Bounded-Metric List Decoding

UCLA ECE MS Project Presentation

Bruce Qu, May.9, 2025

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- Bounded Angle List Decoding (BALD)
- Results
- Application

Introduction

Basics

- BPSK modulation (0->1, 1->-1) and a BI-AWGN Channel
- We use the ELF-TBCC code listed in [1]
- $K = 64$, $m = 12$. Puncturing is applied to maintain rate-1/2, i.e., $N = 128$

[1] R. D. Wesel, A. Antonini, L. Wang, W. Sui, B. Towell, and H. Grissett, “ELF codes: Concatenated codes with an expurgating linear function as the outer code,” 2023 12th International Symposium on Topics in Coding (ISTC), pp. 287–291, June 2023.

Term Notations

- Undetected Error Rate (UER) $P_u = P(\hat{x} \neq x)$
- Detected Error Rate (DER) $P_d = P_{erasure}$
- Total Error Rate (TER) $P_t = P_u + P_d$
- Correct Probability P_c

Introduction

List Decoding

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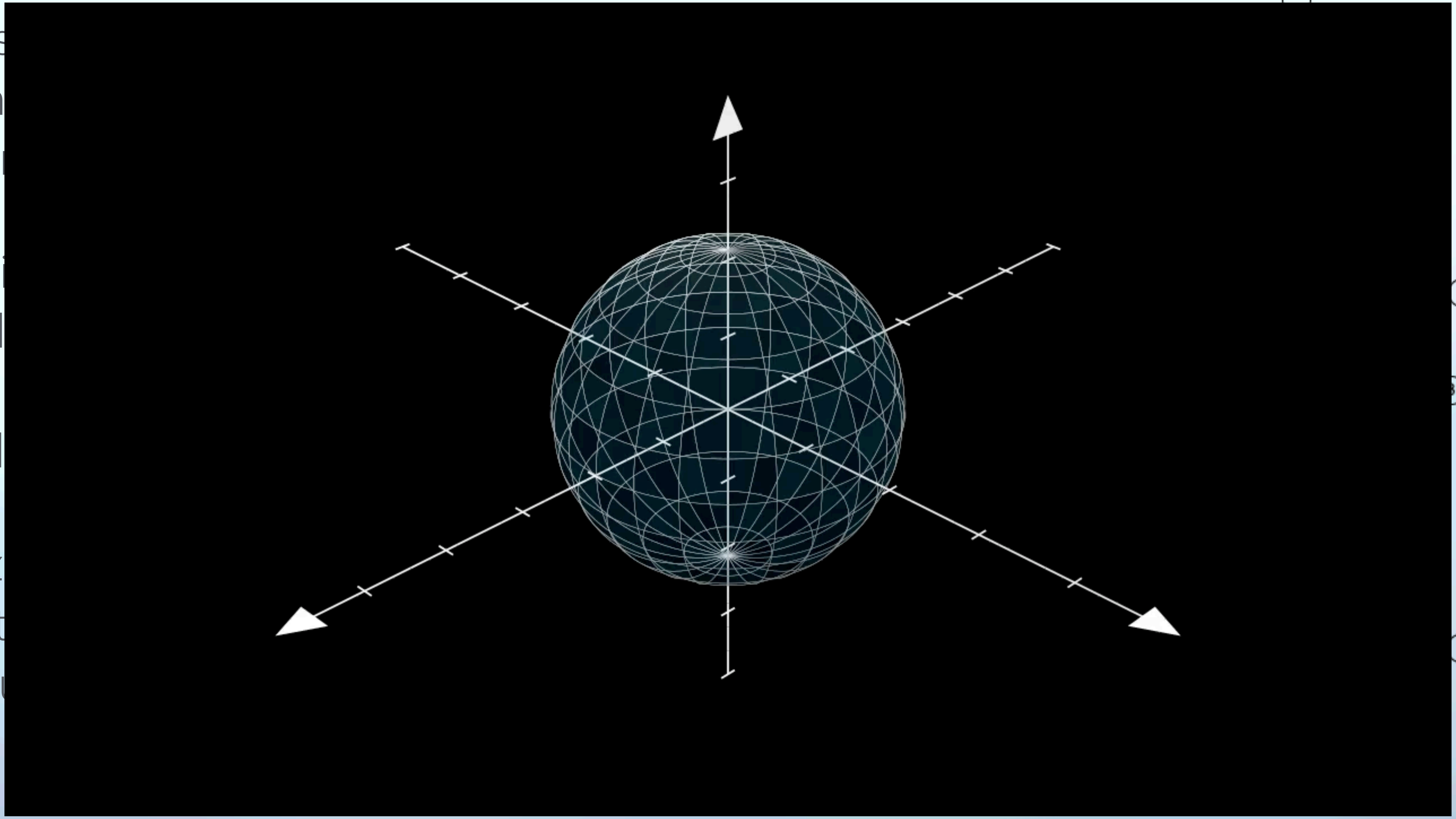
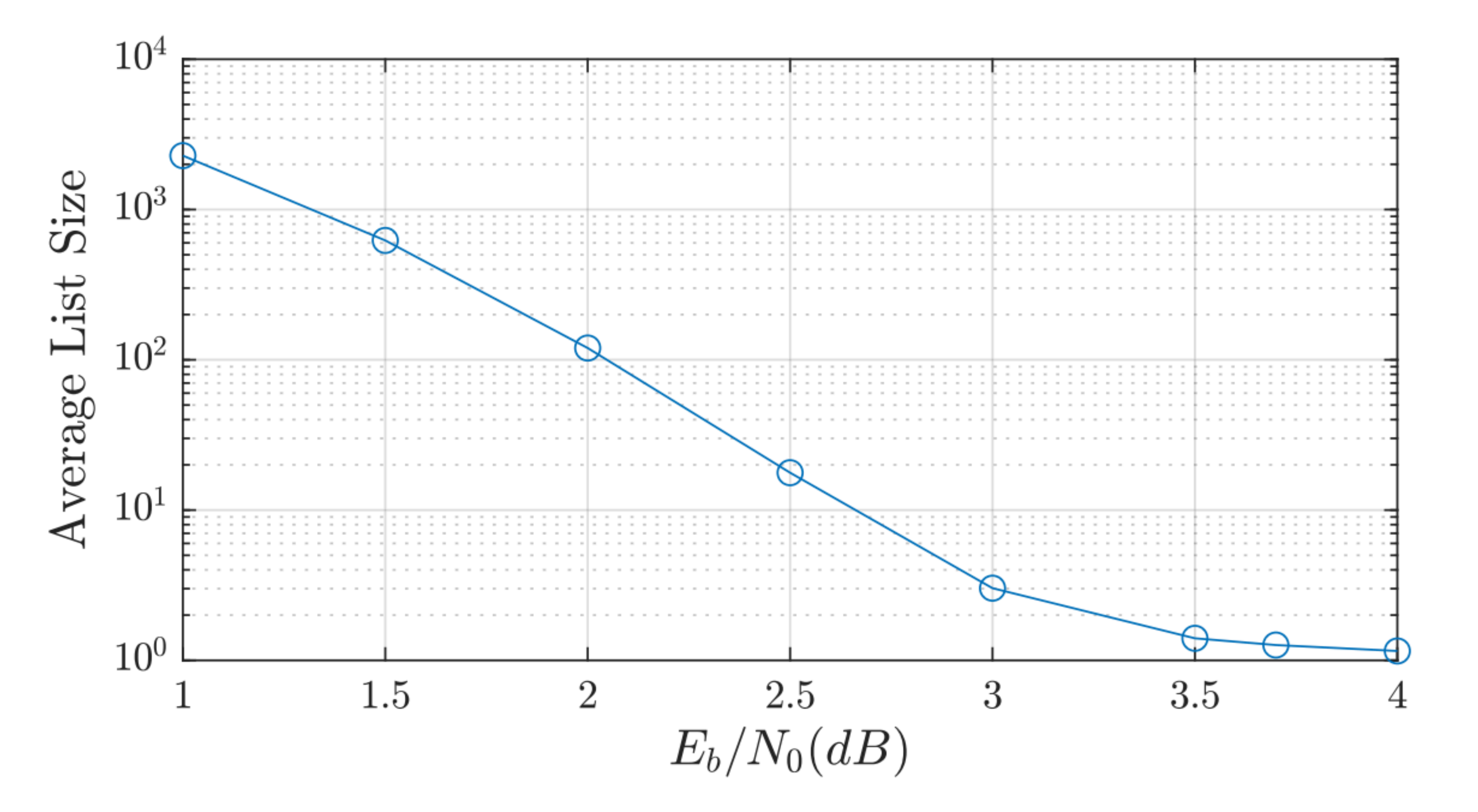


