Matsu notes for Steve: optimized color test

Jim

July 22, 2012

1 What is this document for?

Yesterday, I did a bit of analysis for Project Matsu because I was curious to see how useful Hyperion's many spectral bands would be for classification[1]. The conclusion is that the techniques work, but it isn't clear yet if they will be as useful as I'd hoped. I haven't found a case where an analysis using many bands outperforms an analysis using only two.

Normally, I'd try to turn the work into something useful before writing it up, but I may not get back to this for a while because I'm still busy with the infrastructure of map tilings. Meanwhile, you're actively working on the analysis and maybe you'd find some pieces of what I did useful[2]. This document is intended to provide you with code segments for the steps that I used in the procedure.

Most of the effort was spent trying to figure out how to feed the data into linear algebra packages. For instance, does it want the transpose of what I'm giving it? Is it giving back what I want or the inverse/conjugate/negation of what I want? If you are trying to do the same operations, you can save some time by looking at these code segments, to see a working example of projecting away clouds or rotating to an optimal coordinate frame in spectral space.

All of these examples can be found somewhere in `water_spectrum.py`. My analysis script is a mess and it probably wouldn't even work if you ran it from start to end. (I use Python analysis scripts like a Mathematica notebook, evaluating things as I need them, and only clean it up and make it linear if it works.) It may be useful to refer to this script for copy-pasting, since that's hard to do from a PDF. Its parent directory also has all of the images described here. When I do a `git pull` on the Amazon instance, the files will be there, too.

My workflow is a little different from yours because I wanted to work on my laptop, but I couldn't load the serialized images on it. I was surprised to find that my laptop only has 4 GB of memory. I was forced to load raw TIFF files from disk and serialize my operations (throw

[1] See "Defining optimized colors for Matsu searches," which should be attached to the same e-mail as `optimized_color.pdf`.

[2] I'm trying to avoid the Philosopher's Error: according to Nietzsche, the philosopher normally aims to build an edifice that will withstand every onslaught, but posterity typically only finds value in using some of his building material for other purposes. If he's lucky.

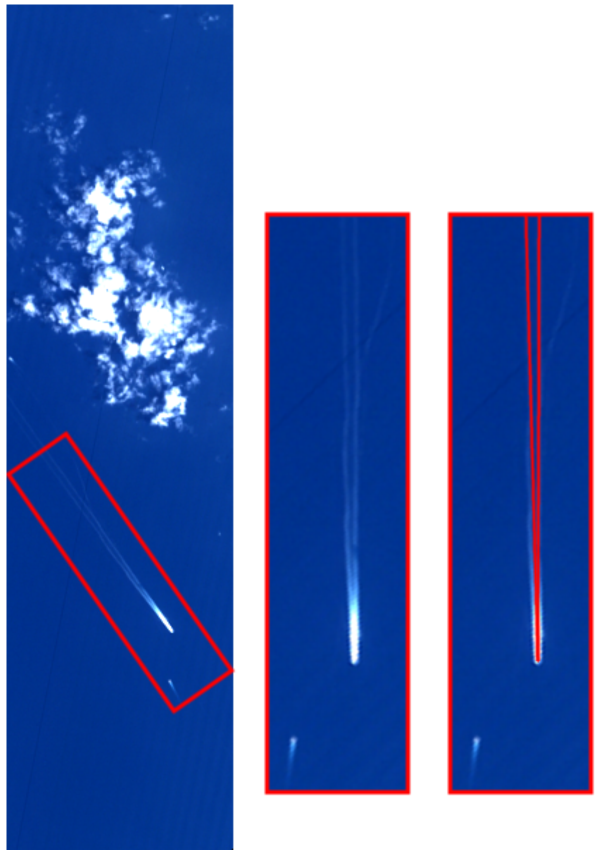


Figure 1: The wake of a ship in Kagoshima Bay with magnification and annotation on the right. You can tell that it's not an airplane contrail because it forms an expanding V.

away a band when I was done with it or work with a small number of bands). But I put the images into a form like the ones you're using: NumPy arrays with a *height:width:bands* shape, just like the ones you're using.

2 A real ship, not an airplane contrail

Kagoshima Bay has a few white streaks that appear to be ships, but we haven't verified that they correlate with AIS data and there's an airport nearby, which means that they might be airplane contrails. In the first few bands that we've looked at, they appear to be the same color as the clouds.

Fortunately for labeled data and supervised learning algorithms, I think I have a proof that one of these is a real ocean-going ship. See Fig. 1: one of these streaks has a V-shaped structure, which I believe can only happen on the surface of the water, not in the air. Airplanes and ships both create wakes that expand behind them, but contrails expand as three-dimensional cones and when there's more than one of them, they grow into each other and merge. Ocean wakes must separate because there's only one cone (the shock wave of the front of the ship) and it is confined to a two dimensional plane (the surface of the water).

This characteristic shape could be useful for automatically identifying ships: we could write a track fitter that searches for V-shaped pairs of line segments in the data. For now, I'll only use the one ship in Fig. 1 to get a labeled dataset. I'm interested in the spectral properties of these pixels, to produce optimized images as input spatial algorithms.

The zoomed-in image was formed by slicing a NumPy array:

```
smallPicture = picture[2200:2800,197:358,:]    # height, width, bands
```

and then selecting bands 29, 23, and 16 (cannonical red, green, and blue). The high contrast was obtained by mapping the 5th to 95th percentiles of the intensity distribution to minimum and maximum intensity, cutting off irrelevant tails. Given `red`, `green`, and `blue` arrays (shape is `2200:2800,197:358`), it can be done like this:

```
import numpy, PIL

# Find the 5th and 95th percentile of the red, green, and blue
# distributions, excluding very dark pixels (less than 10.), which is
# important when the image includes zeros around the tilted rectangle.
mincut = 5.
maxcut = 95.
minvalue = min(numpy.percentile(red[red > 10.], mincut),
               numpy.percentile(green[green > 10.], mincut),
               numpy.percentile(blue[blue > 10.], mincut))
maxvalue = max(numpy.percentile(red[red > 10.], maxcut),
               numpy.percentile(green[green > 10.], maxcut),
               numpy.percentile(blue[blue > 10.], maxcut))

# Map (minvalue, maxvalue) to (0, 255) and truncate any values higher
# or lower than these.
red = numpy.array(numpy.maximum(numpy.minimum((red - minvalue) /
                      (maxvalue - minvalue) * 255., 255), 0),
                  dtype=numpy.uint8)
green = numpy.array(numpy.maximum(numpy.minimum((green - minvalue) /
                      (maxvalue - minvalue) * 255., 255), 0),
                  dtype=numpy.uint8)
blue = numpy.array(numpy.maximum(numpy.minimum((blue - minvalue) /
                      (maxvalue - minvalue) * 255., 255), 0),
                  dtype=numpy.uint8)

# Write this out to an image file to view on the web.
image = numpy.dstack((red, green, blue))
image = PIL.Image.fromarray(image)
image.save("/var/www/quick-look/tmp.png", "PNG", options="optimize")
```

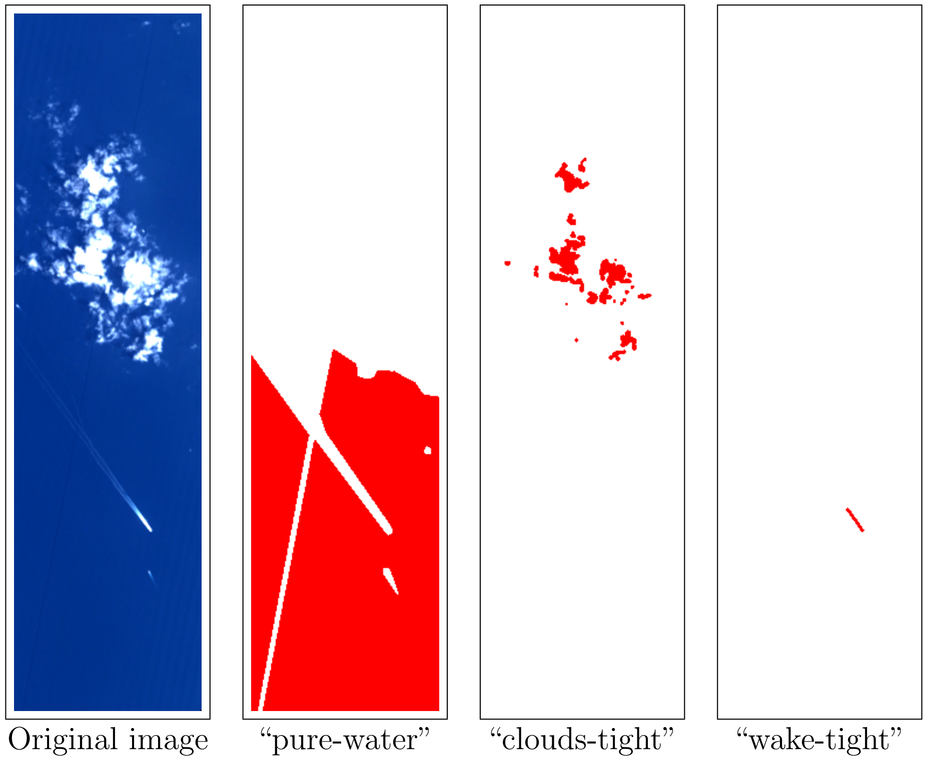


Figure 2: Masks used to define sets of pixels for spectral studies.

