Maths 3C

2D Rigid Analysis in C++

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Stages

INPUT NODES, MEMBERS, FORCES, SUPPORTS

↓

CALCULATE THE DEGREES OF FREEDOM

↓

FORMULATE THE GLOBAL STIFFNESS MATRIX
AND MATRIX INVERSION

↓

WORK OUT THE DEFLECTIONS AND ROTATIONS

↓

IMAGE PRODUCED ON AUTOCAD SHOWING THE
PREDICTED DEFLECTION

Data Input

<u>NODES</u>

The first line is the number of nodes minus 1 (first node is 0)

The second column represents the x co-ordinate

```
n
0  0.1  0.2
1  1.2  1.4
```

The first column represents the node ID

The third column is the y co-ordinate

Data Input (continued)

<u>MEMBERS</u>

n ⟵ Last member ID

Current ID member ⟶ 0 0 1 1 1 1

End 0

End 1

E, A and I values respectively

Data Input (continued)

LOADS

Direction in the y axis

Node number

1 1 -1

-1

Magnitude of the force

End of loads input

SUPPORTS

Direction

Node number

0 0
0 1
0 2
-1

0= fixed in x direction
1= fixed in y direction
2= rotational fixity

End of support input

Degrees of Freedom

$$(a \times b) + c = d$$

where:

a = Member ID

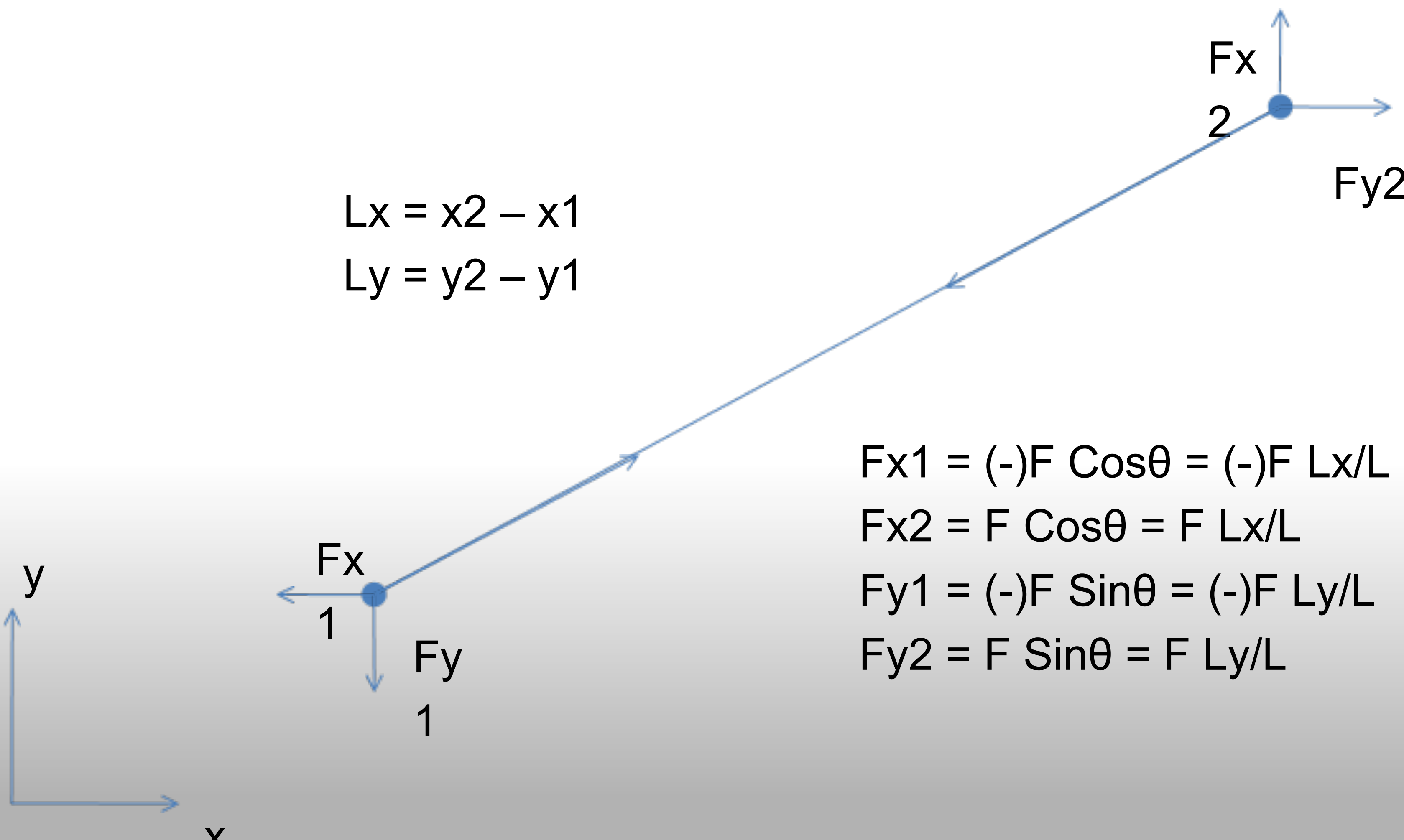
b = Degrees of freedom at one node

c = +0 for x-axis, +1 for y-axis, +2 for rotational

d = Degrees of freedom of the system

Member	0	1	2	3
X	0 x 3 = 0	1 x 3 = 3	2 x 3 = 6	3 x 3 = 9
Y	(0 x 3) + 1 = 1	(1 x 3) + 1 = 4	(2 x 3) + 1 = 7	(3 x 3) + 1 = 10
ϴ	(0 x 3) + 2 = 2	(1 x 3) + 2 = 5	(2 x 3) + 2 = 8	(3 x 3) + 3 = 11

