Curvature of plane curve

$$x = x(\theta)$$

$$y = y(\theta)$$

$$\text{slope of curve} = \tan\lambda = \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

$$s = \text{distance along curve}$$

$$\left(\frac{ds}{d\theta}\right)^2 = \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$$

$$\sec^2\lambda\frac{d\lambda}{d\theta} = \frac{\frac{d^2y}{d\theta^2}\frac{dx}{d\theta} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}}{\left(\frac{dx}{d\theta}\right)^2}$$

$$\frac{d\lambda}{d\theta} = \frac{\frac{d^2y}{d\theta^2}\frac{dx}{d\theta} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}}{\left(1+\tan^2\lambda\right)\left(\frac{dx}{d\theta}\right)^2} = \frac{\frac{d^2y}{d\theta^2}\frac{dx}{d\theta} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}}{\left(1+\frac{\left(\frac{dy}{d\theta}\right)^2}{\left(\frac{dx}{d\theta}\right)^2}\right)\left(\frac{dx}{d\theta}\right)^2} = \frac{\frac{d^2y}{d\theta^2}\frac{dx}{d\theta} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}}{\left(\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right)}$$

$$\kappa = \text{Curvature} = \frac{d\lambda}{ds} = \frac{\frac{d^2y}{d\theta^2}\frac{dx}{d\theta} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}}{\left(\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right)^{\frac{3}{2}}}$$

$$\text{Radius of curvature} = \frac{1}{\kappa}$$