3 Creating labeled datasets

Unfortunately, the ship is travelling at an angle, so we can't select the interesting pixels by a rectangular slice of the array. I want to get spectra for a specific set of pixels, so I opened the image in a paint program (Gimp) and painted the desired pixels red, then loaded my masks into Python and used them to make the selection.

Figure 2 shows the three masks I used the most. The “pure-water” mask avoids the wake of the ship, a possible second ship, and an image defect. The “clouds-tight” includes the thickest parts of the clouds, and “wake-tight” includes the brightest part of the ship's wake. I painted over the original image in a separate layer so that I could remove the original image before saving. I chose to paint in red on a white background, so only the green and blue channels indicate the threshold (red is always 255, green and blue are sometimes 255 and sometimes 0). Here's how I loaded the images into NumPy arrays:

```
import numpy, PIL
masks = {}
for name in ["cloud-shadow", "pure-water", "image-defect",
             "clouds-loose", "clouds-tight", "wake-loose", "wake-tight"]:
    picture = PIL.Image.open(open("mask_%s.png" % name))
    masks[name] = (numpy.array(picture)[:,:,1] < 128)
```

Since these masks were made from the `[2200:2800,197:358,:]` image, it can only be applied to images that have been sliced like that.

Now each `masks[name]` is an array of booleans and it can be used to select pixels from an image using `picture[masks[name]]` where `picture` is a $600 \times 161 \times N_{\rm bands}$ array and the result is a $N_{\rm selected} \times N_{\rm bands}$ array. I applied this to all the bands to produce Fig. 3.

The first thing we learn from this plot is that the wake of the ship and the clouds are not the same: they're both much brighter than flat water, but the clouds have a softer spectrum than the wake. The slope of log(radiance) versus wavelength is steepest (bluest) for flat water, less steep for the ship's wake, and the least steep (reddest) for clouds.

I think this is extraordinary: we're finding differences among the spectra of liquid water, liquid water, and liquid water! The slope must have something to do with the average size of water droplets, since that's the only difference I can think of between flat water (pretty

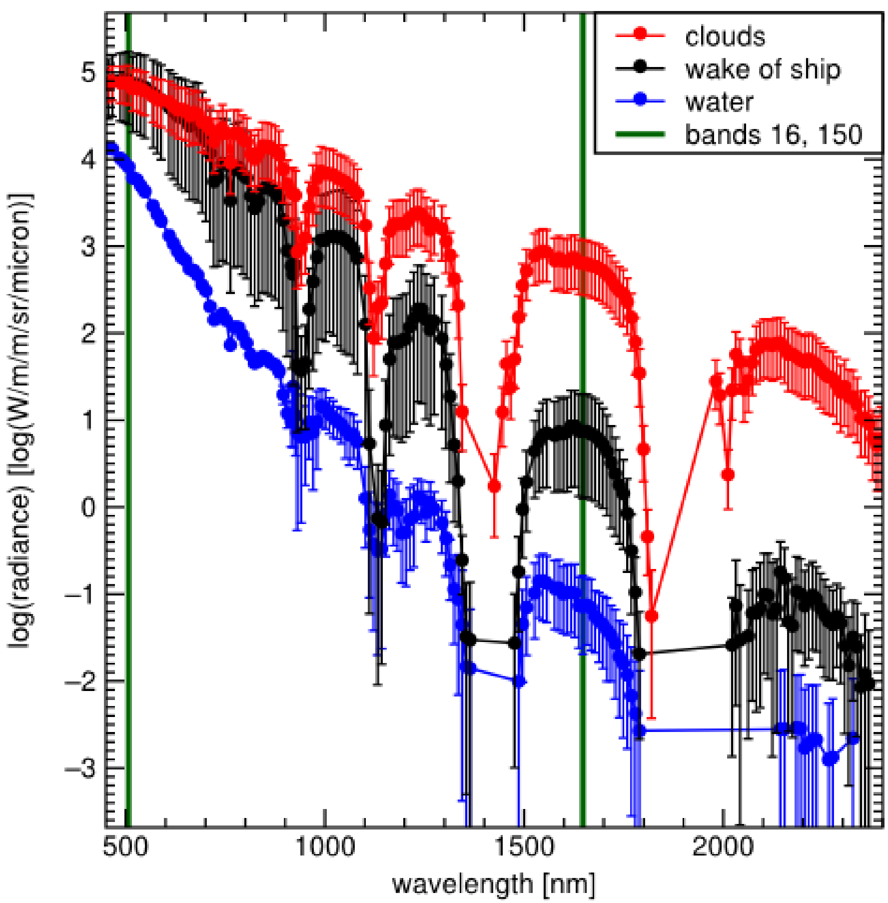


Figure 3: Spectra of water, clouds, and the wake of the ship. Bands 16 and 150 are used in the next section. Error bars indicate the standard deviation of each band over all matching pixels (and *not* the uncertainty in the mean: see the correlation from band to band).

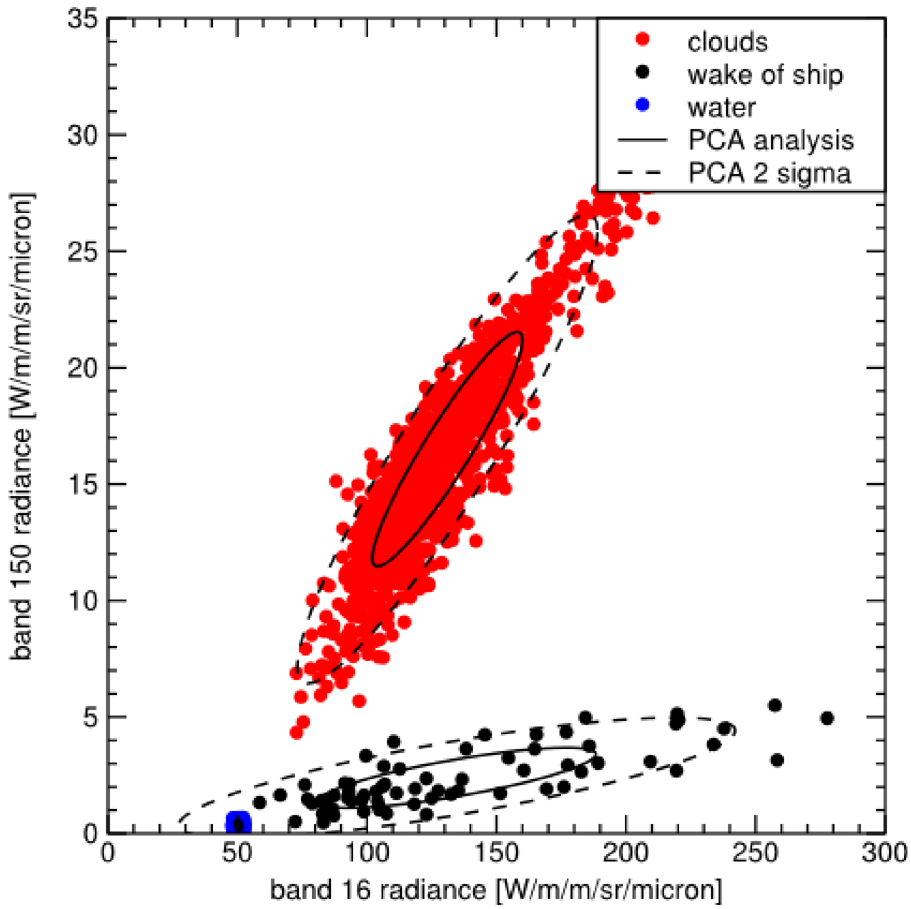


Figure 4: Distributions of band 16 and band 150 radiance in for clouds, the ship's wake, and water. Each point in this plot corresponds to a pixel in the image.

much one big surface), the ship's wake (foam kicked up by the bow of the ship), and clouds (a diffuse aerosol of microscopic droplets). Perhaps each distribution of droplet sizes causes a different fraction of the sun's light to specular-reflect into the camera. The flat sea only specular-reflects at one angle, probably different from the camera's position, but spherical droplets would specular-reflect some fraction into the camera just from geometry.

With only two bands, 16 (cannonical blue) and 150 (near infrared), we can already do some classification. I made Fig. 4 by loading two raw TIFFs and making scatter plots in Cassius:

```
from cassius import *
from osgeo import gdal, gdalconst

# You would get this from the serialized image.
image016 = gdal.Open(glob.glob(inputDir + "/*_B%03d_L1T.TIF" % 16)[0],
```

```
                     gdalconst.GA_ReadOnly)
image150 = gdal.Open(glob.glob(inputDir + "/*_B%03d_L1T.TIF" % %150)[0],
                     gdalconst.GA_ReadOnly)
image016 = image016.GetRasterBand(1).ReadAsArray()[2200:2800,197:358] / 40.
image150 = image150.GetRasterBand(1).ReadAsArray()[2200:2800,197:358] / 80.

water_x, water_y = image016[masks["pure-water"]], \
                   image150[masks["pure-water"]]
wake_x, wake_y = image016[masks["wake-tight"]], \
                 image150[masks["wake-tight"]]
clouds_x, clouds_y = image016[masks["clouds-tight"]], \
                     image150[masks["clouds-tight"]]

waterplot = Scatter(x=water_x, y=water_y, limit=1000, markercolor="blue")
wakeplot = Scatter(x=wake_x, y=wake_y, limit=1000, markercolor="black")
cloudsplot = Scatter(x=clouds_x, y=clouds_y, limit=1000, markercolor="red")

view(Overlay(waterplot, wakeplot, cloudsplot))
```

Flat water occupies a small spot in at (50, 3) because it is mostly uniform in these two bands. Clouds vary in brightness because some pixels correspond to the tops of clouds, some to their sides, and they vary in thickness. However, the brightness in the two bands are strongly correlated because cloud-material has a consistent spectrum. The same can be said for the ship's wake, but with a different slope because of the different spectral shape.

