AP Calculus BC 2025

1 Multiple Choice Questions

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}(3+5x^2)^{1/4} =$

2. if $f'(x) = 6x^2 - 2x + 5$ and $f(1) = -3$, then $f(2) =$

3. The function f is given by $f(x) = \dfrac{\sin x}{x}$. What is the instantaneous rate of change of f at $x = \frac{\pi}{2}$?

4. The table shown gives the speed $s(t)$, in miles per hour, of a car at selected times t, in hours.

t (hours)	1	1.25	1.5	2.25	3	3.5	4
$s(t)$ (mph)	55	50	65	70	55	60	55

   Using a midpoint Riemann sum with subintervals $[1, 1.5]$, $[1.5, 3]$, and $[3, 4]$, what is the approximate distance, in miles, the car traveled from time $t = 1$ to time $t = 4$?

5. Let f be the function defined as shown:

$$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 \le x \le 1 \\ x + 1 & \text{if } x > 1 \end{cases}$$

   Which of the following statements is true?

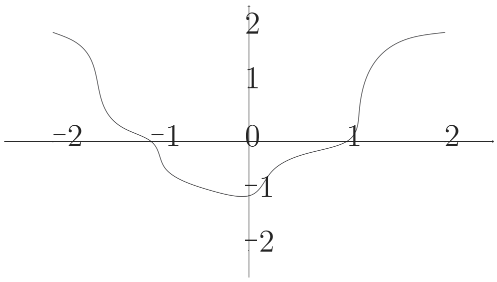
   A. f is continuous and differentiable for all x.

   B. f is continuous for all x and differentiable except at $x = 1$.

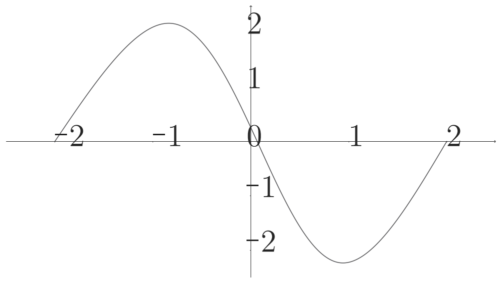
   C. f is continuous for all x and differentiable except at $x = 0$ and $x = 1$.

   D. f is continuous except at $x = 1$.

6. Let f be a differentiable function that is decreasing on the interval $(-1, 1)$ and increasing on the intervals $(-2, -1)$ and $(1, 2)$. Which of the following could be the graph of f', the derivative of f, on the interval $(-2, 2)$?



(A)



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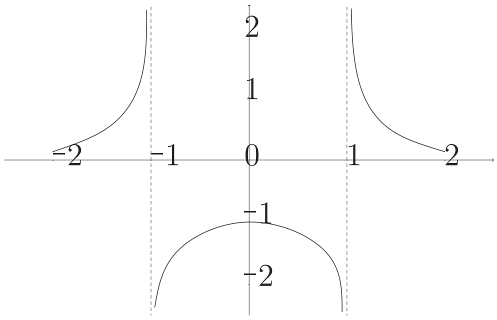

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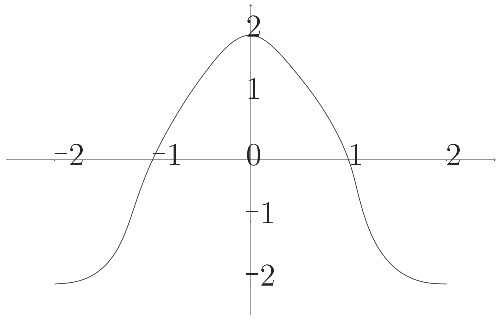


(D)

7. For $t > 0$, a parametric curve in the xy-plane is defined by $x(t) = t^2 - 4t + 5$ and $y(t) = t^2 - 6t$. For what value of t does the curve have a vertical tangent?

8. Let f be a function with second derivative given by $f''(x) = (x-1)^2(x-2)(x-3)^3$. How many points of inflection does the graph of f have?

9. If at any time t the position vector of a particle moving in the xy-plane is $\langle \sin(\pi t), \cos(\pi t)\rangle$, what is the acceleration vector of the particle at time $t = \frac{1}{2}$?

