Hyperbola

Consider the line

$$xe^{-\theta} + ye^{\theta} = a$$

in which a and θ are constants.

The line passes through $x = 0,\ y = ae^{-\theta}$ and $x = ae^{\theta},\ y = 0.$

Consider an adjoining line, in which θ is replaced by $\theta + d\theta$:

$$xe^{-(\theta+d\theta)} + ye^{(\theta+d\theta)} = a$$
$$xe^{-\theta}(1 - d\theta) + ye^{\theta}(1 + d\theta) = a$$

The two lines intersect where

$xe^{-\theta} + ye^{\theta} = a$ and $xe^{-\theta}(1 - d\theta) + ye^{\theta}(1 + d\theta) = a$ so that

$$(-xe^{-\theta} + ye^{\theta})d\theta = 0$$
$$y = xe^{-2\theta}$$

Hence

$$x = ae^{\theta} - ye^{2\theta} = ae^{\theta} - x$$
$$x = \frac{ae^{\theta}}{2}$$
$$y = \frac{ae^{-\theta}}{2}$$
$$xy = \frac{a^2}{4}$$