As described in the first note (`optimized_color.pdf`), I think we can describe the spectra of groups of pixels using a Principal Component Analysis (PCA) of their distribution in a multidimensional space, where each dimension corresponds to a spectral band. As you can already see in Fig. 4, the correlations among bands is important. As a first test, I did a simple PCA of these two bands using a function from Cassius (wraps the NumPy function):

```
from cassius import *
origin, xscale, xbasis, yscale, ybasis =
        principleComponents(numpy.array(zip(water_x, water_y)))
```

The `origin` is the mean of the distribution, `xscale` and `yscale` are the square roots of the largest two eigenvalues (x is the largest), and `xbasis`, `ybasis` are two-component eigenvectors. You can make error ellipses like the ones in Fig. 4 with this:

```
waterellipse = Scatter(
    x=[origin[0] + xscale*xbasis[0]*cos(t) + yscale*ybasis[0]*sin(t)
              for t in numpy.arange(0., 2.*pi + 0.1, 0.1)],
    y=[origin[1] + xscale*xbasis[1]*cos(t) + yscale*ybasis[1]*sin(t)
              for t in numpy.arange(0., 2.*pi + 0.1, 0.1)],
    connector="unsorted", marker=None)
```

(A parametric curve is a scatter plot with no markers and with lines connecting the points.)

4 More than two spectral bands

Next, I decided to try something more realistic: more than two bands, but not (yet?) all the bands. I chose six bands with roughly maximal differences between water, wake, and clouds: 40 (752.39 nm), 54 (895.372 nm), 99 (1132.2 nm), 110 (1243.41 nm), 131 (1455.72 nm), and 155 (1698.36 nm). The plots in Fig. 5 show the motivation for these choices.

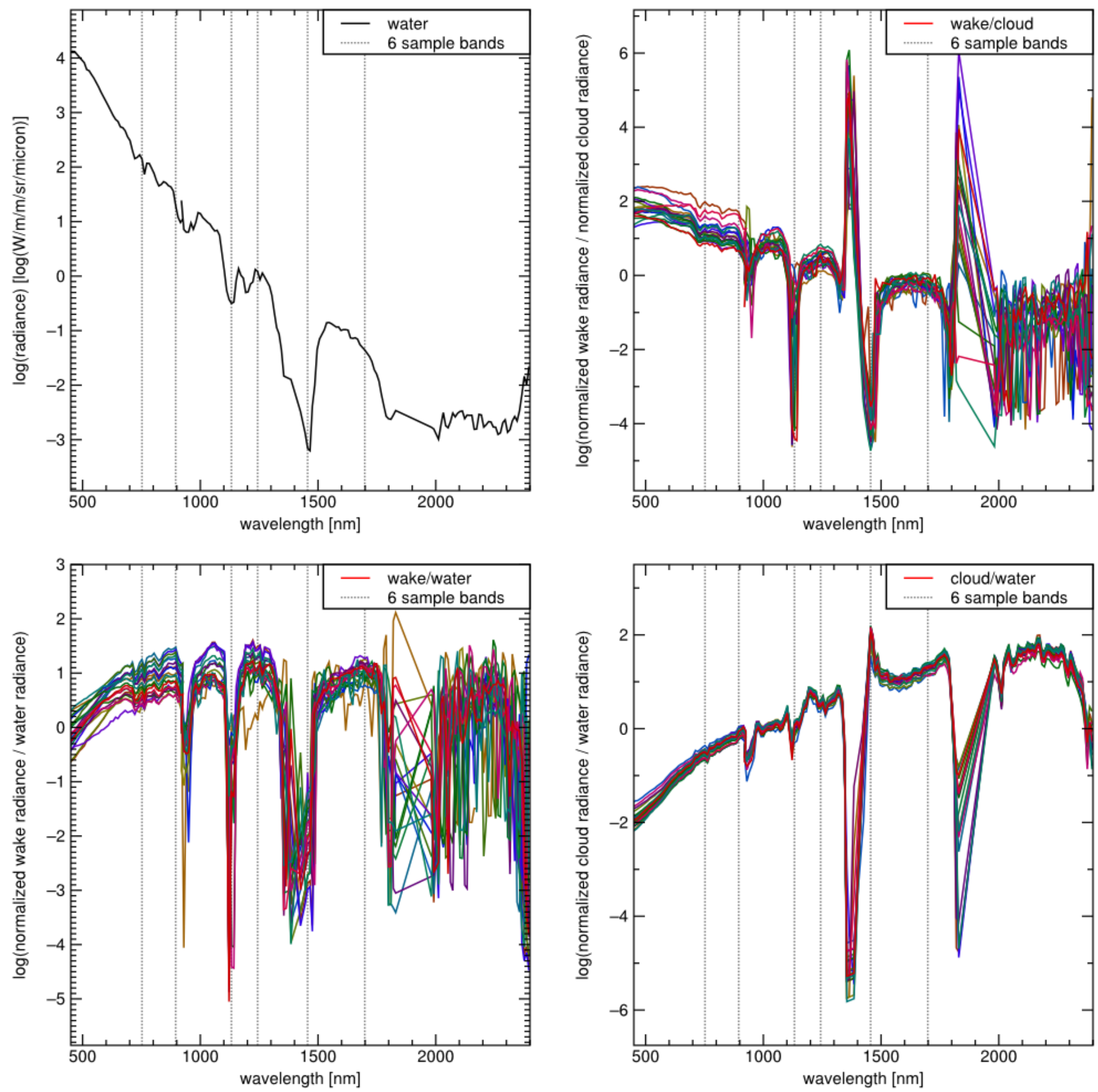


Figure 5: Spectral bands chosen for this study, overlaid on (top-left) the average water spectrum, (top-right) normalized wake pixels divided by cloud (one line-color per pixel, 20 pixels sampled), (bottom-left) same for the ratio of wake over water, (bottom-right) same for the ratio of clouds over water. Wake colors vary more than cloud colors.

A six-dimensional space is still small enough to visualize. Figure 6 shows the distribution of cloud, wake, and water radiances in each pair of the six spectral bands. Much like Fig. 4, these plots show that radiances in different bands are highly correlated. That is, within each category, the color is roughly constant but the brightness varies. Figure 7 shows the same thing in log-log scale.

Most importantly for identifying ships, the three distributions do not completely overlap. We want to manipulate the data to maximize the separation between ship wakes and everything else, so that ships show up as bright spots in the images and everything else is dim or invisible. The algorithms that take advantage of spatial information (for example, a bow-wave "V" finder) could then start with a high ratio of signal to background.

First, let's translate the distributions so that water is centered on zero (it is already near zero). It is as easy to find medians as it is to find means, so I use that to avoid dependence on tails. With `images` as my *height:width:bands* array with six bands,

```
import numpy
watercenter = numpy.median(images[masks["pure-water"]], axis=0)
wakecenter = numpy.median(images[masks["wake-tight"]], axis=0)
cloudscenter = numpy.median(images[masks["clouds-tight"]], axis=0)
```

The results are 1-D, six-element vectors (plotted as white crosses in Fig. 6).

Next, let's remove the clouds. The vector `cloudscenter - watercenter` points from the center of the water distribution to the center of the cloud distribution: I'd like to construct a projection operator that projects along this axis (the null space) onto a plane at the origin, perpendicular to this axis (the range). Figure 8 shows the distributions after this projection: by construction, water and clouds are centered at zero but the ship's wake is not. If we declare zero to be black and distance away from zero to be grey to white, the sea and the clouds would both be black and the ship's wake would be grey to white.

The projection operator is constructed in two steps. First we find an orthonormal basis in which one of the basis vectors points in the direction of `cloudscenter - watercenter`. This basis is not unique, but the choice of the other five basis vectors is not important. Most linear algebra packages provide a QR factorization for producing an orthonormal basis, but they use algorithms that don't guarantee that the first one points along a given axis (for more numerical stability). The following implementation of a simple Gram-Schmidt process guarantees that the first output (`us[0]`) is parallel to the first input (`vs[0]`).

```
import numpy
def gramSchmidt(vs):
    us = []
    for i, v in enumerate(vs):
        u = numpy.array(v)
        for j in xrange(i):
            u -= us[j].dot(v) * us[j]
        u /= numpy.linalg.norm(u)
        us.append(u)
    return us
```

