CG – Assignment 07

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6.1) Color space: Compute the position of the sRGB color (1,0,1) in the CIE-XYZ and CIE-xy color space. Argue why gamma correction in sRGB is irrelevant for the transformation of this particular color.

First, we need to obtain the primary colors and place them on the transformation matrix. We can obtain them from the CIE-xy primaries, we will use the HDTV standard:

TABLE 0.2 CIE chromaticities for ITU-R BT.709 reference primaries and CIE standard illuminant

	Red	Green	Blue	D65
x	0.6400	0.3000	0.1500	0.3127
y	0.3300	0.6000	0.0600	0.3290
z	0.0300	0.1000	0.7900	0.3583

To transform from sRGB to CIE-XYZ we multiply by the transformation matrix:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

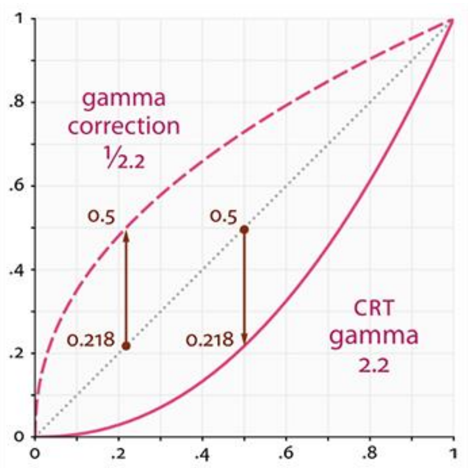
We obtain:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.5929 \\ 0.2848 \\ 0.9698 \end{bmatrix}$$

And for CIE-xy:

$$\begin{cases} x = \dfrac{X}{X+Y+Z} = 0.3209 \\ y = \dfrac{Y}{X+Y+Z} = 0.1542 \end{cases}$$

In the following figure we can see a representation of the gamma correction operation. If we focus on input values for 0 and 1, we see that they are not modified, only the values in between. If we do a transformation per component, R will stay in 1, G will stay in 0 and B will stay in 1.



6.2) Perspective Projection:

a) **Compute the point where two arbitrary parallel lines seem to intersect after being projected by the perspective projection P. For which parallel lines does no such intersection point exist?**

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1 \end{bmatrix}$$

To define a line, we can use one point and one vector, for two lines to be parallel they have to have the same direction vector, so we can define:

$$\begin{cases} p = (p_x, p_y, p_z, 1) \\ q = (q_x, q_y, q_z, 1) \\ v = (v_x, v_y, v_z, 0) \end{cases}$$

But if we want to perform projection transformations, we can't use vectors, so we have to use another method to define lines, using two points:

$$\begin{cases} p = (p_x, p_y, p_z, 1) \\ q = (q_x, q_y, q_z, 1) \\ r = (r_x, r_y, r_z, 1) = (p_x + v_x, p_y + v_y, p_z + v_z, 1) \\ s = (s_x, s_y, s_z, 1) = (q_x + v_x, q_y + v_y, q_z + v_z, 1) \end{cases}$$

Now we apply the transformation to our 4 points, obtaining the 2 new lines:

$$\begin{cases} p' = \left(p_x, p_y, 0, \frac{p_z}{2}\right) = \left(\frac{2p_x}{p_z}, \frac{2p_y}{p_z}, 0,1\right) \\ q' = \left(q_x, q_y, 0, \frac{q_z}{2}\right) = \left(\frac{2q_x}{q_z}, \frac{2q_y}{q_z}, 0,1\right) \\ r' = \left(r_x, r_y, 0, \frac{r_z}{2}\right) = \left(\frac{2r_x}{r_z}, \frac{2r_y}{r_z}, 0,1\right) \\ s' = \left(s_x, s_y, 0, \frac{s_z}{2}\right) = \left(\frac{2s_x}{s_z}, \frac{2s_y}{s_z}, 0,1\right) \end{cases}$$

To determine the point where they cross, we will compute the vector between the two points:

$$\begin{cases} v_1 = (v_{1x}, v_{1y}, v_{1z}, 0) = \left(\frac{2r_x}{r_z} - \frac{2p_x}{p_z}, \frac{2r_y}{r_z} - \frac{2p_y}{p_z}, 0,0\right) \\ v_2 = (v_{2x}, v_{2y}, v_{2z}, 0) = \left(\frac{2s_x}{s_z} - \frac{2q_x}{q_z}, \frac{2s_y}{s_z} - \frac{2q_y}{q_z}, 0,0\right) \end{cases}$$