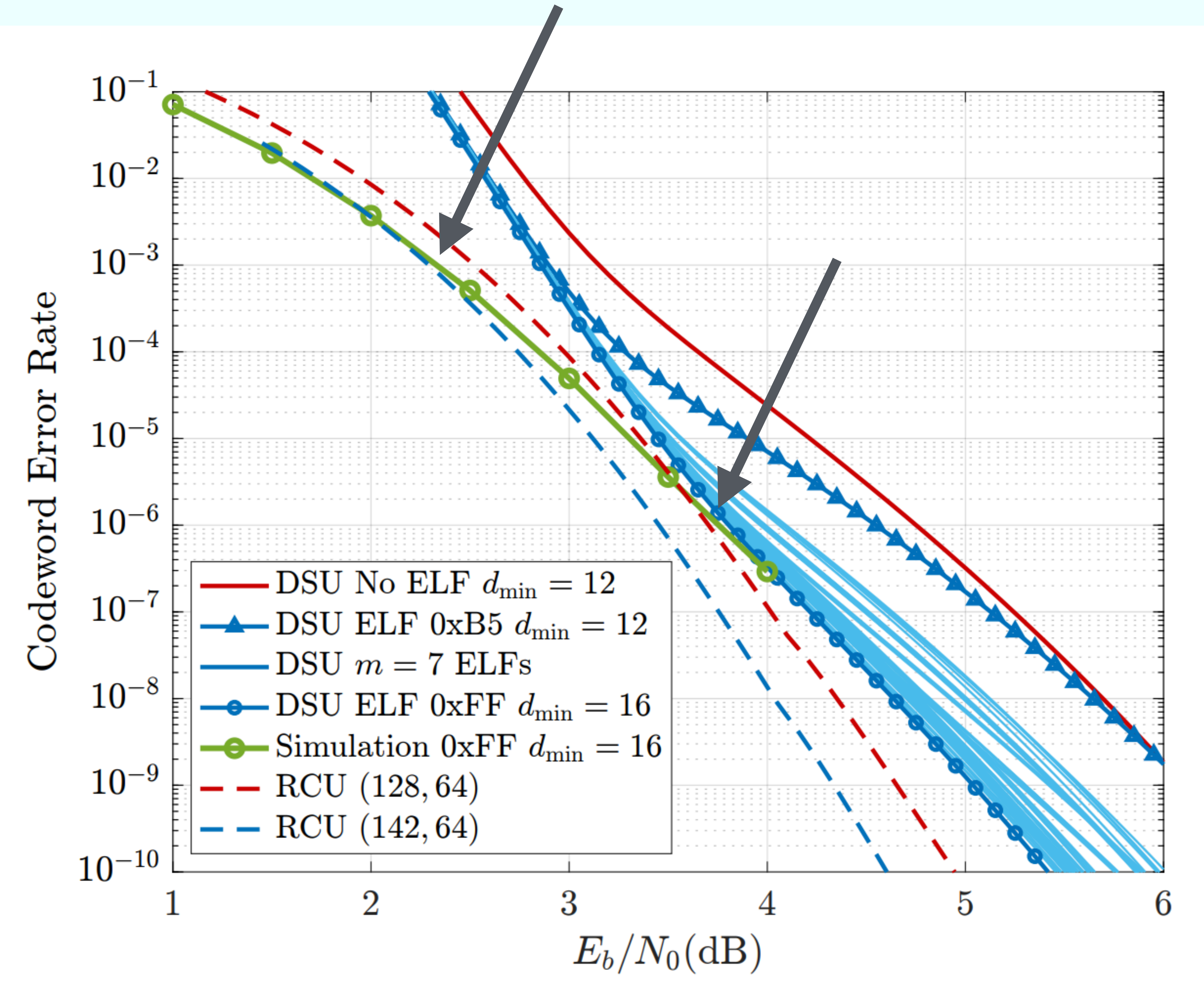
SLVD Performance

List size cap = 2^20

- Average List size is fine at desired SNR, but max list size (max decoding time) can become prohibitively large.

- Higher SNR -> lower complexity





Bounded Distance List Decoding

Motivation

- Problem1: Real systems impose strict **decoding time constraint** on a single decoding
- Problem2: Even if a codeword is found, how much do we **trust** that to be the right answer?
- Solution: What if we put a smaller cap on Max list size?
- Implication: Sacrifice performance, you start to get erasures
- Question: If so, how to find the value of this smaller cap on list size?
- Possible Answer: What about the distance between Transmitted codeword and received word?