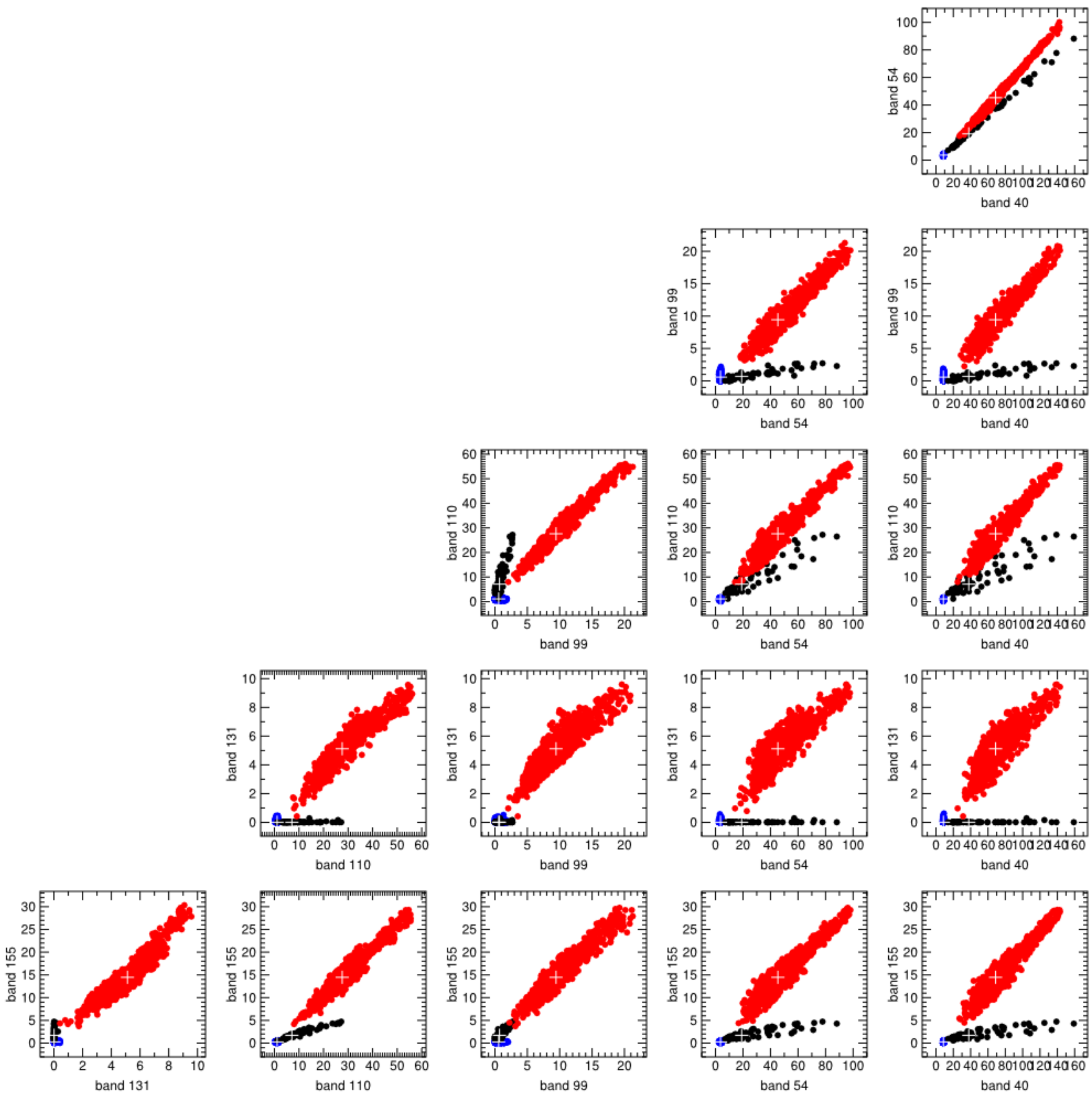


Figure 6: Cloud (red), wake (black), and water (blue) radiances in each of the six sample bands. Every pair of distinct bands is plotted in a separate window, showing how the distributions overlap in all faces of the 6-D hypercube. White crosses are the medians of each distribution.

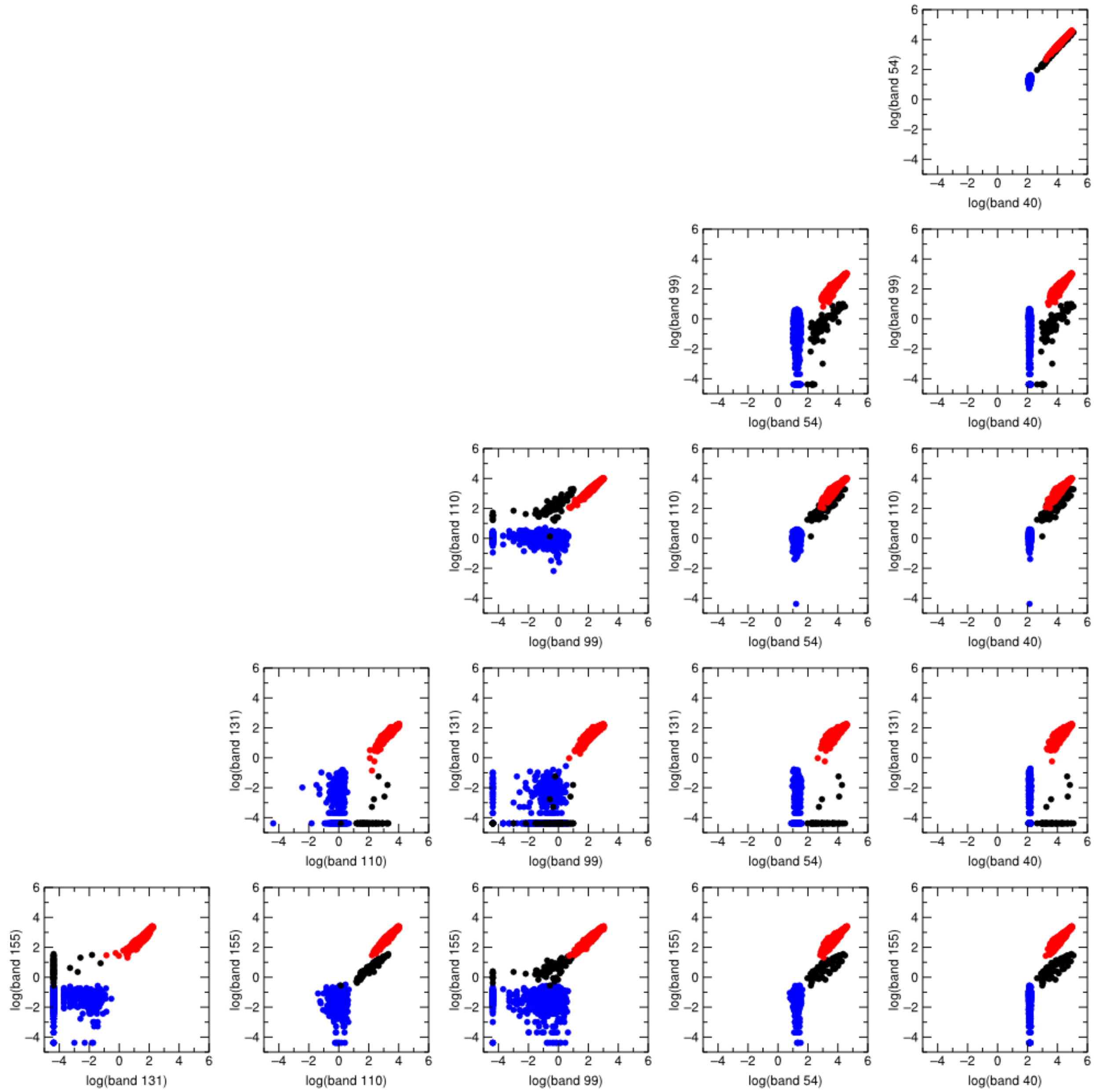


Figure 7: Same as Fig. 6 (cloud is red, wake is black, and water is blue), but all radiances are shown in log scale, providing more detail on the water distribution.

