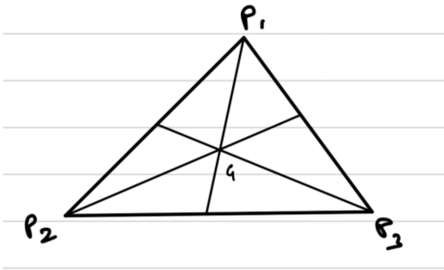
CG – Assignment 02

Group 5: Basavaraj Rajendra Sunagad - 7015644 and Javier Usón Peirón - 7024337

2.1) A triangle T is defined by its 3 vertices P1, P2, P3.

- **Compute the barycentric coordinates of the center of the mass of T.**

The center of mass or centroid is the intersection of all the medians. This point (**a**) subdivides the triangle in three triangles which should have equal areas.



Knowing this, we can follow this deduction:

$$\Delta P_1P_2P_3 = \Delta P_1P_2a + \Delta P_1aP_3 + \Delta aP_2P_3$$
$$\Delta P_1P_2a = \Delta P_1aP_3 = \Delta aP_2P_3 = \frac{1}{3}\Delta P_1P_2P_3$$

So the barycentric coordinates of point **a** are:

$$\boldsymbol{a} = u\boldsymbol{P_1} + v\boldsymbol{P_2} + w\boldsymbol{P_3}\,; \quad u = v = w = \frac{1}{3}$$

- **Compute the barycentric coordinates of the incenter of T, which is the center of the inscribed circle. We define the inscribed circle as the circle whose center is equidistant to each edge of T.**

Let a, b and c be the length of the sides of the triangle P1P2P3 such that
a = |P2-P3|, b= |P1-P3|, c= |P2-P1|.
Let h_a, h_b ,h_c denote the lengths of the altitudes from vertices P1, P2 & P3 respectively.
Let r be the radius of inscribed triangle and G be centre of circle.

$$\text{u} = \frac{\Delta P1P2G}{\Delta P1P2P3} = \frac{0.5 * |P1 - P2| * r}{0.5 * |P1 - P2| * h_a} = \frac{r}{h_a}$$

similarly

$$\text{v} = \frac{\Delta P2P3G}{\Delta P1P2P3} = \frac{0.5 * |P2 - P3| * r}{0.5 * |P2 - P3| * h_b} = \frac{r}{h_b}$$

$$\text{w} = \frac{\Delta P3P1G}{\Delta P1P2P3} = \frac{0.5 * |P3 - P1| * r}{0.5 * |P3 - P1| * h_c} = \frac{r}{h_c}$$

we know that

$$\frac{1}{2} * ah_a = \frac{1}{2} * bh_b = \frac{1}{2} * ch_c = k$$

$$a = \frac{2k}{h_a}, b = \frac{2k}{h_b}, c = \frac{2k}{h_c}$$

Since 2k and r are constants we can write

$$u : v : w = a : b : c$$

And we know u, v and w should sum up to 1.

$$\begin{cases} \frac{u}{v} = \frac{a}{b} \\ \frac{u}{v} = \frac{b}{c} \\ u + v + w = 1 \end{cases}$$

Solving we arrive at the conclusion:

$$u = \frac{a}{a+b+c};\ v = \frac{b}{a+b+c}; w = \frac{c}{a+b+c}$$

2.2) On the lecture we define an infinite plane by an arbitrary point a on the plane and a normal n. This is however not the only way to define a plane.
Derive the values a and n when the plane is defined as:

- **The set of all points p = (x,y,z) ∈ R^3 satisfying equation Ax+By+Cz+D = 0 with A,B,C,D ∈ R The constructed normal n is unique except for its sign.**

In this case the plane is given to us as the equation in general form, so we know that the normal vectors to the plane are:

$$\boldsymbol{n} = (A, B, C); or\ \boldsymbol{n} = -(A, B, C)$$

To compute a point we just have to use the expression given above: **a** = (x,y,z) ∈ R^3 satisfying equation Ax+By+Cz+D. One example can be:

$$\boldsymbol{a} = \left(\frac{1}{A}, \frac{-1}{B}, \frac{-D}{C}\right)$$

- **As a set of 3 points p1, p2, p3 lying on the plane defined in counter-clockwise order with respect to intended surface normal direction. In contrary to the first subtask, there exists only one n this time.**

Now we can use any of the given points, so **a** = **p1**, **p2** or **p3**.

For the normal vector, we can compute 2 vectors joining the 3 points, and then calculate the cross product to obtain a perpendicular vector to both of them.