Bounded Distance List Decoding

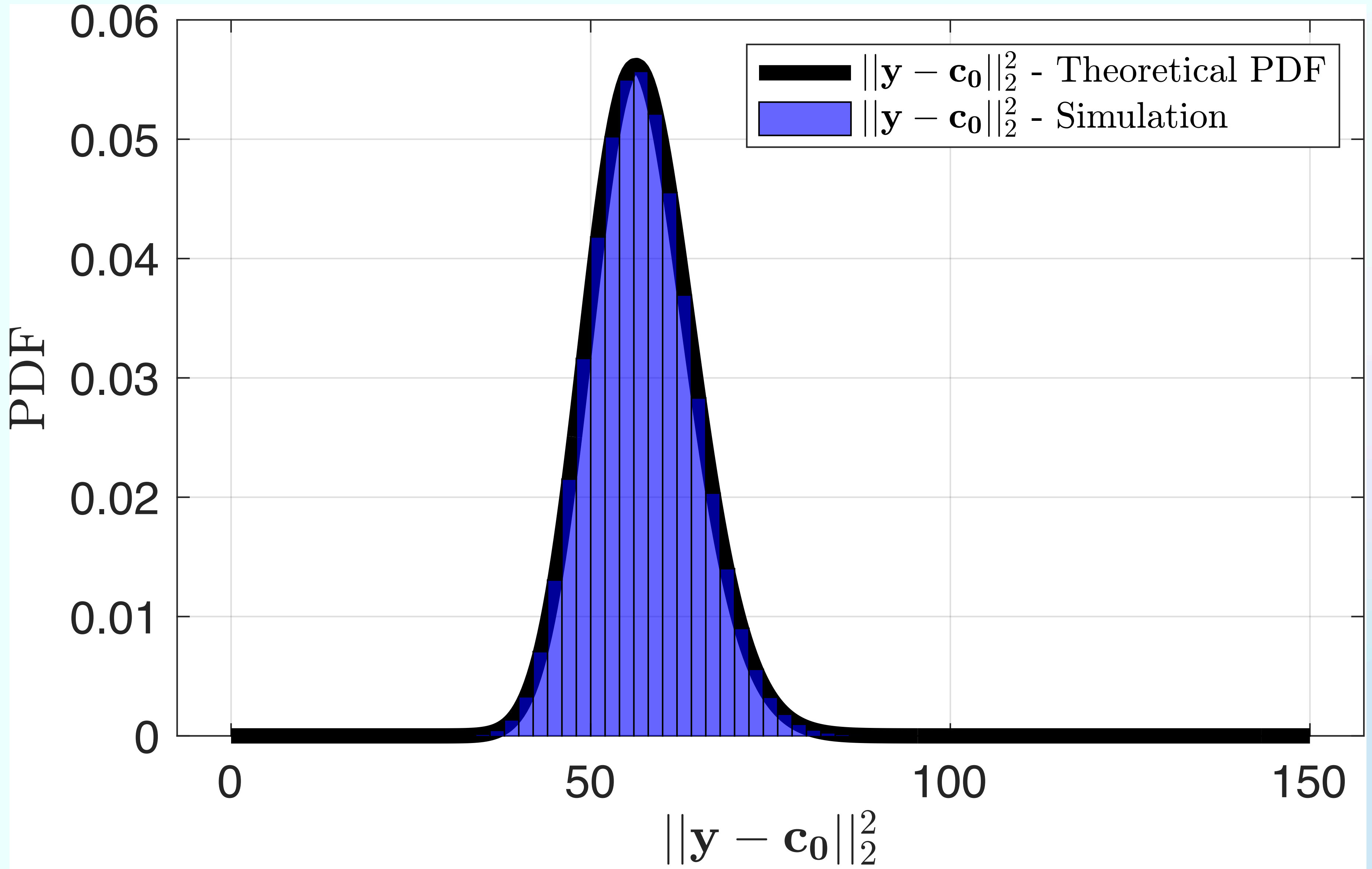
Squared Distance from the transmitted codeword $\mathbf{c_0}$

- $\mathbf{c_0} = [c_0, c_1, c_2, \ldots, c_{127}]$, $\mathbf{n} = [n_0, n_1, n_2, \ldots, n_{127}]$, and $\mathbf{y} = \mathbf{c_0} + \mathbf{n}$

- $n_i \sim \mathcal{N}(0, N_0/2) \implies \sum_{i=0}^{N-1} n_i^2 \sim \chi_N^2 \implies \sum_{i=0}^{N-1} n_i^2 \sim \Gamma(\alpha = N/2, \theta = N_0)$

- Simulation result confirms that the pdf of $||\mathbf{y} - \mathbf{c_0}||^2$ follows a Gamma distribution.

- This allows us to compute the probability that the squared Euclidean distance between noisy word $\mathbf{y}$ and transmitted codeword $\mathbf{c_0}$ is further than some distance d.