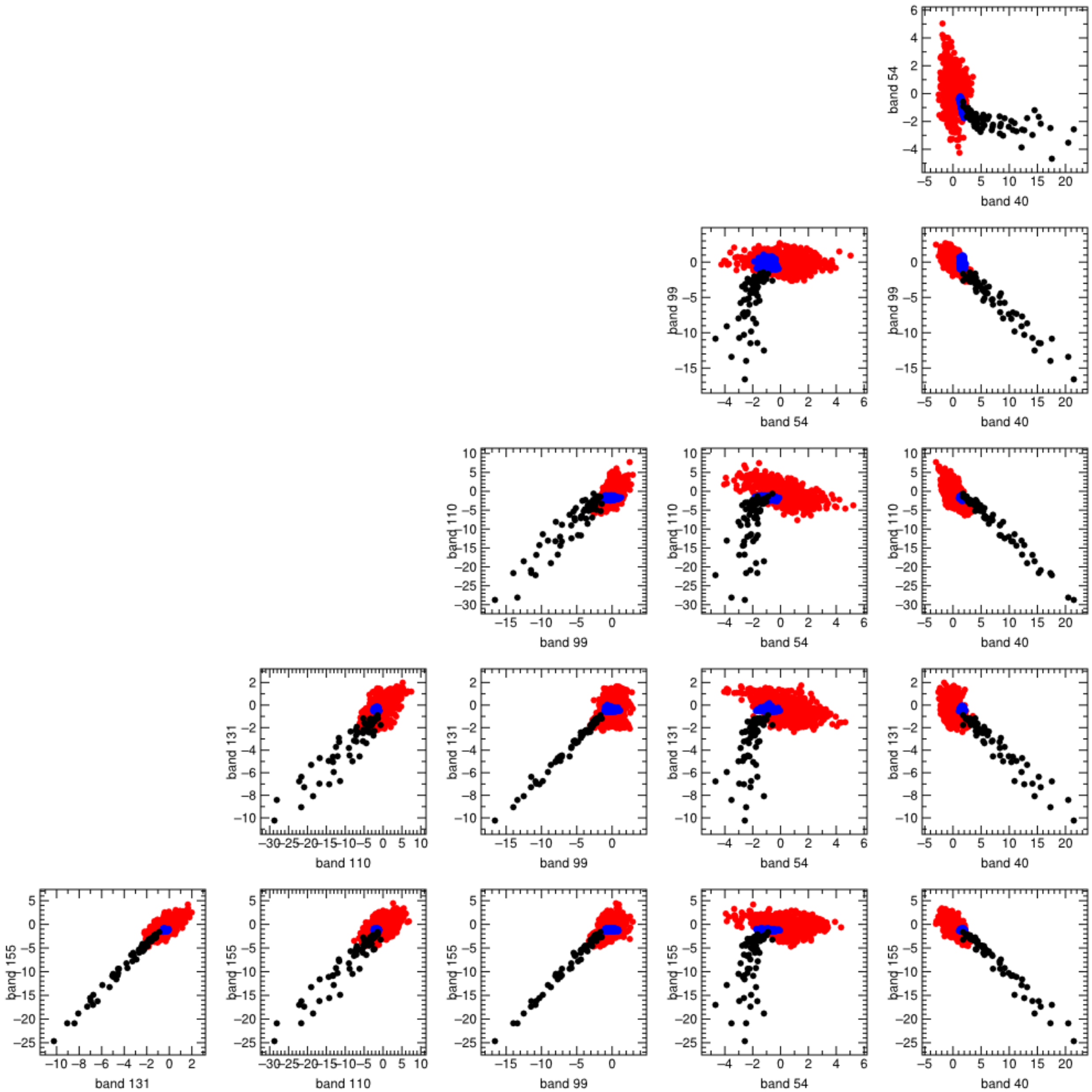


Figure 8: Same as Fig. 6 (cloud is red, wake is black, and water is blue), but after translating the origin to the median of water and projecting away the axis pointing from the median of water through the median of clouds. Water and clouds are centered on zero by construction; wake is maximally separated from both.

Since we don't care about the distribution of the other five basis vectors (`us[1:6]`), we can feed it a random set of input vectors. (The unit-normal distribution from `numpy.random.randn` minimizes numerical instability, which is a problem for the Gram-Schmidt algorithm.)

```
basis = gramSchmidt([cloudscenter - watercenter] +
                    zip(*numpy.random.randn(6, 5)))
```

Now `basis` is a list of six 6-D unit vectors, the first is parallel to `cloudscenter - watercenter` and the rest are perpendicular to `cloudscenter - watercenter` and each other.

The second step is to build the projection operator. Ideally, this should be a vectorized NumPy procedure so that it can be applied to a huge dataset (such as a large image) without any Python calls in the loop. I was lazy and just wrote a lambda expression, so it's a lot slower than it needs to be. If we scale this up, I should rethink the implementation.

```
projection = lambda x: sum([w * w.dot(x) for w in basis[1:]])
```

In symbols, this is

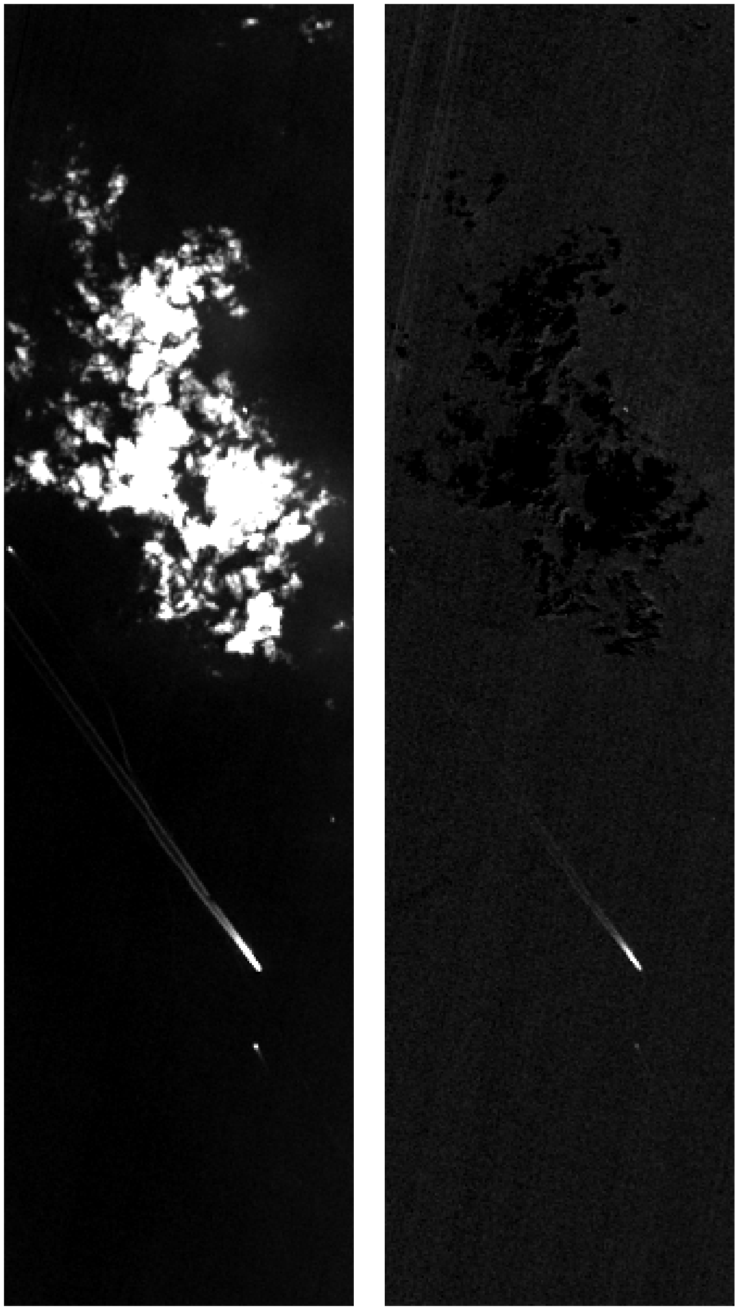
$$\texttt{projection}_W(\vec{x}) = \sum_{i=1}^{5} (\vec{x} \cdot \vec{w}_i)\, \vec{w}_i \tag{1}$$

where $\vec{w}_i \in W$ is the space orthogonal to `basis[0]` and $\vec{x}$ is any vector.

To see what it looks like to remove clouds, we can apply the projection operator to an image, band 40 (closest to visible light in our `sampleBands`).

```
import numpy, PIL

for i in xrange(len(sampleBands)):
    image[:,:,i] -= watercenter[i]     # center on water

# Linearize height and width, apply projection, then restore the structure
projimage = numpy.reshape(image,
                          (image.shape[0]*image.shape[1], image.shape[2]))
projimage = numpy.array(map(projection, projimage))
projimage = numpy.reshape(projimage,
                          (image.shape[0], image.shape[1], image.shape[2]))

band = projimage[:,:,sampleBands.index(40)]  # select band 40

minvalue = 1.    # cuts out clouds and water, both centered at zero
maxvalue = 10.   # rough maximum for this band
band = numpy.array(numpy.maximum(numpy.minimum((band - minvalue) /
                   (maxvalue - minvalue) * 255., 255), 0), dtype=numpy.uint8)

image = numpy.dstack((band, band, band))
image = PIL.Image.fromarray(image)
image.save("/var/www/quick-look/tmp.png", "PNG", options="optimized")
```

The resulting image is shown in Fig. 9.



Before After

Figure 9: The image in band 40, before and after cloud-removal. The cloud and the water become dark without sacrificing too much intensity in the ship's wake. The `minvalue` threshold set by hand in the code controls how much of the cloud the human eye detects.

5 Background likelihood

Now that we have a way of making the clouds look like the background (flat water), we want to use the six-dimensional information to color the pixels in a way that maximizes the difference between signal and background. The example shown in Fig. 9 uses only one band (40) and a threshold that was set by hand (`minvalue`). I'd like to find an automated solution to get better results with less arbitraryness, but I haven't yet found one that outperforms Fig. 9.

The idea described in `optimized_color.pdf` is to treat the background distribution as a Gaussian blob with correlations. That is, do a PCA analysis on everything (which is mostly background) to get the parameters of a best-fit Gaussian error ellipse. I take this as the likelihood distribution and color pixels by their unlikelihood. By construction, the ship's wake pixels contain the least likely combination of the six bands and hence should stand out.

The PCA analysis is (1) find the covariance of the set of 6-D vectors, one for each pixel in the image, (2) find the eigenvalues and eigenvectors of that covariance matrix, and (3) remove the smallest one. Step (3) is only needed because our data is effectively five-dimensional, now that we've projected away the clouds. It shows up in the eigenvalues as a nearly-zero eigenvalue (`1e-16`, machine precision). Since we no longer have any information along this direction (we threw it all away), this axis of the error ellipse should be infinite, so I set it to a large number (`1e10`, but since our data has been projected, any number larger than `1e-16` would do).

The eigenvalues and eigenvectors together define a matrix that transforms our dataset to a spherically symmetric unit Gaussian (plus any unmodeled tails). The likelihood in that space is $\exp(-|\vec{x}|^2)$, so we get the likelihood in our space by untransforming.

```
from math import *
import numpy

# Linearize height and width and apply projection
projimage = numpy.array(map(projection,
    numpy.reshape(image, (image.shape[0]*image.shape[1], image.shape[2]))))

# Perform the PCA analysis.
eigenvalues, eigenvectors = numpy.linalg.eig(numpy.cov(projimage.T))
smallestIndex = numpy.argsort(eigenvalues)[0]
eigenvalues[smallestIndex] = 1e10

# Define a likelihood function using the eigenbasis.
def likelihood(p0, p1, p2, p3, p4, p5):
    return exp(-sum(numpy.square(numpy.array(
        numpy.matrix([[p0, p1, p2, p3, p4, p5]]) * eigenvectors).flatten())
        / eigenvalues))
```

Figure 10 shows slices through the likelihood function overlaid by wake pixels.

