CG – Assignment 03

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3.1) Consider a tree with a constant arity A. This means that every internal node has exactly A children. Derive a relation between the number of internal nodes I and leaf nodes L.

For a binary tree we can see that every Internal node (I) spawn 2 leaves (L), and to get more L we have to transform one of the current L to an I. This clearly means that for A=2:

$$L = I + 1$$

We can see that what is happening here is that the total number of nodes is twice the number of I, we add 1 for the origin node:

$$Total = I * 2 + 1$$

And by definition:

$$Total = I + L$$

So, to obtain L we just have to subtract I from the total:

$$L = I * 2 + 1 - I = I + 1$$

This is obviously generalized as:

$$L = I * (A - 1) + 1$$

3.2) Planar Surface Area Heuristic: When sliding the plane from one end to the other of the bounding box, the number of primitives on the left and right side will change. Show that the SAH cost has to be computed only at those locations to find the global minimum.

We know that the SAH cost is calculated using the following formula:

$$Cost(c) = Cost_{trav} + Prov(L)Cost_L + Prov(R)Cost_R$$

With $Cost_{trav}$ being the cost of traversing the node, Prov(L) the probability of hitting the left node and $Cost_L$ the cost of traversing the geometry inside the left node. Same for the right node. We know we can rewrite this and make it depend on the surface area (SA) and the number of shapes inside (Tricount).

$$Cost(c) = Cost_{trav} + \frac{SA(L)}{SA(c)}TriCount(L) + \frac{SA(R)}{SA(c)}TriCount(R)$$

If we want to minimize the energy now, the necessary condition is that the derivate equals to zero. Here we have to take derivates over the parameter we can control, the position of x_p.

We can try to derivate every term of the equation and get rid of any term that does not depend on the position of x_p.

$$\frac{dCost(c)}{dx_p} = 0 + \frac{1}{SA(c)}\frac{dSA(L)}{dx_p}\frac{dTriCount(L)}{dx_p} + \frac{1}{SA(c)}\frac{dSA(R)}{dx_p}\frac{dTriCount(R)}{dx_p}$$
$$\frac{dCost(c)}{dx_p} = \frac{1}{SA(c)}\left(\frac{dSA(L)}{dx_p}\frac{dTriCount(L)}{dx_p} + \frac{dSA(R)}{dx_p}\frac{dTriCount(R)}{dx_p}\right)$$

So, our necessary condition is:

$$\frac{dSA(L)}{dx_p}\frac{dTriCount(L)}{dx_p} + \frac{dSA(R)}{dx_p}\frac{dTriCount(R)}{dx_p} = 0$$

Here we face a problem, we can't calculate the derivate of the TriCount(x) because it depends on the geometry. We will continue with the derivates of SA(x). We know that the total area of c can be calculated as:

$$SA(c) = 2\left[(B_{x1} - B_{x0})(B_{y1} - B_{y0}) + (B_{y1} - B_{y0})(B_{z1} - B_{z0}) + (B_{x1} - B_{x0})(B_{z1} - B_{z0})\right]$$

And this can be easily extended to the child nodes as:

$$\begin{cases} SA(L) = 2\left[(x_p - B_{x0})(B_{y1} - B_{y0}) + (B_{y1} - B_{y0})(B_{z1} - B_{z0}) + (x_p - B_{x0})(B_{z1} - B_{z0})\right] \\ SA(R) = 2\left[(B_{x1} - x_p)(B_{y1} - B_{y0}) + (B_{y1} - B_{y0})(B_{z1} - B_{z0}) + (B_{x1} - x_p)(B_{z1} - B_{z0})\right] \end{cases}$$

Now we take derivates for both of them:

$$\begin{cases} \dfrac{dSA(L)}{dx_p} = 2\left[(B_{y1} - B_{y0}) + (B_{z1} - B_{z0})\right] \\ \dfrac{dSA(R)}{dx_p} = -2\left[(B_{y1} - B_{y0}) + (B_{z1} - B_{z0})\right] \end{cases}$$

We can see that they are equal, except for the sign. if we add these expressions to our necessary condition, only the sign will survive as it is equal to 0. The result is:

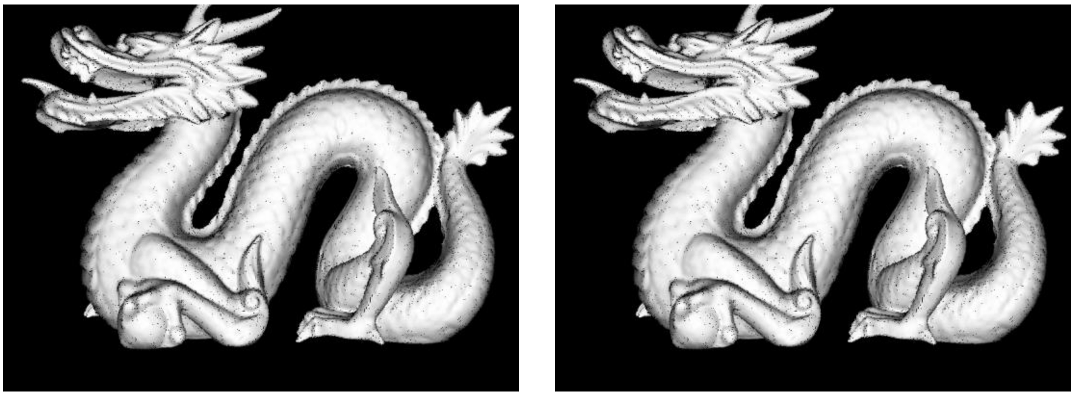
$$\frac{dTriCount(L)}{dx_p} - \frac{dTriCount(R)}{dx_p} = 0$$

Now the formula only depends on the number of polygons in each node. This means that placing the splitting point x_p on any position where the same geometry remains on both sides will yield the same cost. So, to find the minimum cost we only have to take into account the points where the number of primitives on the left and right side will change.

3.5) Surface Area Heuristic

We implemented SAH and tested it on the model dragon.obj, which has a polygon count of 871306, much higher that the cow.obj that was given to us. This will help us get higher rendering times, so it is easier to compare the differences between. Using SAH or split-in-the-middle.

We then run the test using this model, and we measure the time taken to render it, the results are:



a3-dragon-noSAH.png on the left, a3-dragon-SAH.png on the right

Time taken to render a3-dragon-noSAH.png: 6533.96 ms
Time taken to render a3-dragon-SAH.png: 6144.56 ms

As we expected, the visual results are equal, and we get a time reduction of 389.4 ms in this case.