

Deep RL with Q-Functions

CS 294-112: Deep Reinforcement Learning

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Class Notes


1. Homework 2 is due Wednesday
2. Project proposal deadline in four weeks...

Today's Lecture

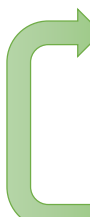
1. How we can make Q-learning work with deep networks
 2. A generalized view of Q-learning algorithms
 3. Tricks for improving Q-learning in practice
 4. Continuous Q-learning methods
- Goals:
 - Understand how to implement Q-learning so that it can be used with complex function approximators
 - Understand how to extend Q-learning to continuous actions

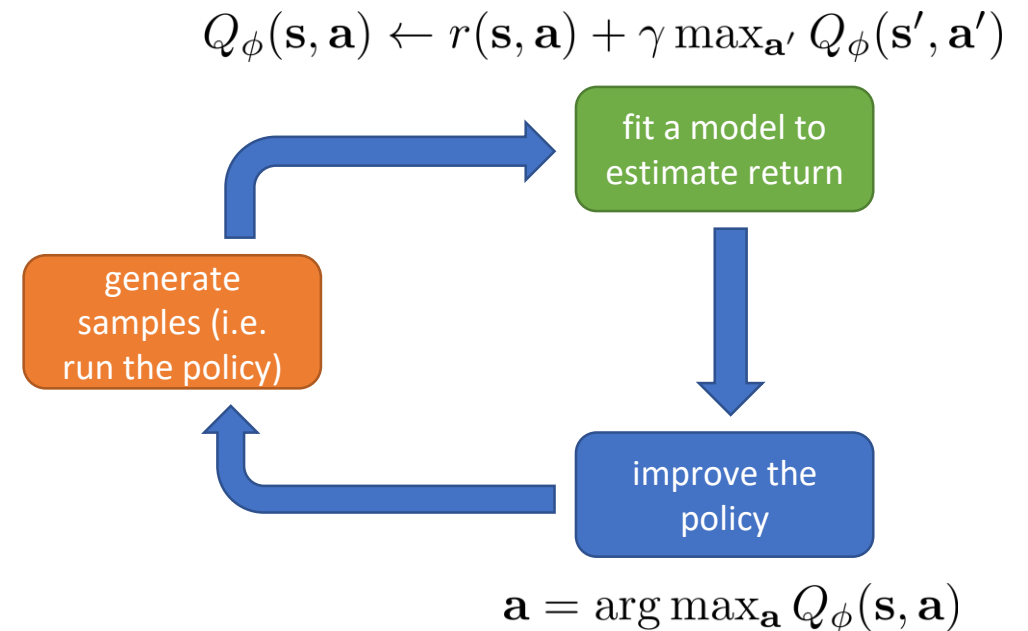
Recap: Q-learning

full fitted Q-iteration algorithm:

- 
1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
 2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
 3. set $\phi \leftarrow \arg \min_{\phi} \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$
- $K \times$

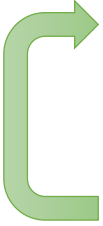
online Q iteration algorithm:

- 
1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
 2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$ target value means what the regression of the Q-fn goes towards
 3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$



What's wrong?

online Q iteration algorithm:

- 
1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
 2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
 3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$
- these are correlated!
- isn't this just gradient descent? that converges, right?

Q-learning is *not* gradient descent!

$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - (r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)))$$

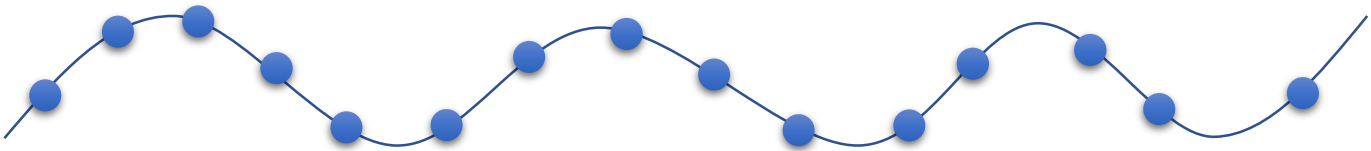
no gradient through target value

Correlated samples in online Q-learning

online Q iteration algorithm:

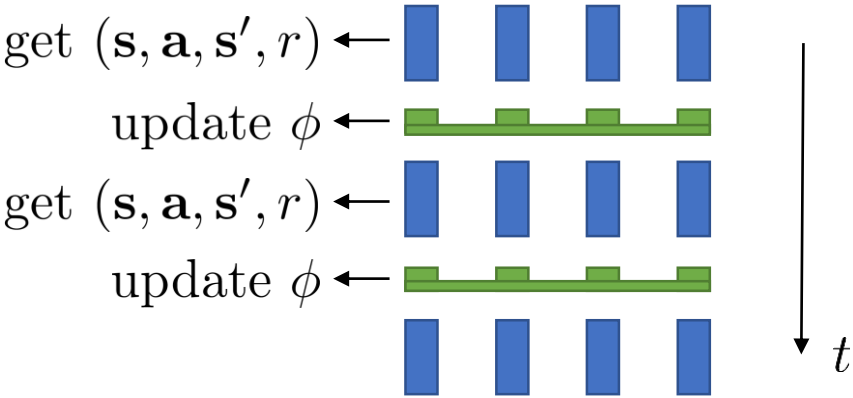
1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

- sequential states are strongly correlated
- target value is always changing

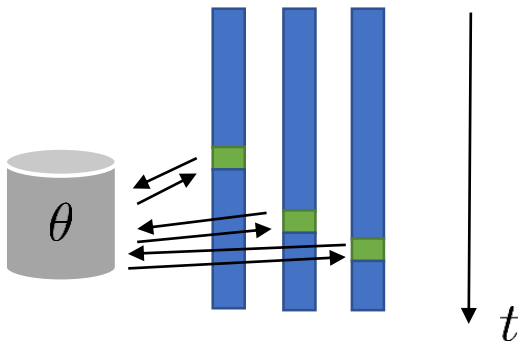


overfitting in local regions is highly likely.

synchronized parallel Q-learning




asynchronous parallel Q-learning




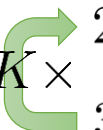
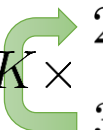
Another solution: replay buffers

online Q iteration algorithm:

special case with $K = 1$, and one gradient step

- 
1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
 2. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}')])$

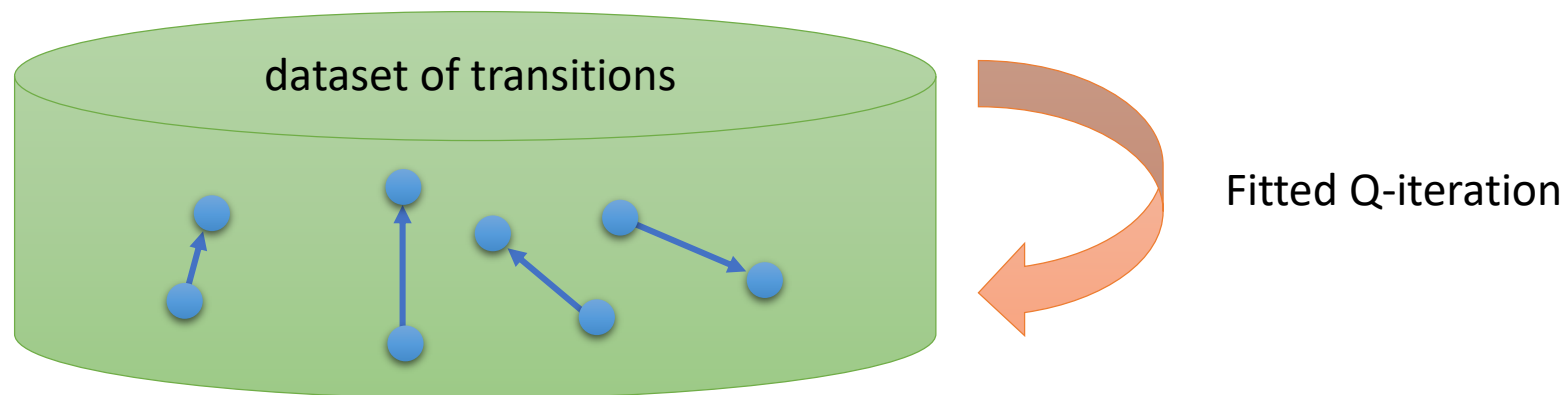
full fitted Q-iteration algorithm:

- 
- ~~1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy~~
 -  2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}')$
 - $K \times$  3. set $\phi \leftarrow \arg \min_{\phi} \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

any policy will work! (with broad support)

just load data from a buffer here

still use one gradient step



Another solution: replay buffers

Q-learning with a replay buffer:

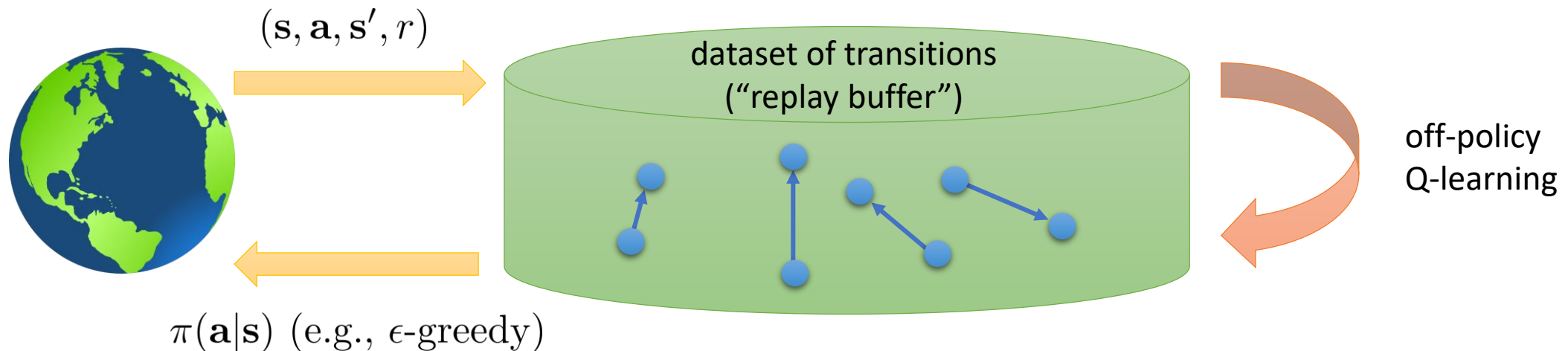
1. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
2. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i) (Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

+ samples are no longer correlated

+ multiple samples in the batch (low-variance gradient)

but where does the data come from?

need to periodically feed the replay buffer...

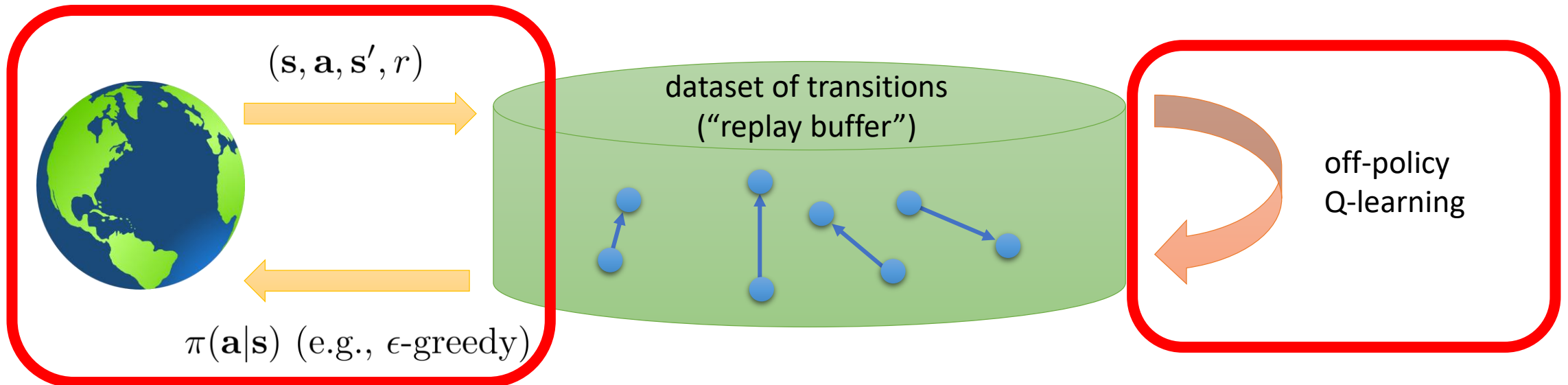


Putting it together

full Q-learning with replay buffer:

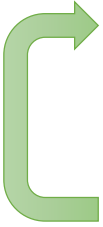
1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to \mathcal{B}
2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

**K = 1 is common, though
larger K more efficient**



What's wrong?

online Q iteration algorithm:

- 
1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$
 2. $\mathbf{y}_i = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
 3. $\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i)$
- ~~these are correlated!~~
use replay buffer

Q-learning is *not* gradient descent!

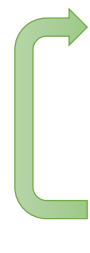
$$\phi \leftarrow \phi - \alpha \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - (r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)))$$

no gradient through target value

This is still a problem!


Q-Learning and Regression

full Q-learning with replay buffer:

- 
1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to \mathcal{B}
 2. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
 3. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i) (Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)])$

one gradient step, moving target

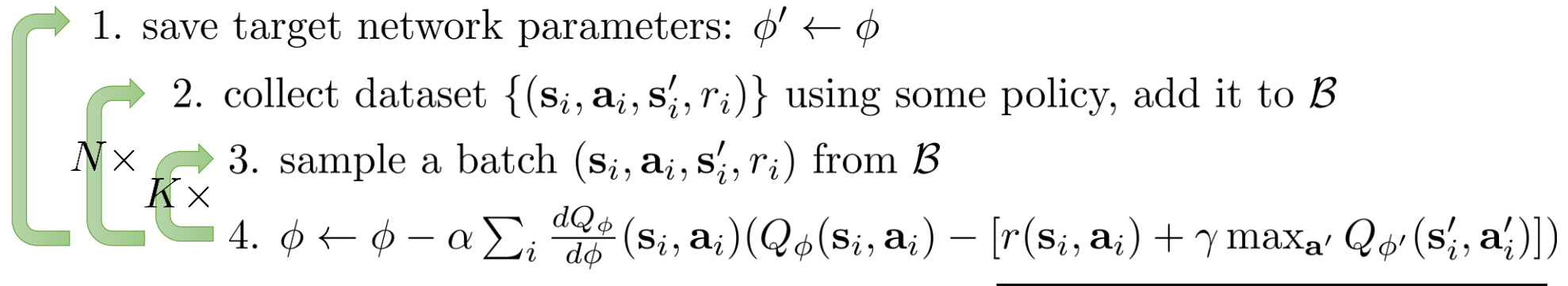
full fitted Q-iteration algorithm:

- 
1. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy
 2. set $\mathbf{y}_i \leftarrow r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'_i} Q_\phi(\mathbf{s}'_i, \mathbf{a}'_i)$
 3. set $\phi \leftarrow \arg \min_\phi \frac{1}{2} \sum_i \|Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \mathbf{y}_i\|^2$

perfectly well-defined, stable regression

Q-Learning with target networks

Q-learning with replay buffer and target network:

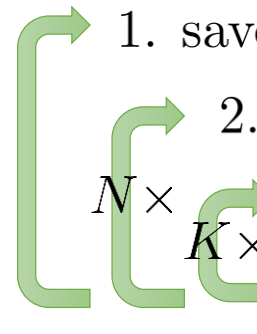
- 
1. save target network parameters: $\phi' \leftarrow \phi$
 2. collect dataset $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add it to \mathcal{B}
 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i) (Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - \underbrace{[r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])}_{\text{targets don't change in inner loop!}})$

supervised regression

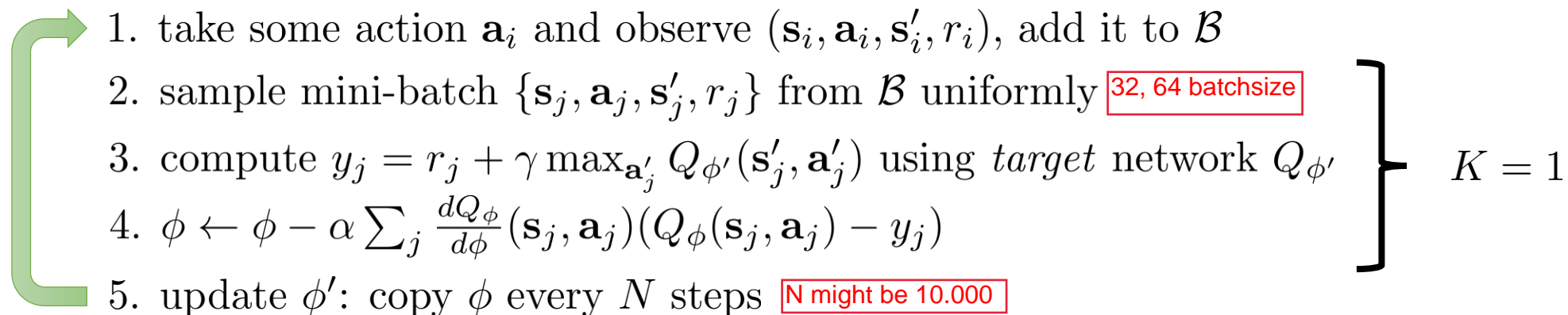
slow process, e.g.
Initilize $Q = 0$
Every 10.000 steps sample and fit a batch
1st 10.000 steps only memorizing rewards
2nd 10.000 steps memorizing rewards plus first value
Trading of speed for better convergence
can be understood as a damper

“Classic” deep Q-learning algorithm (DQN)

Q-learning with replay buffer and target network:

- 
1. save target network parameters: $\phi' \leftarrow \phi$
 2. collect dataset $\{(s_i, a_i, s'_i, r_i)\}$ using some policy, add it to \mathcal{B}
 3. sample a batch (s_i, a_i, s'_i, r_i) from \mathcal{B}
 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(s_i, a_i)(Q_\phi(s_i, a_i) - [r(s_i, a_i) + \gamma \max_{a'} Q_{\phi'}(s'_i, a'_i)])$

“classic” deep Q-learning algorithm:

- 
1. take some action a_i and observe (s_i, a_i, s'_i, r_i) , add it to \mathcal{B}
 2. sample mini-batch $\{s_j, a_j, s'_j, r_j\}$ from \mathcal{B} uniformly 32, 64 batchsize
 3. compute $y_j = r_j + \gamma \max_{a'_j} Q_{\phi'}(s'_j, a'_j)$ using *target* network $Q_{\phi'}$
 4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(s_j, a_j)(Q_\phi(s_j, a_j) - y_j)$
 5. update ϕ' : copy ϕ every N steps N might be 10.000

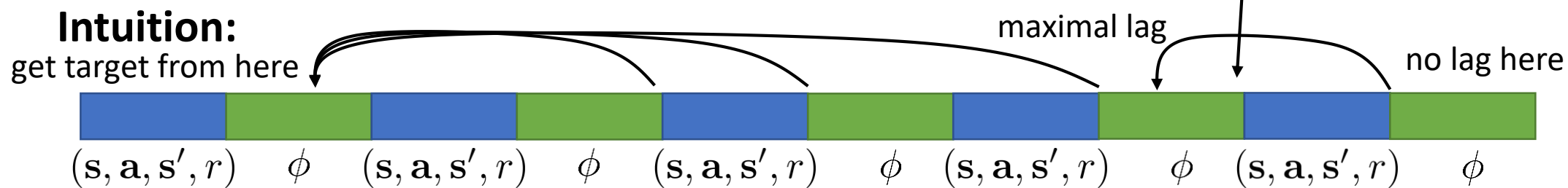
High N can lead to more stable algorithm and convergence
Low N can lead to faster learning process but instability rises

You'll implement this in HW3!

Alternative target network

“classic” deep Q-learning algorithm:

1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to \mathcal{B}
2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from \mathcal{B} uniformly
3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$ using *target* network $Q_{\phi'}$
4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
5. update ϕ'



Feels weirdly uneven, can we always have the same lag?

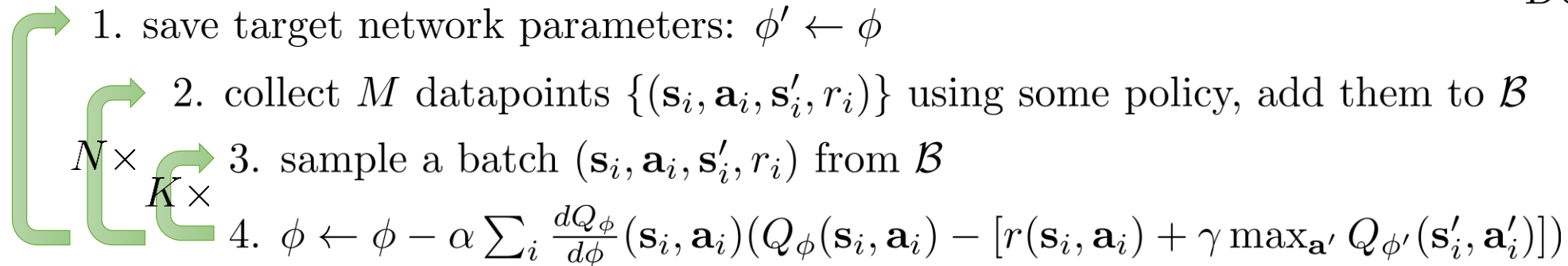
Popular alternative (similar to Polyak averaging):

5. update ϕ' : $\phi' \leftarrow \tau \phi' + (1 - \tau)\phi$ $\tau = 0.999$ works well

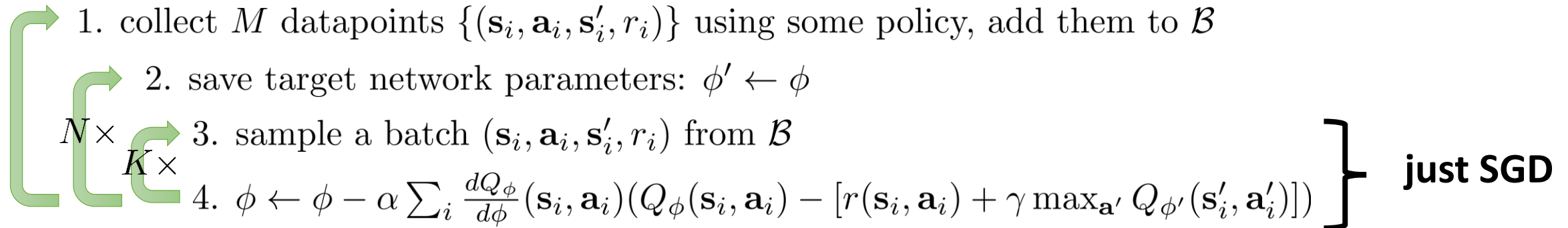
Fitted Q-iteration and Q-learning

Q-learning with replay buffer and target network:

DQN: $N = 1, K = 1$

- 
1. save target network parameters: $\phi' \leftarrow \phi$
 2. collect M datapoints $\{(s_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to \mathcal{B}
 - $N \times$ 3. sample a batch $(s_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
 - $K \times$ 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(s_i, \mathbf{a}_i)(Q_\phi(s_i, \mathbf{a}_i) - [r(s_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$

Fitted Q-learning (written similarly as above):

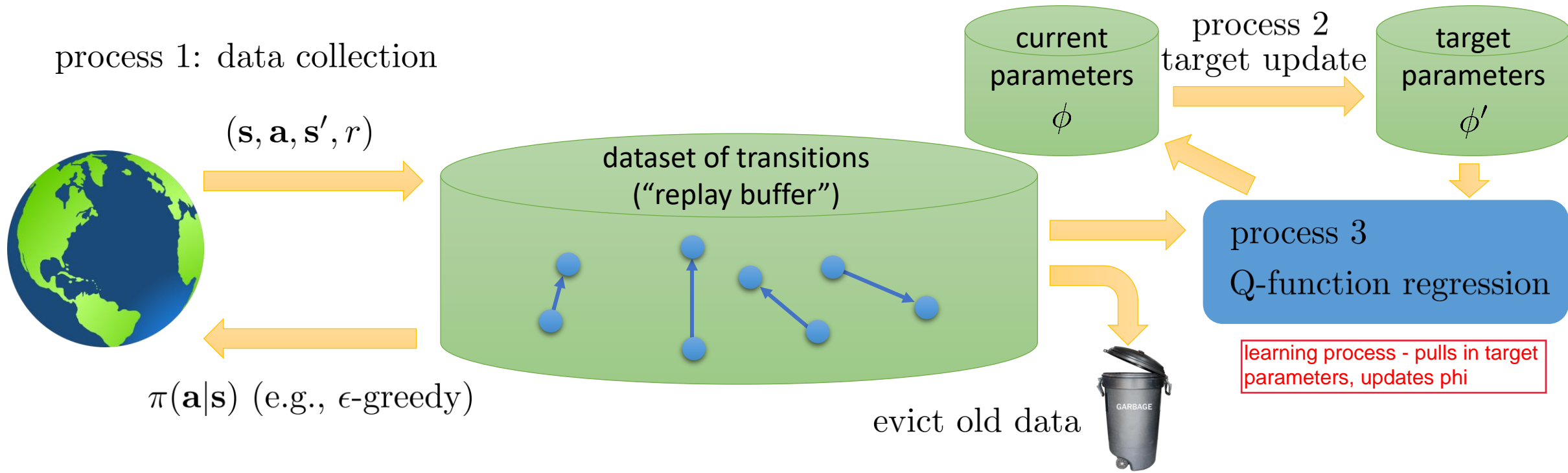
- 
1. collect M datapoints $\{(s_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to \mathcal{B}
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 - $N \times$ 3. sample a batch $(s_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
 - $K \times$ 4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(s_i, \mathbf{a}_i)(Q_\phi(s_i, \mathbf{a}_i) - [r(s_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$
- } just SGD

A more general view

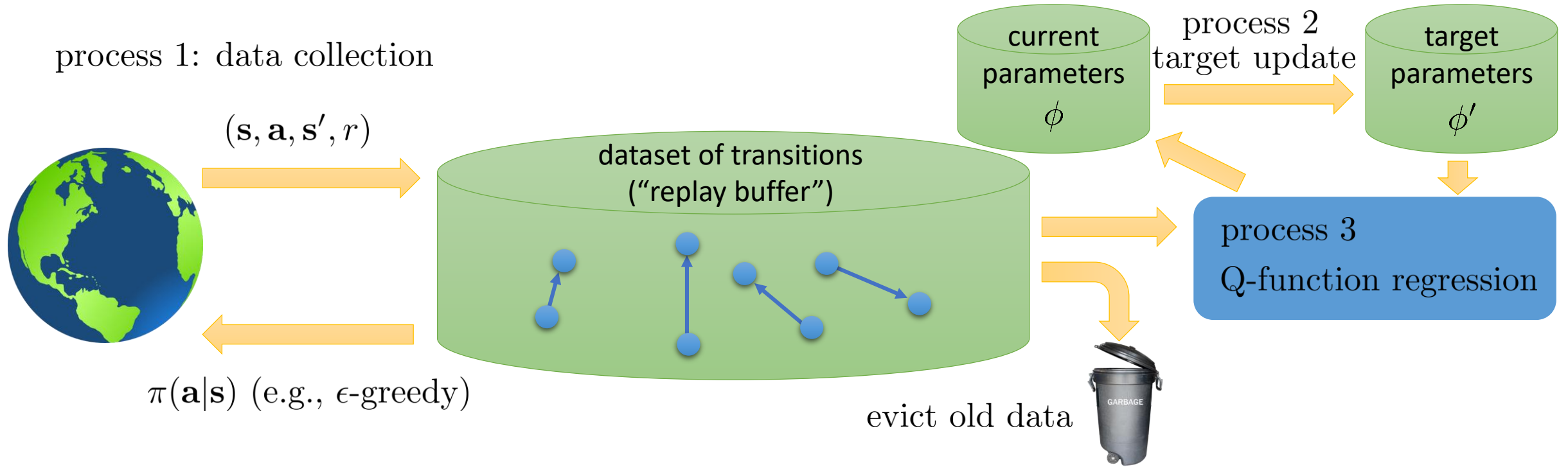
Q-learning with replay buffer and target network:

1. save target network parameters: $\phi' \leftarrow \phi$
2. collect M datapoints $\{(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)\}$ using some policy, add them to \mathcal{B}
- $N \times$
 $K \times$ 3. sample a batch $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$ from \mathcal{B}
4. $\phi \leftarrow \phi - \alpha \sum_i \frac{dQ_\phi}{d\phi}(\mathbf{s}_i, \mathbf{a}_i)(Q_\phi(\mathbf{s}_i, \mathbf{a}_i) - [r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}'_i, \mathbf{a}'_i)])$ low frequency