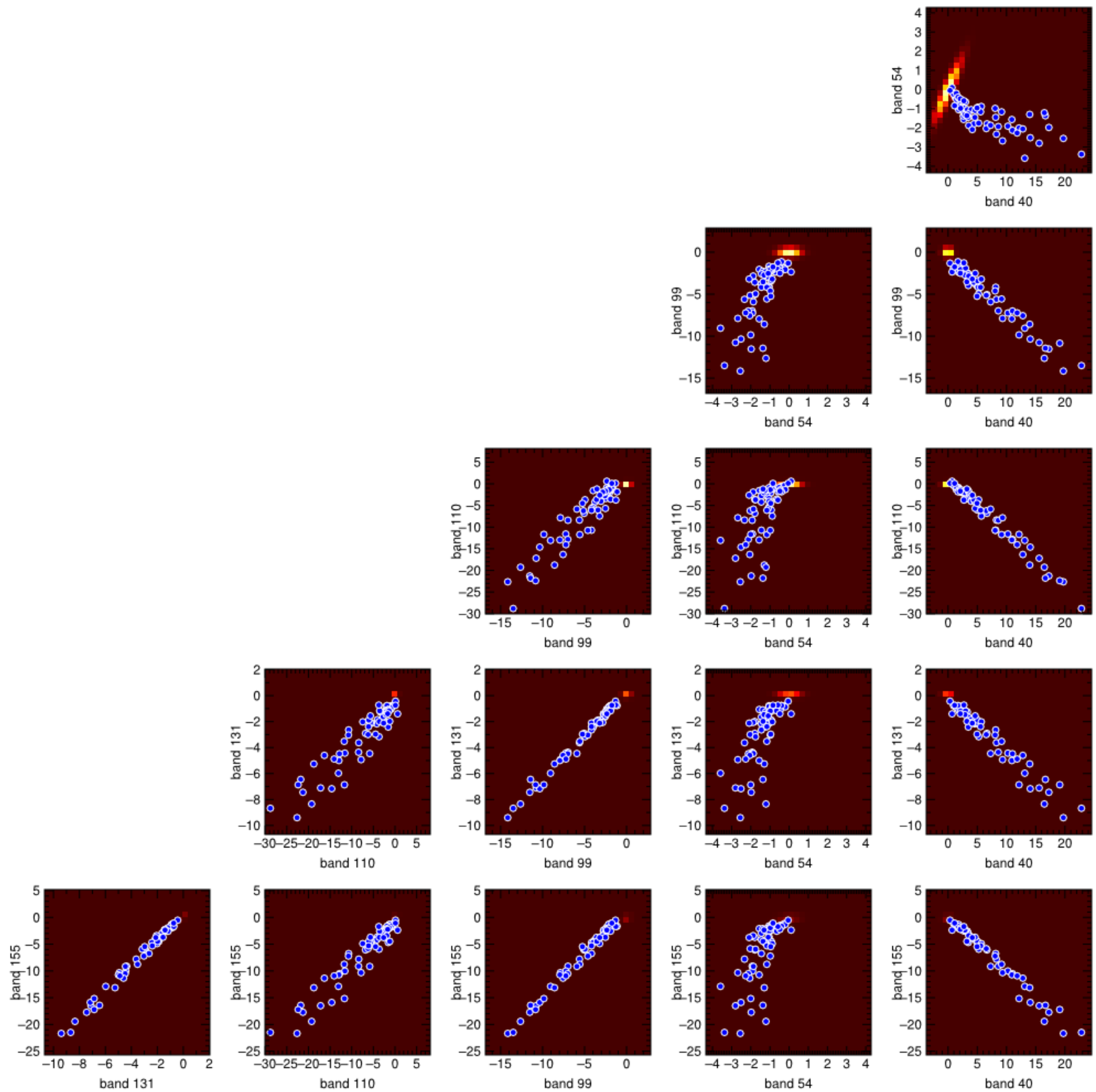


Figure 10: The color field is the likelihood function, sliced through each pair of bands. Yellow is maximum likelihood, magenta is minimum. The blue points are the ship's wake data, which extend beyond the first few sigma in likelihood.

To turn the likelihood values into pixel colors, we need to transform the likelihood's very broad interval to a manageable interval like (0, 1). Note that the negative log-likelihood,

```
def loglikelihood(p0, p1, p2, p3, p4, p5):
    return sum(numpy.square(numpy.array(
        numpy.matrix([[p0, p1, p2, p3, p4, p5]]) * eigenvectors).flatten())
        / eigenvalues)
```

is just a χ^2. (The usual formula for χ^2 is $\sum_i^N (y_i - y(x_i))^2/{\sigma_i}^2$, but here $y(x_i) = 0$ and $\sigma_i = 1$ because we've transformed to the space in which the model is a unit Gaussian.) Therefore, we can use a χ^2 cumulative distribution function to map `loglikelihood` to (0, 1). SciPy provides such a function:

```
from scipy.stats import chi2 as chi2prob
def normalize(x):
    return chi2prob.cdf(x, 5)    # 5 d.o.f
```

Once the distribution is in the unit interval, we map to (0, 255) the normal way (see code fragments involving `minvalue` and `maxvalue`).

The result in Fig. 11 is somewhat disappointing: this method found the ship's wake, but it also brought out image defects that were only faintly visible in the normal view and it hasn't suppressed the cloud. My conclusion is that defects are non-Gaussian tails and the cloud is non-Gaussian as well. Even if I cheat and apply the PCA algorithm to the cloud pixels only, the edges of the cloud are still visible because the distribution just isn't round.

We can suppress the tails by exaggerating the unlikelihood function. For instance, once the values are mapped to the unit interval, we can apply `numpy.power(unitIntervalData, 10)` to bring out highly unlikely features at the expense of less unlikely features. The result is a dot at the ship's wake with no image defects or clouds, but the dot loses most of its spatial distribution. The bow of the wake is at an extreme of the unlikelihood, but not the two separating waves.

6 Another idea: Naïve Bayes

Since the tails of the distribution are important, I tried a Naïve Bayes approach. I originally didn't consider Naïve Bayes because correlations among parameters are so important. To add a simple handling of correlations, I first transformed by the matrix of eigenvectors from the PCA, then applied Naïve Bayes in the eigenspace. That way, the marginal distributions would at least be aligned with the shape of the blob.

After subtracting the water and projecting away the clouds, I transformed the distributions like this:

```
def transformation((p0, p1, p2, p3, p4, p5)):
    return numpy.array(numpy.matrix([[p0, p1, p2, p3, p4, p5]])
        * eigenvectors).flatten() / numpy.sqrt(eigenvalues)
projimage_trans = numpy.array(map(transformation, projimage))
projwake_trans = numpy.array(map(transformation, projwake))
```

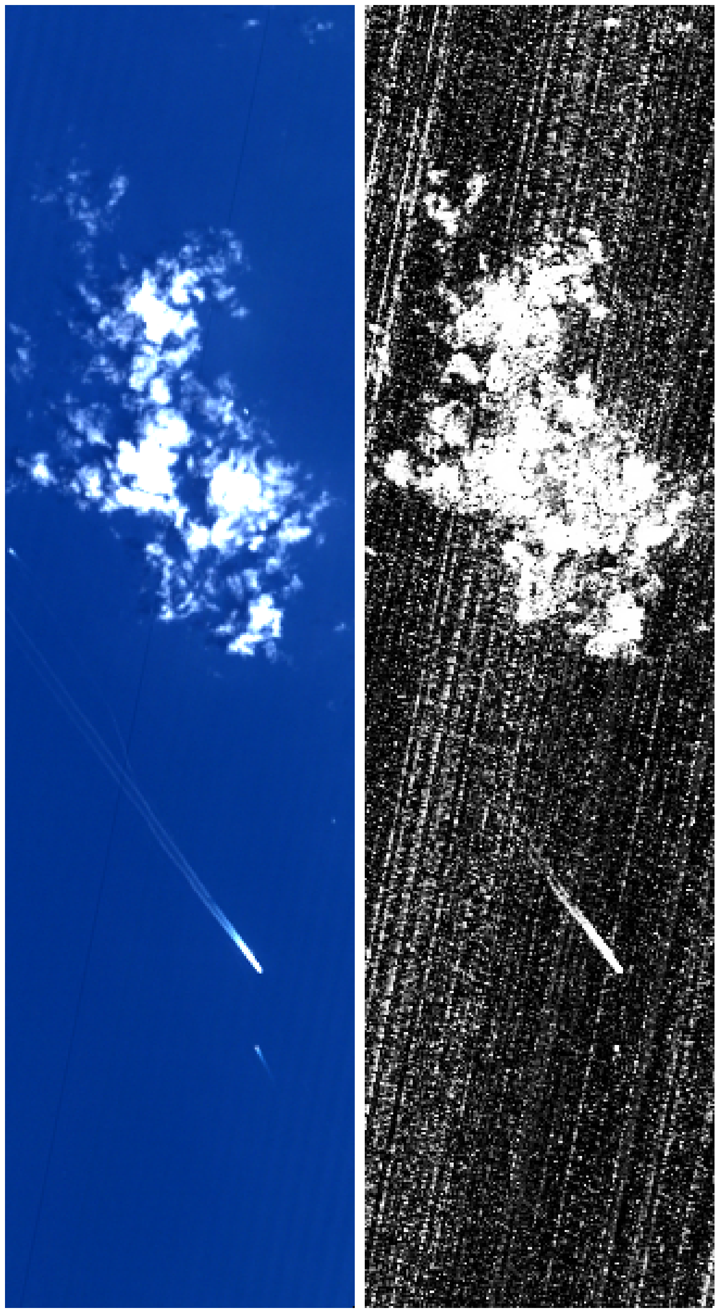


Figure 11: Original RGB image (left) and image produced by a Gaussian likelihood (right). The ship was easier to see before we did anything!