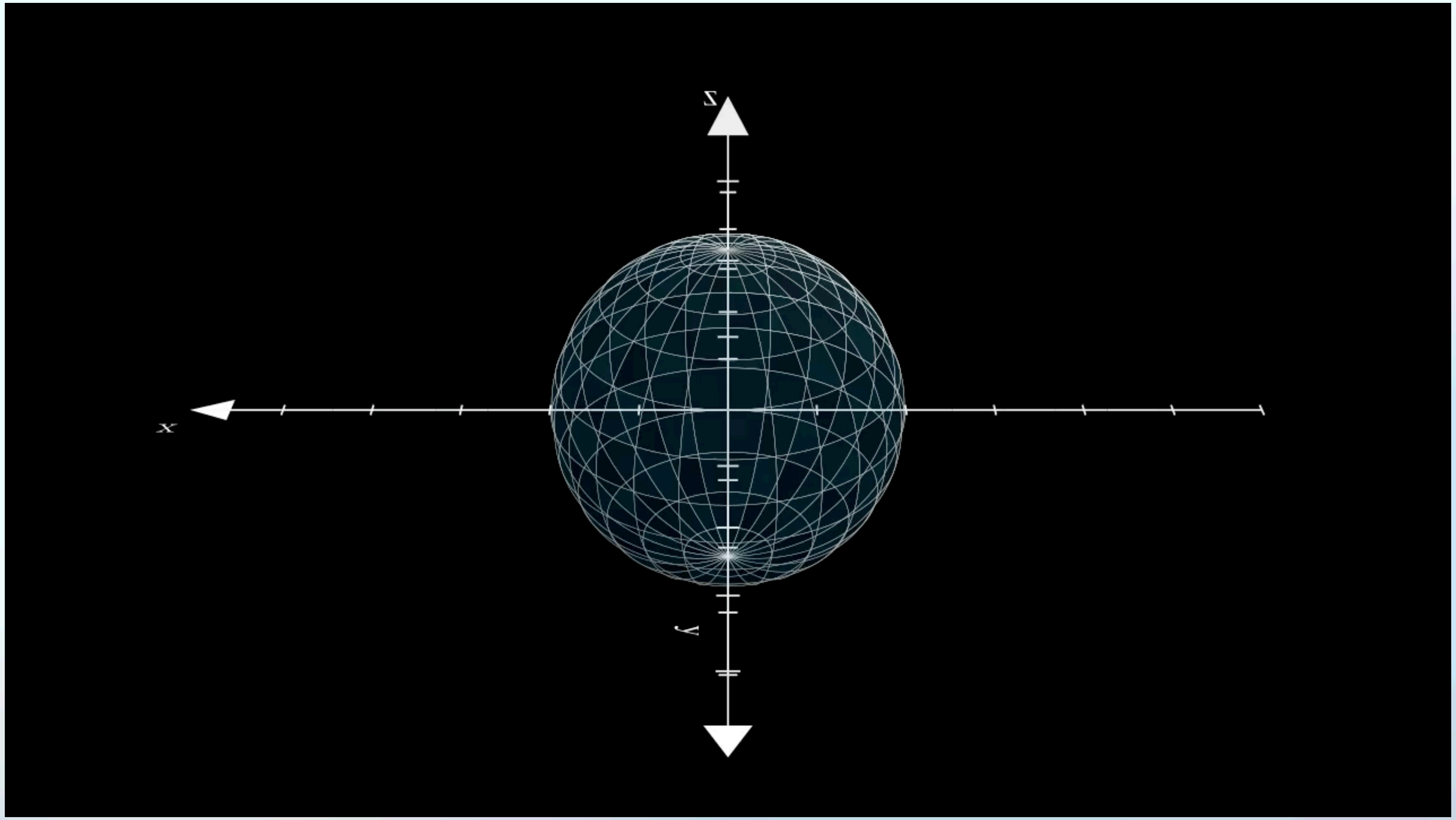
Bounded Distance List Decoding

Problems

- Throws away codewords that are pushed far away radially.

- Can we do better?

- Projecting the point back to the codeword sphere



Bounded Angle List Decoding

Motivation

- Bounded **Angle** List Decoding is the same as Bounded **Projected-distance** List Decoding.

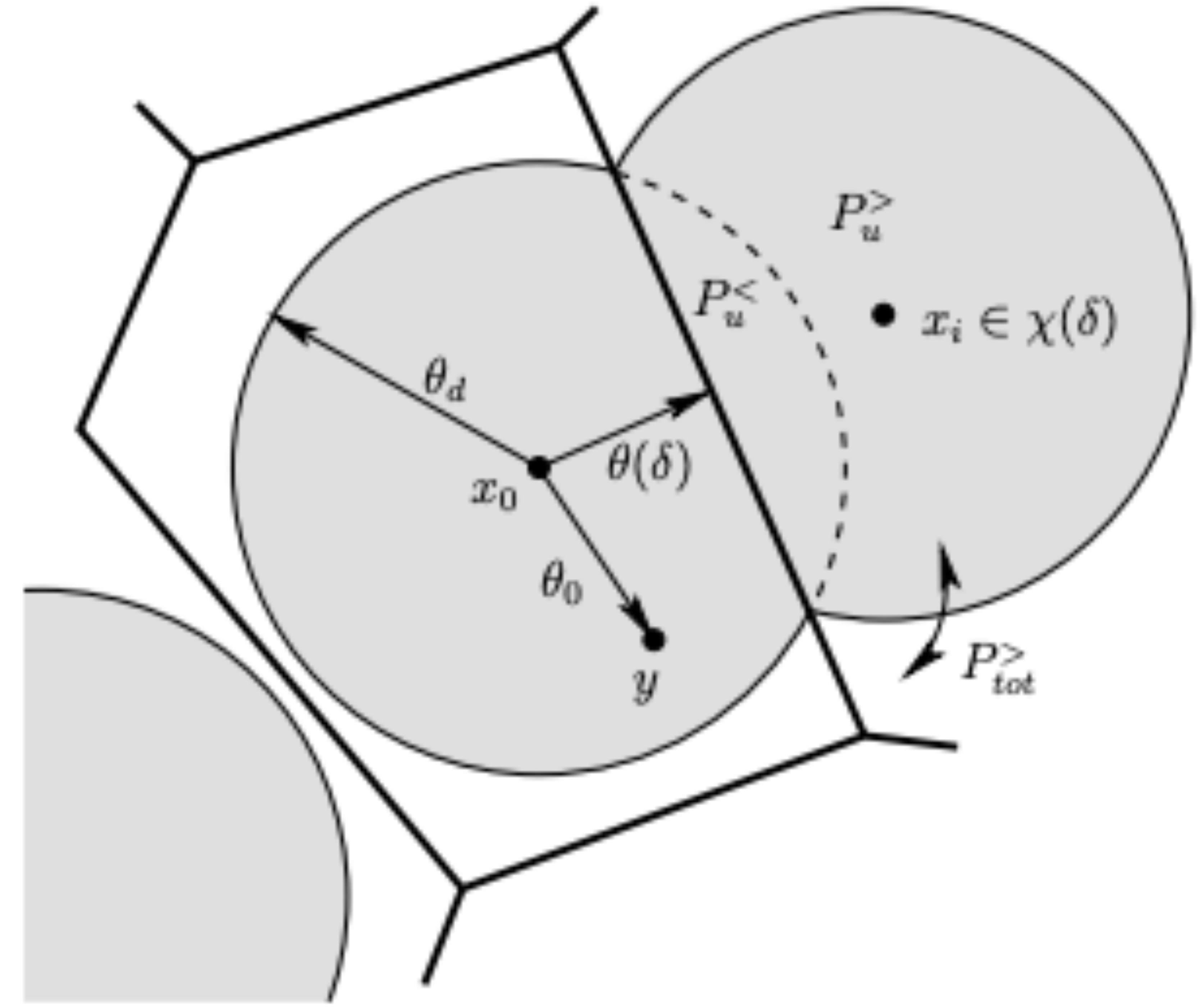


Figure 2: Illustration of the conical angle geometry underlying our analysis of BA-ML decoding.

Bounded Angle List Decoding

Terminology

- $P_{tot}^{>} = P_u^{>} + P_d$
- $P_{tot}^{<} = P_c + P_u^{<}$
- UER: $P_u = P_u^{>} + P_u^{<}$
- TER: $P_t = P_u^{<} + P_u^{>} + P_d$
- If we can compute $P_{tot}^{>}$, then I have an upper bound on P_d and a lower bound on P_t.