10. What are all values of p for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{4p-1}}$ converges?

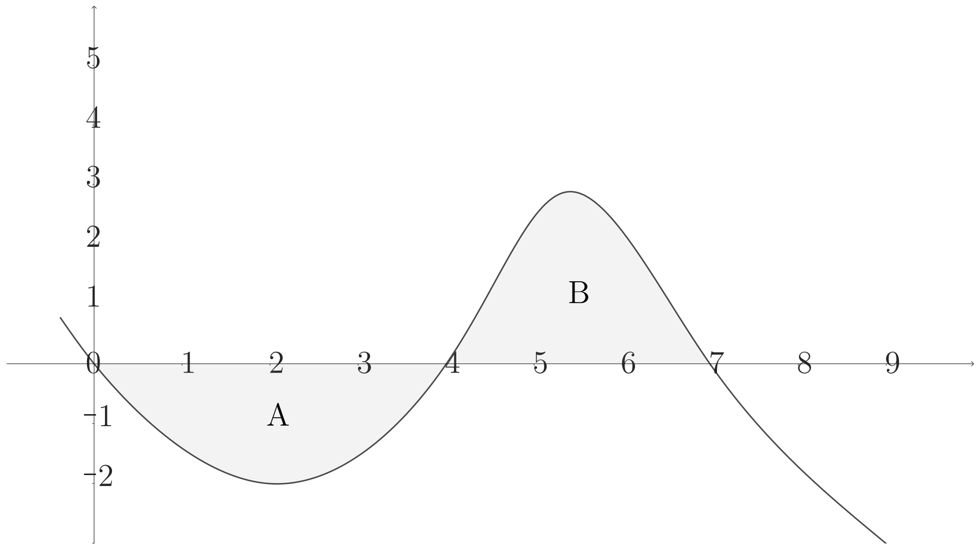
11. If 10 people have heard the news at time $t = 0$, how many people have heard the news when the rate at which the news is spreading changes from increasing to decreasing?

    The number of people at a business who have heard a piece of news at time t days is modeled by the function P. The rate at which the news is spreading is modeled by the logistic differential equation $\dfrac{\mathrm{d}P}{\mathrm{d}t} = 144P\left(1 - \dfrac{P}{4800}\right)$.

12. If $x^2 - xy - 3y^2 = 6$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} =$

13. What is the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3nx^n}{(n+2)^2}$?

14. If the area of each region is 3, what is the value of $\displaystyle\int_0^4 f(x)\,dx + \int_0^7 f(x)\,dx$?



15. If $y = \sin^{-1}(x-1)$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} =$

16. Which of the following statements about the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n}{n^3+3n^2-1}$ is true?

    A. The series converges because $\displaystyle\lim_{n\to\infty} \frac{n^2+n}{n^3+3n^2-1} = 0$.

    B. The series converges by limit comparison to the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$.

    C. The series diverges by the ratio test.

    D. The series diverges by limit comparison to the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

17. Which of the following satisfies the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(\dfrac{\pi}{2}x^2\right)$ with the initial condition $y(0) = 2$?

    A. $y = 2 + \sin\left(\dfrac{\pi}{2}x^2\right)$

    B. $y = 2 + \dfrac{1}{\pi x}\sin\left(\dfrac{\pi}{2}x^2\right)$

    C. $y = \displaystyle\int_2^x \cos\left(\frac{\pi}{2}t^2\right)\, dt$

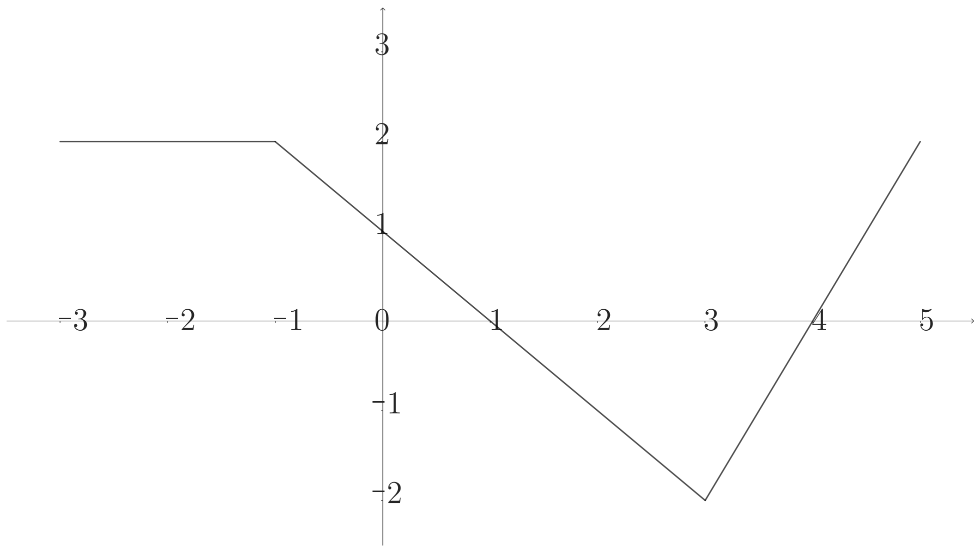
    D. $y = 2 + \displaystyle\int_0^x \cos\left(\frac{\pi}{2}t^2\right)\, dt$

18. The second and third terms of the geometric series $\displaystyle\sum_{n=1}^{\infty} a_n$ are $a_2 = -\frac{3}{4}$ and $a_3 = \frac{1}{4}$, respectively. What is the sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$?

19. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = 3y - 4x$ with initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler’ s method is used, starting at $x = 1$ with a step size of 0.5?

20. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$. At what value of x in the closed interval $[-3, 5]$ does g have an absolute minimum?

The graph of the function f on the closed interval $[-3, 5]$ is shown.



21.

$$\int_2^3 \frac{4}{(x+3)(x-1)}\,dx =$$

22. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = 3x^2y$ with initial condition $f(0) = 6$. Which of the following is an expression for $f(x)$?

