

New analytical models to describe bar resonant zones in galactic discs

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Introduction

Thanks to the recent increases in the quality and quantity of the kinematic data, as well as on the methods used to analyse it, we now have a precise measurement of each moving group across a large volume around the Sun (e.g., Ramos et al. 2018, Bernet et al. 2022, 2023 – **do not miss his talk on Friday afternoon!!**).

These moving groups appear as over-densities in velocity space and most of them are thought to be caused by the bar and the resonances it induces on the stellar disc.

Despite having measured the gradients in R and ϕ for most moving groups, we still lack the theoretical tools to extract the parameters of the bar and the rotation curve encoded within them.



Analytical Model

The **goal** is to describe analytically the region of phase-space where stars are susceptible of being affected by the resonant induced by the bar.

First, we define the **resonant condition** as usual:

$$\frac{\Omega - \Omega_{bar}}{\kappa} = b$$

To find the position in phase-space of the stars that are in the resonance zone, we need the **equations of motion**. We take the improved epicyclic approximation (Dehnen 1999) because it accounts for the coupling between the radial action (J_R) and the angular momentum (L_z).

$$R(\eta) = R_g [1 - e \cos(\eta)]^{\frac{1}{\gamma}}, \quad e = \sqrt{1 - \left(\frac{L_z}{L_{circ}} \right)^2},$$

where $\eta \approx kt$ represents the phase along the orbit, and e is the eccentricity.

When we impose the resonant condition on these orbits, we obtain the equation that relates the properties of the Galaxy ($v_c \equiv$ rot. curve) with the **observable phase-space coordinates of the stars in the resonance zone**:

$$V_\phi^{res}(R, V_R) = \frac{L_z^{res}}{R} \sqrt{2 \left(\frac{R}{r_{res}} \right)^{\frac{2}{\gamma}} - \left(\frac{R}{r_{res}} \right)^{\frac{4}{\gamma}} \left[1 + \left(\frac{V_R}{v_c(R)} \right)^2 \right]}.$$

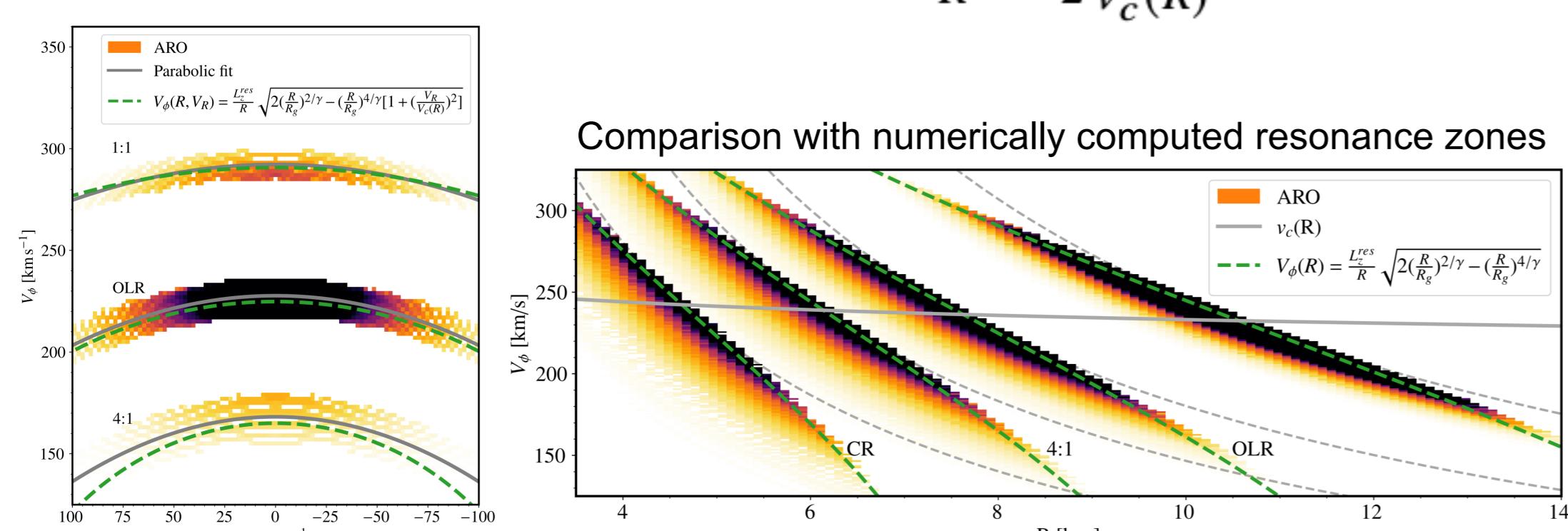
We refer to this equation as the *Analytical Simplified Envelope* (ASE) and it presents a couple of interesting properties:

A) The maximum slope of the ridges ($R - V_\phi$ over-densities, see Antoja+18) is *almost* independent of the gravitational potential

$$\frac{\delta V_\phi^{res}(R = r_{res}, V_R = 0)}{\delta R} = -\frac{L_z^{res}}{r_{res}^2} = -\frac{\Omega_{bar} \gamma}{\gamma - 2b},$$

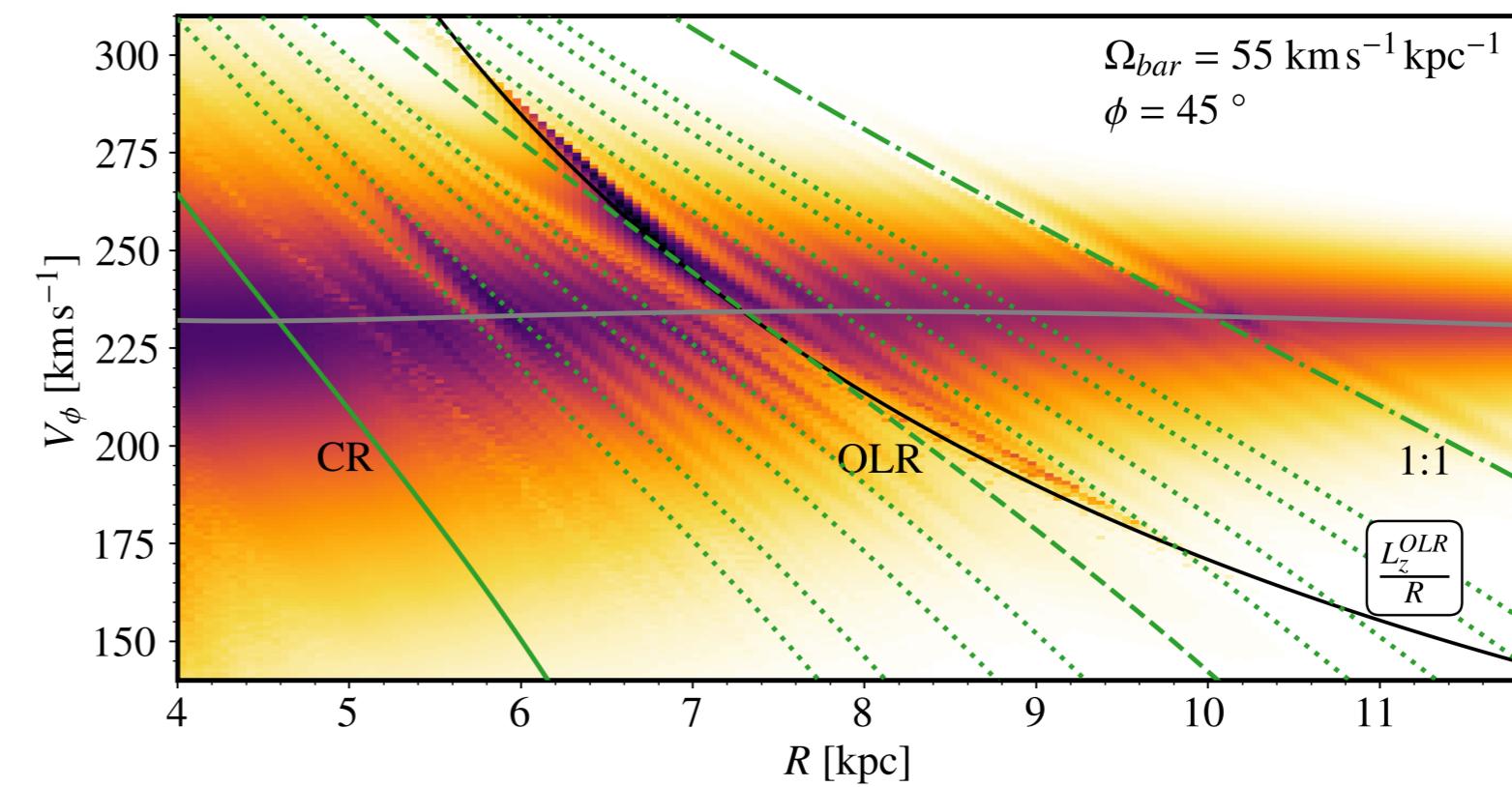
B) The shape of the resonant zone in velocity space is *almost* parabolic

$$V_\phi^{res}(R \sim r_{res}, V_R \sim 0) \approx \frac{L_z^{res}}{R} - \frac{1}{2} \frac{V_R^2}{v_c^2(R)}.$$