process 1: data collection



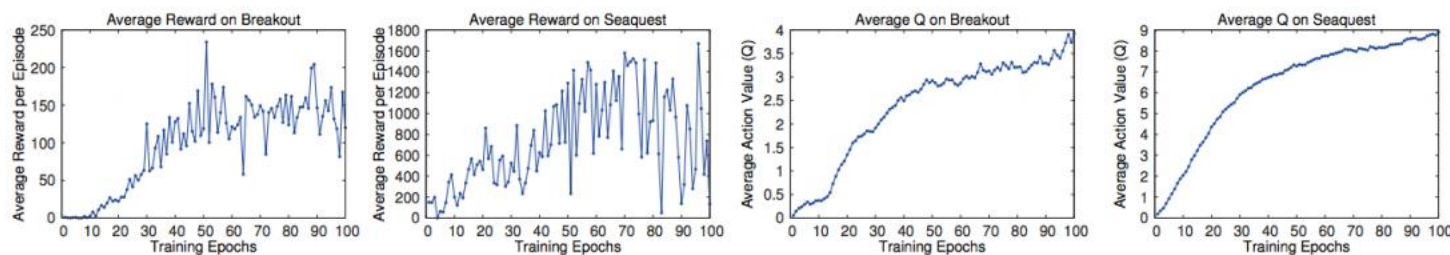
A more general view



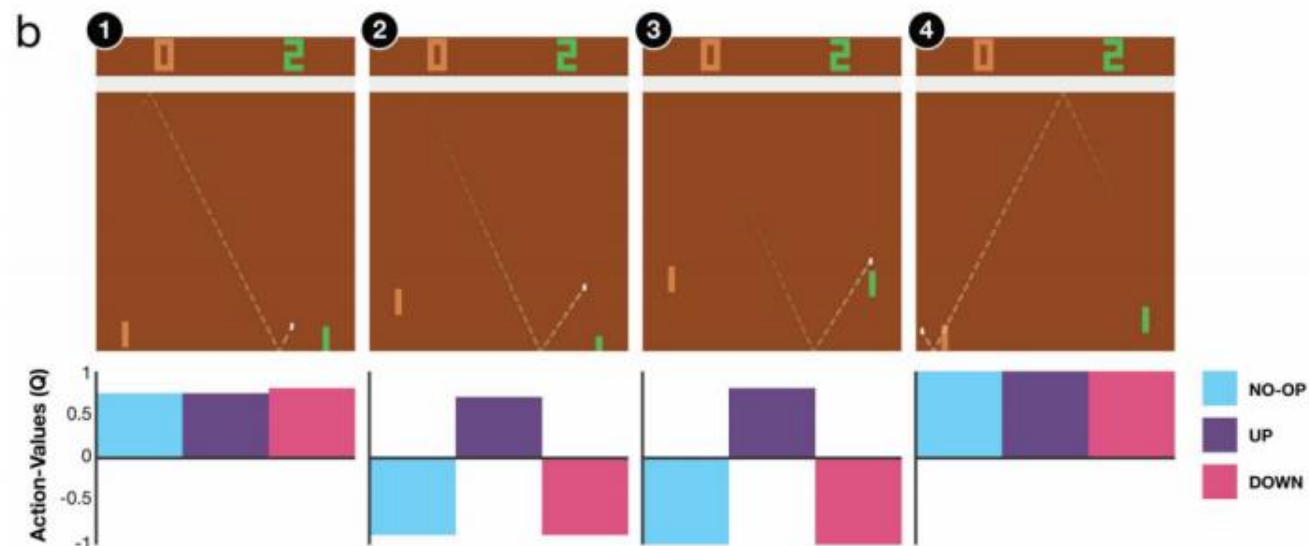
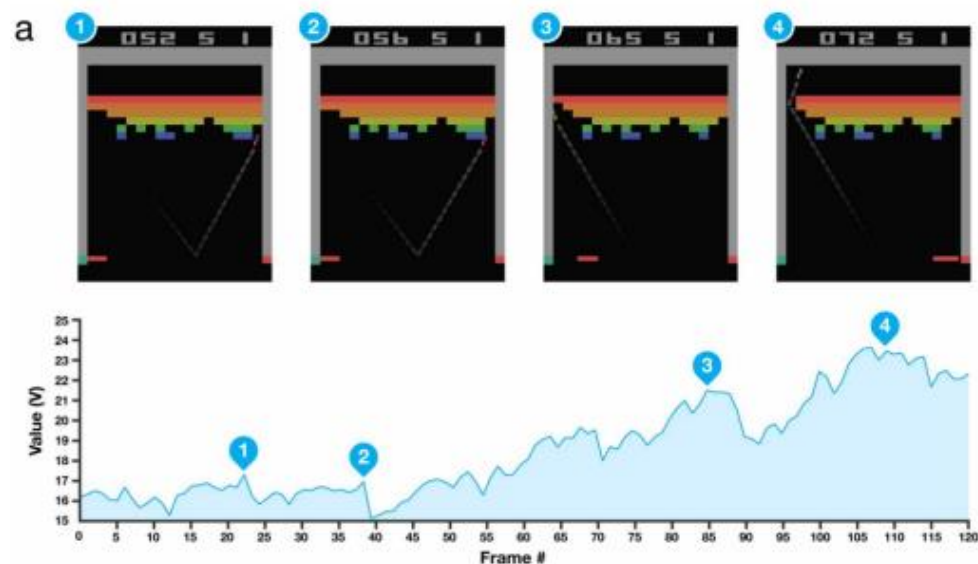
- Online Q-learning (last lecture): evict immediately, process 1, process 2, and process 3 all run at the same speed
- DQN: process 1 and process 3 run at the same speed, process 2 is slow
- Fitted Q-iteration: process 3 in the inner loop of process 2, which is in the inner loop of process 1

Break

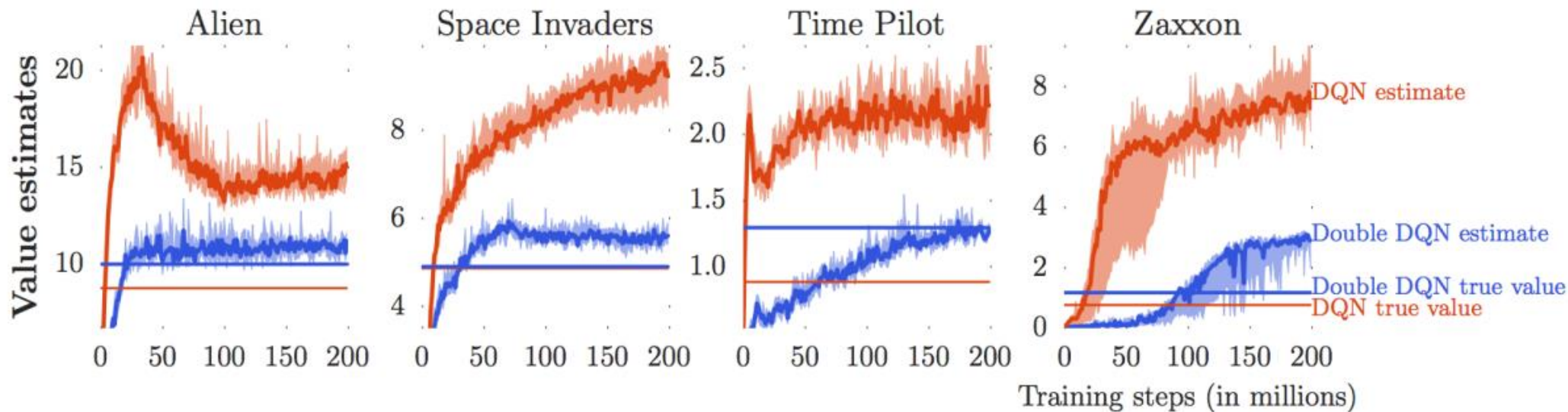
Are the Q-values accurate?



As predicted Q increases, so does the return



Are the Q-values accurate?



Q-fn overestimating the reward...

Overestimation in Q-learning

target value $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$

← this last term is the problem

max over actions causes overestimation

imagine we have two random variables: X_1 and X_2

amplifying the noise here

$$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$$

$Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ is not perfect – it looks “noisy”

hence $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}')$ *overestimates* the next value!

note that $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}') = \underline{Q_{\phi'}(\mathbf{s}', \arg \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}'))}$

value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

Double Q-learning

$$E[\max(X_1, X_2)] \geq \max(E[X_1], E[X_2])$$

note that $\max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}') = \underline{Q_{\phi'}(\mathbf{s}', \arg \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}'))}$

value *also* comes from $Q_{\phi'}$ action selected according to $Q_{\phi'}$

if the noise in these is decorrelated, the problem goes away!

idea: don't use the same network to choose the action and evaluate value!