I then built a likelihood on the transformed distributions using Cassius histograms (just because they were readily available).

```
from math import *
from cassius import *

projimage_0 = Histogram(100, -50., 50., data=projimage_trans[:,0])
projimage_1 = Histogram(100, -20., 20., data=projimage_trans[:,1])
projimage_2 = Histogram(100, -20., 20., data=projimage_trans[:,2])
projimage_3 = Histogram(100, -20., 20., data=projimage_trans[:,3])
projimage_4 = Histogram(100, -1., 1., data=projimage_trans[:,4])
projimage_5 = Histogram(100, -10., 10., data=projimage_trans[:,5])

wake_0 = Histogram(100, -50., 50., data=projwake_trans[:,0])
wake_1 = Histogram(100, -20., 20., data=projwake_trans[:,1])
wake_2 = Histogram(100, -20., 20., data=projwake_trans[:,2])
wake_3 = Histogram(100, -20., 20., data=projwake_trans[:,3])
wake_4 = Histogram(100, -1., 1., data=projwake_trans[:,4])
wake_5 = Histogram(100, -10., 10., data=projwake_trans[:,5])

def loglikelihood(x):
    x = transformation(x)
    denom_loglikelihood = 0.
    numer_loglikelihood = 0.
    for i, (denomhist, numerhist) in enumerate([
            (projimage_0, wake_0), (projimage_1, wake_1), (projimage_2, wake_2),
            (projimage_3, wake_3), (projimage_5, wake_5)]):
            # 4 is excluded because that's the cloud removal eigenvector
        index = denomhist.index(x[i])
        if index is None:
            denom_loglikelihood += log(1. / denomhist.entries)
            numer_loglikelihood += log(1. / numerhist.entries)
        else:
            denomvalue = denomhist.values[index] + 1e-5
            numervalue = numerhist.values[index] + 1e-5
            denom_loglikelihood += log(denomvalue / denomhist.entries)
            numer_loglikelihood += log(numervalue / numerhist.entries)
    return numer_loglikelihood / denom_loglikelihood
    # Last minute note: shouldn't the division in the last line be subtraction???
```

These histograms are shown in Fig. 12.

7 Applying the algorithm to whole images

Figure 14 shows what the algorithm looks like when applied to whole images, rather than just the training sample. Land features are unlikely in the training data, so they show up as

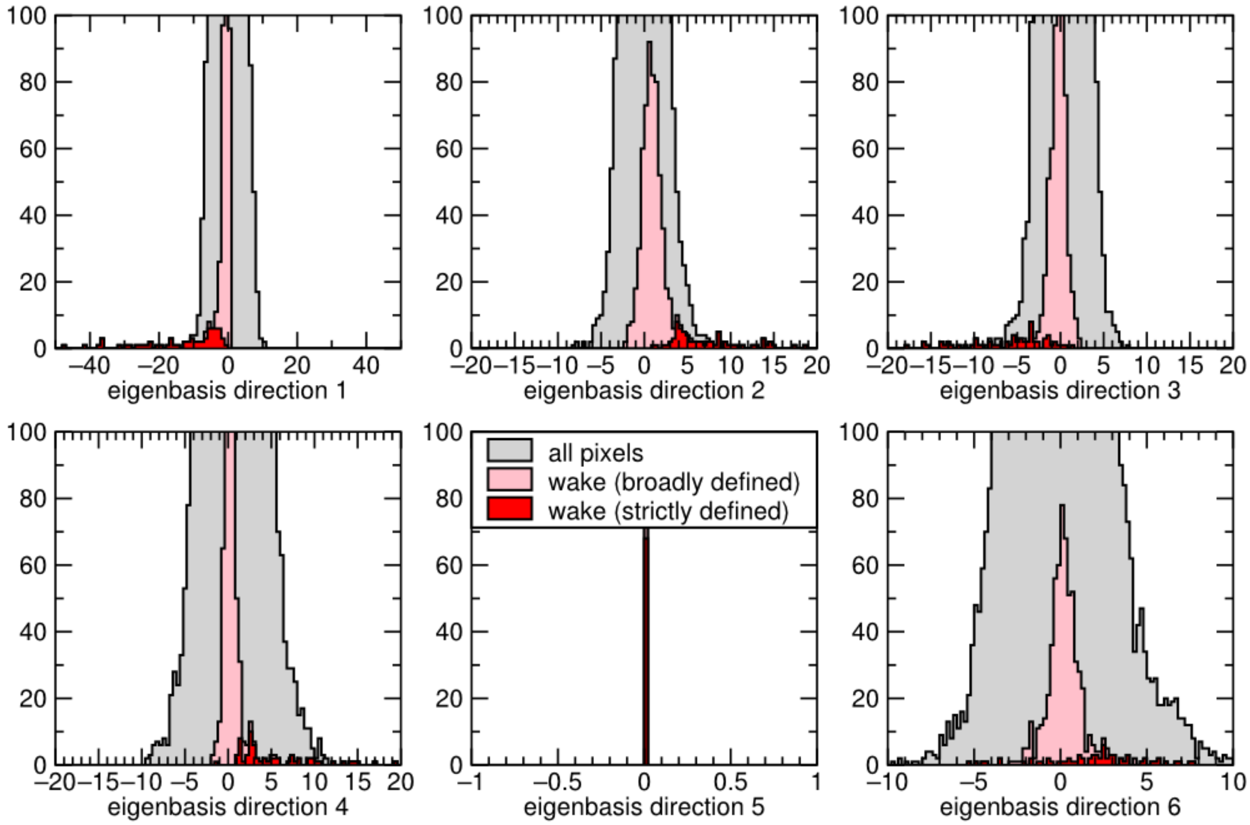
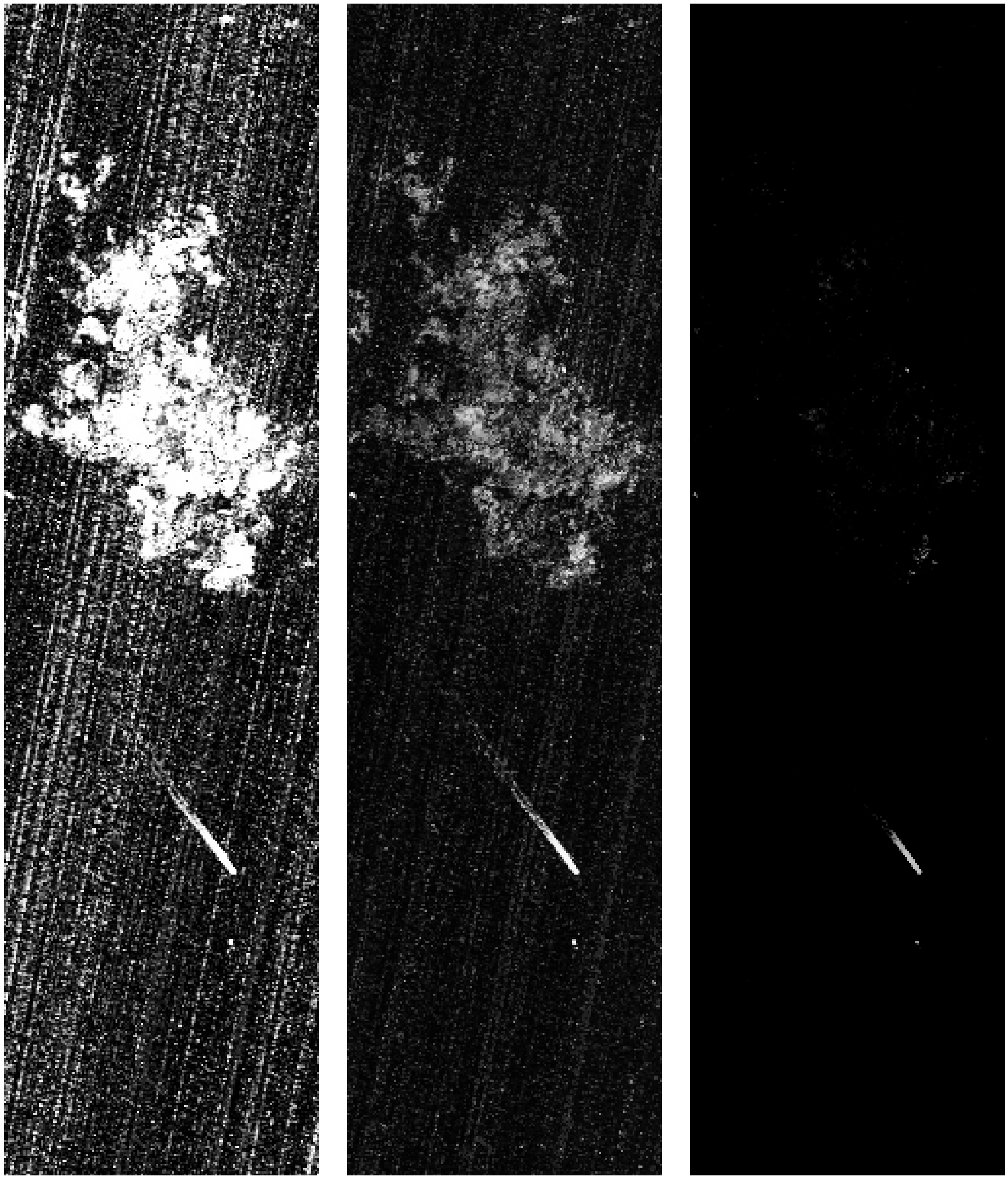


Figure 12: Distributions used to define likelihood for the Naïve Bayes analysis. Strictly defined wake (red) is the definition I've been using in most of this document; loosely defined wake (pink) is a superset including a few more of the surrounding pixels (highly contaminated by water spectra). Eigenbasis direction 5 is the axis that was projected to remove clouds (skipped in the analysis).



