

Introduction to Reinforcement Learning

CS 294-112: Deep Reinforcement Learning

Sergey Levine

Class Notes

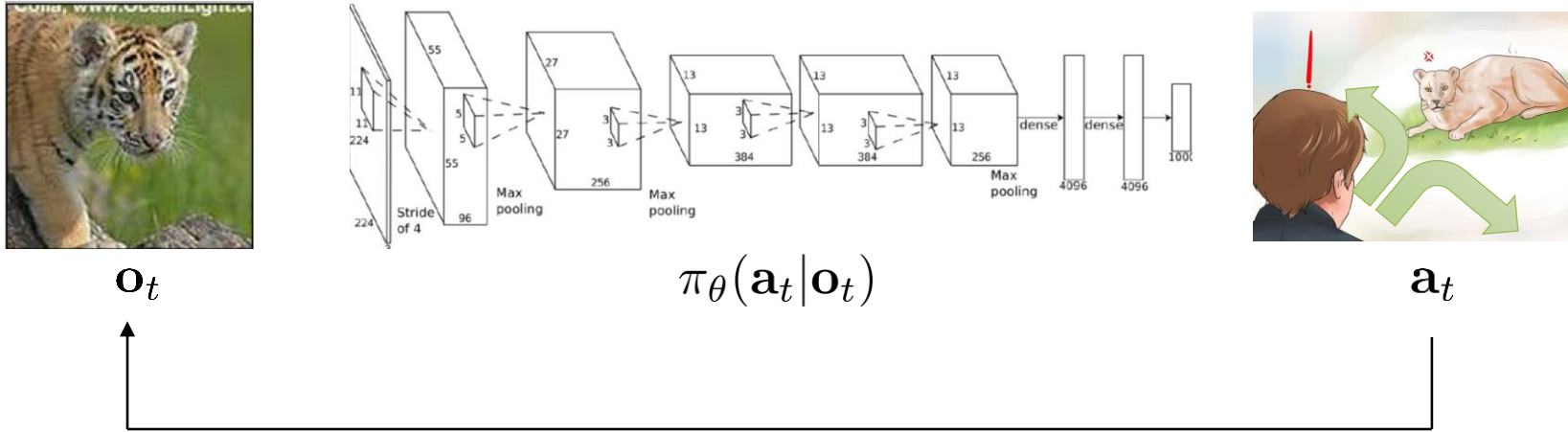
1. Homework 1 milestone in one week!
 - Don't be late!
2. Remember to start forming final project groups
3. MuJoCo license was e-mailed to you

Today's Lecture

1. Definition of a Markov decision process
 2. Definition of reinforcement learning problem
 3. Anatomy of a RL algorithm
 4. Brief overview of RL algorithm types
- Goals:
 - Understand definitions & notation
 - Understand the underlying reinforcement learning objective
 - Get summary of possible algorithms

Definitions

Terminology & notation



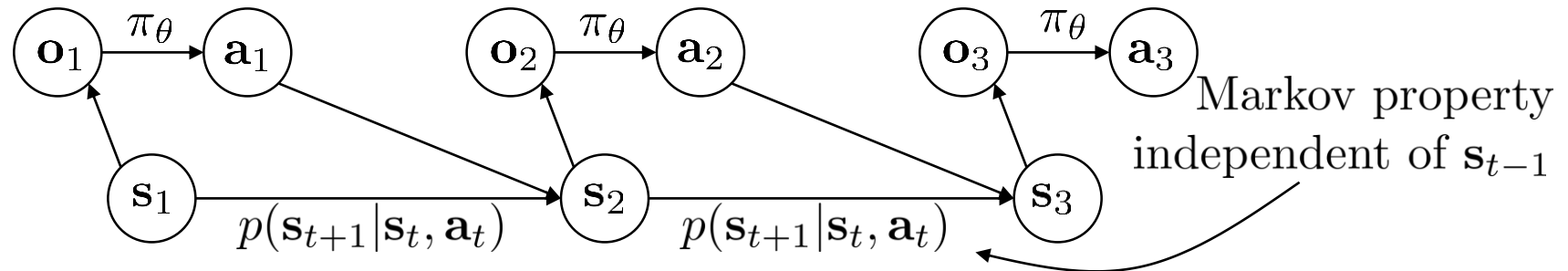
\mathbf{s}_t – state

\mathbf{o}_t – observation

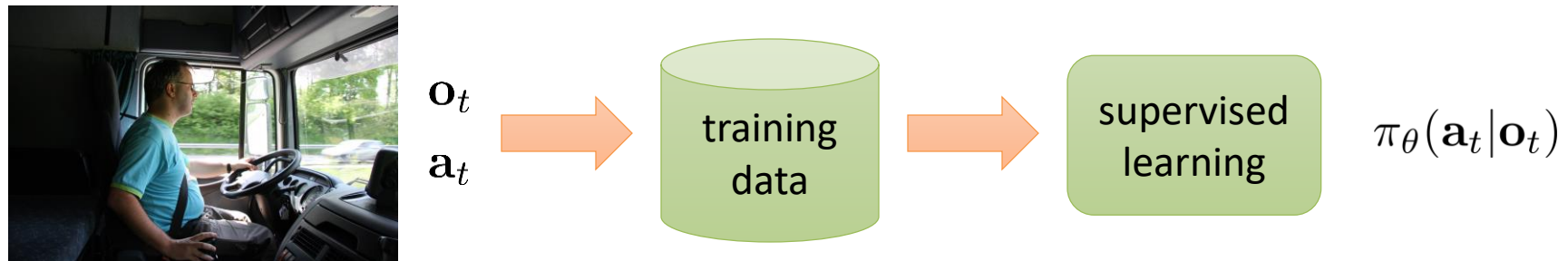
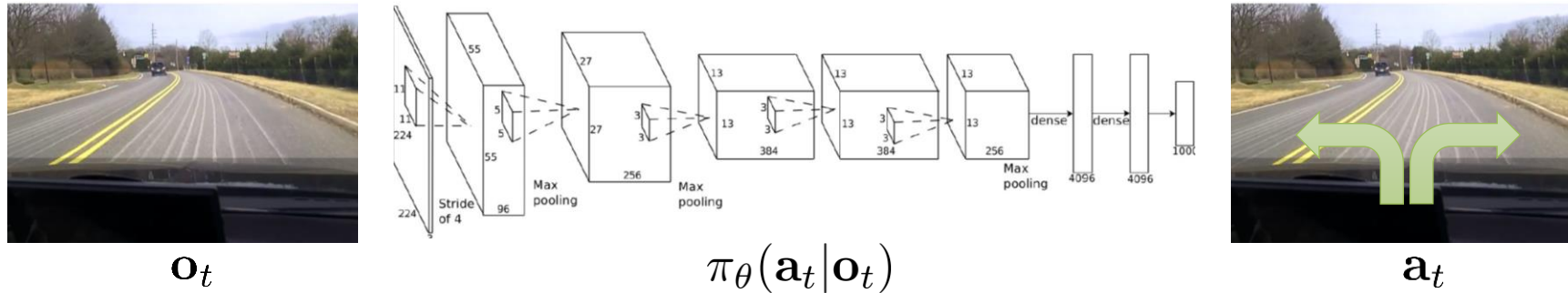
\mathbf{a}_t – action

$\pi_{\theta}(\mathbf{a}_t | \mathbf{o}_t)$ – policy

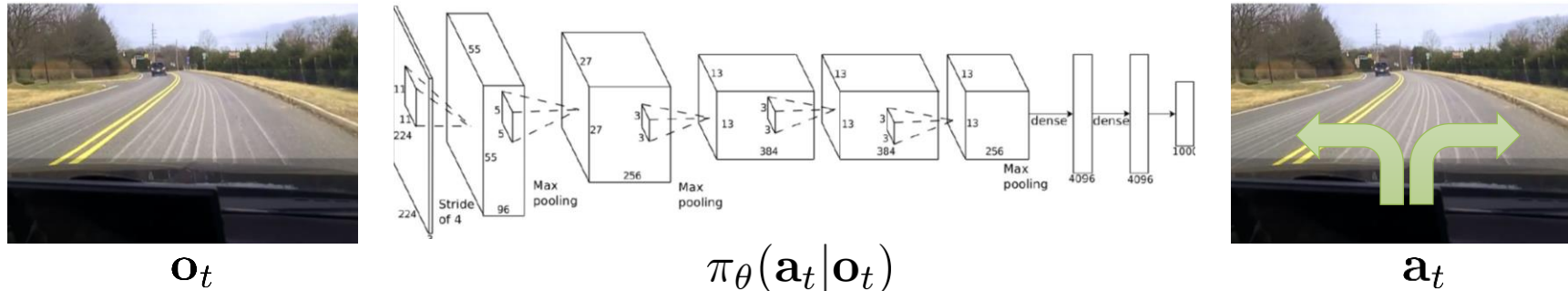
$\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$ – policy (fully observed)



Imitation Learning



Reward functions



which action is better or worse?