A. e^{x^3}

B. $e^{x^3} + 5$

C. $e^{x^3} + 6$

D. $6e^{x^3}$

23. The line $y = 2$ is a horizontal asymptote to the graph of which of the following functions?

A. $y = \frac{x + 2\sin x}{x - 2}$

B. $y = \frac{2x^2 + \sin x}{x^2}$

C. $y = \frac{2x^2 + \sin x}{4 - x^2}$

D. $y = \frac{2^x + 2\sin x}{2^x}$

24. What is the perimeter of the region enclosed by the graphs of $y = 3$ and $y = x^2 - 2x$?

25. If F is an antiderivative of a continuous function f, then $\int_{-1}^{2} f(3x+4)\,dx =$

26.

$$\lim_{x\to\frac{\pi}{4}} \frac{\displaystyle\int_{\pi/4}^{x} \tan(t)\,dt}{x^2 - \frac{\pi^2}{16}} =$$

27.

$$\int \frac{x^2+4x-1}{x+2}\,dx =$$

28.

$$\int_1^{\infty} xe^{-x}\,dx =$$

29. What is the slope of the line tangent to the polar curve $r = \theta^2$ at the point where $\theta = \pi$?

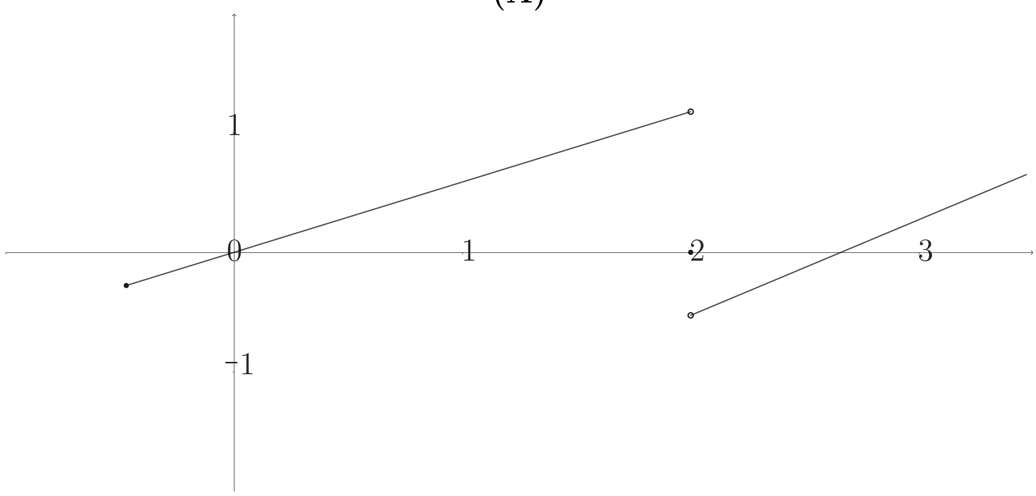
30. What is the sum of the series shown?

$$1 - \frac{\left(\frac{\pi}{3}\right)^2}{2!} + \frac{\left(\frac{\pi}{3}\right)^4}{4!} - \frac{\left(\frac{\pi}{3}\right)^6}{6!} + \cdots + \frac{(-1)^n\left(\frac{\pi}{3}\right)^{2n}}{(2n)!} + \cdots$$

2 Multiple Choice Questions (Graphing Calculator Allowed)

1. Which of the following is the graph of a function f such that $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ both exist, but $\lim_{x\to 2} f(x)$ does not exist?

(A)



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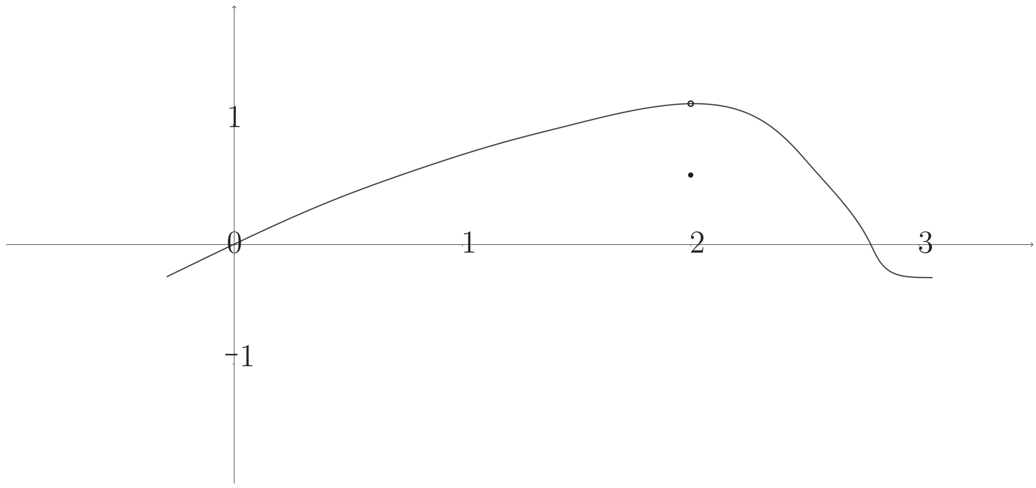