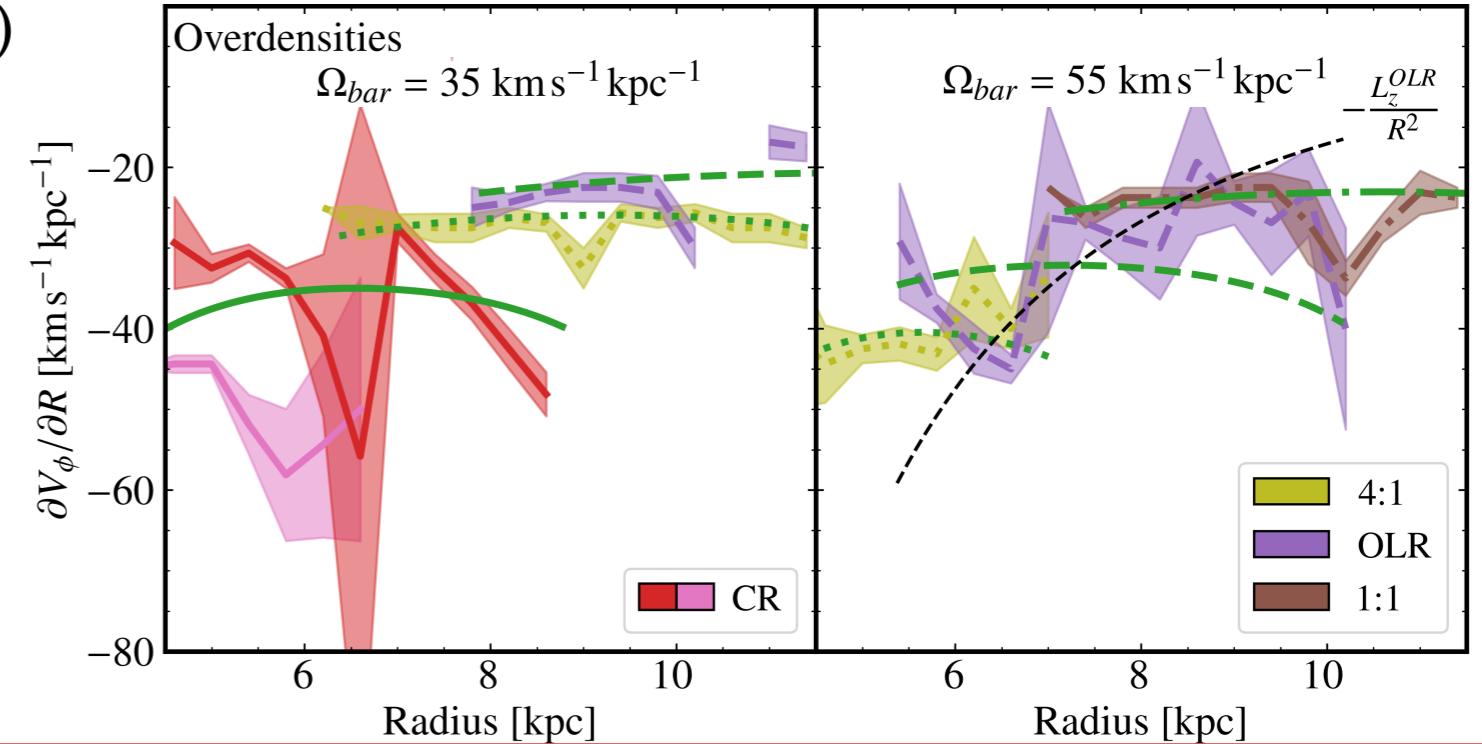
Comparison with test particle simulations

We use the McMillan et al. 2017 gravitational potential with its corresponding self-consistent Distribution Function (DF) for the stellar disc to run a Backwards Integration of a growing bar (Pichardo et al. 2004) for 15 bar rotations. This allows us to simulate a high-resolution snapshot of the perturbed phase-space that we can then compare with.



The results show that, even though we are describing the resonance zone (and not the perturbed DF), **our analytical formula describes accurately the high order resonances** (both in velocity space and the ridges). For scattering resonances, the ASE traces the gap; for trapping resonances, it traces the over-densities. Low order resonances, especially the Outer Lindblad Resonance 2:1 (OLR), present the well-known complexity that makes them so challenging to describe analytically. Nevertheless, our equations do trace well enough the space between the over- and under-densities created by the OLR.