$r(\mathbf{s}, \mathbf{a})$: reward function

tells us which states and actions are better

\mathbf{s} , \mathbf{a} , $r(\mathbf{s}, \mathbf{a})$, and $p(\mathbf{s}' | \mathbf{s}, \mathbf{a})$ define

Markov decision process



high reward



low reward

Definitions

Markov chain

first order here. 2nd order Markov chain consists of two states depending on the last ones

$$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$$

\mathcal{S} – state space

states $s \in \mathcal{S}$ (discrete or continuous)

\mathcal{T} – transition operator

$$p(s_{t+1}|s_t)$$

mu is the probability of being in state i, in time step t

why “operator”?

$$\text{let } \mu_{t,i} = p(s_t = i)$$

$\vec{\mu}_t$ is a vector of probabilities

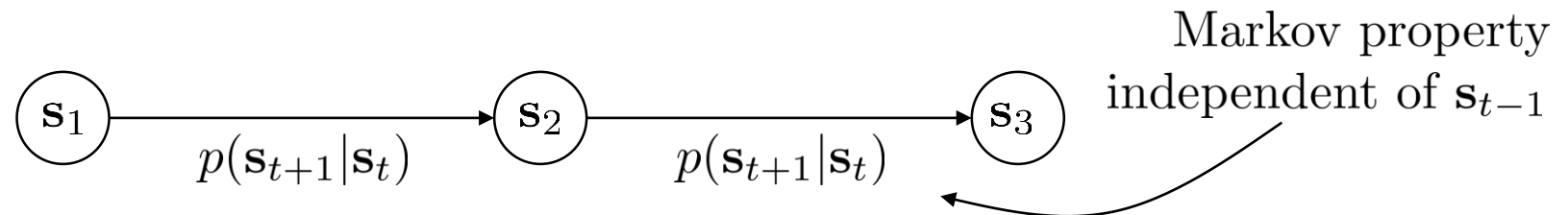
$T_{i,j}$ is the probability transitioning in state i given you are in state j

$$\text{let } \mathcal{T}_{i,j} = p(s_{t+1} = i | s_t = j)$$

$$\text{then } \vec{\mu}_{t+1} = \mathcal{T} \vec{\mu}_t$$



Andrey Markov



Definitions

Markov decision process

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$$

\mathcal{S} – state space

states $s \in \mathcal{S}$ (discrete or continuous)

\mathcal{A} – action space

actions $a \in \mathcal{A}$ (discrete or continuous)

\mathcal{T} – transition operator (now a tensor!)

depends on mu and eps

let $\mu_{t,j} = p(s_t = j)$

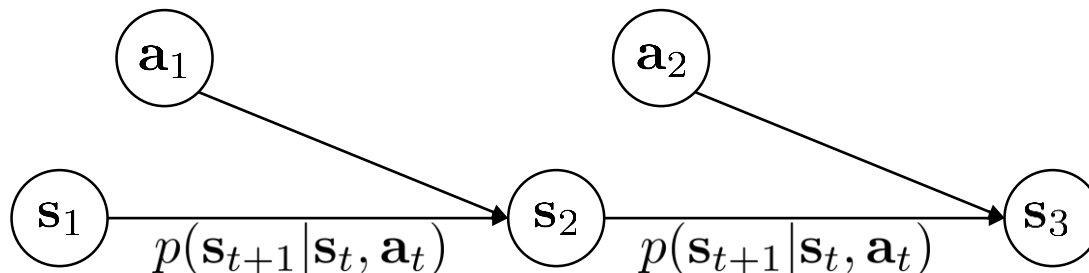
let $\xi_{t,k} = p(a_t = k)$

probability of taking action k in time step t

let $\mathcal{T}_{i,j,k} = p(s_{t+1} = i | s_t = j, a_t = k)$

$$\mu_{t,i} = \sum_{j,k} \mathcal{T}_{i,j,k} \mu_{t,j} \xi_{t,k}$$

mu_t+1,i is correct



Andrey Markov



Richard Bellman

Definitions

Markov decision process

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$$

\mathcal{S} – state space

states $s \in \mathcal{S}$ (discrete or continuous)

\mathcal{A} – action space

actions $a \in \mathcal{A}$ (discrete or continuous)

\mathcal{T} – transition operator (now a tensor!)

r – reward function

$$r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$

$r(s_t, a_t)$ – reward



Andrey Markov



Richard Bellman

Definitions

if the observation is the space the partially observed MDP becomes the regular MDP

partially observed Markov decision process

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{O}, \mathcal{T}, \mathcal{E}, r\}$$

\mathcal{S} – state space

states $s \in \mathcal{S}$ (discrete or continuous)

\mathcal{A} – action space

actions $a \in \mathcal{A}$ (discrete or continuous)

\mathcal{O} – observation space

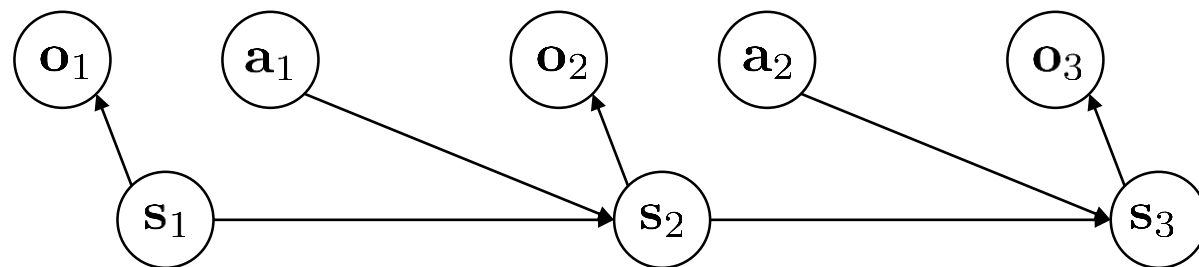
observations $o \in \mathcal{O}$ (discrete or continuous)

\mathcal{T} – transition operator (like before)

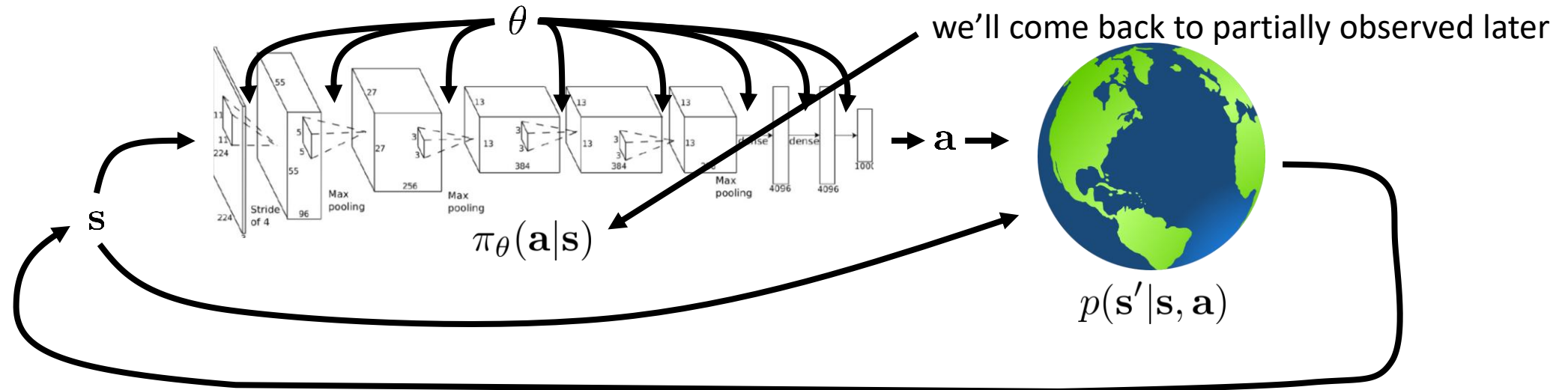
\mathcal{E} – emission probability $p(o_t|s_t)$

r – reward function

$$r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$



The goal of reinforcement learning



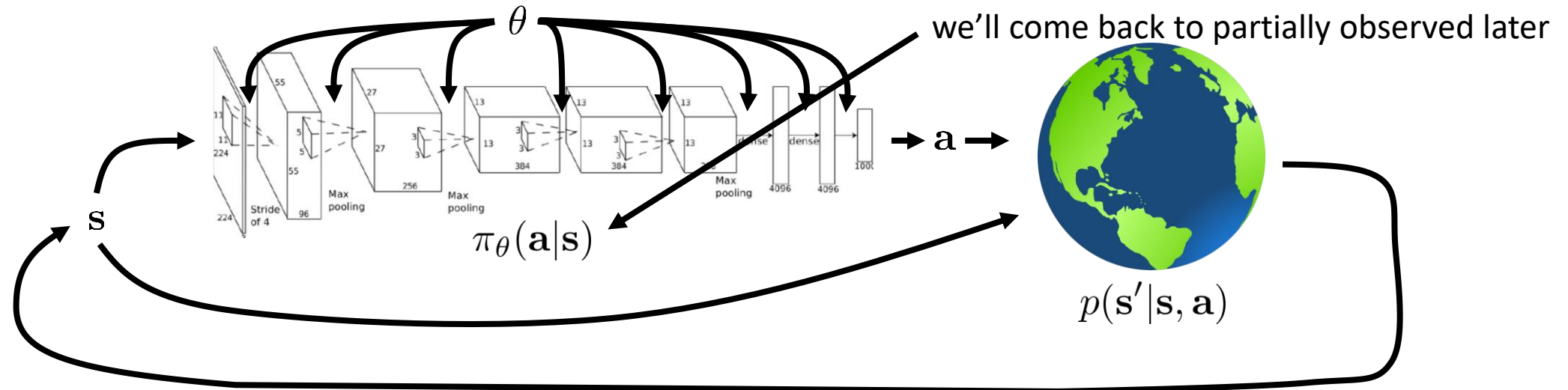
$$\underbrace{p_\theta(\mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)}_{\pi_\theta(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^T \pi_\theta(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

find parameters θ that maximize the sum of expected rewards

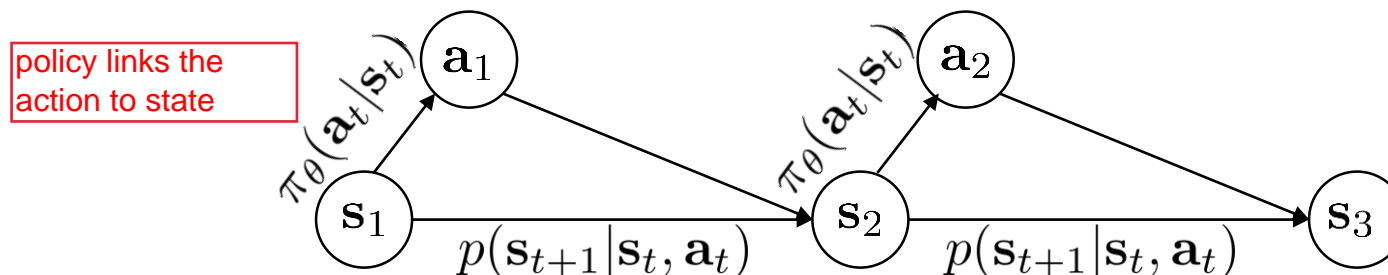
$$\theta^* = \arg \max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$\tau \sim p_{\theta}(\tau)$
expectation with respect to τ ,
distributed according to $p_{\theta}(\tau)$

