牛頓推導

泰勒展開 $$f(x) \approx f(x_k) + \underbrace{\nabla f(x_k)^T}_{f'(x_k)^T}(x-x_k) + \frac{1}{2}(x-x_k)^T \underset{\text{Hessian Metrix}}{\underset{\uparrow}{Hf}}(x_k)(x-x_k)$$

$\downarrow$ 求最佳

$$f(x_k) + \nabla f(x_k)^T(x-x_k) + \frac{1}{2}(x-x_k)^T H(x_k)(x-x_k) = 0$$

$$H(x_k)(x-x_k) = -f'(x_k)$$

$$x - x_k = -H(x_k)^{-1} f'(x_k)$$

$$x_{k+1} = x_k - H(x_k)^{-1} \nabla f(x_k)$$

$$|Ax-b|^2 = (Ax-b)^T(Ax-b)$$

$$\nabla \quad '' \quad = 2A^T(Ax-b)$$

$$x_{k+1} = x_k - H(x_k)^{-1} \nabla f(x_k)$$

$$= x_k - (2A^TA)^{-1} \underbrace{\nabla f(x_k)}_{(2A^T(Ax_k-b))}$$

$$= \quad -(2A^TA)^{-1} \cdot 2(A^TAx_k - A^Tb)$$

$$\underline{x_1 = x_0 - (2A^TA)^{-1} \cdot 2A^T(Ax_k - b)} \quad \#$$