“double” Q-learning: use two networks:

$$Q_{\phi_A}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_B}(\mathbf{s}', \arg \max_{\mathbf{a}'} Q_{\phi_A}(\mathbf{s}'))$$

$$Q_{\phi_B}(\mathbf{s}, \mathbf{a}) \leftarrow r + \gamma Q_{\phi_A}(\mathbf{s}', \arg \max_{\mathbf{a}'} Q_{\phi_B}(\mathbf{s}'))$$

noise is no longer amplified

if the two Q's are noisy in *different* ways, there is no problem

Double Q-learning in practice

where to get two Q-functions?

just use the current and target networks!

standard Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg \max_{\mathbf{a}'} Q_{\phi'}(\mathbf{s}', \mathbf{a}'))$

double Q-learning: $y = r + \gamma Q_{\phi'}(\mathbf{s}', \arg \max_{\mathbf{a}'} Q_{\phi}(\mathbf{s}', \mathbf{a}'))$

just use current network (not target network) to evaluate action

still use target network to evaluate value!

decorrelation cannot be proven
but in practice it works

Multi-step returns

Q-learning target: $y_{j,t} = r_{j,t} + \gamma \max_{\mathbf{a}_{j,t+1}} Q_{\phi'}(\mathbf{s}_{j,t+1}, \mathbf{a}_{j,t+1})$

Q-fn is fit well already

these are the only values that matter if $Q_{\phi'}$ is bad! these values are important if $Q_{\phi'}$ is good

where does the signal come from?

Q-learning does this: max bias, min variance

remember this?

Actor-critic:
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left(r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t+1}) - \hat{V}_{\phi}^{\pi}(\mathbf{s}_{i,t}) \right)$$

+ lower variance (due to critic)
- not unbiased (if the critic is not perfect)

Policy gradient:
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left(\left(\sum_{t'=t}^T \gamma^{t'-t} r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'}) \right) - b \right)$$

+ no bias
- higher variance (because single-sample estimate)

can we construct multi-step targets, like in actor-critic?

$$y_{j,t} = \sum_{t'=t}^{t+N-1} r_{j,t'} + \gamma^N \max_{\mathbf{a}_{j,t+N}} Q_{\phi'}(\mathbf{s}_{j,t+N}, \mathbf{a}_{j,t+N})$$

N -step return estimator

Q-learning with N-step returns

$$y_{j,t} = \sum_{t'=t}^{t'+N-1} r_{j,t'} + \gamma^N \max_{\mathbf{a}_{j,t+N}} Q_{\phi'}(\mathbf{s}_{j,t+N}, \mathbf{a}_{j,t+N})$$

this is supposed to estimate $Q^\pi(\mathbf{s}_{j,t}, \mathbf{a}_{j,t})$ for π

$$\pi(\mathbf{a}_t | \mathbf{s}_t) = \begin{cases} 1 & \text{if } \mathbf{a}_t = \arg \max_{\mathbf{a}_t} Q_\phi(\mathbf{s}_t, \mathbf{a}_t) \\ 0 & \text{otherwise} \end{cases}$$

+ less biased target values when Q-values are inaccurate

+ typically faster learning, especially early on

- only actually correct when learning on-policy

why?

we need transitions $\mathbf{s}_{j,t'}, \mathbf{a}_{j,t'}, \mathbf{s}_{j,t'+1}$ to come from π for $t' - t < N - 1$

(not an issue when $N = 1$)

how to fix?

- ignore the problem
 - often works very well
- cut the trace – dynamically choose N to get only on-policy data
 - works well when data mostly on-policy, and action space is small
- importance sampling

For more details, see: “Safe and efficient off-policy reinforcement learning.” Munos et al. ‘16

Q-learning with continuous actions

What's the problem with continuous actions?

$$\pi(\mathbf{a}_t | \mathbf{s}_t) = \begin{cases} 1 & \text{if } \mathbf{a}_t = \arg \max_{\mathbf{a}_t} Q_{\phi}(\mathbf{s}_t, \mathbf{a}_t) \\ 0 & \text{otherwise} \end{cases}$$

this max

$$\text{target value } y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mathbf{a}'_j)$$

this max
particularly problematic (inner loop of training)

How do we perform the max?

Option 1: optimization

- gradient based optimization (e.g., SGD) a bit slow in the inner loop
- action space typically low-dimensional – what about stochastic optimization?

Q-learning with stochastic optimization

Simple solution:

$$\max_{\mathbf{a}} Q(\mathbf{s}, \mathbf{a}) \approx \max \{Q(\mathbf{s}, \mathbf{a}_1), \dots, Q(\mathbf{s}, \mathbf{a}_N)\}$$

$(\mathbf{a}_1, \dots, \mathbf{a}_N)$ sampled from some distribution (e.g., uniform)

+ dead simple

+ efficiently parallelizable

- not very accurate

but... do we care? How good does the target need to be anyway?

More accurate solution:

**works OK, for up to about 40
dimensions**

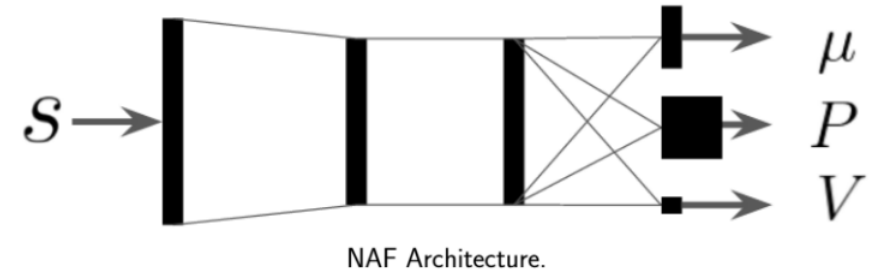
- cross-entropy method (CEM)
 - simple iterative stochastic optimization
- CMA-ES
 - substantially less simple iterative stochastic optimization

Easily maximizable Q-functions

Option 2: use function class that is easy to optimize

$$Q_{\phi}(\mathbf{s}, \mathbf{a}) = -\frac{1}{2}(\mathbf{a} - \mu_{\phi}(\mathbf{s}))^T P_{\phi}(\mathbf{s})(\mathbf{a} - \mu_{\phi}(\mathbf{s})) + V_{\phi}(\mathbf{s})$$

vector μ ,
scalar value V ,
matrix P



NAF: Normalized Advantage Functions

$$\arg \max_{\mathbf{a}} Q_{\phi}(\mathbf{s}, \mathbf{a}) = \mu_{\phi}(\mathbf{s}) \quad \max_{\mathbf{a}} Q_{\phi}(\mathbf{s}, \mathbf{a}) = V_{\phi}(\mathbf{s})$$

- + no change to algorithm
- + just as efficient as Q-learning
- loses representational power

Q-learning with continuous actions

Option 3: learn an approximate maximizer

DDPG (Lillicrap et al., ICLR 2016)

“deterministic” actor-critic
(really approximate Q-learning)

$$\max_{\mathbf{a}} Q_{\phi}(\mathbf{s}, \mathbf{a}) = Q_{\phi}(\mathbf{s}, \arg \max_{\mathbf{a}} Q_{\phi}(\mathbf{s}, \mathbf{a}))$$

idea: train another network $\mu_{\theta}(\mathbf{s})$ such that $\mu_{\theta}(\mathbf{s}) \approx \arg \max_{\mathbf{a}} Q_{\phi}(\mathbf{s}, \mathbf{a})$

how? just solve $\theta \leftarrow \arg \max_{\theta} Q_{\phi}(\mathbf{s}, \mu_{\theta}(\mathbf{s}))$

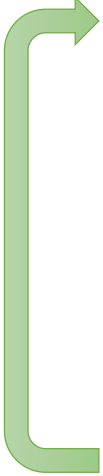
$$\frac{dQ_{\phi}}{d\theta} = \frac{d\mathbf{a}}{d\theta} \frac{dQ_{\phi}}{d\mathbf{a}}$$

new target $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mu_{\theta}(\mathbf{s}'_j))$

Q-learning with continuous actions

Option 3: learn an approximate maximizer

DDPG:

- 
1. take some action \mathbf{a}_i and observe $(\mathbf{s}_i, \mathbf{a}_i, \mathbf{s}'_i, r_i)$, add it to \mathcal{B}
 2. sample mini-batch $\{\mathbf{s}_j, \mathbf{a}_j, \mathbf{s}'_j, r_j\}$ from \mathcal{B} uniformly
 3. compute $y_j = r_j + \gamma \max_{\mathbf{a}'_j} Q_{\phi'}(\mathbf{s}'_j, \mu_{\theta'}(\mathbf{s}'_j))$ using *target* nets $Q_{\phi'}$ and $\mu_{\theta'}$
 4. $\phi \leftarrow \phi - \alpha \sum_j \frac{dQ_\phi}{d\phi}(\mathbf{s}_j, \mathbf{a}_j)(Q_\phi(\mathbf{s}_j, \mathbf{a}_j) - y_j)$
 5. $\theta \leftarrow \theta + \beta \sum_j \frac{d\mu}{d\theta}(\mathbf{s}_j) \frac{dQ_\phi}{d\mathbf{a}}(\mathbf{s}_j, \mathbf{a})$
 6. update ϕ' and θ' (e.g., Polyak averaging)

Simple practical tips for Q-learning

- Q-learning takes some care to stabilize
 - Test on easy, reliable tasks first, make sure your implementation is correct

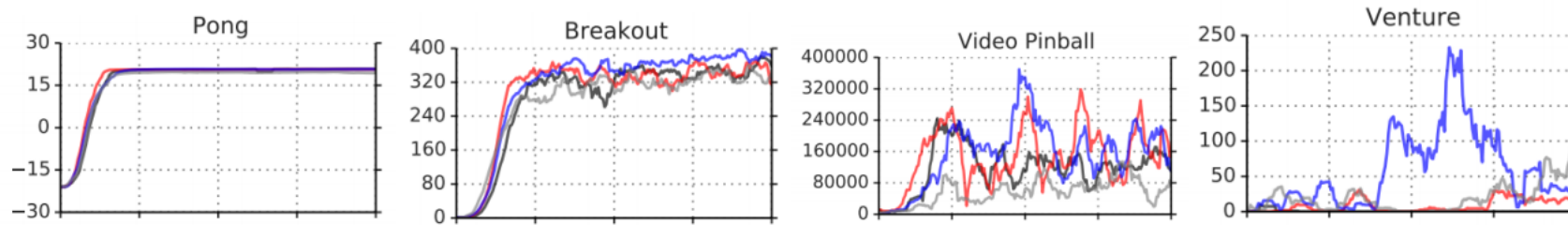


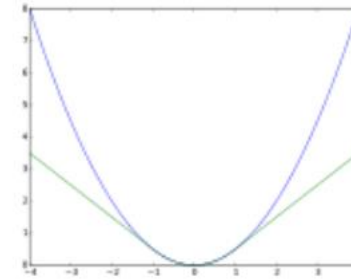
Figure: From T. Schaul, J. Quan, I. Antonoglou, and D. Silver. “Prioritized experience replay”. *arXiv preprint arXiv:1511.05952* (2015), Figure 7

- Large replay buffers help improve stability
 - Looks more like fitted Q-iteration
- It takes time, be patient – might be no better than random for a while
- Start with high exploration (epsilon) and gradually reduce

Advanced tips for Q-learning

- Bellman error gradients can be big; clip gradients or use Huber loss

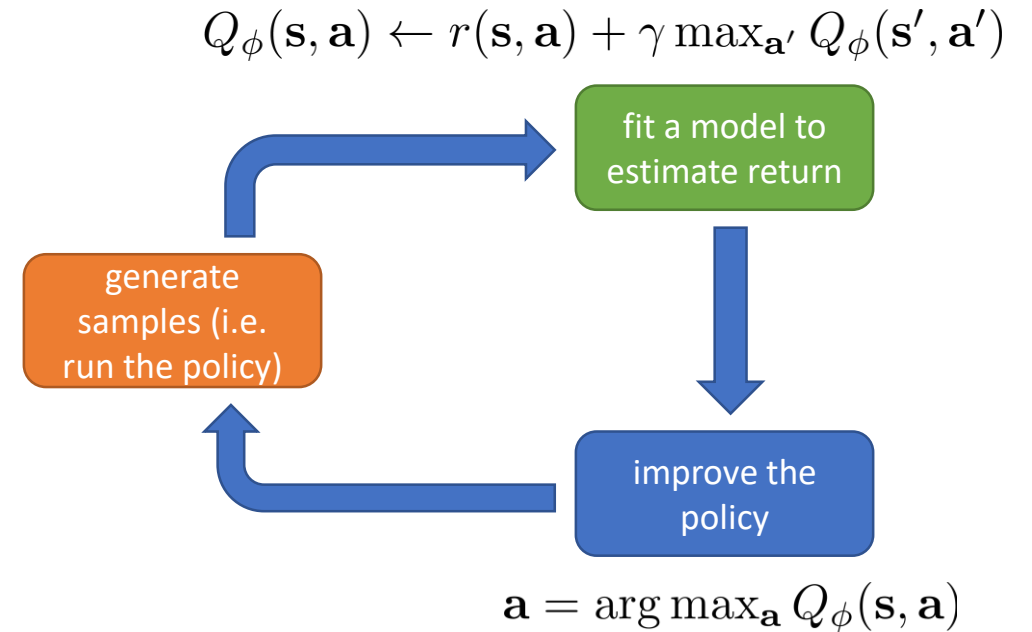
$$L(x) = \begin{cases} x^2/2 & \text{if } |x| \leq \delta \\ \delta|x| - \delta^2/2 & \text{otherwise} \end{cases}$$



- Double Q-learning helps *a lot* in practice, simple and no downsides
- N-step returns also help a lot, but have some downsides
- Schedule exploration (high to low) and learning rates (high to low), Adam optimizer can help too
- Run multiple random seeds, it's very inconsistent between runs

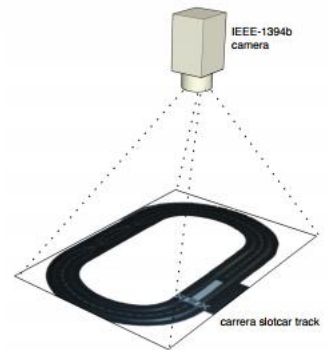
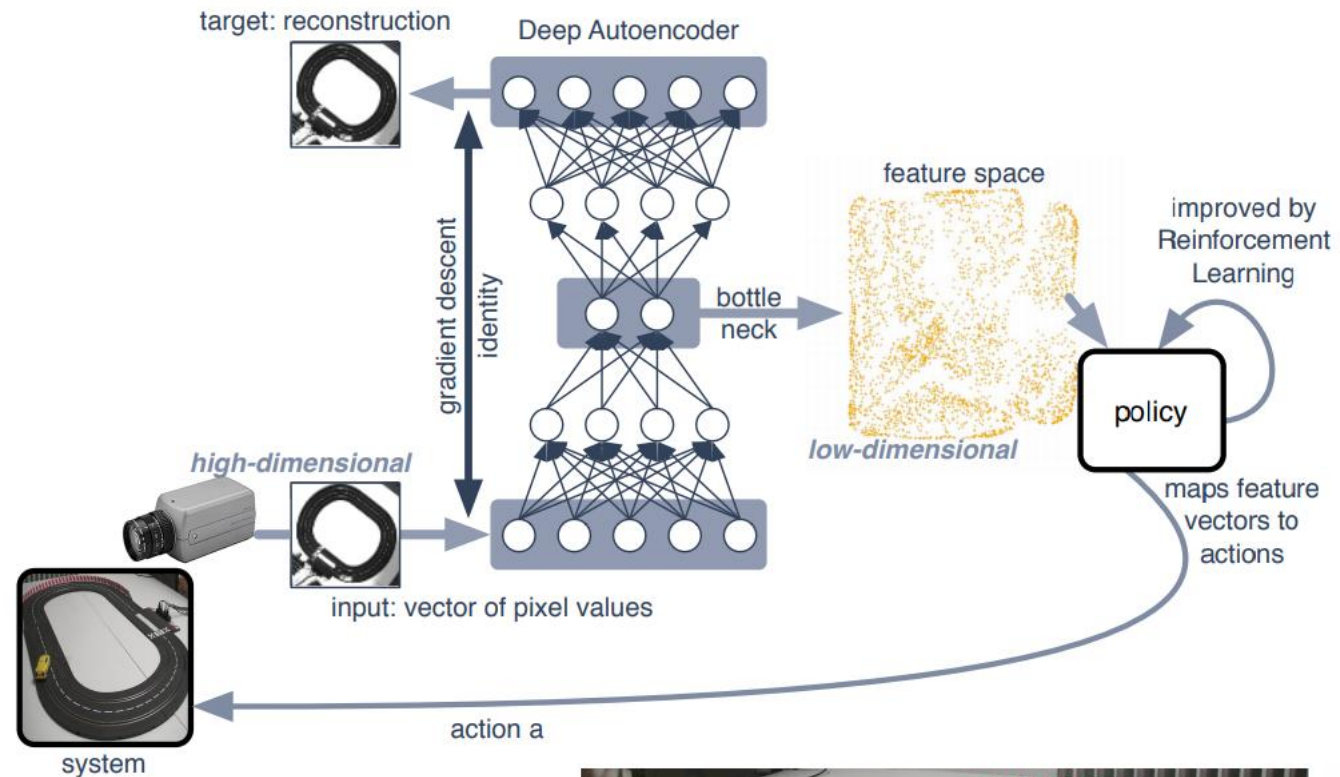
Review