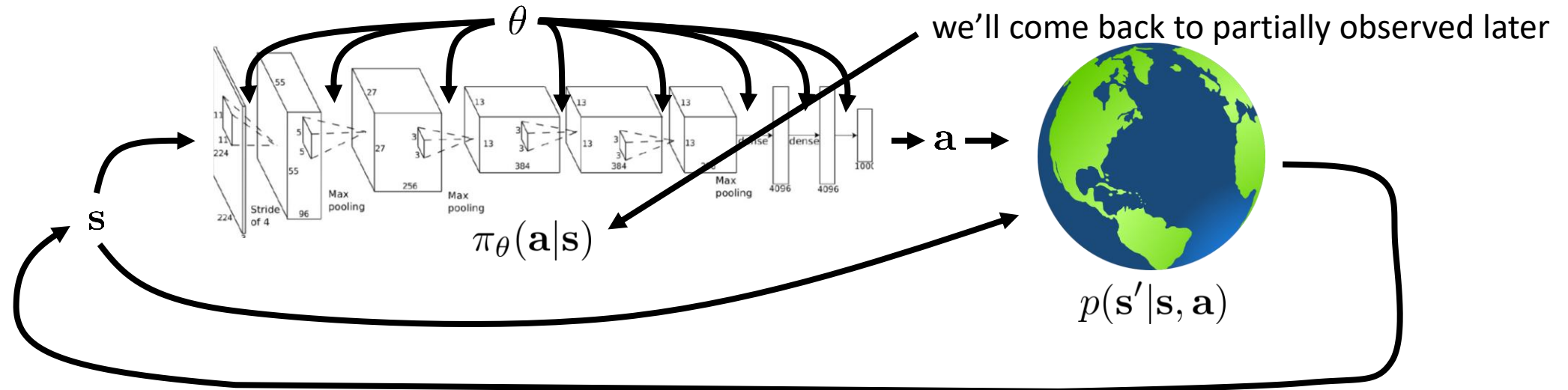
The goal of reinforcement learning



$$\underbrace{p_\theta(s_1, a_1, \dots, s_T, a_T)}_{\pi_\theta(\tau)} = p(s_1) \prod_{t=1}^T \underbrace{\pi_\theta(a_t|s_t)p(s_{t+1}|s_t, a_t)}_{\text{Markov chain on } (s, a)}$$

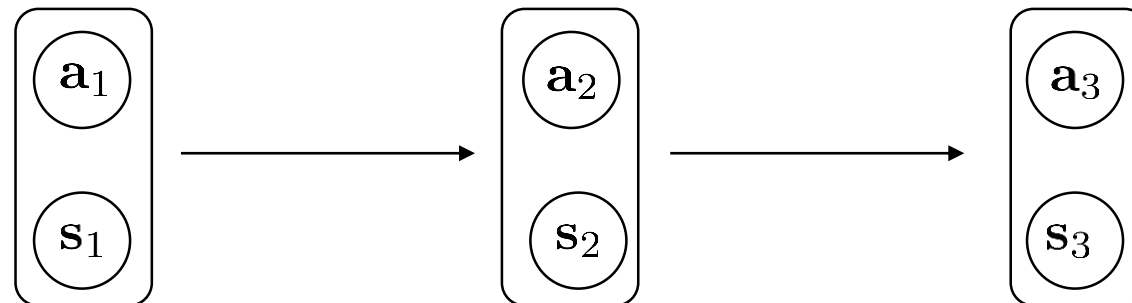


The goal of reinforcement learning



$$\underbrace{p_\theta(s_1, a_1, \dots, s_T, a_T)}_{\pi_\theta(\tau)} = p(s_1) \prod_{t=1}^T \underbrace{\pi_\theta(a_t|s_t)p(s_{t+1}|s_t, a_t)}_{\text{Markov chain on } (s, a)}$$

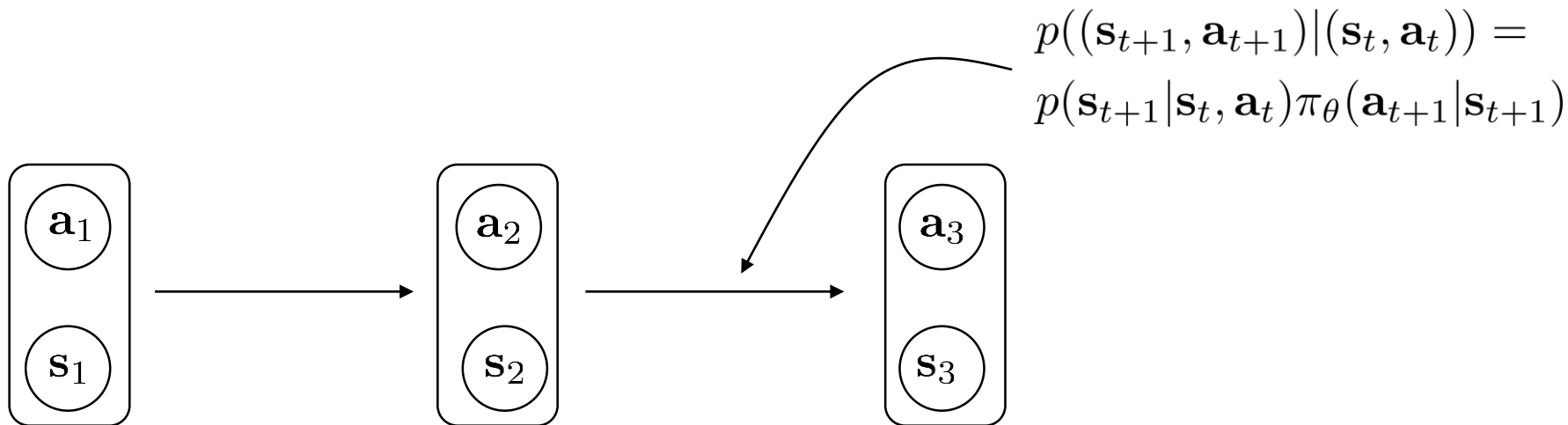
$$p((s_{t+1}, a_{t+1})|(s_t, a_t)) = p(s_{t+1}|s_t, a_t)\pi_\theta(a_{t+1}|s_{t+1})$$



Finite horizon case: state-action marginal

$$\theta^* = \arg \max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$= \arg \max_{\theta} \sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)] \quad p_{\theta}(\mathbf{s}_t, \mathbf{a}_t) \quad \text{state-action marginal}$$



Infinite horizon case: stationary distribution

$$\theta^* = \arg \max_{\theta} \sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]$$

what if $T = \infty$?

does $p(\mathbf{s}_t, \mathbf{a}_t)$ converge to a *stationary* distribution?

$$\mu = \mathcal{T} \mu$$

stationary = the
same before and
after transition

$$(\mathcal{T} - \mathbf{I})\mu = 0$$

μ is eigenvector of \mathcal{T} with eigenvalue 1!

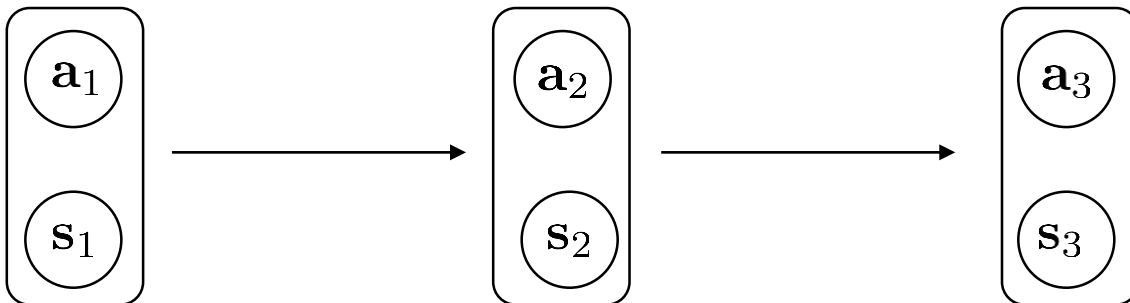
(always exists under some regularity conditions)

$$\mu = p_{\theta}(\mathbf{s}, \mathbf{a}) \quad \text{stationary distribution}$$

state-action transition operator

k \rightarrow inf, linear operation

$$\begin{pmatrix} \mathbf{s}_{t+1} \\ \mathbf{a}_{t+1} \end{pmatrix} = \mathcal{T} \begin{pmatrix} \mathbf{s}_t \\ \mathbf{a}_t \end{pmatrix} \quad \begin{pmatrix} \mathbf{s}_{t+k} \\ \mathbf{a}_{t+k} \end{pmatrix} = \mathcal{T}^k \begin{pmatrix} \mathbf{s}_t \\ \mathbf{a}_t \end{pmatrix}$$



Infinite horizon case: stationary distribution

$$\theta^* = \arg \max_{\theta} \frac{1}{T} \sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)] \rightarrow E_{(\mathbf{s}, \mathbf{a}) \sim p_{\theta}(\mathbf{s}, \mathbf{a})} [r(\mathbf{s}, \mathbf{a})]$$

(in the limit as $T \rightarrow \infty$)