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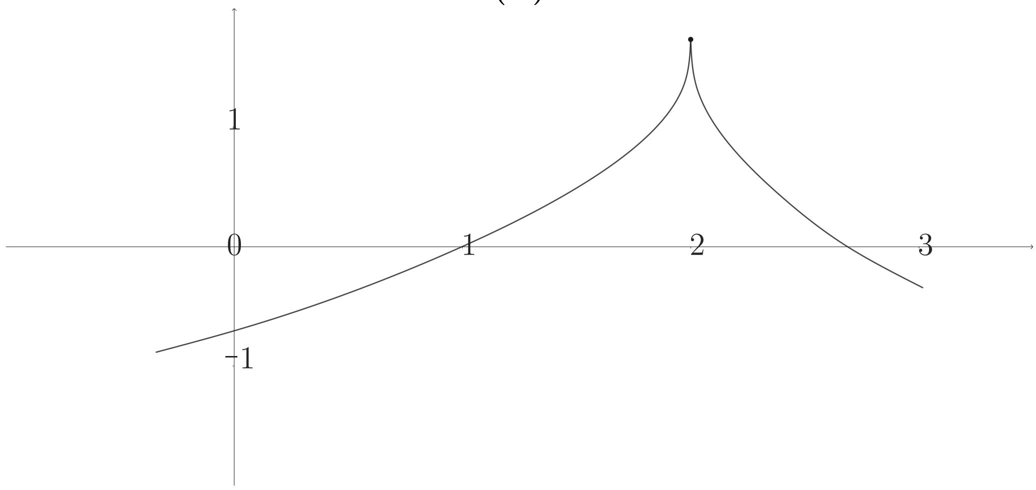

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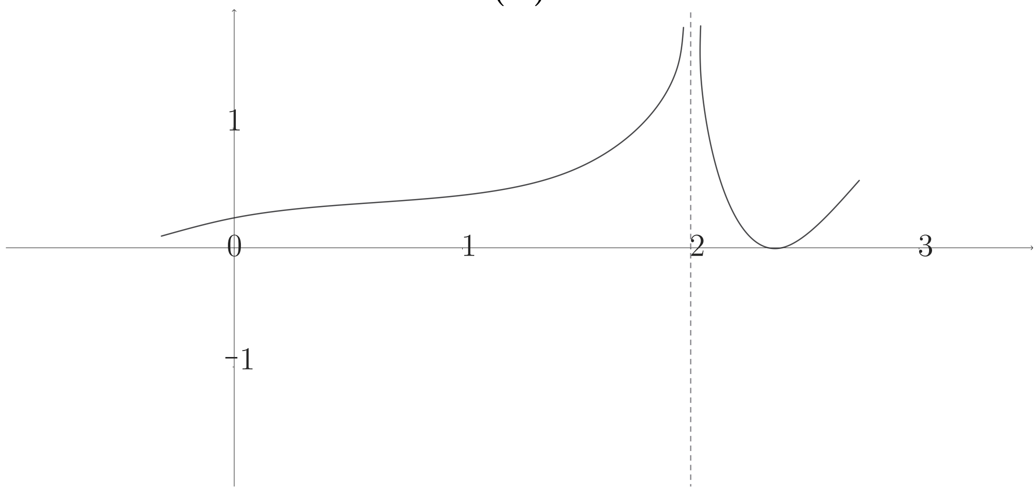
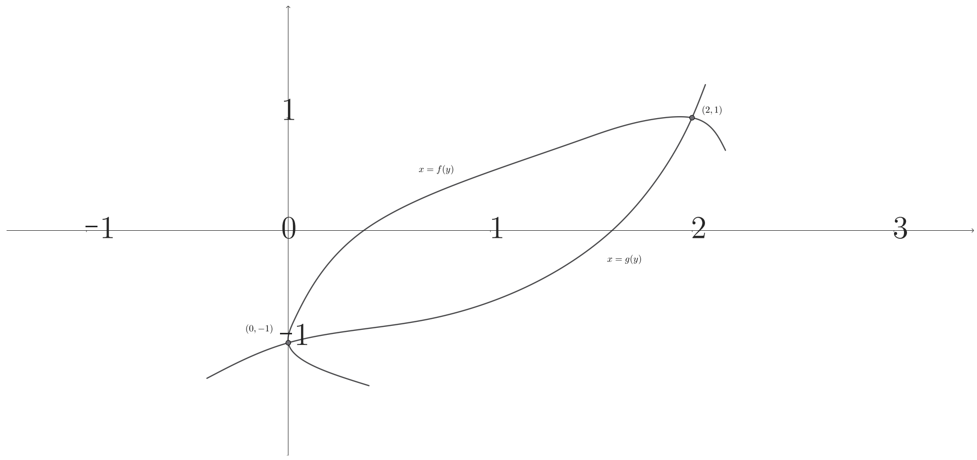


2. Which of the following gives the area of R?

   Let R be the region bounded on the left by the graph of $x = f(y)$ and on the right by the graph of $x = g(y)$, as shaded in the figure.