- From p1 to p2 → **u** = (x_{P2} - x_{P1}, y_{P2} - y_{P1}, z_{P2} - z_{P1})
- From p1 to p3 → **v** = (x_{P3} - x_{P1}, y_{P3} - y_{P1}, z_{P3} - z_{P1})

$$(\boldsymbol{u}\ x\ \boldsymbol{v}) = \boldsymbol{i}\big((y_{P2}-y_{P1})(z_{P3}-z_{P1})-(z_{P2}-z_{P1})(y_{P3}-y_{P1})\big)$$
$$-\boldsymbol{j}\big((x_{P2}-x_{P1})(z_{P3}-z_{P1})-(z_{P2}-z_{P1})(x_{P3}-x_{P1})\big)$$
$$+\boldsymbol{k}\big((x_{P2}-x_{P1})(y_{P3}-y_{P1})-(y_{P2}-y_{P1})(x_{P3}-x_{P1})\big)$$

2.3) Given a ray R(t) = O + t · D and quadric
F(x,y,z)=Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0.

- **Compute the values t for which the ray intersects the quadric.**
- **Derive the ray-sphere intersection formula from it, as a special case.**

a) Compute the values t for which the ray intersects the quadric
Substitute ray equation in F(x,y,z)
F(ox+tdx, oy+tdy, oz+tdz)=

A (ox+tdx)^2 + B (oy+tdy)^2 +C (oz+tdz)^2 + D(ox+tdx) (oy+tdy) + E(ox+tdx) (oz+tdz) + F(oy+tdy) (oz+tdz) + G(ox+tdx) + H(oy+tdy) +I(oz+tdz) + J = 0

A (ox^2+t^2dx^2+2oxtdx) + B(oy^2+t^2dy^2+2oytdy)+ C (oz^2+t^2dz^2+2oztdz)+
D(oxoy+oxtdy+tdxoy+t^2dxdy) + E(oxoz+oxtdz+tdxoz+t^2dxdz) +
F(oyoz+oytdz+tdyoz+t^2dydz) + G(ox+tdx) + H(oy+tdy) + I(oz+tdz) + J = 0

Aox^2 + At^2dx^2 + A2oxtdx + Boy^2 + Bt^2dy^2 + B2oytdy + C oz^2 + Ct^2dz^2 + C2oztdz + Doxoy + Doxtdy + Dtdxoy + Dt^2dxdy + Eoxoz + Eoxtdz + Etdxoz + Et^2dxdz + Foyoz + Foytdz + Ftdyoz + Ft^2dydz + Gox + Gtdx + Hoy + Htdy + Ioz + Itdz + J = 0

At^2dx^2 + Bt^2dy^2 + Ct^2dz^2 + Dt^2dxdy + Et^2dxdz + Ft^2dydz +
A2oxtdx + B2oytdy + C2oztdz + Eoxtdz + Etdxoz+ Doxtdy + Dtdxoy + Foytdz + Ftdyoz +
Gtdx+ Htdy + Itdz + Aox^2 + Boy^2 + C oz^2 + Doxoy + Eoxoz + Foyoz + Gox + Hoy + Ioz + J = 0

(Adx^2 + Bdy^2 + Cdz^2 + Ddxdy + Edxdz + Fdydz)t^2 +
(2Aoxdx + 2Boydy + 2Cozdz + E(oxdz + dxoz)+ D(oxdy + dxoy) + F(oydz + dyoz) + Gdx+ Hdy + Idz)t + Aox^2 + Boy^2 + C oz^2 + Doxoy + Eoxoz + Foyoz + Gox + Hoy + Ioz + J = 0

Aqt^2 + Bqt + Cq = 0

Where:
Aq = Adx^2 + Bdy^2 + Cdz^2 + Ddxdy + Edxdz + Fdydz
Bq = 2Aoxdx + 2Boydy + 2Cozdz + E(oxdz + dxoz)+ D(oxdy + dxoy) + F(oydz + dyoz) + Gdx+ Hdy + Idz
Cq = Aox^2 + Boy^2 + C oz^2 + Doxoy + Eoxoz + Foyoz + Gox + Hoy + Ioz + J

Aqt^2 + Bqt + Cq = 0 forms a quadratic equation whose roots be t_0 & t_1
t_0 = (- Bq - ((Bq^2 - 4AqCq))^0.5)/ 2Aq
t_1 = (- Bq + ((Bq^2 - 4AqCq))^0.5)/ 2Aq
t = t_0 or t_1 (which ever is real)

b) Derive the ray-sphere intersection formula from it, as a special case

quadric is given by $Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$.

sphere is of the form $x^2 + y^2 + z^2 + j = 0$
$=> A= B = C=1$
$=> D = E = F = G = H = I = 0$

$Aq = dx^2 + dy^2 + dz^2$
$Bq = 2oxdx + 2oydy + 2ozdz$
$Cq = ox^2 + oy^2 + oz^2 + J$

$t_0 = (- Bq - ((Bq^2 - 4AqCq))$^0.5)/ 2Aq
$t_1 = (- Bq + ((Bq^2 - 4AqCq))$^0.5)/ 2Aq
$t = t_0$ or t_1 (which ever is real)