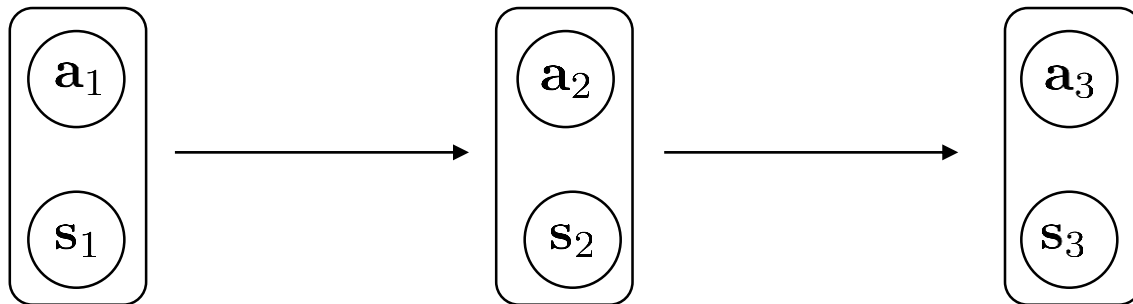
what if $T = \infty$?

does $p(\mathbf{s}_t, \mathbf{a}_t)$ converge to a *stationary* distribution?

$$\mu = \mathcal{T} \mu \quad (\mathcal{T} - \mathbf{I})\mu = 0 \quad \mu = p_{\theta}(\mathbf{s}, \mathbf{a}) \quad \text{stationary distribution}$$

stationary = the same before and after transition

μ is eigenvector of \mathcal{T} with eigenvalue 1!
(always exists under some regularity conditions)



state-action transition operator

$$\begin{pmatrix} \mathbf{s}_{t+1} \\ \mathbf{a}_{t+1} \end{pmatrix} = \mathcal{T} \begin{pmatrix} \mathbf{s}_t \\ \mathbf{a}_t \end{pmatrix} \quad \begin{pmatrix} \mathbf{s}_{t+k} \\ \mathbf{a}_{t+k} \end{pmatrix} = \mathcal{T}^k \begin{pmatrix} \mathbf{s}_t \\ \mathbf{a}_t \end{pmatrix}$$

Expectations and stochastic systems

$$\theta^* = \arg \max_{\theta} E_{(\mathbf{s}, \mathbf{a}) \sim p_{\theta}(\mathbf{s}, \mathbf{a})} [r(\mathbf{s}, \mathbf{a})]$$

infinite horizon case

$$\theta^* = \arg \max_{\theta} \sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]$$

finite horizon case

In RL, we almost always care about *expectations*



$r(\mathbf{s}, \mathbf{a})$ – *not* smooth

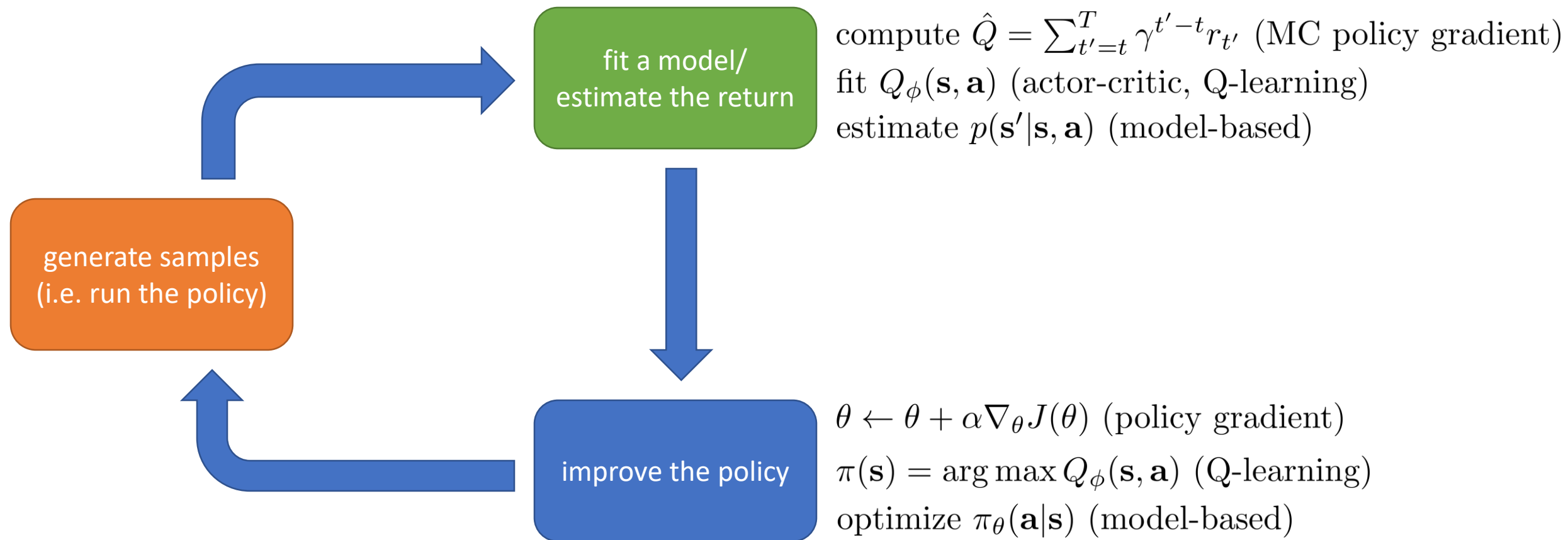
ψ – probability of falling

$E_{(\mathbf{s}, \mathbf{a}) \sim p_{\psi}(\mathbf{s}, \mathbf{a})} [r(\mathbf{s}, \mathbf{a})]$ – *smooth* in ψ !

This allows to use gradient descent algorithms.
Over infinite number of trials, expectation becomes smooth, even with not smooth reward

Algorithms

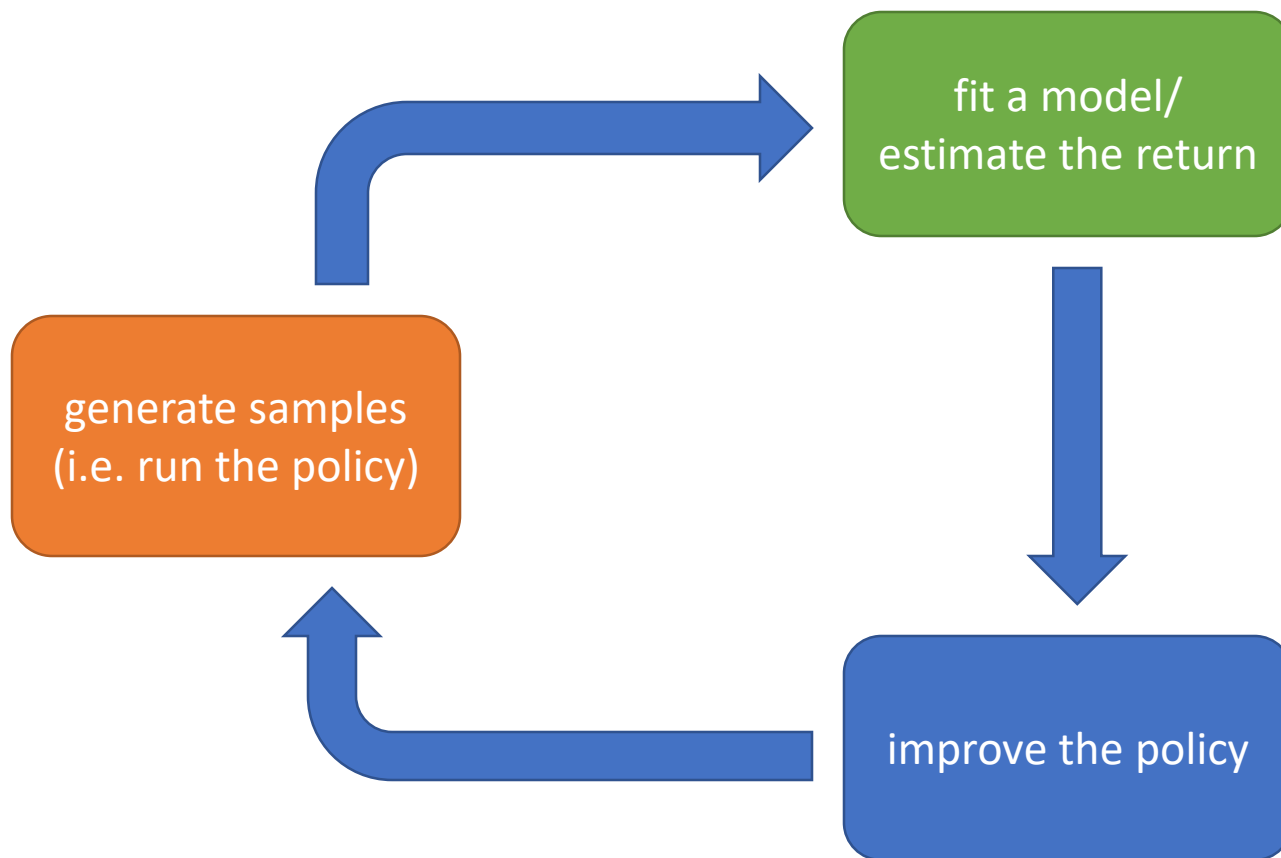
The anatomy of a reinforcement learning algorithm



Which parts are expensive?