A. $\displaystyle\int_{-1}^{1} (f(y) - g(y))\, dy$

B. $\displaystyle\int_{-1}^{1} (g(y) - f(y))\, dy$

C. $\displaystyle\int_{0}^{2} (f(y) - g(y))\, dy$

D. $\displaystyle\int_{0}^{2} (g(y) - f(y))\, dy$

3. Let f be a function that is continuous on the closed interval $[0, 2]$ with $f(0) = -2$, $f(1) = 5$, and $f(2) = 0$. Which of the following statements must be true?

   A. The maximum value of f on the closed interval $[0, 2]$ is 5.

   B. There is a number c, $0 < c < 2$, with $f(c) = -1$.

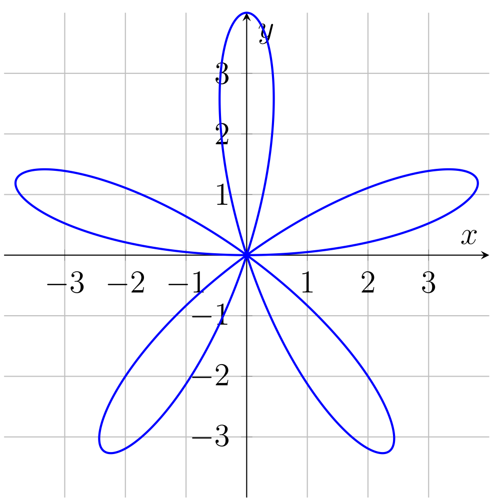
   C. There is a number c, $0 < c < 2$, with $f(c) = 7$.

   D. There is a number c, $0 < c < 2$, with $f'(c) = 1$.

4. Let f be the function with first derivative defined by $f'(x) = 3\cos(e^{2x})$ on the interval $0 < x < 1.5$. What is the smallest value of x at which f has a relative minimum on $0 < x < 1.5$?

5. What is the total area of the region enclosed by the curve?

   The figure shows the polar curve $r = 3\sin(5\theta)$ for $0 \leq \theta \leq \pi$.



6. Consider the series $\sum_{n=1}^{\infty} a_n$, where $a_n > 0$ for all n. Which of the following statements must be true?

   A. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   B. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

   C. If $\lim_{n\to\infty} a_n = \frac{1}{2}$, then $\sum_{n=1}^{\infty} a_n$ converges.

   D. If $\lim_{n\to\infty} a_n = \frac{1}{2}$, then $\sum_{n=1}^{\infty} a_n$ diverges.

7. Which of the following equations describes the situation?

   The rate at which people register for a new social-networking site is given by $g(t)$, where t is measured in minutes and $g(t)$ is measured in people per minute. The average rate of registration during the first 60 minutes is equal to the rate of registration at time $t = 30$ minutes.

   A. $\int_0^{60} g(t)\,dt = g'(30)$

   B. $\int_0^{60} g'(t)\,dt = g'(30)$

   C. $\frac{1}{60}\int_0^{60} g(t)\,dt = g(30)$

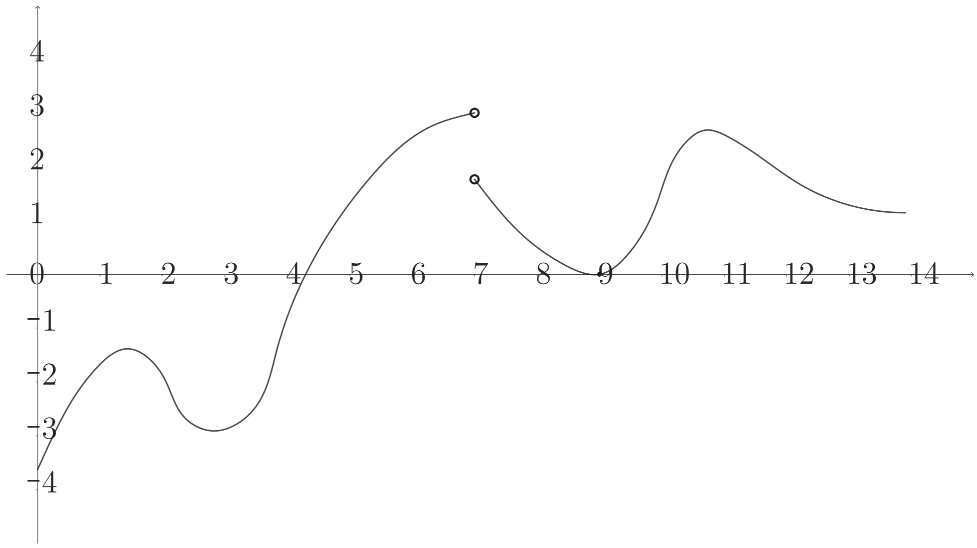
   D. $\frac{1}{60}\int_0^{60} g'(t)\,dt = g(30)$

8. At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \langle \sin(2t),\, e^t \rangle$. What is the total distance the particle travels over the interval $0 \leq t \leq 1$?