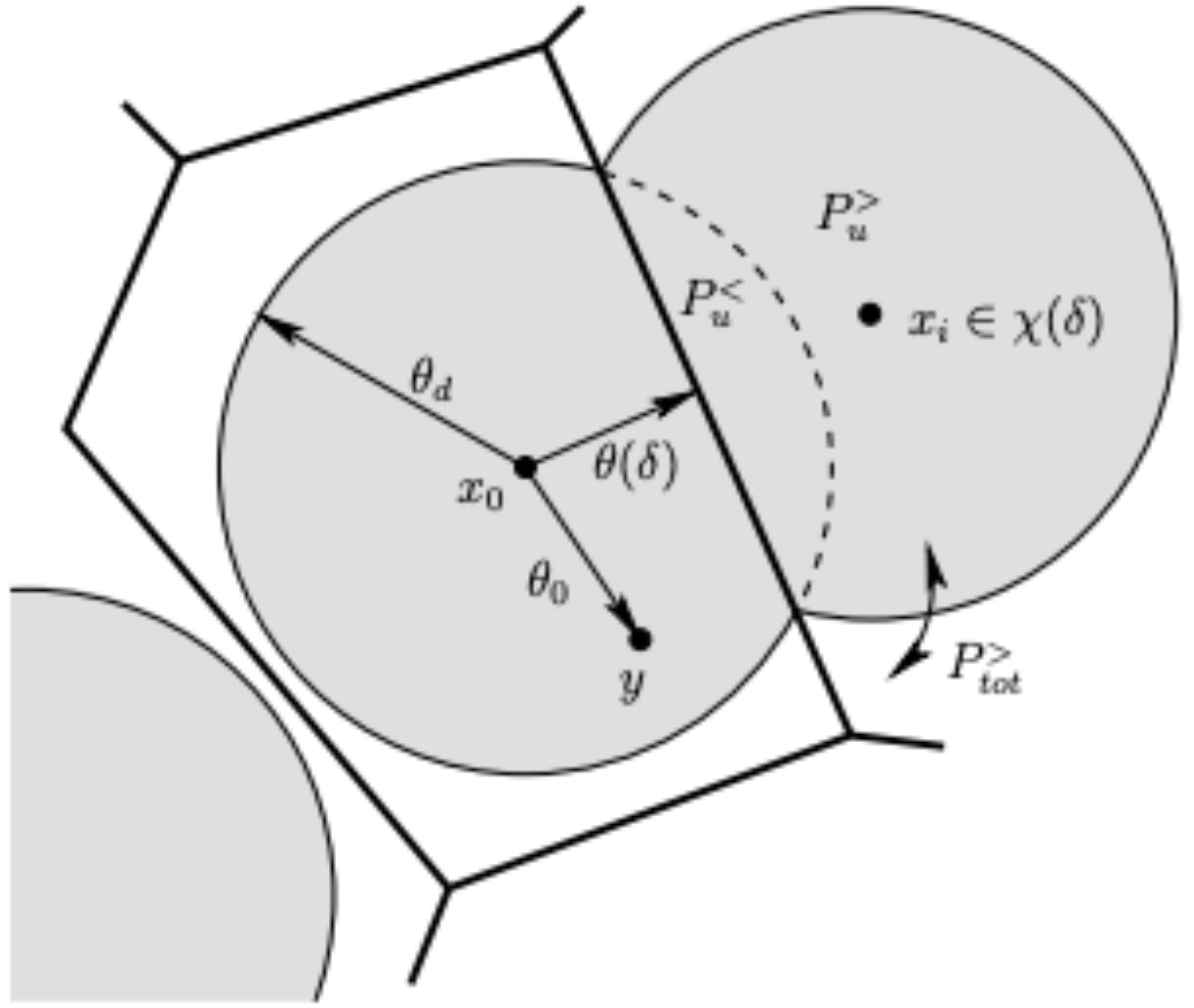


Figure 2: Illustration of the conical angle geometry underlying our analysis of BA-ML decoding.

[2] S. Dolinar, K. Andrews, F. Pollara, and D. Divsalar, “Bounds on error probability of block codes with bounded-angle maximum-likelihood incomplete decoding,” in 2008 International Symposium on Information Theory and Its Applications, December 2008, pp. 1–6.

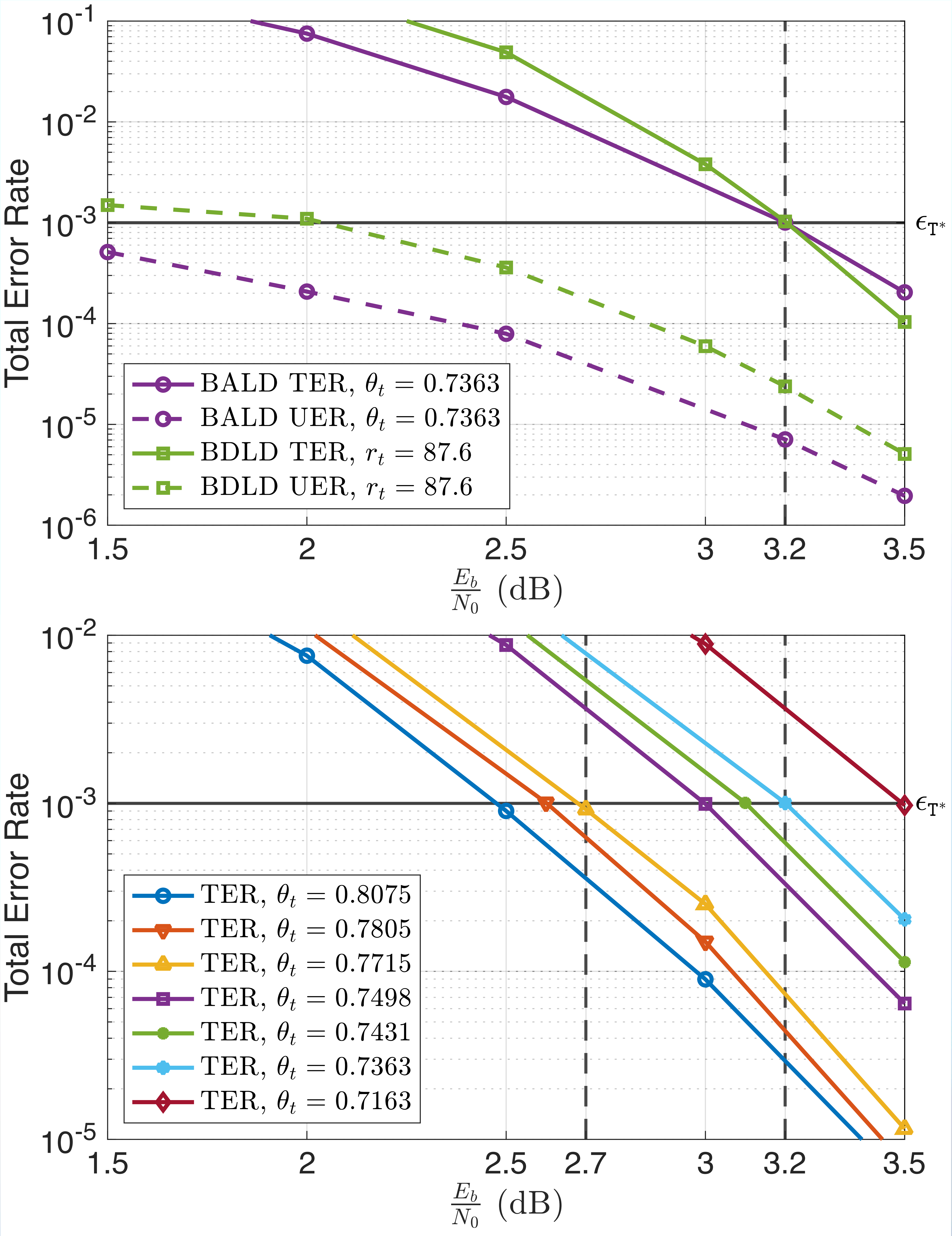
Results

BALD vs. BDLD

- Step1: Gamma distribution and Shannon's Sphere Packing Bound provides the threshold values for $P_{tot}^{>} = 10^{-3}$.