real robot/car/power
grid/whatever:
1x real time, until we
invent time travel

MuJoCo simulator:
up to 10000x real time



$$\hat{Q} = \sum_{t'=t}^T \gamma^{t'-t} r_{t'}$$

trivial, fast

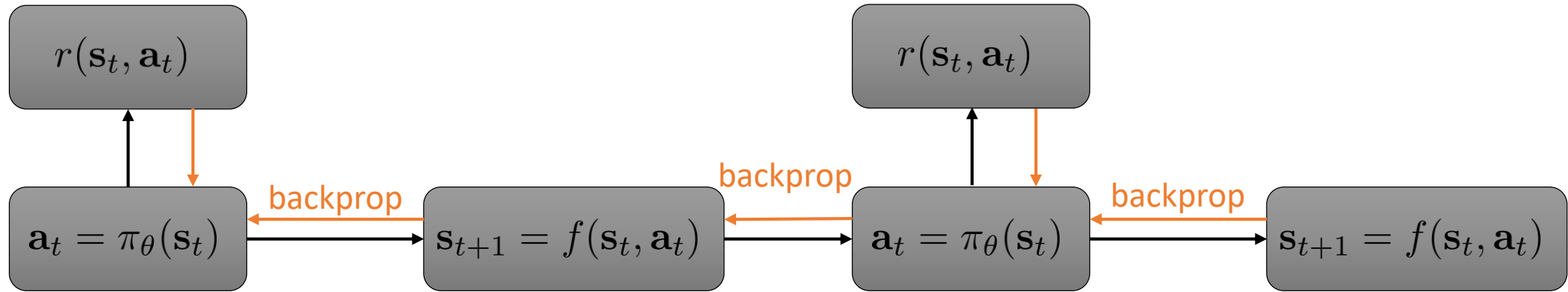
fit $Q_{\phi}(\mathbf{s}, \mathbf{a})$
expensive, but non-
trivial to parallelize

$$\pi(\mathbf{s}) = \arg \max Q_{\phi}(\mathbf{s}, \mathbf{a})$$

trivial, nothing to do

optimize $\pi_{\theta}(\mathbf{a}|\mathbf{s})$ (model-based)
expensive, but non-
trivial to parallelize

Simple example: RL by backprop

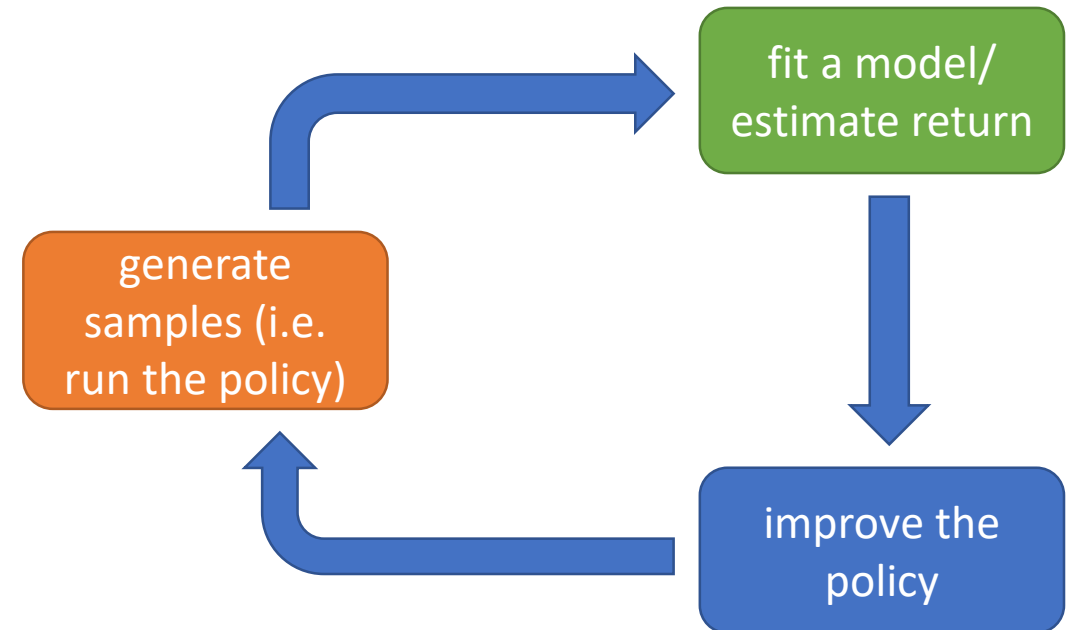


collect data

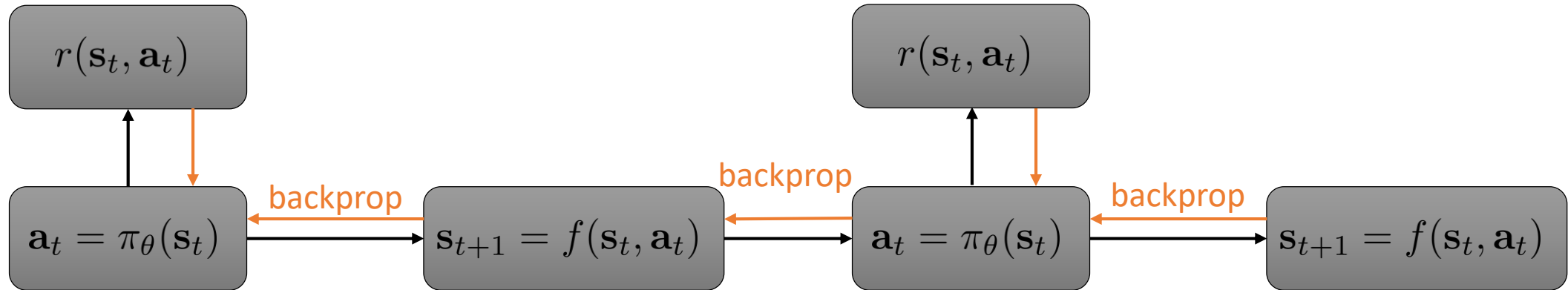
update the model f

forward pass evaluate the reward/return

backward pass & gradient step



Why is this not enough?



- Only handles deterministic dynamics
- Only handles deterministic policies
- Only continuous states and actions
- Very difficult optimization problem
- We'll talk about this more later!

How can we work with *stochastic* systems?

Conditional expectations

$$\sum_{t=1}^T E_{(\mathbf{s}_t, \mathbf{a}_t) \sim p_{\theta}(\mathbf{s}_t, \mathbf{a}_t)} [r(\mathbf{s}_t, \mathbf{a}_t)]$$

$$E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)}$$



what if we knew this part?

$Q(\mathbf{s}_1, \mathbf{a}_1) = r(\mathbf{s}_1, \mathbf{a}_1) + \dots$

$$Q(\mathbf{s}_1, \mathbf{a}_1) = r(\mathbf{s}_1, \mathbf{a}_1) + E_{\mathbf{s}_2 \sim p(\mathbf{s}_2 | \mathbf{s}_1, \mathbf{a}_1)} [E_{\mathbf{a}_2 \sim \pi(\mathbf{a}_2 | \mathbf{s}_2)} [r(\mathbf{s}_2, \mathbf{a}_2) + \dots | \mathbf{s}_2] | \mathbf{s}_1, \mathbf{a}_1]$$

$$E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)} [E_{\mathbf{a}_1 \sim \pi(\mathbf{a}_1 | \mathbf{s}_1)} [Q(\mathbf{s}_1, \mathbf{a}_1) | \mathbf{s}_1]]$$



easy to modify $\pi_{\theta}(\mathbf{a}_1 | \mathbf{s}_1)$ if $Q(\mathbf{s}_1, \mathbf{a}_1)$ is known!

example: $\pi(\mathbf{a}_1 | \mathbf{s}_1) = 1$ if $\mathbf{a}_1 = \arg \max_{\mathbf{a}_1} Q(\mathbf{s}_1, \mathbf{a}_1)$

Definition: Q-function

Q-fn depending on policy π

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$: total reward from taking \mathbf{a}_t in \mathbf{s}_t

Definition: value function

value-fn depending on policy π

$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t]$: total reward from \mathbf{s}_t

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$

$E_{\mathbf{s}_1 \sim p(\mathbf{s}_1)} [V^\pi(\mathbf{s}_1)]$ is the RL objective!

Using Q-functions and value functions

Idea 1: if we have policy π , and we know $Q^\pi(\mathbf{s}, \mathbf{a})$, then we can *improve* π :

set $\pi'(\mathbf{a}|\mathbf{s}) = 1$ if $\mathbf{a} = \arg \max_{\mathbf{a}} Q^\pi(\mathbf{s}, \mathbf{a})$

this policy is at least as good as π (and probably better)!

and it doesn't matter what π is

Idea 2: compute gradient to increase probability of good actions \mathbf{a} :

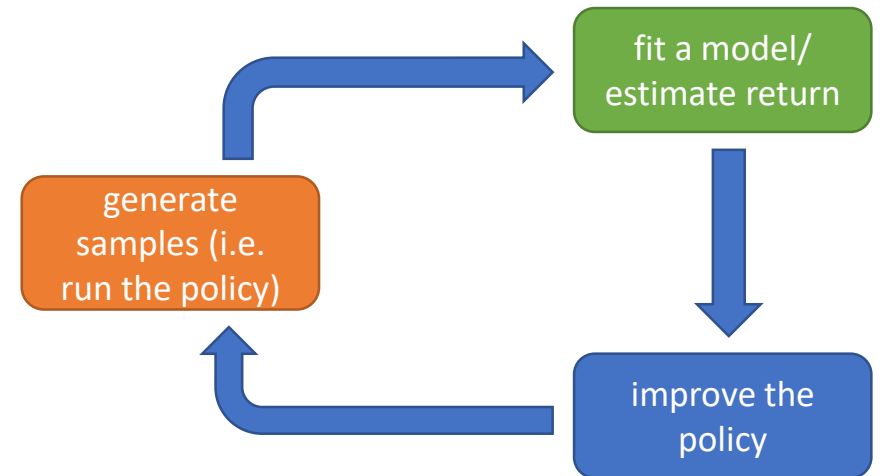
if $Q^\pi(\mathbf{s}, \mathbf{a}) > V^\pi(\mathbf{s})$, then \mathbf{a} is *better than average* (recall that $V^\pi(\mathbf{s}) = E[Q^\pi(\mathbf{s}, \mathbf{a})]$ under $\pi(\mathbf{a}|\mathbf{s})$)

modify $\pi(\mathbf{a}|\mathbf{s})$ to increase probability of \mathbf{a} if $Q^\pi(\mathbf{s}, \mathbf{a}) > V^\pi(\mathbf{s})$

These ideas are *very* important in RL; we'll revisit them again and again!