Forces



$Lx = x2 – x1$

$Ly = y2 – y1$

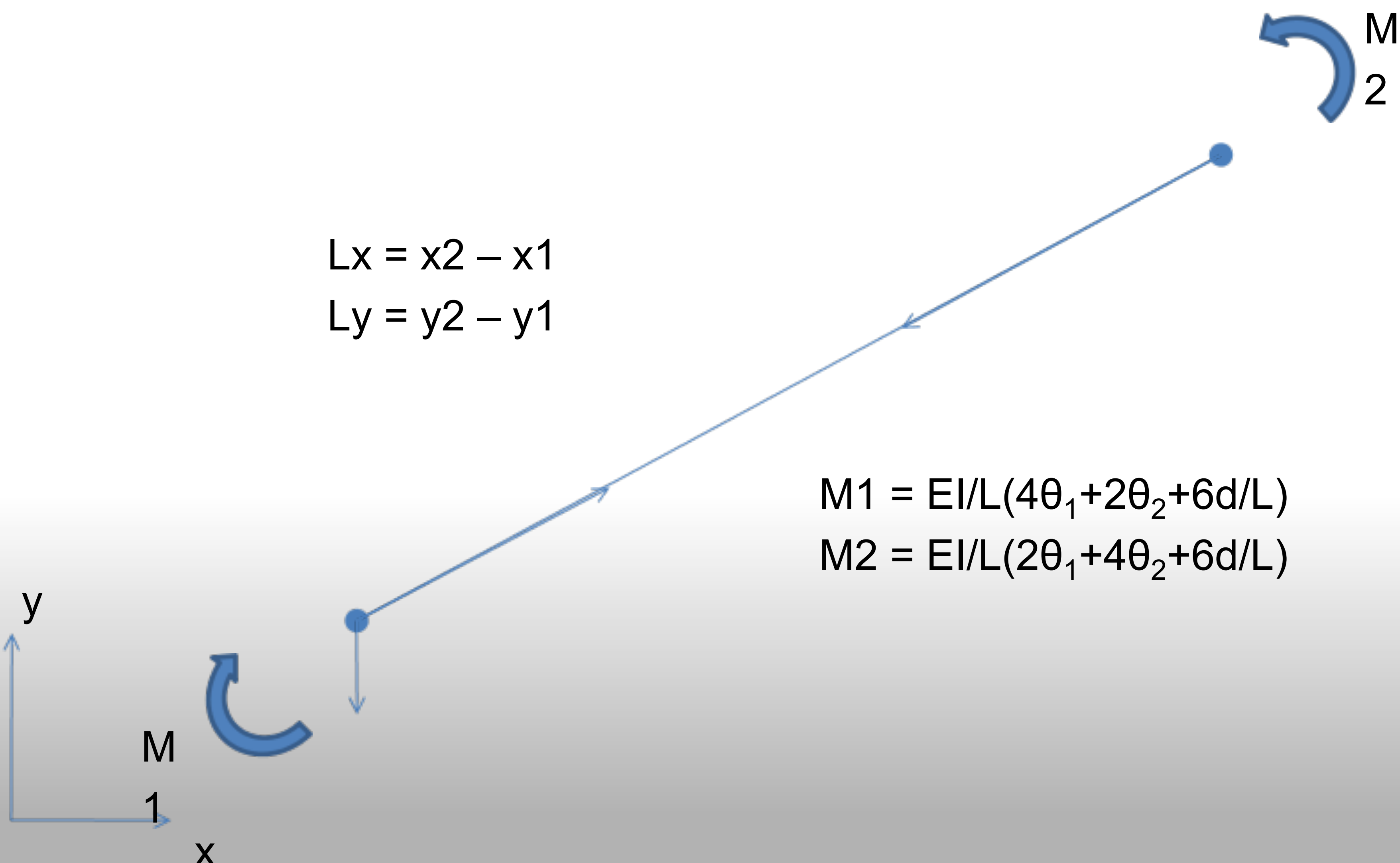
$Fx1 = (-)F \cos\theta = (-)F\ Lx/L$

$Fx2 = F \cos\theta = F\ Lx/L$

$Fy1 = (-)F \sin\theta = (-)F\ Ly/L$

$Fy2 = F \sin\theta = F\ Ly/L$

Moments



$Lx = x2 - x1$

$Ly = y2 - y1$

$M1 = EI/L(4\theta_1+2\theta_2+6d/L)$

$M2 = EI/L(2\theta_1+4\theta_2+6d/L)$

Account for supports

$$
\begin{bmatrix}
k_{x_1x_1} & k_{x_1y_1} & k_{x_1\theta_1} & \cdots & k_{x_1x_n} & k_{x_1y_n} & k_{x_1\theta_n} \\
k_{y_1x_1} & k_{y_1y_1} & k_{y_1\theta_1} & \cdots & k_{y_1x_n} & k_{y_1y_n} & k_{y_1\theta_n} \\
k_{\theta_1x_1} & k_{\theta_1y_1} & k_{\theta_1\theta_1} & \cdots & k_{\theta_1x_n} & k_{\theta_1y_n} & k_{\theta_1\theta_n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
k_{x_ny_1} & k_{x_ny_1} & k_{x_n\theta_1} & \cdots & k_{x_nx_n} & k_{x_ny_n} & k_{x_n\theta_n} \\
k_{y_ny_1} & k_{y_ny_1} & k_{y_n\theta_1} & \cdots & k_{y_nx_n} & k_{y_ny_n} & k_{y_n\theta_n} \\
k_{\theta_ny_1} & k_{\theta_ny_1} & k_{\theta_n\theta_1} & \cdots & k_{\theta_nx_n} & k_{\theta_ny_n} & k_{\theta_n\theta_n}
\end{bmatrix}
$$

If node 1 is fixed in the y axis, the following would be introduced...

Account for supports (continued)

$$
\begin{bmatrix}
k_{x_1x_1} & k_{x_1y_1}\;0 & k_{x_1\theta_1} & \cdots & k_{x_1x_n} & k_{x_1y_n} & k_{x_1\theta_n} \\
k_{y_1x_1}\;0 & k_{y_1y_1}\;1 & k_{y_1\theta_1}\;0 & \cdots & k_{y_1x_n}\;0 & k_{y_1y_n}\;0 & k_{y_1\theta_n}\;0 \\
k_{\theta_1x_1} & k_{\theta_1y_1}\;0 & k_{\theta_1\theta_1} & \cdots & k_{\theta_1x_n} & k_{\theta_1y_n} & k_{\theta_1\theta_n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
k_{x_ny_1} & k_{x_ny_1}\;0 & k_{x_n\theta_1} & \cdots & k_{x_nx_n} & k_{x_ny_n} & k_{x_n\theta_n} \\
k_{y_ny_1} & k_{y_ny_1}\;0 & k_{y_n\theta_1} & \cdots & k_{y_nx_n} & k_{y_ny_n} & k_{y_n\theta_n} \\
k_{\theta_ny_1} & k_{\theta_ny_1}\;0 & k_{\theta_n\theta_1} & \cdots & k_{\theta_nx_n} & k_{\theta_ny_n} & k_{\theta_n\theta_n}
\end{bmatrix}
$$

Global Stiffness Matrix

$$\mathbf{K} = \left(\frac{EA}{L} - \frac{12EI}{L^3}\right)\begin{bmatrix} c^2 & cs & 0 & -c^2 & -cs & 0 \\ cs & s^2 & 0 & -cs & -s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -c^2 & -cs & 0 & c^2 & cs & 0 \\ -cs & -s^2 & 0 & cs & s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{12EI}{L^3}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+\frac{6EI}{L^2}\begin{bmatrix} 0 & 0 & -s & 0 & 0 & s \\ 0 & 0 & c & 0 & 0 & -c \\ -s & c & 0 & s & -c & 0 \\ 0 & 0 & s & 0 & 0 & -s \\ 0 & 0 & -c & 0 & 0 & c \\ s & -c & 0 & -s & c & 0 \end{bmatrix} + \frac{2EI}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}$$