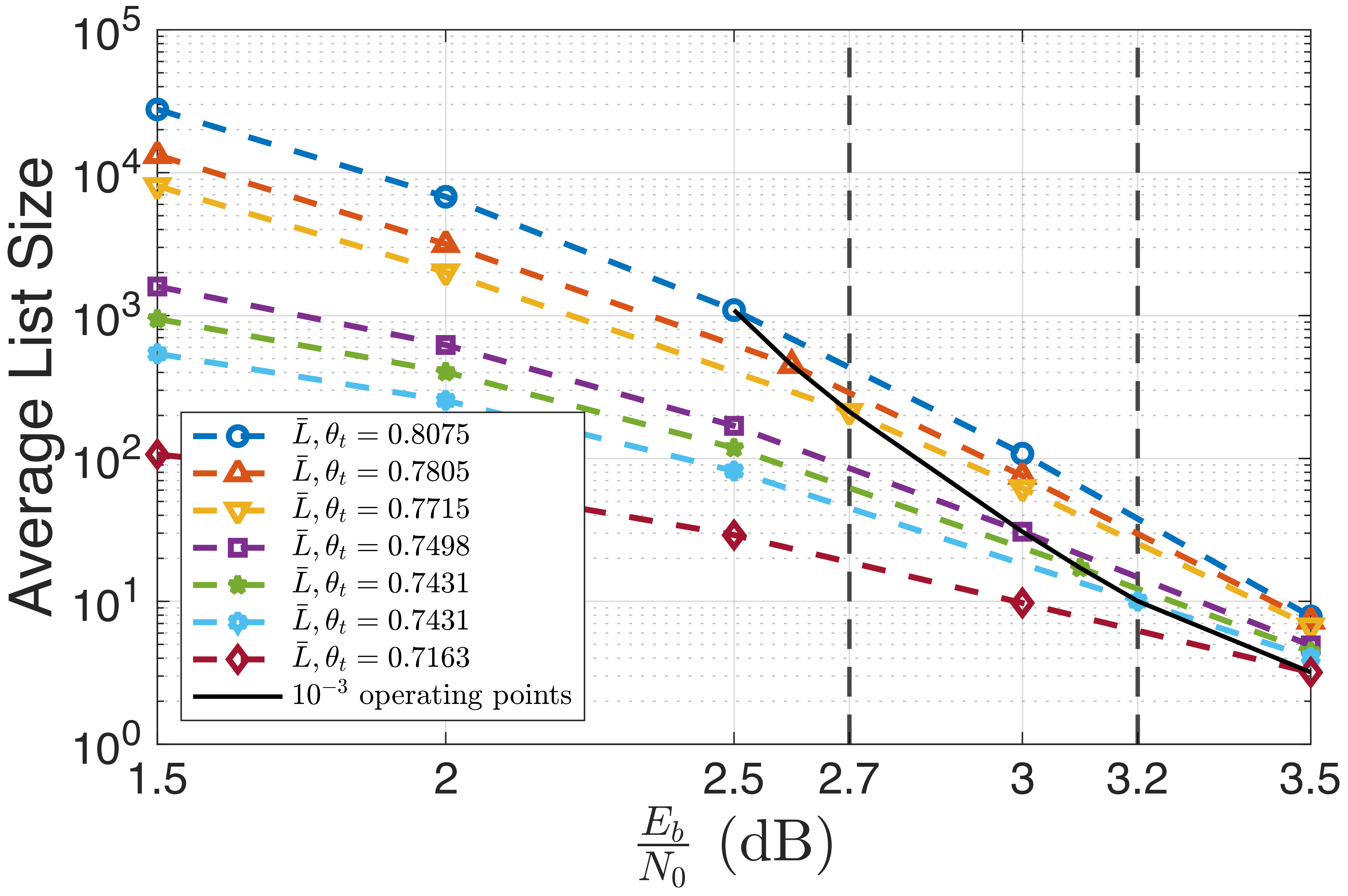
- Step2: When BDLD and BALD have the same TER, BALD has a lower UER.

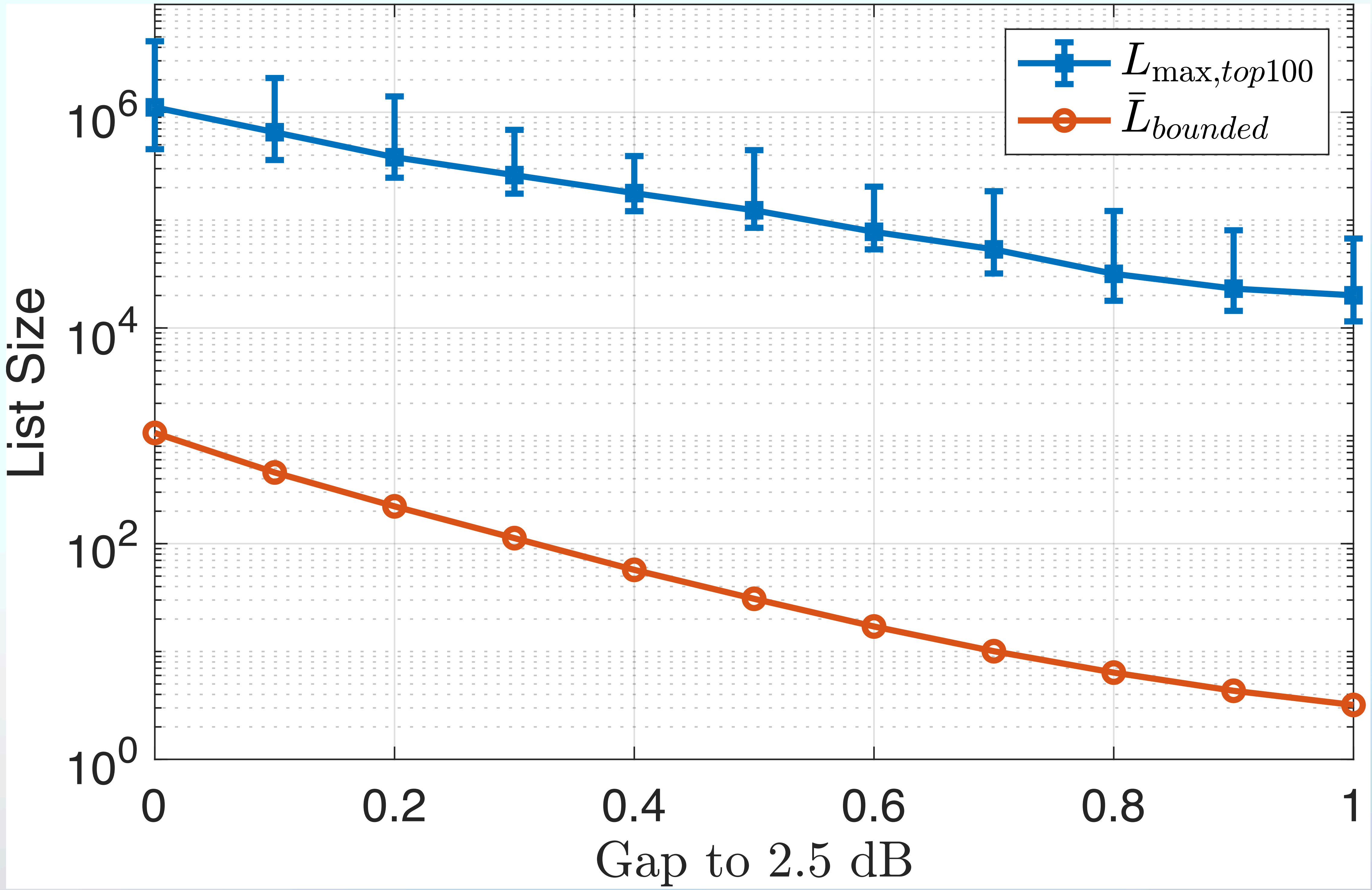
- Sometimes, we can afford higher SNR for lower decoding complexity.