9. What is the volume of the solid formed when the region bounded by the graphs of $f(x) = \frac{1}{2}x^2 - x + 1$ and $g(x) = 5\sin\left(\frac{x}{3}\right)$ is revolved about the x-axis?

10. How many critical points does the function f have on the interval $0 < x < 14$?

    The function f is continuous on $0 < x < 14$. Shown in the figure is the graph of f', the derivative of f, on that interval. The graph of f' is tangent to the x-axis at $x = 9$.



11. A particle moves along a straight line with velocity given by $v(t) = t^2 - 3t + e^{-t/2}$ and acceleration given by $a(t) = 2t - 3 - \frac{1}{2}e^{-t/2}$ for time $0 \le t \le 4$. On which open intervals is the speed of the particle increasing?

12. Using the third-degree Taylor polynomial for f about $x = 1$, what is the approximation for $f(0.7)$?

    Let f be a function with derivatives of all orders for all values of x. Selected values of f and its first three derivatives are given in the table:

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	0.2	0	0	6
1	1	2.2	3	-2.3

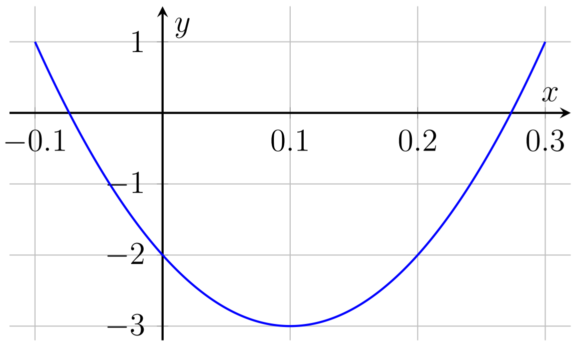
13. If $\displaystyle\int_a^b 2f(x)\,dx + \int_c^b f(x)\,dx = 5$ and $\displaystyle\int_a^c f(x)\,dx = 1$, then $\displaystyle\int_a^b f(x)\,dx =$

14. At what rate is the volume of the cylinder changing, in cubic inches per minute, at that instant?

    The volume of a cylinder with radius r inches is given by $V = \pi r^2\big(4r + \sin(0.2r^2)\big)$. At the instant when the radius is 2.7 inches, the radius is increasing at a rate of 0.75 inch per minute.

15. If $T_3(0.3)$ is used to approximate $f(0.3)$, what is the maximum possible error guaranteed by the Lagrange error bound?

    Let f be a function that has derivatives of all orders for all real numbers, and let $T_3(x)$ be the third-degree Taylor polynomial for f about $x = 0$. The graph of $y = f^{(4)}(x)$, the fourth derivative of f, is shown for $0 \le x \le 0.3$.



3 Free Response Questions (Graphing Calculator Allowed)

1. A tank contains 30 liters of water at time $t = 0$. For time $0 \leq t \leq 20$ minutes, water flows into the tank at a rate modeled by the function $f(t) = 2\ln(t+3) - 1.6$. For time $20 < t \leq 45$ minutes, water flows into the tank at a rate modeled by the function $g(t) = \dfrac{7.5}{\ln t} + 2.2$. Both $f(t)$ and $g(t)$ are measured in liters per minute.

   (a) Find the rate of change of g at time $t = 23$ minutes. Show the setup for your calculations, and indicate units of measure.

   (b) How many liters of water are in the tank at time $t = 20$ minutes? Show the setup for your calculations.

   (c) Let w be the piecewise function

   $$w(t) = \begin{cases} f(t), & 0 \leq t \leq 20, \\ g(t), & 20 < t \leq 45. \end{cases}$$

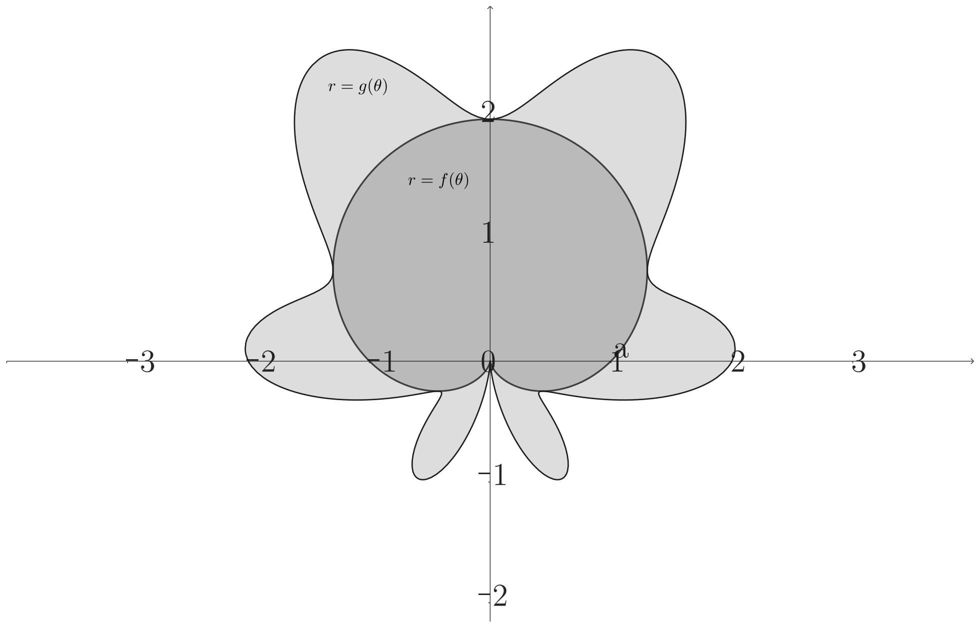
   Find $\lim_{t \to 20^+} w(t)$. Is w continuous at $t = 20$? Use the definition of continuity to justify your answer.