EA/L - 12EI/L^3

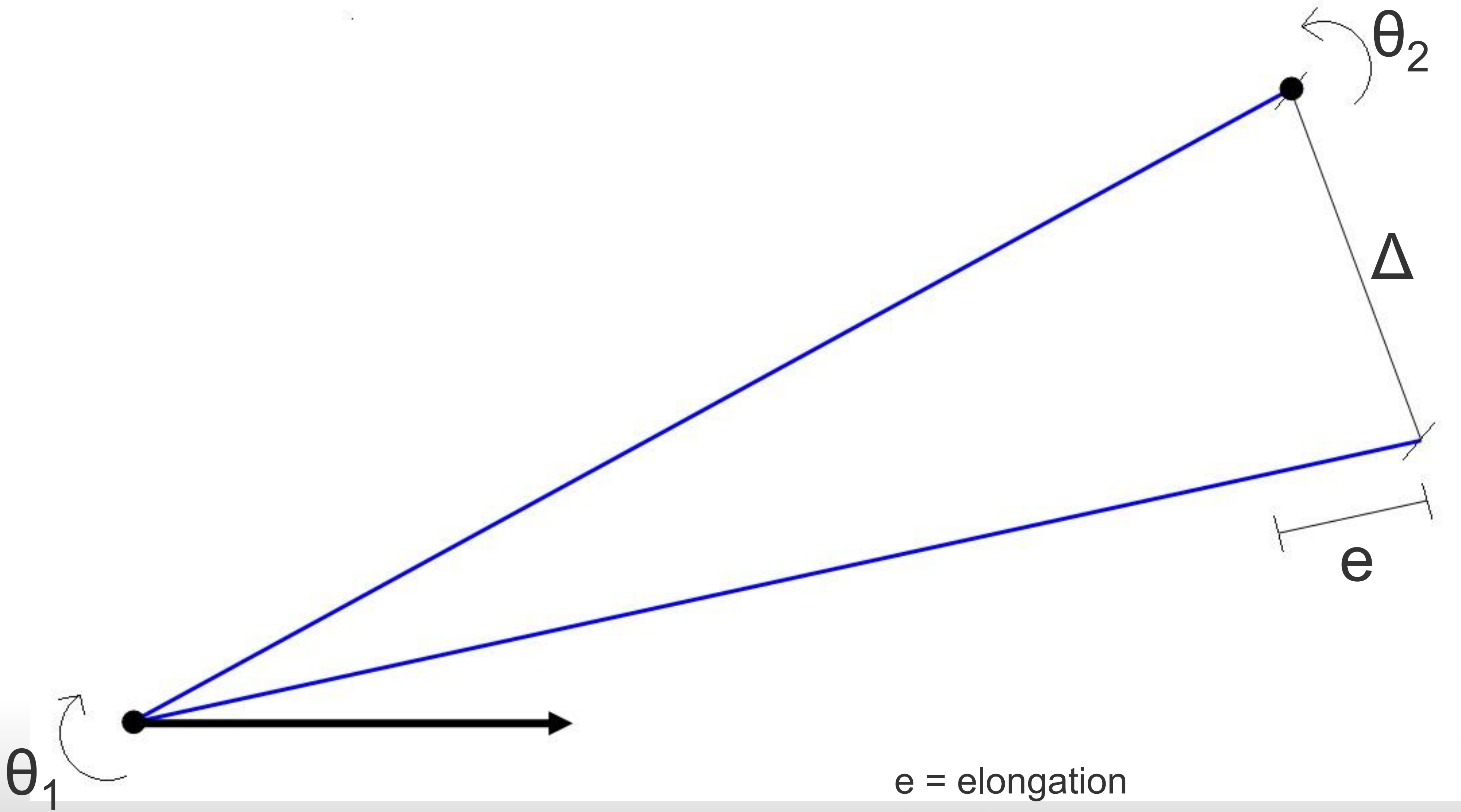
```
Loop ThisAxis from 0 to 1 {
    Loop ThatAxis from 0 to 1 {
        MemberStiffness = EA/L - 12EI/L^3 for ThisAxis and ThatAxis
            Loop ThisQuadrant from 0 to 1 {

                Loop ThatQuadrant from 0 to 1 {
                     Determine This and That position in the global stiffness
matrix
                     If leading diagonal, add to the global stiffness matrix
                    Else, subtract from the global stiffness matrix
                 }

            }

    }
}
```

Deflection, rotation, forces and moments



e = elongation
Δ = displacement
θ_1, θ_2 = rotation at each end

Deflection, rotation, forces and moments

$$\begin{bmatrix} T \\ F \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{2EI}{L} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} e \\ \Delta \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

T = tension
F = shear force
M_1, M_2 = moments at each end

$T = e(EA/L)$
$F = \Delta(12EI/L^3) + \theta_1(6EI/L^2) + \theta_2(6EI/L^2)$
$M_1 = \Delta(6EI/L^2) + \theta_1(4EI/L) + \theta_2(2EI/L)$
$M_2 = \Delta(6EI/L^2) + \theta_1(2EI/L) + \theta_2(4EI/L)$

Output

Stiffness.txt Log of how the stiffnesses are compiles

Data.txt Forces, moment and deflections and rotations for each member

Drawing.dwg This is the file that contains the AutoCAD drawing

Program Demonstration!

Any questions??

Thanks!

Hope that made sense =]