a) without padding

$$I = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$k = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Applying k on I,

$$O = \begin{bmatrix} ((1\times1)+(1\times0)+(1\times1)+(0\times0)+(1\times1)+(1\times0)+(0\times1)+(0\times0)+(1\times1)) & ((1\times1)+(1\times0)+(0\times1)+(1\times0)+(1\times1)+(1\times0)+(0\times1)+(1\times0)+(1\times1)) & ((1\times1)+(0\times0)+(0\times1)+(1\times0)+(1\times1)+(0\times0)+(1\times1)+(1\times0)+(1\times1)) \\ ((0\times1)+(1\times0)+(1\times1)+(0\times0)+(0\times1)+(1\times0)+(0\times1)+(0\times0)+(1\times1)) & ((1\times1)+(1\times0)+(1\times1)+(0\times0)+(1\times1)+(1\times0)+(0\times1)+(1\times0)+(1\times1)) & ((1\times1)+(1\times0)+(0\times1)+(1\times0)+(1\times1)+(1\times0)+(1\times1)+(1\times0)+(0\times1)) \\ ((0\times1)+(0\times0)+(1\times1)+(0\times0)+(0\times1)+(1\times0)+(0\times1)+(1\times0)+(1\times1)) & ((0\times1)+(1\times0)+(1\times1)+(0\times0)+(1\times1)+(0\times1)+(1\times1)+(1\times0)+(0\times1)) & ((1\times1)+(1\times0)+(1\times1)+(1\times0)+(1\times1)+(0\times1)+(1\times1)+(0\times0)+(0\times1)) \end{bmatrix}$$

$$O = \begin{bmatrix} 4 & 3 & 4 \\ 2 & 4 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

b) with padding.

Since after applying the convolution filter k on image I, the output image size decreases.

By padding zeroes, we try to keep the output image size consistent with the input image size.

So, image $I = 5 \times 5$

i.e., $m = 5$ & $n = 5$

filter $k = 3 \times 3$

i.e., $a = 3$, $b = 3$

output image size desired $= 5 \times 5$

i.e., $p = 5$, $q = 5$

padding for height:

$= (m-a+1)$

$= (5-3+1)$

$= 3$

Since $(m-a+1) = 3 < p = 5$

we have to pad by $\dfrac{|(m-a+1-p)|}{2} = \dfrac{2}{2} = 1$

similarly, padding for width $= 1$

$$I_p = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying filter k on I_p,

$$O_p = \begin{bmatrix} 2 & 2 & 3 & 1 & 1 \\ 1 & 4 & 3 & 4 & 1 \\ 1 & 2 & 4 & 3 & 3 \\ 1 & 2 & 3 & 4 & 1 \\ 0 & 2 & 2 & 1 & 1 \end{bmatrix}$$

Q2

$$I = \begin{bmatrix} 3 & 4 & 2 & 5 \\ 1 & 9 & 6 & 3 \\ 8 & 3 & 5 & 3 \\ 2 & 6 & 5 & 0 \end{bmatrix}$$

Applying maxpooling filter of 2×2 & stride $= 2$,

$$O = \begin{bmatrix} \max(3, 4, 1, 9) & \max(2, 5, 6, 3) \\ \max(8, 3, 2, 6) & \max(5, 3, 5, 0) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 6 \\ 8 & 5 \end{bmatrix}$$

Q3

we have R, B, & G channel

Applying convolution filter K on each channel without padding, stride = 1,

$$K = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$O_R = \begin{bmatrix} ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) & ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) & ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) \\ ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) & ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) & ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) \\ ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) & ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) & ((1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)+(1\times1)) \end{bmatrix}$$

$$O_R = \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$$

Similarly

O_B will be addition of all 5's in each cell of the stride

$$\therefore O_B = \begin{bmatrix} 45 & 45 & 45 \\ 45 & 45 & 45 \\ 45 & 45 & 45 \end{bmatrix}$$

$$O_G = \begin{bmatrix} ((2\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)) & ((1\times1)+(0\times1)+(1\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)+(3\times1)) & ((0\times1)+(1\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)+(3\times1)+(1\times1)) \\ ((2\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)) & ((1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)+(3\times1)+(1\times1)+(0\times1)+(4\times1)) & ((0\times1)+(2\times1)+(1\times1)+(0\times1)+(3\times1)+(1\times1)+(0\times1)+(4\times1)+(1\times1)) \\ ((2\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)+(2\times1)+(1\times1)+(0\times1)) & ((1\times1)+(0\times1)+(3\times1)+(1\times1)+(0\times1)+(4\times1)+(1\times1)+(0\times1)+(5\times1)) & ((0\times1)+(3\times1)+(1\times1)+(0\times1)+(4\times1)+(1\times1)+(0\times1)+(5\times1)+(1\times1)) \end{bmatrix}$$

$$O_G = \begin{bmatrix} 9 & 9 & 9 \\ 9 & 12 & 12 \\ 9 & 15 & 15 \end{bmatrix}$$