   (d) Consider the function w defined in part C. Find the average value of w from time $t = 15$ minutes to $t = 30$ minutes. Show the setup for your calculations.

2. The graph shows the polar curves $r = f(\theta) = 1 + \sin\theta$ and $r = g(\theta) = 1 + \sin\theta + \cos^2(3\theta)$.

   The derivative of g with respect to θ is $g'(\theta) = \cos\theta - 3\sin(6\theta)$.

   Note: Your calculator should be in radian mode.



(a) Let R be the shaded region that is inside the curve $r = g(\theta)$ with $0 \leq \theta \leq 2\pi$. Find the area of region R. Show the setup for your calculations.

(b) For each θ in the interval $0 \leq \theta \leq 2\pi$, the ray from the origin with angle θ intersects the curve $r = f(\theta)$ and the curve $r = g(\theta)$. For any value of θ, the distance between the curves is the difference between the r-values of the outer curve and the inner curve. Find the average distance between the two curves for $0 \leq \theta \leq 2\pi$. Show the setup for your calculations.

(c) Find the value of θ in the interval $\pi/4 \leq \theta \leq \pi/2$ that corresponds to the point on the curve $r = g(\theta)$ with greatest distance from the origin. Justify your answer.

(d) There is a point P on the curve $r = g(\theta)$ where the slope of the line tangent to the curve at P is equal to $\sqrt{2} - 1$. At point P, the rate of change of the x-coordinate with respect to θ is $\sqrt{2}/2$. Find the rate of change of the y-coordinate with respect to θ at point P. Show the work that leads to your answer.

4 Free Response Questions (Graphing Calculator Not Allowed)

1. Let f be a continuous function defined on the interval $0 \leq x \leq 7$. The function f and its derivatives have the properties indicated in the table shown below, where *DNE* indicates the x-values at which the derivatives of f do not exist.

	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 7$	$x = 7$
$f(x)$	-1	Negative	-4	Negative	0	Negative	-2
$f'(x)$	DNE	Negative	DNE	Positive	0	Positive	DNE
$f''(x)$	DNE	Positive	DNE	Negative	0	Positive	DNE

(a) On what open intervals contained in $0 < x < 7$, if any, is the graph of f both *increasing* and *concave down*? Give a reason for your answer.

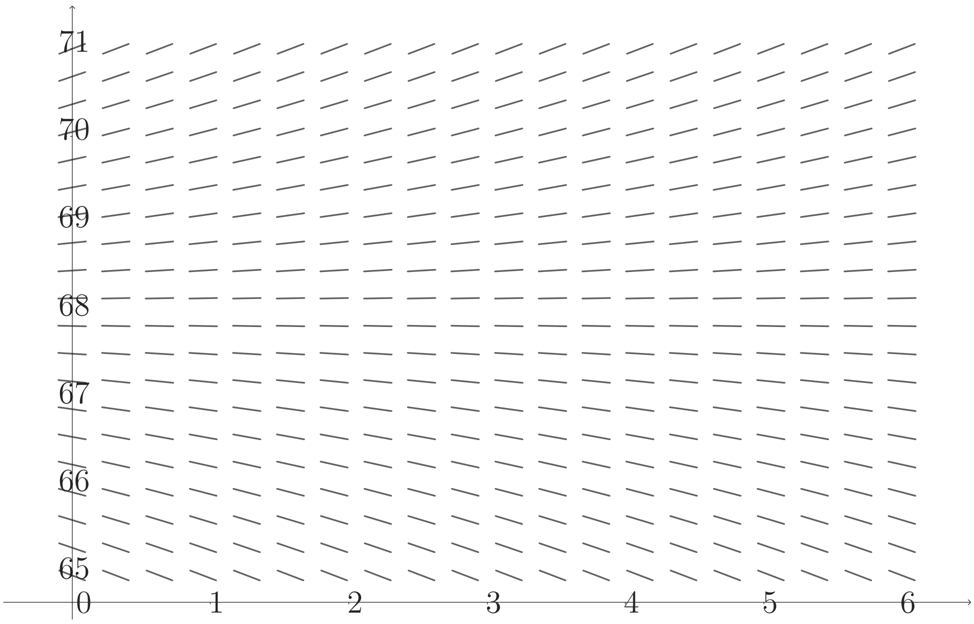
(b) Find the average rate of change of f over the interval $0 \leq x \leq 4$. Can the Mean Value Theorem be applied to f on $0 \leq x \leq 4$ to guarantee a value of c, for $0 < c < 4$, such that $f'(c)$ is equal to this average rate of change? Give a reason for your answer.