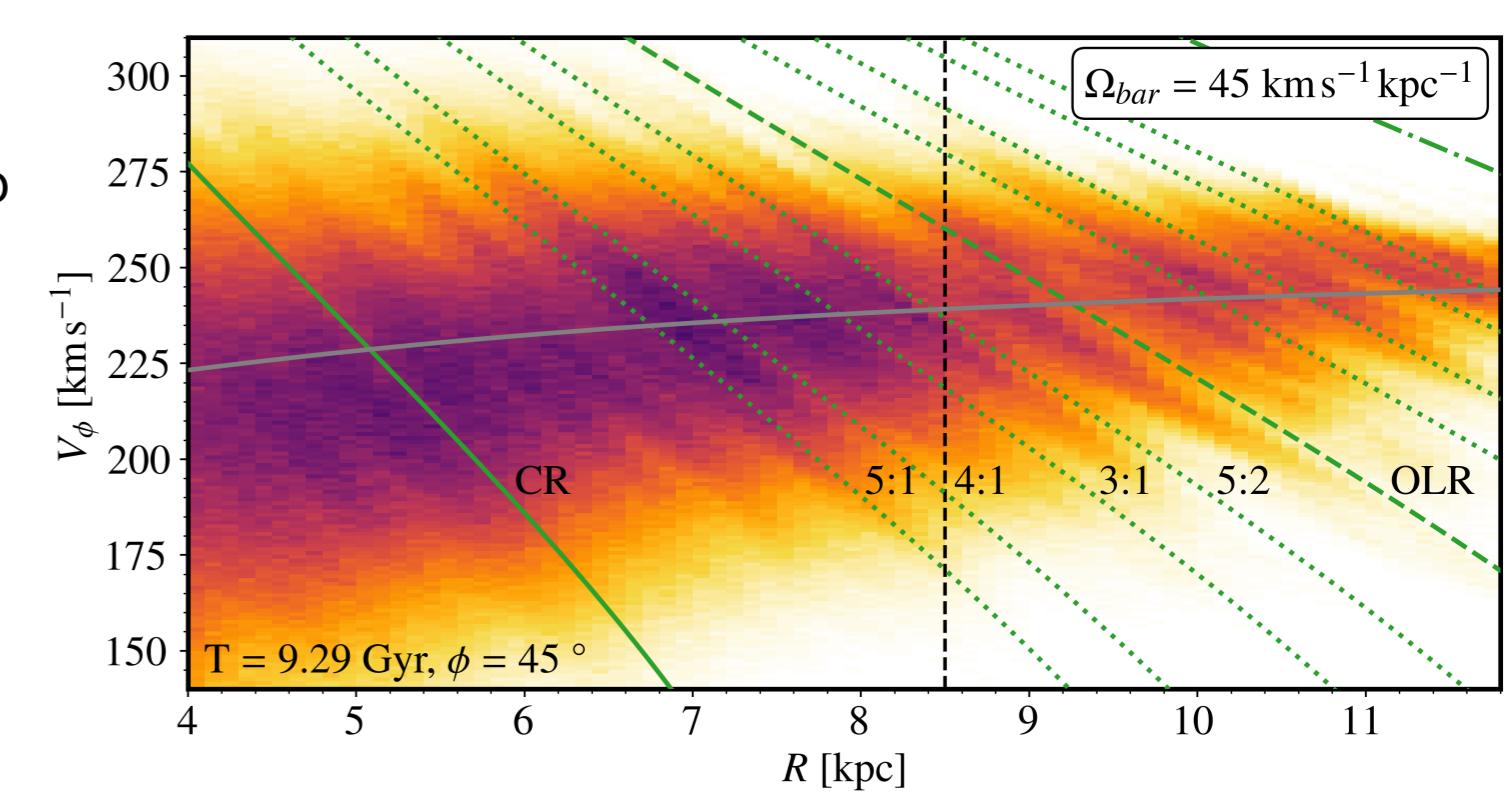
We note that **the slope of the ridges is a robust tracer of the resonances**, and our equations can reproduce them (except the OLR, which follows a line of cte. L_z)



Comparison with N-body simulations

We also compare the ASE with a couple snapshots of the simulation selected by Asano et al. 2022 as the best match to Gaia data, from the suit ran by Fujii et al. 2019.

Despite the lack or resolution inherent to N-bodies, we report a good qualitative agreement between the ASE and the ridges of the simulation at different times.



Conclusion

Even though our analytical model is purely axisymmetric (i.e., it cannot account for the azimuthal variations that we observe in reality) it offers a simple parametric equation to describe the shapes and gradients that we have measured, and a way to link them with the fundamental properties of the Milky Way.

In the future, we plan on improving the model by studying the azimuthal and temporal variations of the kinematic structures in simulations and finding a way to include them in the analytic description of the resonances.

