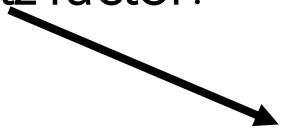
PRACTICE PROBLEM 1: THOUGHT EXPERIMENTS

- a) I'm travelling across the galaxy (100,000 light years across as measured from Earth) in a spaceship really fast – so fast that I get some cool time dilation effects and from my perspective it only takes me 10 years to cross the entire galaxy. Did I break the speed of light?
- b) There are two car accidents in New York and London at the same time according to the watches on both sets of drivers involved (who are now stationary). I'm on a flight that's halfway between New York to London on a new super fast jet with powerful cameras such that I can watch both accidents happen from the plane – from my perspective which happens first, and why?

Think about and justify your answers conceptually only, no math allowed!

MATH – THE LORENTZ FACTOR

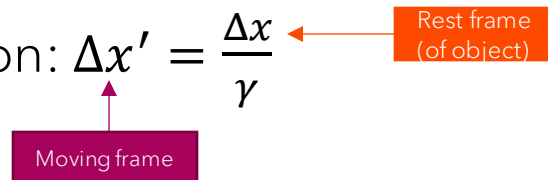
To calculate the change in distance / time from one reference frame to another we use the Lorentz factor:


$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time dilation: $\Delta t' = \gamma \Delta t$



Length contraction: $\Delta x' = \frac{\Delta x}{\gamma}$



Practice problem 2:

- If it takes me 10 years to cross the galaxy in my spaceship, but people on Earth think it took 100,000 years, what is γ ? How big is the galaxy from my perspective?
- Mark Kelly holds the record for longest consecutive time spent in space at 215 days. He lived on the ISS, which orbits the Earth at $\sim 8,000$ m/s. If you travelled in a spaceship at this speed for 215 days (from Earth's perspective) how much of a discrepancy between clocks on Earth and clocks on your spaceship would there be?

VISUALIZING SPACETIME: MINKOWSKI DIAGRAMS

How is the coordinate system defined in a Minkowski spacetime diagram? It's relative to each observer!

If c is the universal speed limits, photons are fastest travelling things through spacetime:

$$\frac{\Delta x}{\Delta t} = v = c \rightarrow \Delta x = c\Delta t$$

If everyone agrees on speed of light, they must also agree on "space time distance", i.e. for a photon:

$$c^2\Delta t^2 - \Delta x^2 = 0$$

Note: this doesn't mean they have to agree on Δt or Δx !

Thus the special relativistic "distance" metric is defined in terms of light:

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2$$

Questions for you:

1. If $\Delta s^2 = 0$ for light, what does it mean for Δs^2 to be positive? Negative?
2. When looking at a spacetime diagram, how do you determine what events appear to happen at the same time and which happen at the same place?

MINKOWSKI PART 2: COMPARING FRAMES

Events that are simultaneous for one observer **are not necessarily simultaneous for any other**. Why? $\Delta t' = \gamma \Delta t$ and $\Delta x' = \frac{\Delta x}{\gamma}$ – observers **don't need to agree** on Δt and Δx , just the spacetime distance Δs !

Seeing motion through space faster = seeing motion through time slower.

Coordinate frames are related with $\tan \alpha = \frac{v}{c}$

Question: What do lines of simultaneity look like? What events are the simultaneous for every observer?