(c) Use a *left* Riemann sum with the three sub-intervals indicated by the data in the table to approximate the value of $\int_0^7 f(x)\,dx$.

(d) Let g be the function defined by $g(x) = x \cdot f(x)$. Is g increasing or decreasing at $x = 1$? Give a reason for your answer.

2. For $0 \leq t \leq 120$, the temperature of a bowl of soup at time t is modeled by the function S that satisfies the differential equation $\dfrac{dS}{dt} = -\dfrac{1}{20}\big(S - 68\big)$, where $S(t)$ is measured in degrees Fahrenheit (°F) and t is measured in minutes. The temperature of the bowl of soup is always greater than 68°F and is 148°F at time $t = 0$.

(a) Explain why the following could *not* be a slope field for the differential equation $\dfrac{dS}{dt} = -\frac{1}{20}(S - 68)$.

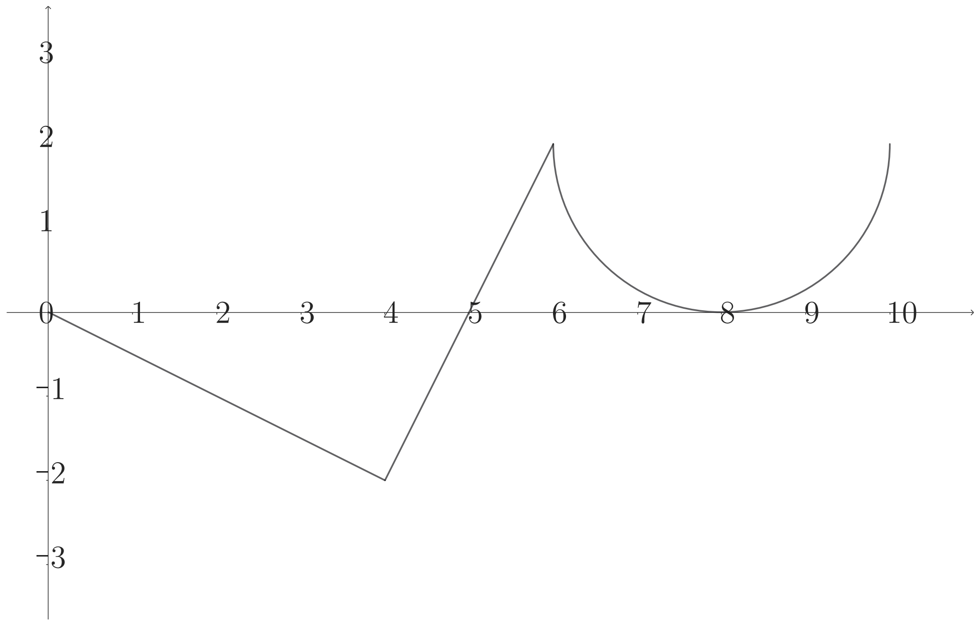


(b) Use the line tangent to the graph of S at $t = 0$ to approximate the temperature of the soup at time $t = 5$ minutes. Show your work.

(c) According to the model, is the approximation found in part B an *overestimate* or an *underestimate* of the temperature of the soup at time $t = 5$ minutes? Give a reason for your answer.

(d) Using separation of variables, find the particular solution $y = S(t)$ to the differential equation $\dfrac{dS}{dt} = -\frac{1}{20}(S - 68)$ with initial condition $S(0) = 148$.

3. Let x be a differentiable function of t for $0 \leq t \leq 10$. The graph of $x'(t)$, the derivative of $x(t)$, consists of two line segments and a semicircle, as shown in the figure below.



(a) For each of $x''(2)$ and $x''(4)$, find the value *or* explain why it does not exist.

(b) On what open intervals contained in $6 < t < 10$, if any, is the graph of $x(t)$ concave down? Give a reason for your answer.

(c) A particle moving in the xy-plane has position $\big(x(t), y(t)\big)$ with velocity vector $\big(x'(t), y'(t)\big)$ for $0 \le t \le 10$, where $x'(t)$ is defined by the given graph and $y'(t) = te^{5t}$. The particle is at position $(5, 1)$ at time $t = 0$. Find the y-coordinate of the particle at time $t = 10$. Show the work that leads to your answer.

(d) For the particle defined in part C, find the *speed* of the particle at time $t = 2$. Show the work that leads to your answer.

4. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^n\, x^{2n}}{(2n+1)!} = 1 - \frac{2}{3!}x^2 + \frac{4}{5!}x^4 - \cdots + \frac{(-1)^n\, 2^n\, x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x for which the series converges.

(a) Use the Ratio Test to determine the interval of convergence of the power series for f.

(b) Determine whether f has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

(c) The first two non-zero terms of the power series for f are used to approximate $f\big(\frac{1}{2}\big)$. Use the Alternating Series Error Bound to determine an upper bound on the error of the approximation.

(d) Write the first three non-zero terms and the general term for an infinite series that represents $\displaystyle\int_0^x f(t)\,dt$.