Tradeoff between SNR and complexity

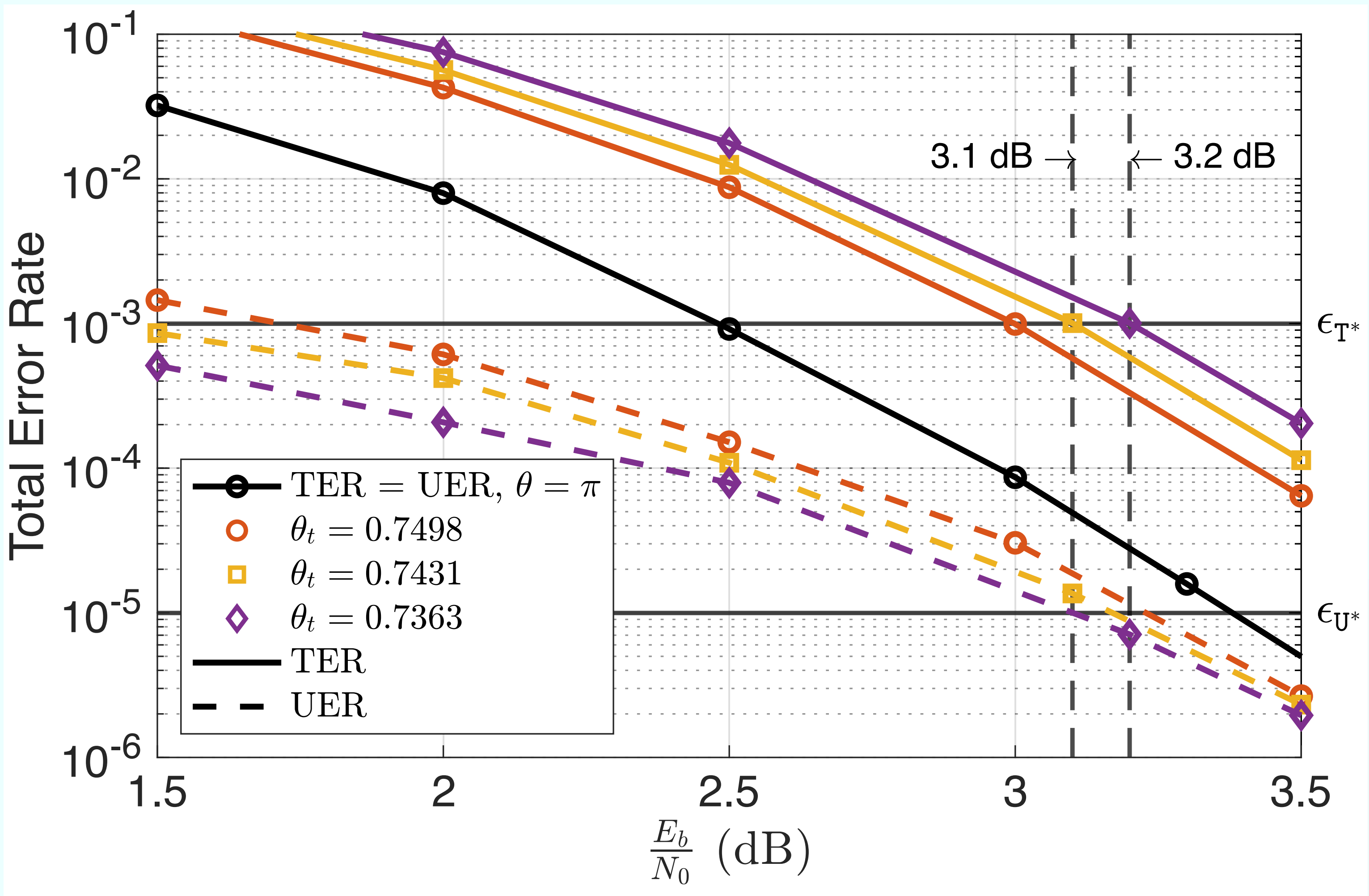
- Idea: Higher complexity if the SNR point is lower