Review

- Definitions
 - Markov chain
 - Markov decision process
- RL objective
 - Expected reward
 - How to evaluate expected reward?
- Structure of RL algorithms
 - Sample generation
 - Fitting a model/estimating return
 - Policy Improvement
- Value functions and Q-functions



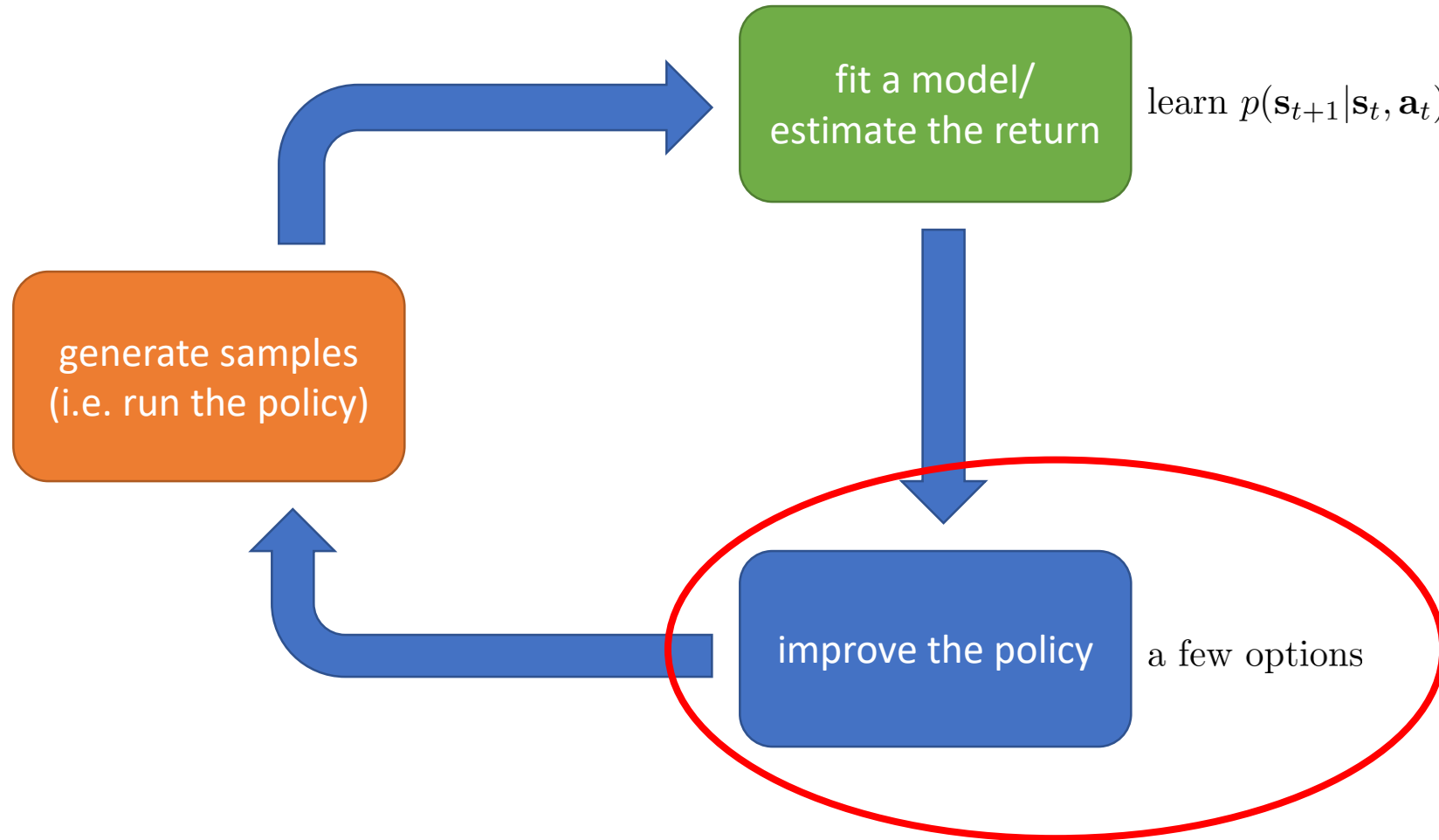
Break

Types of RL algorithms

$$\theta^* = \arg \max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[\sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

- Policy gradients: directly differentiate the above objective
- Value-based: estimate value function or Q-function of the optimal policy (no explicit policy)
- Actor-critic: estimate value function or Q-function of the current policy, use it to improve policy "Policy gradients + Value-based = actor-critic"
- Model-based RL: estimate the transition model, and then...
 - Use it for planning (no explicit policy)
 - Use it to improve a policy
 - Something else