Let us substitute the definitions for r and s and try to simplify them:

$$\begin{cases} v_1 = \left(2\left(\frac{p_x + v_x}{p_z + v_z} - \frac{p_x}{p_z}\right), 2\left(\frac{p_y + v_y}{p_z + v_z} - \frac{p_y}{p_z}\right), 0,0\right) \\ v_2 = \left(2\left(\frac{q + v_x}{q_z + v_z} - \frac{q_x}{q_z}\right), 2\left(\frac{q_y + v_y}{q_z + v_z} - \frac{q_y}{q_z}\right), 0,0\right) \end{cases}$$

To make this task easier let us focus on v_{1x} and then apply the same process on every component:

$$v_{1x} = 2\left(\frac{p_x + v_x}{p_z + v_z} - \frac{p_x}{p_z}\right) = \frac{2}{p_z(p_z + v_z)}\left(p_z(p_x + v_x) - p_x(p_z + v_z)\right)$$
$$= \frac{2}{p_z(p_z + v_z)}(p_x p_z + v_x p_z - p_x p_z - p_x v_z) = \frac{2(v_x p_z - p_x v_z)}{p_z(p_z + v_z)}$$

$$\begin{cases} v_1 = \dfrac{2}{p_z(p_z + v_z)}\Big((v_x p_z - p_x v_z), \big(v_y p_z - p_y v_z\big), 0,0\Big) \\ v_2 = \dfrac{2}{q_z(q_z + v_z)}\Big((v_x q_z - q_x v_z), \big(v_y q_z - q_y v_z\big), 0,0\Big) \end{cases}$$

And finally, to find the intersection, we force a point on both lines to be the same:

$$\begin{cases} \dfrac{2p_x}{p_z} + \alpha\dfrac{2(v_x p_z - p_x v_z)}{p_z(p_z + v_z)} = \dfrac{2q_x}{q_z} + \beta\dfrac{2(v_x q_z - q_x v_z)}{q_z(q_z + v_z)} \\ \dfrac{2p_y}{p_z} + \alpha\dfrac{2\big(v_y p_y - p_y v_z\big)}{p_z(p_z + v_z)} = \dfrac{2q_y}{q_z} + \beta\dfrac{2\big(v_y q_z - q_y v_z\big)}{q_z(q_z + v_z)} \end{cases}$$

If we obtain alpha and beta, we can express the intersection point as:

$$\begin{cases} i_x = \dfrac{2p_x}{p_z} + \alpha\dfrac{2(v_x p_z - p_x v_z)}{p_z(p_z + v_z)} = \dfrac{2q_x}{q_z} + \beta\dfrac{2(v_x q_z - q_x v_z)}{q_z(q_z + v_z)} \\ i_y = \dfrac{2p_y}{p_z} + \alpha\dfrac{2\big(v_y p_y - p_y v_z\big)}{p_z(p_z + v_z)} = \dfrac{2q_y}{q_z} + \beta\dfrac{2\big(v_y q_z - q_y v_z\big)}{q_z(q_z + v_z)} \\ i_z = 0 \end{cases}$$

For which parallel lines does no such intersection point exist?

If the lines already live in the projection plane, the transformation will not change them. Something similar will happen if they live in a plane parallel to the projection plane, in this case they will be translated to said projection plane, but their parallelism wont change.

b) Compute the center of projection of the following perspective projection Q.

$$Q = \begin{bmatrix} 3/4 & 1 & -1/4 & -1/4 \\ 0 & 1 & 1/4 & 5/4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 5/4 \end{bmatrix}$$

The center of projection is the same notion as the origin in the perspective camera that we use for the ray tracer. To find it we can project 2 points, then get the line joining each point and its projection. The intersection between those lines will be the center we are looking for:

$$\begin{cases} p = (5,2,4,1) \\ q = (1,3,9,1) \end{cases} \rightarrow \begin{cases} p' = (2,1,-1.78,1) \\ q' = (0.36,1,0,1) \end{cases}$$

The vectors for the lines uniting each point and its projection are:

$$\begin{cases} v_p = (3,5,1.78,0) \\ v_q = (0.64,2,9,0) \end{cases}$$

Now we can follow the same procedure as last exercise, find a point belonging to both lines and that should be the center of the projection. The result is:

$$O = (-0.48, -1.61, -11.76, 1)$$

c) Compute the projection plane of projection Q'

$$Q' = \begin{bmatrix} 1 & {}^{1}/_{4} & -{}^{1}/_{4} & 0 \\ 0 & {}^{3}/_{4} & {}^{1}/_{4} & 0 \\ 0 & -{}^{1}/_{4} & {}^{5}/_{4} & 0 \\ 0 & {}^{1}/_{4} & -{}^{1}/_{4} & 1 \end{bmatrix}$$

