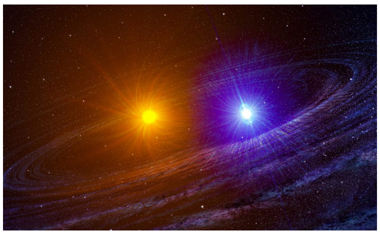
600035/661852: Numerical Modelling & Simulation - Assignment

Orbiting binary star system

INTRODUCTION



Binary star systems are extremely common in our Universe and a recent estimate suggest that 80% of stars might actually be a binary system. In this assignment, you will perform a simulation of a simplified 2-D version of a binary star system that only contains 2 stars: a sun-like G class star and a smaller K class star. The aim of the assignment is to determine the orbital period of the system in Earth years by simulating the time evolution of the binary system. In addition, you will be able to check how close your numerical calculation matches the analytical model of Kepler's equation.

REQUIREMENTS

- You will need to create a short **individual** report in the form of a styled Jupyter notebook (including sections and comments) and upload the report to Canvas by **2pm on the specified deadline**.
- Note that your notebook will be checked to see if it **runs correctly on the University jupyterhub server (jupyterhub.hull.ac.uk)**, so do double-check that using "restart and run all" provides you with the correct answer on that server.
- *Non-functioning notebooks will attract a fixed penalty of 35% regardless of the correctness of the presented solutions.*
- You MUST annotate/comment your code throughout to indicate an understanding of the approach used and facilitate readability.
- *Notebooks without annotations/comments will attract a further fixed penalty of 35% regardless of the correctness of the presented solutions.*

TASKS

Our binary star model is made of two stars, one G and one K class star. It can be shown, without loss of generality, that a two-body problem is always evolving in a two-dimensional orbital plane. The force that drives the motion of the two stars in this model is the gravitational force.

Therefore, the force of an object of mass M at location (x_M, y_M) acting on another object of mass m at location (x_m, y_m) is given by the following two formulae (one for each coordinate, x and y):

$$F_x = G\left(\frac{M \cdot m}{R^2}\right)\frac{r_x}{R}$$
$$F_y = G\left(\frac{M \cdot m}{R^2}\right)\frac{r_y}{R}$$

where $G = 6.67408 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the gravitational constant, R is the distance between each objects given as:

$$R = \sqrt{r_x^2 + r_y^2} \tag{1}$$

and $r_x = x_M - x_m$ and $r_y = y_M - y_m$, the *signed* relative positions between the two objects.

The acceleration resulting from this force acting on object m at location (x_m, y_m) is then given by:

$$a_x = \tfrac{dv_x}{dt} = G\left(\frac{M}{R^2}\right)\frac{r_x}{R} \tag{2}$$
$$a_y = \tfrac{dv_y}{dt} = G\left(\frac{M}{R^2}\right)\frac{r_y}{R} \tag{3}$$

These two equation enable us to determine the change in velocity of each object, subject to gravitational forces.

Starting from a set of initial positions and velocities for each object (given in table below), we can use equations 2 and 3 to compute the change of velocity. The change in position can then be obtained from the velocity information, so that:

$$x(t) \approx x_0 + v_x(t) \cdot \Delta t \tag{4}$$
$$y(t) \approx y_0 + v_y(t) \cdot \Delta t \tag{5}$$

Using the fact that $v = \frac{dx}{dt}$.

This is best performed using a differential equation solver, such as the one we used in the zombie invasion simulations (note that here you have technically 4 differential equations per object). The suggested length of the simulation should be 2 Earth years, with a time step of one day (less if you encounter issues with orbit stability).

Remember that all equations are in SI units (i.e. using meters, kilograms and seconds), but you might find it more convenient to use years and astronomical units for your graphs.

Starting conditions

Object	(x, y) [AU]	(v_x, v_y) [m/s]	Mass [kg]
Star 1 (G class)	$(0, 0)$	$(0, 0)$	1.989×10^{30}
Star 2 (K class)	$(1.00269, 0)$	$(0, 36434.5)$	$0.5 \times m_{\text{Star 1}}$

1 AU = $1.495978707 \times 10^{11}$ m

The steps below are there to guide you through your task:

1. Start by computing the forces on both stars and solve the equations of motion. Check if the two stars orbit each other in a stable fashion. One way of showing this it to plot the path of each star, either on an x, y diagram (i.e. orbit diagram) or as two separate position–time plots (one for each coordinate: x-position vs time and y-position vs time). Beware that using too large time steps is likely to cause stars to crash into each other.

   [20 marks]

2. Use the data from your model to determine the orbital period of the binary star system. This can be computed using, for example, a graph of the x-position of either star as a function of time. Please provide a numerical answer for the period in Earth years and a graphical illustration.

   [15 marks]

3. This type of motion can also be described using Kepler's equation below:

$$\frac{\bar{p}^2}{\bar{a}^3} = \frac{4\pi^2}{G(m_1 + m_2)} \tag{6}$$

   where $\bar{p}$ is the average orbital period and $\bar{a}$ is the average star separation over the orbit. Use the data from your simulation to compute the average period predicted by Kepler's equation. Discuss any agreement/disagreement with the orbital period you determined earlier.

   [15 marks]

MARKING CRITERIA

This assignment enables you to demonstrate the following skills: analysis of a physical problem, basic programming skills, ability to use a computer program to solve differential equations, graphing abilities.

During marking, particular attention will be paid to:

1. a clear presentation of your model and any assumptions you make
2. an appropriate choice of techniques
3. a solid evaluation of the outcome of the simulations

NOTE THAT THE REQUIRED FORMAT FOR THIS ASSIGNMENT IS A JUPYTER NOTEBOOK UNLESS AGREED OTHERWISE.