Model-based RL algorithms



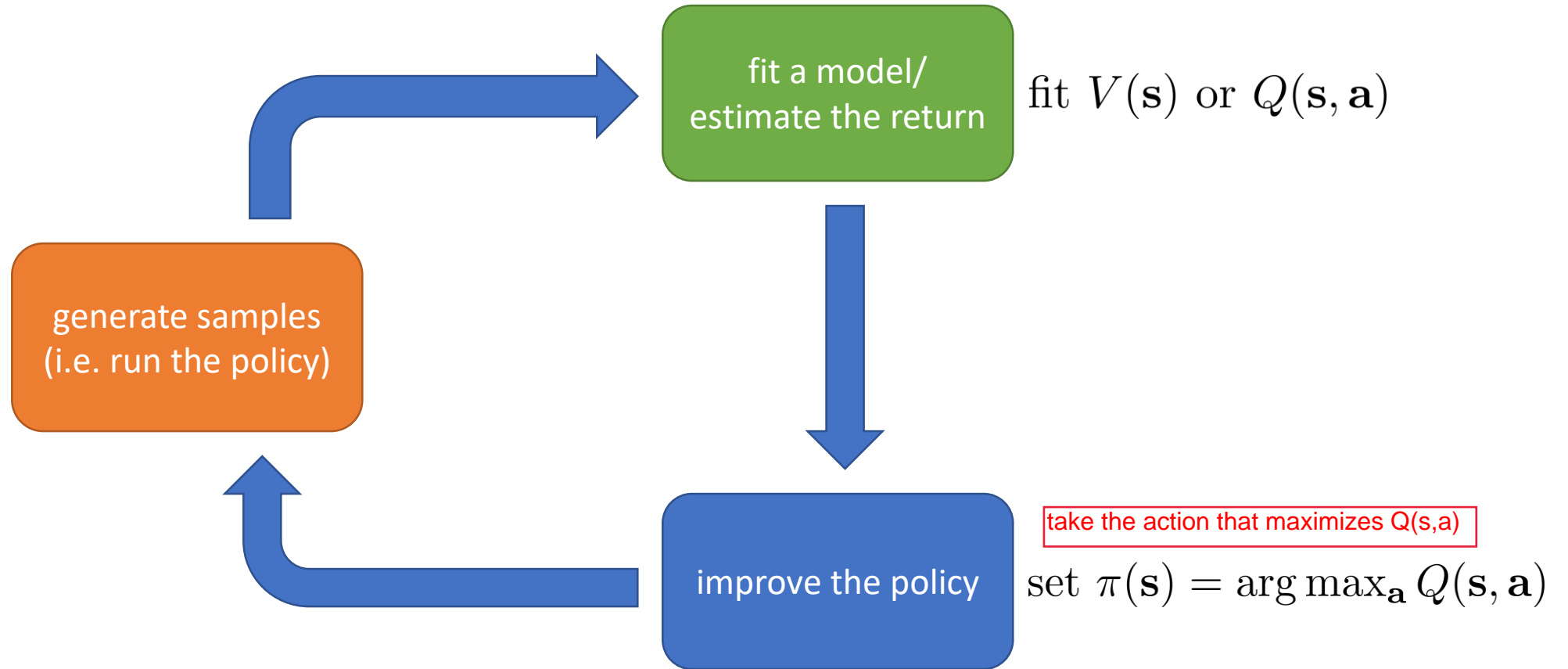
Model-based RL algorithms

improve the policy

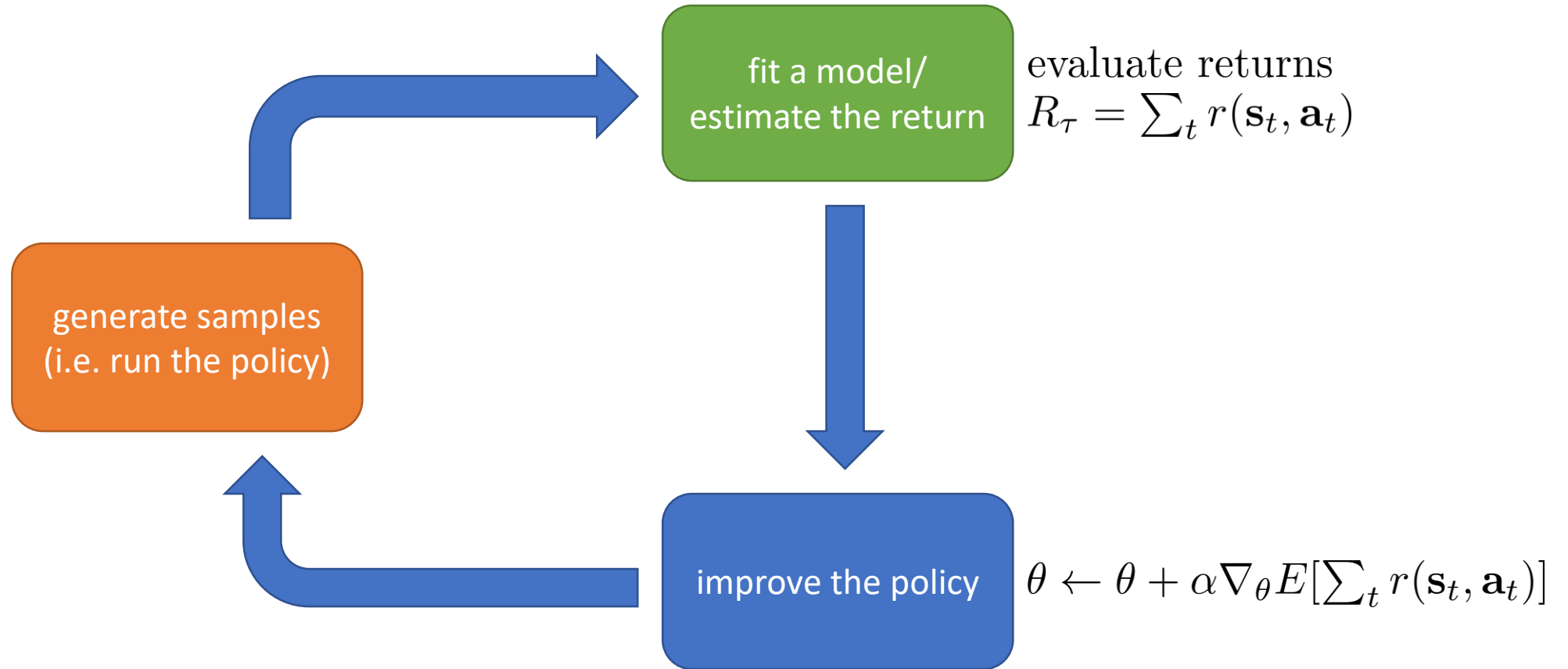
a few options

1. Just use the model to plan (no policy)
 - Trajectory optimization/optimal control (primarily in continuous spaces) – essentially backpropagation to optimize over actions
 - Discrete planning in discrete action spaces – e.g., Monte Carlo tree search
2. Backpropagate gradients into the policy
 - Requires some tricks to make it work
3. Use the model to learn a value function
 - Dynamic programming
 - Generate simulated experience for model-free learner (Dyna)

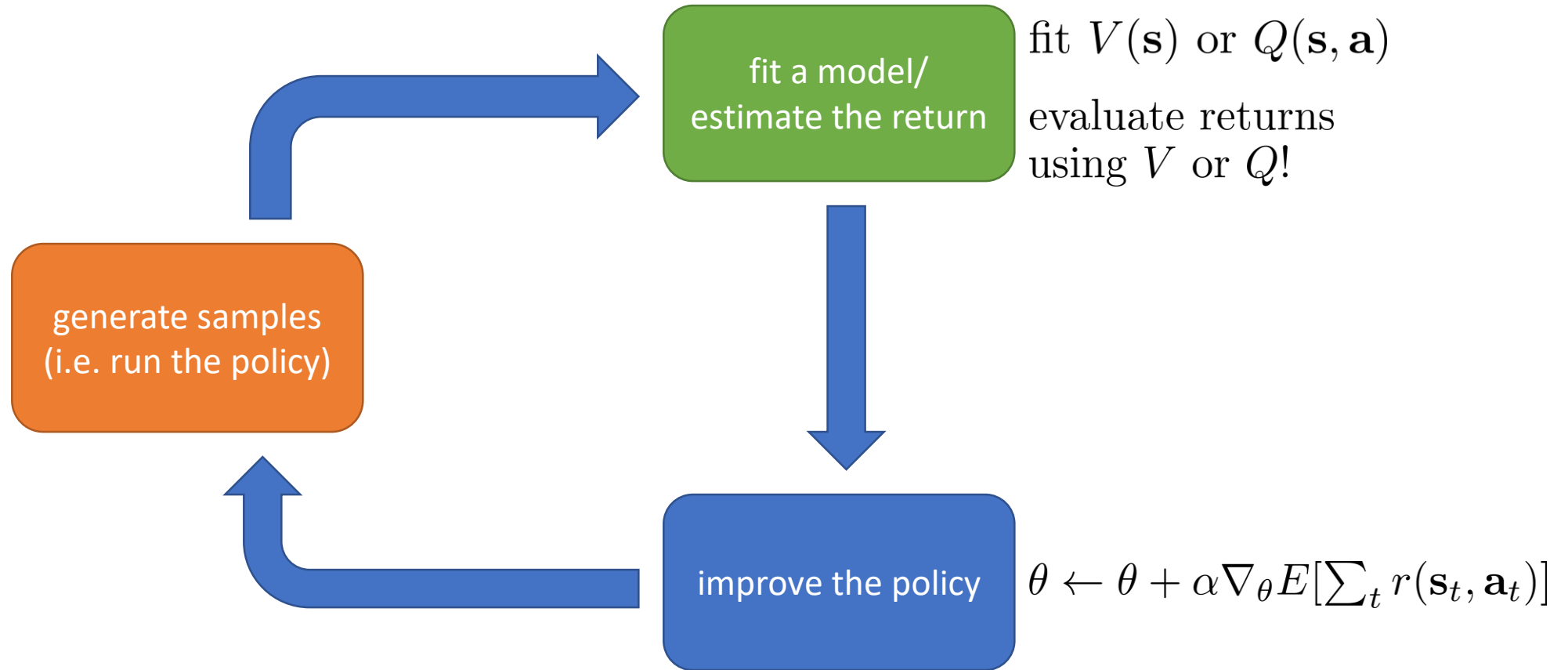
Value function based algorithms



Direct policy gradients



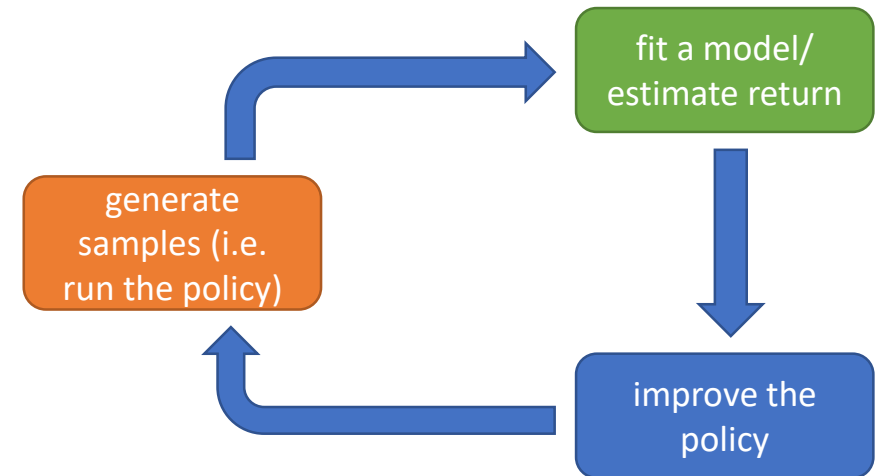
Actor-critic: value functions + policy gradients



Tradeoffs

Why so many RL algorithms?

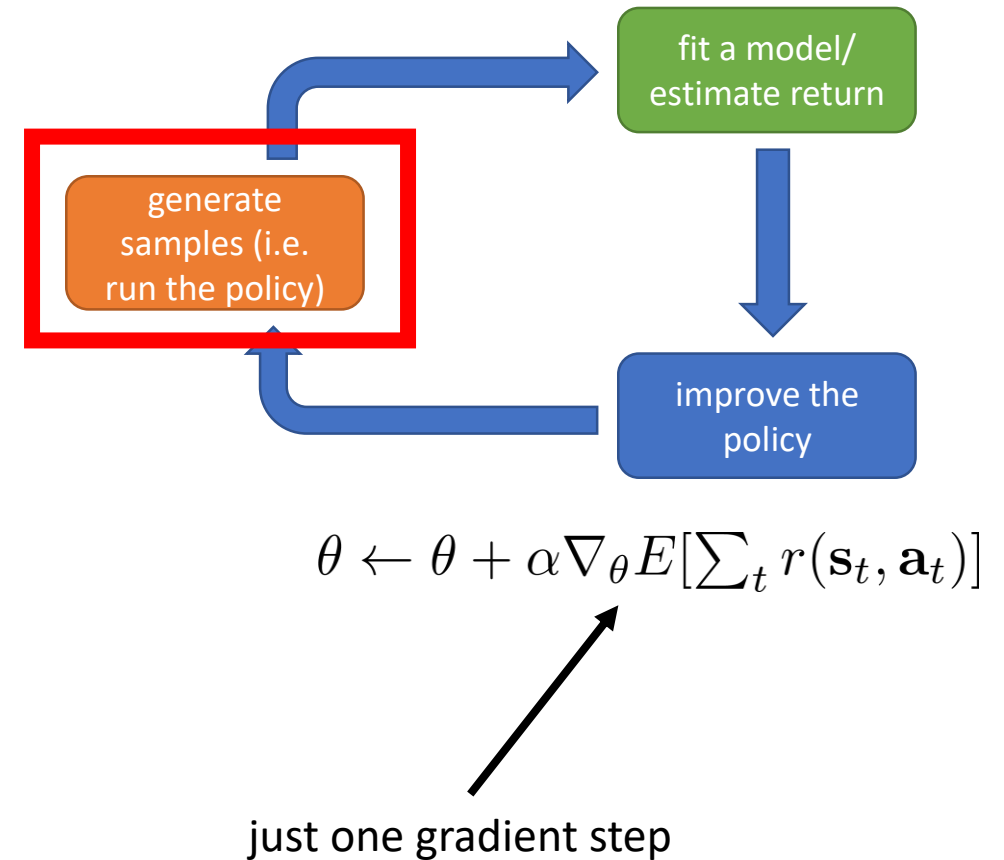
- Different tradeoffs
 - Sample efficiency
 - Stability & ease of use how difficult is it to choose the hyperparameters?
- Different assumptions
 - Stochastic or deterministic?
 - Continuous or discrete?
 - Episodic or infinite horizon? T is finite vs T \rightarrow inf
- Different things are easy or hard in different settings
 - Easier to represent the policy?
 - Easier to represent the model?



