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CS 536 HW1

1. Course Basics

Task 1.1

a) Testing provides more of a guarantee that a program is correct than verification because you actually run the program.

Ans: False. As the professor explained in his 1st class that testing a program is more like a random sampling from a set while verification covers the entire set. Even though if a person tests multiple inputs, he/she might not be able to cover unexpected inputs, unexpected human behavior, concurrency errors, changes in code, and changes in requirements.
He even mentioned about the example of Therac 25 where the testing failed to consider the unexpected input of changing the modes within 8 seconds.
Hence, it would be false to say that “Testing provides more of a guarantee that a program is correct than verification, because you actually run the program.”

b) It is useful to do both testing and verification on a program before deploying it.

Ans: True. Testing and Verification both go hand in hand. Verification is the task of formally checking if the program is correct or not. Testing confirms this formal checking with some inputs. Once, we are done with formal checking (i.e., Verification) and informal checking (i.e., Testing) of the program as per the specification, we can deploy the program.
Hence, it would be true to say that “It is useful to do both testing and verification on a program before deploying it.”

Task 1.2

a) Proposition A: $p \wedge q \vee r$
Proposition B: $(p \wedge q) \vee r$

Ans: Propositions A and B are syntactically equal (I). As explained by the professor in the 2nd lecture that syntactic equality is about how to write a program (up to, e.g., Parentheses). Here, as well, even if we write or not write the parentheses, the propositions are the same. This can also be verified by the below truth table.

p	q	r	$p \wedge q \vee r$	$(p \wedge q)$	$(p \wedge q) \vee r$
T	T	T	$T \wedge T \vee T = T$	T	T
T	T	F	$T \wedge T \vee F = T$	T	T
T	F	T	$T \wedge F \vee T = T$	F	T
T	F	F	$T \wedge F \vee F = F$	F	F
F	T	T	$F \wedge T \vee T = T$	F	T

F	T	F	$F \wedge T \vee F = F$	F	F
F	F	T	$F \wedge F \vee T = T$	F	T
F	F	F	$F \wedge F \vee F = F$	F	F

b) Proposition A: $p \rightarrow q$
   Proposition B: $\neg p \rightarrow \neg q$

Ans: Propositions A and B are neither syntactically equal not semantically equal, so option (III). This can also be verified from the below table.

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

c) Proposition A: $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
   Proposition B: T

Ans: Propositions A and B are semantically equal, so option (II). This can also be verified from the below truth table.

p	q	$p \rightarrow q$ (X)	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$ (Y)	(X) $\leftrightarrow$ (Y)
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

d) Proposition A: $\neg p$
   Proposition B: $p \rightarrow q$

Ans: Propositions A and B are neither syntactically equal nor semantically equal, so option (III). This can be verified from the below truth table.

p	q	$\neg p$	$p \rightarrow q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

e) Proposition A: $p \wedge q \vee r$
   Proposition B: $p \wedge (q \vee r)$

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Ans: Propositions A and B are neither syntactically equal nor semantically equal, so option (III). This can be verified from the below truth table.

p	q	r	$p \wedge q \vee r$	$(q \vee r)$	$p \wedge (q \vee r)$
T	T	T	$T \wedge T \vee T = T$	T	$T \wedge T = T$
T	T	F	$T \wedge T \vee F = T$	T	$T \wedge T = T$
T	F	T	$T \wedge F \vee T = T$	T	$T \wedge T = T$
T	F	F	$T \wedge F \vee F = F$	F	$T \wedge F = F$
F	T	T	$F \wedge T \vee T = T$	T	$F \wedge T = F$
F	T	F	$F \wedge T \vee F = F$	T	$F \wedge T = F$
F	F	T	$F \wedge F \vee T = T$	T	$F \wedge T = F$
F	F	F	$F \wedge F \vee F = F$	F	$F \wedge F = F$

