Earthquake 1 degree of freedom model

$$x = \text{ground movement}$$
$$\theta = \text{column rotation (including effect of column bending)}$$
$$h = \text{height of column}$$
$$m = \text{mass of roof}$$
$$mh^2 = \text{moment of inertia}$$
$$M = \text{total bending moment at tops of columns}$$
$$k = \text{rotational stiffness}$$

If $-M_{\text{yield}} < M < M_{\text{yield}}$

$$M = k\left(\theta - \theta_{\text{plastic}}\right)$$

otherwise

$$M = M_{\text{yield}} \text{ or } M = -M_{\text{yield}}$$

$$\theta_{\text{plastic}} = \theta - \frac{M}{k}$$

Horizontal acceleration of mass is equal to

$$\frac{d^2}{dt^2}\left(x + h\sin\theta\right) = \frac{d}{dt}\left(\dot{x} + h\cos\theta\dot{\theta}\right) = \ddot{x} + h\cos\theta\ddot{\theta} - h\sin\theta\dot{\theta}^2$$

Vertical acceleration of mass (including gravity) is equal to

$$\frac{d^2}{dt^2}\left(h\cos\theta\right) + g = \frac{d}{dt}\left(-h\sin\theta\dot{\theta}\right) = -h\sin\theta\ddot{\theta} - h\cos\theta\dot{\theta}^2 + g$$

Therefore

$$M = -m\left(\ddot{x} + h\cos\theta\ddot{\theta} - h\sin\theta\dot{\theta}^2\right)h\cos\theta + m\left(-h\sin\theta\ddot{\theta} - h\cos\theta\dot{\theta}^2 + g\right)h\sin\theta$$
$$= -m\ddot{x}h\cos\theta - mh^2\ddot{\theta} + mgh\sin\theta$$

so that

$$\ddot{\theta} = \frac{1}{mh^2}\left(-mh\ddot{x}\cos\theta + mgh\sin\theta - M\right)$$