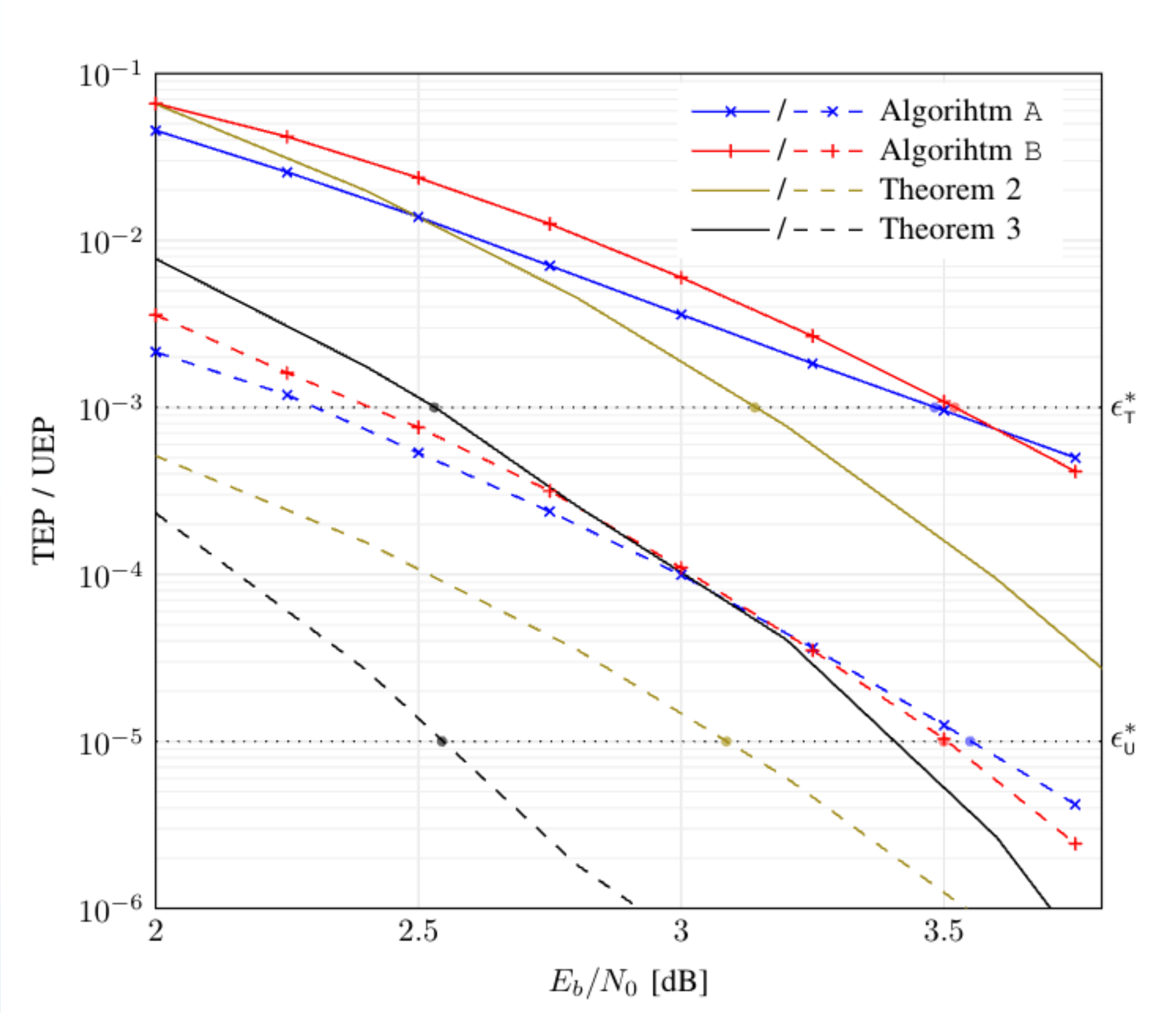




Constraints on TER and UER

- Certain applications require two constraints, one on TER $\epsilon_T^* = 10^{-3}$ and another on UER $\epsilon_U^* = 10^{-5}$.
- Using SLVD, we can meet both requirements at SNR wherever $P_{SLVD} = \epsilon_U^*$.
- We can do better than that by terminating early and therefore convert "unsure" decodings to detected errors.
- Our solution outperforms the (128,64) polar code solution.

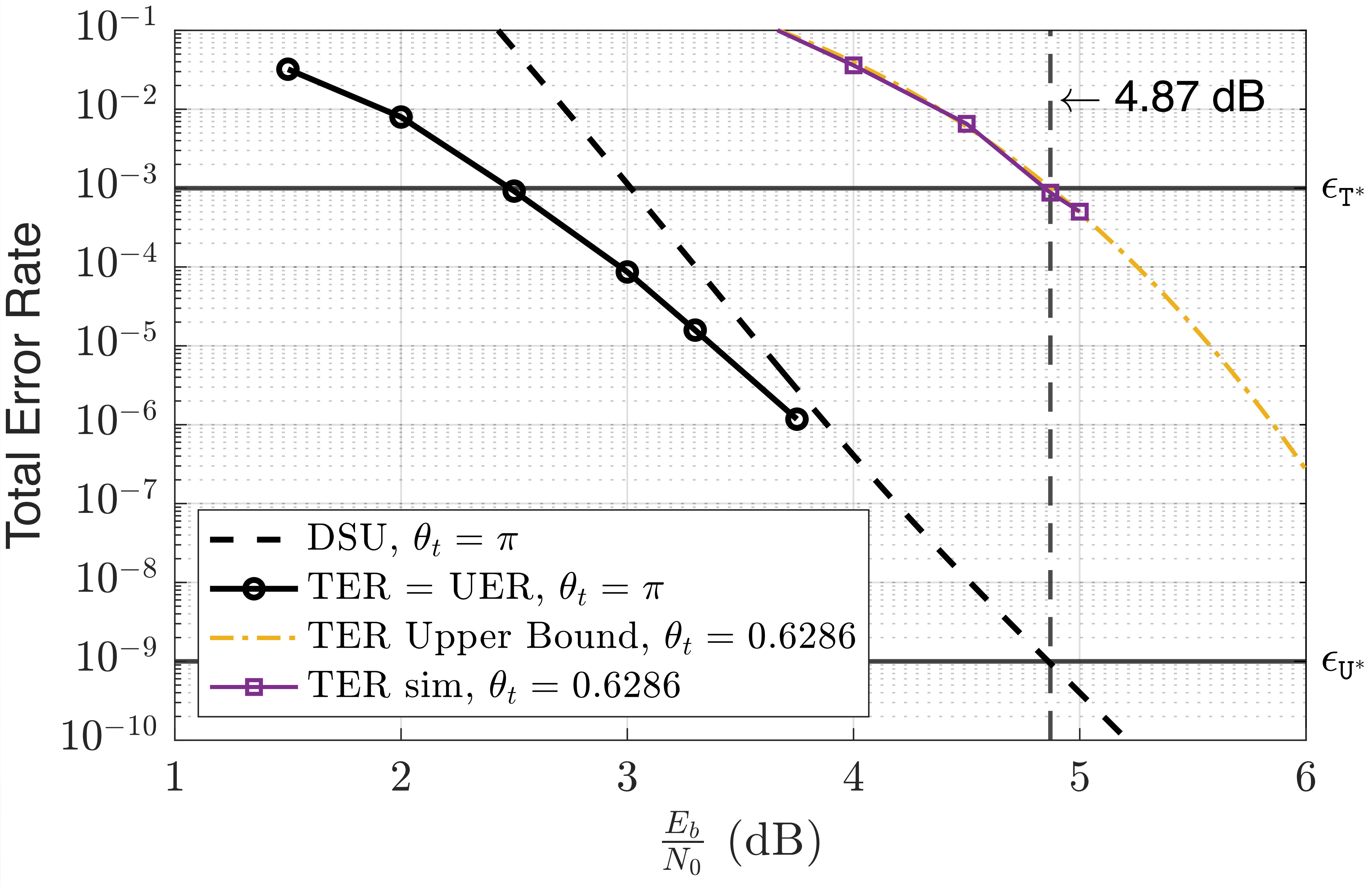




CCSDS Requirements

Consultive Committee for Space Data Systems

- $\epsilon_T^* = 10^{-3}, \epsilon_U^* = 10^{-9}$
- Since we cannot fine-tune the θ values, the SNR value where $P_{SLVD} = \epsilon_U^* = 10^{-9}$ provides an upper bound on the SNR needed to meet both TER and UER requirements.



Thank you!