Evolute II

$$x = a\left(\cos\theta + \theta\sin\theta\right)$$
$$y = a\left(\sin\theta - \theta\cos\theta\right)$$
$$x' = \frac{dx}{d\theta} = a\left(-\sin\theta + \sin\theta + \theta\cos\theta\right) = a\left(\theta\cos\theta\right)$$
$$x'' = \frac{d^2x}{d\theta^2} = a\left(\cos\theta - \theta\sin\theta\right)$$
$$y' = \frac{dy}{d\theta} = a\left(\cos\theta - \cos\theta + \theta\sin\theta\right) = a\left(\theta\sin\theta\right)$$
$$y'' = \frac{d^2y}{d\theta^2} = a\left(\sin\theta + \theta\cos\theta\right)$$

The curvature,

$$\kappa = \frac{\frac{d^2y}{d\theta^2}\frac{dx}{d\theta} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}}{\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]^{\frac{3}{2}}}$$
$$= \frac{a\left(\sin\theta + \theta\cos\theta\right)a\left(\theta\cos\theta\right) - a\left(\theta\sin\theta\right)a\left(\cos\theta - \theta\sin\theta\right)}{\left[a^2\left(\theta\cos\theta\right)^2 + a^2\left(\theta\sin\theta\right)^2\right]^{\frac{3}{2}}}$$
$$= \frac{a^2\theta^2}{a^3\theta^3} = \frac{1}{a\theta}$$

and therefore the radius of curvature is $a\theta$.