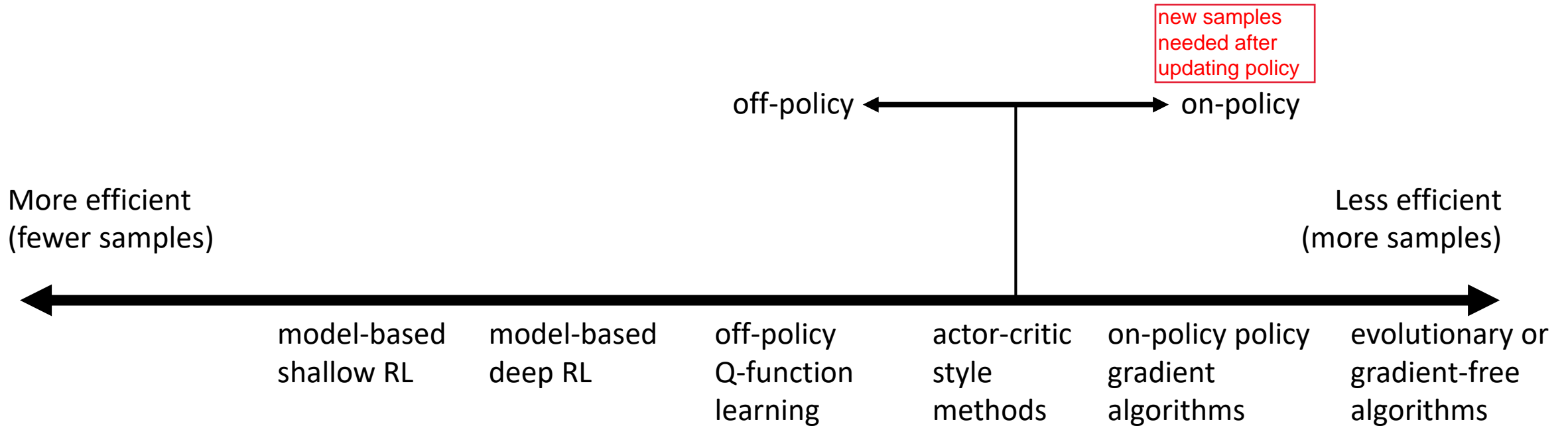
Comparison: sample efficiency

- Sample efficiency = how many samples do we need to get a good policy?
- Most important question: is the algorithm *off policy*?
- Off policy: able to improve the policy without generating new samples from that policy
- On policy: each time the policy is changed, even a little bit, we need to generate new samples

can you use historical data to improve the policy? -> DQN



Comparison: sample efficiency



Why would we use a *less* efficient algorithm?

Wall clock time is not the same as efficiency!

Comparison: stability and ease of use

- Does it converge?
- And if it converges, to what?
- And does it converge every time?

Why is any of this even a question???

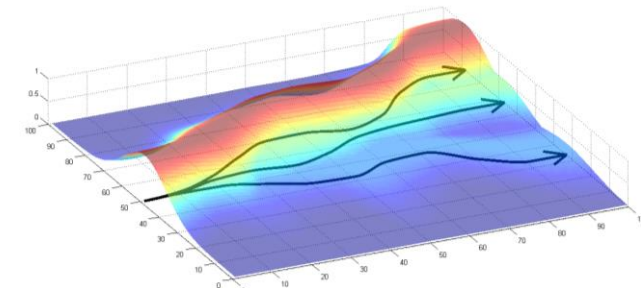
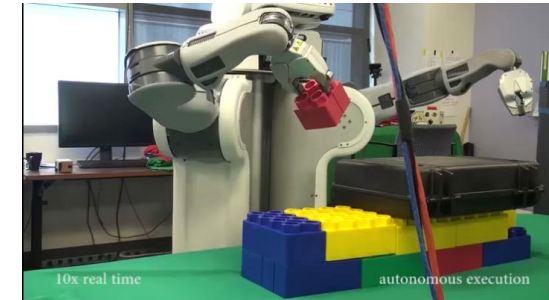
- Supervised learning: almost *always* gradient descent
- Reinforcement learning: often *not* gradient descent
 - Q-learning: fixed point iteration
 - Model-based RL: model is not optimized for expected reward
 - Policy gradient: *is* gradient descent, but also often the least efficient!

Comparison: stability and ease of use

- Value function fitting
 - At best, minimizes error of fit (“Bellman error”)
 - Not the same as expected reward
 - At worst, doesn’t optimize anything
 - Many popular deep RL value fitting algorithms are not guaranteed to converge to *anything* in the nonlinear case
- Model-based RL
 - Model minimizes error of fit
 - This will converge
 - No guarantee that better model = better policy
- Policy gradient
 - The only one that actually performs gradient descent (ascent) on the true objective

Comparison: assumptions

- Common assumption #1: full observability
 - Generally assumed by value function fitting methods
 - Can be mitigated by adding recurrence
- Common assumption #2: episodic learning
 - Often assumed by pure policy gradient methods
 - Assumed by some model-based RL methods
- Common assumption #3: continuity or smoothness
 - Assumed by some continuous value function learning methods
 - Often assumed by some model-based RL methods



Examples of specific algorithms

- Value function fitting methods
 - Q-learning, DQN
 - Temporal difference learning
 - Fitted value iteration
- Policy gradient methods
 - REINFORCE
 - Natural policy gradient
 - Trust region policy optimization
- Actor-critic algorithms
 - Asynchronous advantage actor critic (A3C)
- Model-based RL algorithms
 - Dyna
 - Guided policy search

We'll learn about most of these in the next few weeks!

Example 1: Atari games with Q-functions

- Playing Atari with deep reinforcement learning, Mnih et al. '13
- Q-learning with convolutional neural networks



Example 2: robots and model-based RL

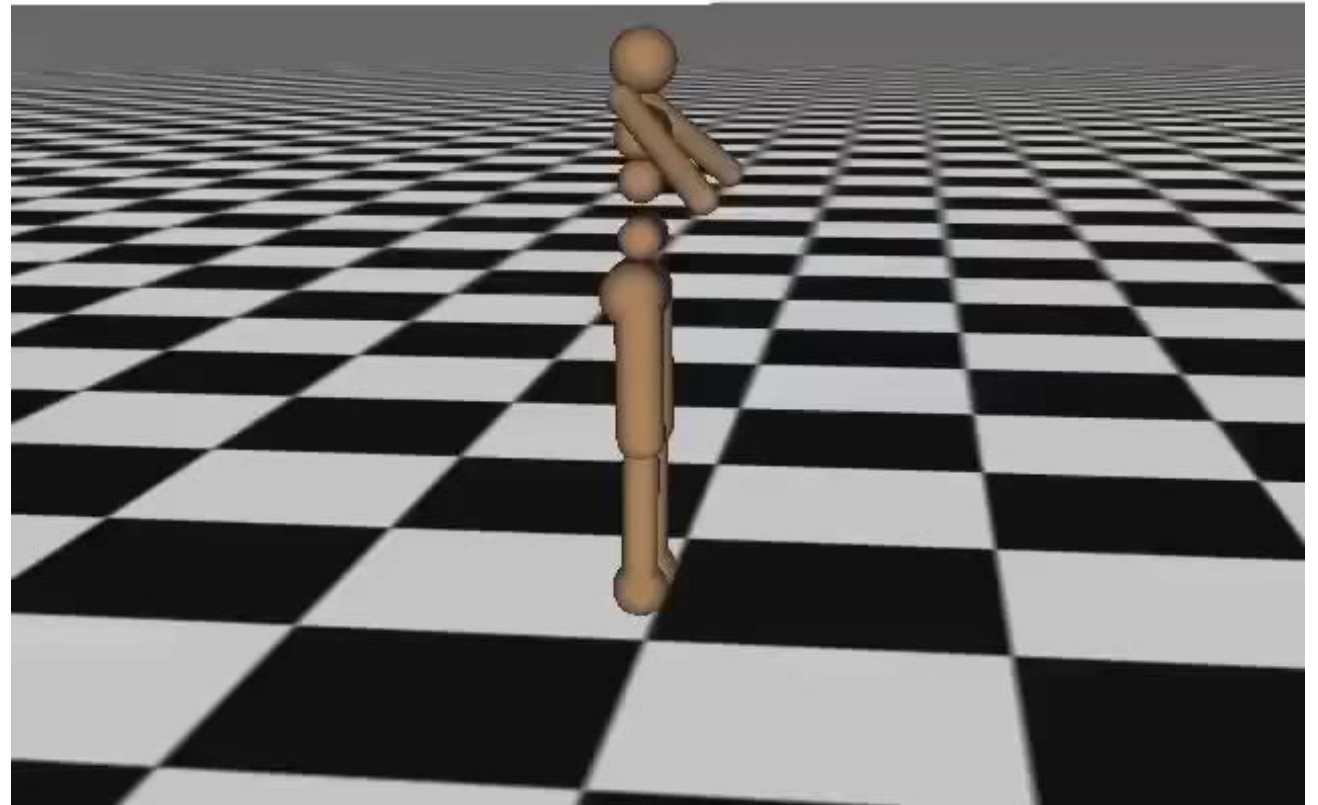
- End-to-end training of deep visuomotor policies, L.* , Finn* '16
- Guided policy search (model-based RL) for image-based robotic manipulation

Various Experiments
Including the policy input

Example 3: walking with policy gradients

- High-dimensional continuous control with generalized advantage estimation, Schulman et al. '16
- Trust region policy optimization with value function approximation

Iteration 0



Next time: model-free RL methods

- Week 3: policy gradient algorithms
 - You'll need these for Homework 2!
- Week 4: actor-critic and value function learning
- Week 5: advances value function algorithms for Q-learning
 - You'll need these for Homework 3!
- Week 6: model-based reinforcement learning