Solutions and Notes to Michael Spivak's Calculus

Community

1 Basic Properties of Numbers

1.1

Prove the following.

1.1.1

If ax = a for some number $a \neq 0$, then x = 1.

$$ax = a$$
$$x = \frac{a}{a}$$
$$x = 1$$

1.1.2

$$x^{2} - y^{2} = (x - y)(x + y)$$

$$x^{2} - y^{2} = x^{2} + xy - xy - y^{2}$$

$$x^{2} - y^{2} = x^{2} - y^{2}$$

1.2

What is wrong with the following "proof"? Let x = y.

$$(x+y)(x-y) = y(x-y)$$
$$x+y=y$$

We divide by (x-y), which given x=y equals 0. We cannot divide by zero.