2. Propositional Logic

Task 2.1

$(P \to Q) \to (\neg P \to \neg Q)$

Proof by truth table:

P	Q	$P \to Q$ (X)	$\neg P$	$\neg Q$	$\neg P \to \neg Q$ (Y)	(X) $\to$ (Y)
T	T	T	F	F	T	T
T	F	F	F	T	T	T
F	T	T	T	F	F	F
F	F	T	T	T	T	T

As there exists states σ_1 and σ_2 such that $\sigma_1 \vDash (P \to Q) \to (\neg P \to \neg Q)$ and $\sigma_2 \nvDash (P \to Q) \to (\neg P \to \neg Q)$, we can say that it is a contingency.

Task 2.2

It is programmed and provided in the file “tauto.log” along with this file on Blackboard.

Task 2.3

$T \Rightarrow P$ means $\vDash T \to P$

When we proved $T \Rightarrow P$ in Task 2.2, we showed that P is a Tautology.
It is of the form $P \Rightarrow Q$ which states, “if whenever P is True, Q is also True”.

This when we apply to T $\Rightarrow$ P means that "if whenever T is True, P is also True". By definition, T is True. This indicates that P is also True for all states.

P $\Rightarrow$ T means $\vDash$ P $\rightarrow$ T

Just because, T $\Rightarrow$ P is a tautology, it doesn't show that P $\Rightarrow$ T is also a Tautology.

Now, when we apply the same definition from above here, P $\Rightarrow$ T means that "if whenever P is True, T is also True". But it does not necessarily mean that P is true for all states here.

P	T	P $\rightarrow$ T
T	T	T
F	T	T

P might or might not be a Tautology. P just implies a Tautology.

Task 2.4

It is programmed and provided in the file "uncurry.log" along with this file on Blackboard.

3. Predicate Logic

Task 3.1

Considering $\mathbb{Z}$ as the set of integers and M as the set of prime numbers.
Given fact: $M \subset \mathbb{Z}$

Goldbach's conjecture states, "Every even integer greater than 2 is the sum of two prime numbers."

Predicate Logic Notation:

$$\forall n \in \mathbb{Z}. ((n > 2 \wedge P(n))(\exists x \in M. \exists y \in M. (x + y = n)))$$

Where P(n) denotes that n is an even number.

Understanding: For all n which belongs to the set of integers where n is an even number there exists two numbers which belongs to the set of prime numbers x & y such that when these two numbers are added it gives back the same number n.

Task 3.2

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a) $\forall x \in \mathbb{Z}. \forall y \in \mathbb{Z}. x < y$

Ans: False, because one can consider multiple states where this does not hold. For example, x = 10 and y = 9. In this example, both x and y $\in \mathbb{Z}$ but $x \nless y$.
Hence, this is a proof of counterexample.

b) $\forall x \in \mathbb{Z}. \neg(\exists a \in \mathbb{Z}. \exists b \in \mathbb{Z}. a > 1 \wedge b > 1 \wedge a * b = x)$

Ans: False, because one can consider multiple states where this does not hold. For example, x = 6, a = 3, b = 2. In this example, x, a, and b $\in \mathbb{Z}$ but there exists a and b where a>1 and b>1 and a*b ≠ x. Working: 3>1 and 2>1 and 3*2 = 6.
Hence, this is a proof of counterexample.

c) $\forall x \in \mathbb{Z}. \exists y \in \mathbb{Z}. y = x * 2$

Ans: True, because for every integer x, there exists an integer y which holds the condition y = x*2.
One can consider multiple states and observe that this holds for all values of x.
For example: if x = 2 which $\in \mathbb{Z}$, there exists a value y = 4 which is 2*2 and $\in \mathbb{Z}$.
Hence, this is provable.

Task 3.3

It is programmed and provided in the file “pred.log” along with this file on Blackboard.

4. One more wrap-up question

Task 4.1

I spent one hour on each section. In total, I spent 3 hours on this assignment.