- Q-learning in practice
 - Replay buffers
 - Target networks
- Generalized fitted Q-iteration
- Double Q-learning
- Multi-step Q-learning
- Q-learning with continuous actions
 - Random sampling
 - Analytic optimization
 - Second “actor” network



Fitted Q-iteration in a latent space

- “Autonomous reinforcement learning from raw visual data,” Lange & Riedmiller ‘12
- Q-learning on top of latent space learned with autoencoder
- Uses fitted Q-iteration
- Extra random trees for function approximation (but neural net for embedding)



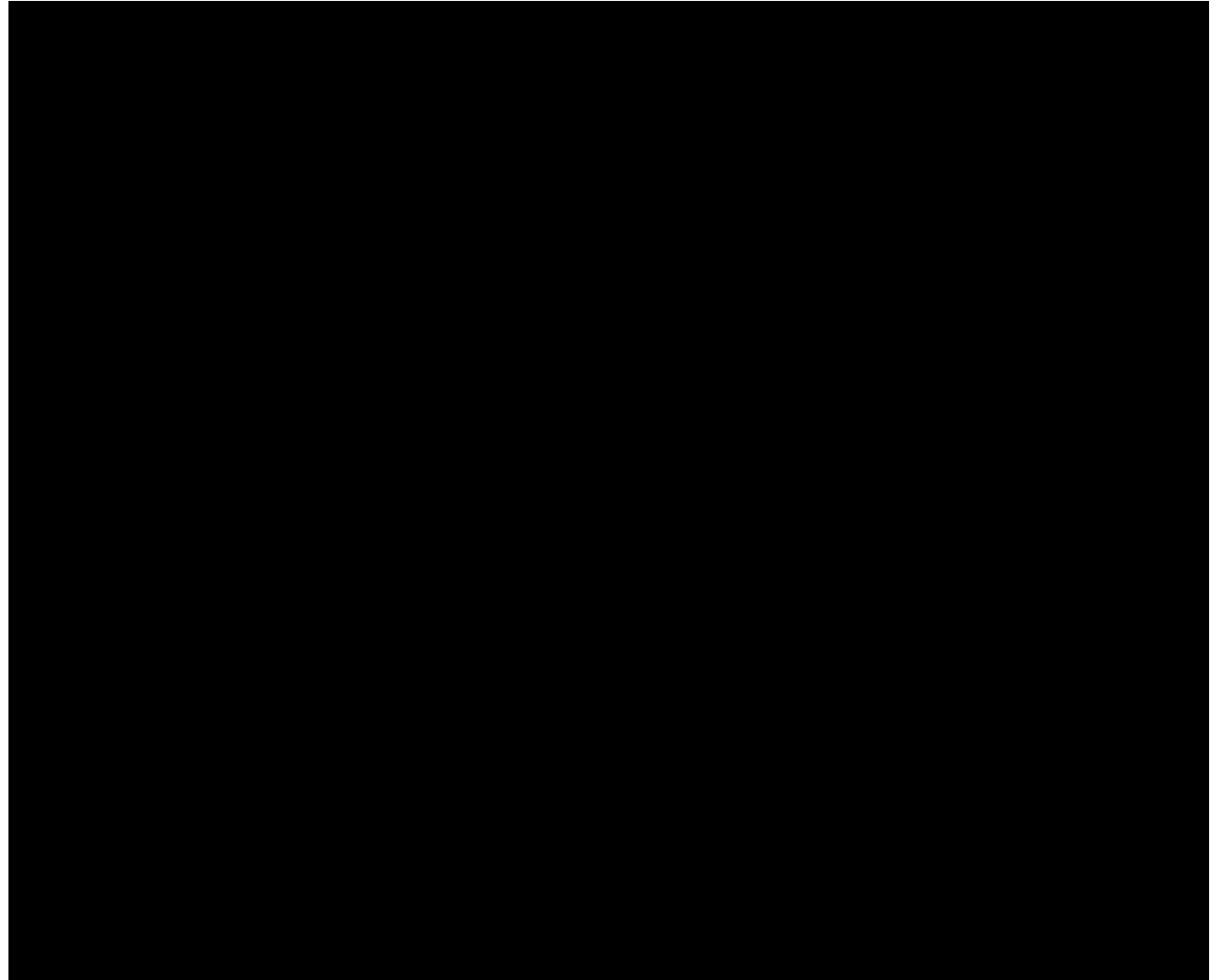
Q-learning with convolutional networks

- “Human-level control through deep reinforcement learning,” Mnih et al. ‘13
- Q-learning with convolutional networks
- Uses replay buffer and target network
- One-step backup
- One gradient step
- Can be improved a lot with double Q-learning (and other tricks)



Q-learning with continuous actions

- “Continuous control with deep reinforcement learning,” Lillicrap et al. ‘15
- Continuous actions with maximizer network
- Uses replay buffer and target network (with Polyak averaging)
- One-step backup
- One gradient step per simulator step



Q-learning on a real robot

- “Robotic manipulation with deep reinforcement learning and ...,” Gu*, Holly*, et al. ‘17
- Continuous actions with NAF (quadratic in actions)
- Uses replay buffer and target network
- One-step backup
- Four gradient steps per simulator step for efficiency
- Parallelized across multiple robots



Q-learning suggested readings

- Classic papers
 - Watkins. (1989). Learning from delayed rewards: introduces Q-learning
 - Riedmiller. (2005). Neural fitted Q-iteration: batch-mode Q-learning with neural networks
- Deep reinforcement learning Q-learning papers
 - Lange, Riedmiller. (2010). Deep auto-encoder neural networks in reinforcement learning: early image-based Q-learning method using autoencoders to construct embeddings
 - Mnih et al. (2013). Human-level control through deep reinforcement learning: Q-learning with convolutional networks for playing Atari.
 - Van Hasselt, Guez, Silver. (2015). Deep reinforcement learning with double Q-learning: a very effective trick to improve performance of deep Q-learning.
 - Lillicrap et al. (2016). Continuous control with deep reinforcement learning: continuous Q-learning with actor network for approximate maximization.
 - Gu, Lillicrap, Stuskever, L. (2016). Continuous deep Q-learning with model-based acceleration: continuous Q-learning with action-quadratic value functions.
 - Wang, Schaul, Hessel, van Hasselt, Lanctot, de Freitas (2016). Dueling network architectures for deep reinforcement learning: separates value and advantage estimation in Q-function.