To get a plane we only need 3 points, so we can easily get the projection plane by projecting 3 arbitrary points:

$$\begin{cases} p = (1,2,4,1) \\ q = (2,5,1,1) \\ r = (3,4,3,1) \end{cases} \rightarrow \begin{cases} p' = (1,5,9,1) \\ q' = (1.5,2,0,1) \\ r' = (3.6,3,2.2,1) \end{cases}$$

The vectors joining them:

$$\begin{cases} v_{pq} = (0.5, -3, -9, 0) \\ v_{pr} = (1.6, -2, -6.8, 0) \end{cases}$$

And getting the 3D cross product we obtain the normal. With the normal and a point, we can get the plane definition:

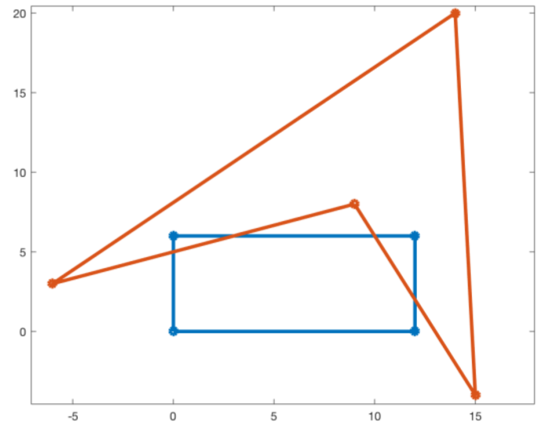
$$\begin{cases} n = (2.4, -11, 3.8) \\ p' = (1,5,9) \end{cases} \rightarrow 2.4 * 1 - 11 * 5 + 3.8 * 9 + D = 0 \rightarrow D = -18.4$$

So:

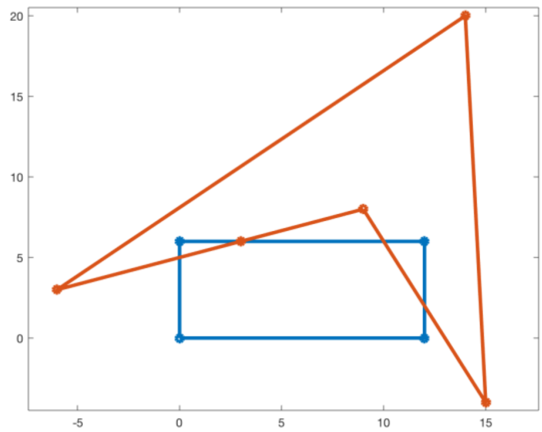
$$2.4X - 11Y + 3.8Z - 18.4 = 0$$

7.3) Clipping: Use the Sutherland-Hodgeman algorithm to perform the first step of the algorithm by clipping the polygon against the top edge of the clipping window only. Describe each step of the algorithm.

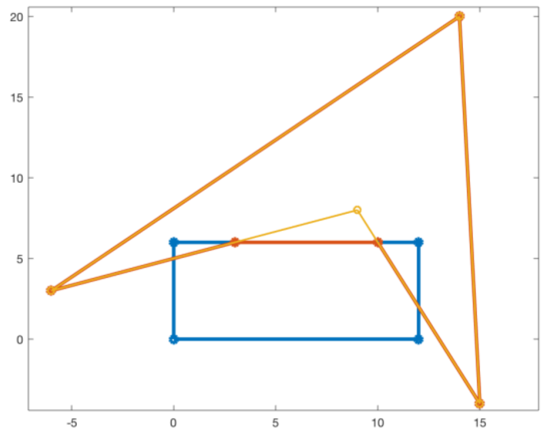
Let us plot the polygon and the window to visualize how the algorithm is going to work. For this explanation, when we say inside, we mean that the y coordinate is below 6.



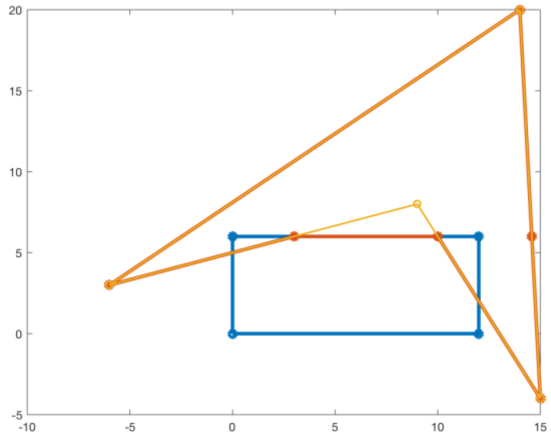
We will start on the vertexes (-6,3) and (9,8). One is inside and the following is outside, so applying the algorithm, we obtain an intersection point p1 that we will add to the resulting polygon:



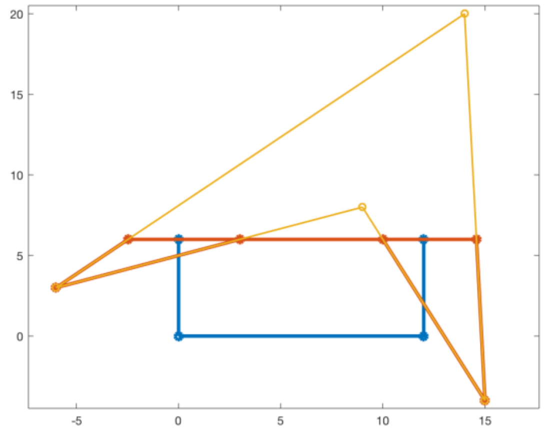
Now we clip the line defined by (9,8) and (15,-4), this time the first is outside and the next one is inside. This results in a point we will call p2, we will add p2 and then (15,-4) to the resulting polygon.



Next, (15,-4) and (14,20). Same situation as the first segment, we obtain the intersection p3 and we add it to the polygon.



Finally, (14,20) and (-6,3). The first one is outside and the second one inside, so we add the intersection p4 and (-6,3). Now, there are no original vertexes left to try, the resulting polygon is: { (-6,3), p1, p2, (15,-4), p3, p4, (-6,3) }



7.4) Hermite Spline:

a) **Compute the coefficients a,b,c, and d of the polynomials such that p(0) = (0,1,0), p(1) = (1,0,1), dp(0) = (1,0,1), dp(1) = (0,−1,0). You have to write down the constraints and solve a system of equations.**

$$\boldsymbol{p}(t) = \boldsymbol{a}t^3 + \boldsymbol{b}t^2 + \boldsymbol{c}t + \boldsymbol{d}$$
$$\frac{d\boldsymbol{p}}{dt}(t) = 3\boldsymbol{a}t^2 + 2\boldsymbol{b}t + \boldsymbol{c}$$

We have the expressions, let us apply the conditions specified:

$$\begin{cases} \boldsymbol{p}(0) = \boldsymbol{d} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \boldsymbol{p}(1) = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\ \dfrac{d\boldsymbol{p}}{dt}(0) = \boldsymbol{c} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\ \dfrac{d\boldsymbol{p}}{dt}(1) = 3\boldsymbol{a} + 2\boldsymbol{b} + \boldsymbol{c} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \end{cases}$$

We already obtained **c** and **d**, now we go for **a** and **b**

$$\begin{cases} \boldsymbol{a} + \boldsymbol{b} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \\ 3\boldsymbol{a} + 2\boldsymbol{b} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \end{cases} ; \begin{cases} \boldsymbol{b} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} - \boldsymbol{a} \\ 3\boldsymbol{a} + \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} - 2\boldsymbol{a} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \end{cases} ; \boldsymbol{a} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} ; \boldsymbol{b} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

The result is:

$$\boldsymbol{a} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix};\ \boldsymbol{b} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix};\ \boldsymbol{c} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix};\ \boldsymbol{d} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

b) Compute the same coefficients by using the Hermite basis.

We repeat the same process but this time we use:

$$\boldsymbol{p}(t) = \boldsymbol{a}H_0^3(t) + \boldsymbol{b}H_1^3(t) + \boldsymbol{c}H_2^3(t) + \boldsymbol{d}H_3^3(t)$$
$$\boldsymbol{p}(t) = \boldsymbol{a}(t-1)^2(1+2t) + \boldsymbol{b}t(t-1)^2 - \boldsymbol{c}t^2(1-t) + \boldsymbol{d}(3-2t)t^2$$
$$\frac{d\boldsymbol{p}}{dt}(t) = \boldsymbol{a}(6t^2 - 6t) + \boldsymbol{b}(3t^2 - 4t + 1) - \boldsymbol{c}(3t^2 - 2t) + \boldsymbol{d}(6t - 2t^2)$$

Applying the conditions:

$$\begin{cases} \boldsymbol{p}(0) = \boldsymbol{a} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ \boldsymbol{p}(1) = \boldsymbol{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\ \dfrac{d\boldsymbol{p}}{dt}(0) = \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\ \dfrac{d\boldsymbol{p}}{dt}(1) = \boldsymbol{c} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \end{cases}$$

We can clearly see that once we know the points for the beginning and the end of the curve, and the tangent at said points, we can multiply them by the Hermite basis and trivially obtain the desired curve.