Gaussian likelihood Naïve Bayes likelihood Raised to the 10^{th} power

Figure 13: Final comparison of images: the Gaussian likelihood (left) re-shown from Fig. 11, Naïve Bayes (middle) is the product of this analysis, and raised to the 10^{th} power applies `numpy.power(unitIntervalData, 10)` to bring out highly unlikely features.

well as the ships' wakes. When aligned and overlaid on Google Maps images, this won't be a problem for a human because the demarcation between land and sea would be obvious. For an automatic ship-finder, the land would be at least as confusing as clouds— if the spatial algorithm can handle land, then we might as well leave the clouds in, too.

In the final images, we can see the ship's wake that we used for training (just barely), and possibly two more in the third image.

Figure 14: The Naïve Bayes raised to the tenth power method, applied to all three Kagoshima Bay images. The training image is on the left.

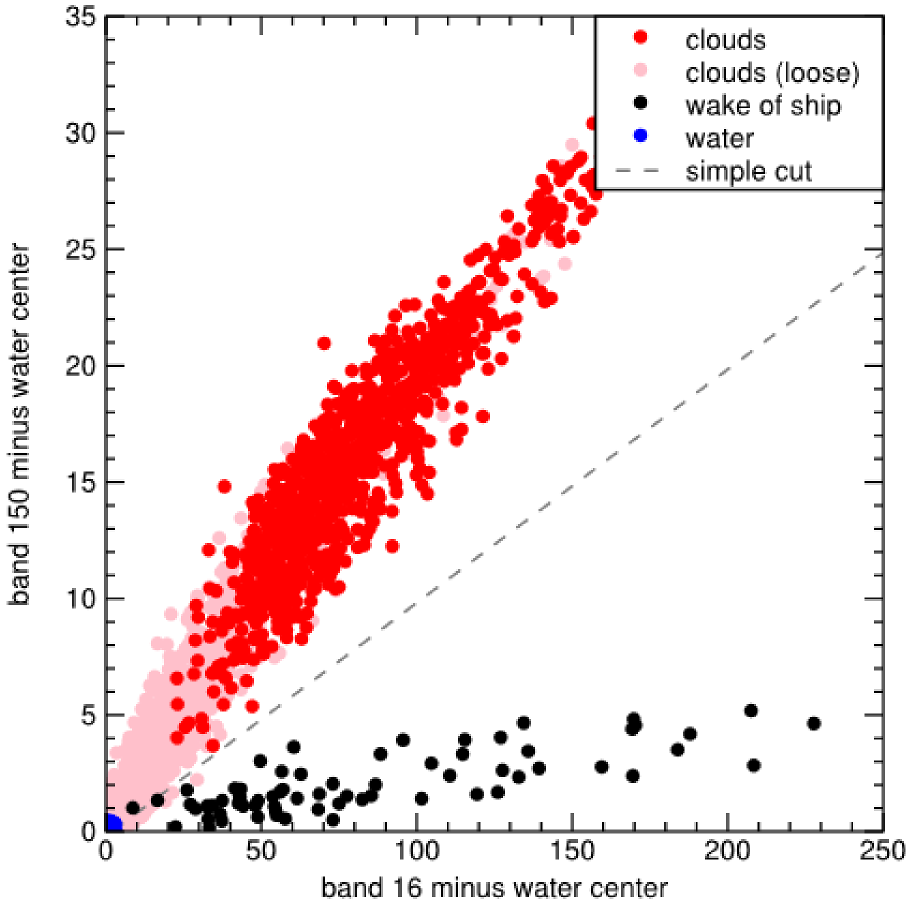


Figure 15: Clouds (tight and loose), wake, and water distributions with a cut in the two-dimensional space of band 16 (cannonical blue) and band 150 (near infrared) to separate clouds from wake.

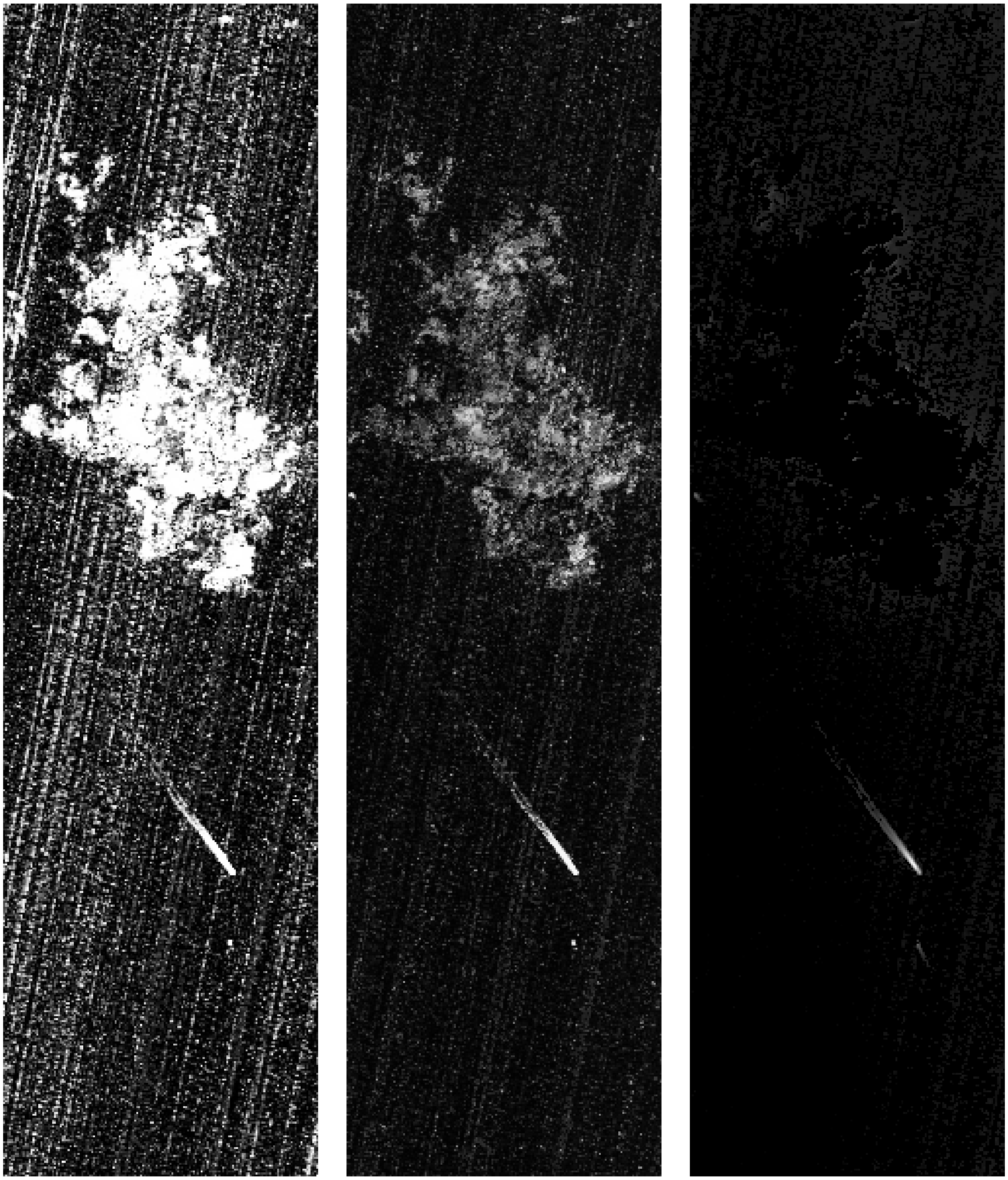
8 A much simpler algorithm

I still don't understand why it's so hard to see the ship's wake in the fully processed images when wake, cloud, and water distributions are easily seperable in Fig. 4. For comparison, I did a much simpler, cuts-based analysis. Figure 15 shows the same data as Fig. 4 with

```
cut = (image016 - 2. > (1000. - 2.)/100.*image150)
```

That is, a line starting 2 units to the right of the origin with slope 100. This should cleanly separate between clouds and wake.

We then define brightness in the image simply by band 16 (cannonical blue) to make Fig. 16. The wake is bright and the clouds are suppressed.



Gaussian likelihood　　Naïve Bayes　　Simple cut

Figure 16: Final images from the three analyses. The simple cut in two bands does everything we need it to. But is it overtrained? Can the hand-selected cut values be automated reliably?

While the simple cut method is much better at identifying the known ship in the training data, it does worse than Naïve Bayes on different datasets. (Neither method was re-trained for different datasets— Naïve Bayes is somehow more stable against changes in conditions.)

Figure 17: The simple cut method, applied to all three Kagoshima Bay images. The training image is on the left. While this method does not highlight the ground, it is too sensitive to changes in conditions. See Fig. 14 for a comparison to Naïve Bayes.

9 Conclusion

None of the resulting methods are satisfactory, but hopefully these code samples will be useful in constructing an optimal method.

Perhaps I've been too focused on eliminating clouds. Do we need to eliminate clouds? None of the methods described here would reveal whatever is below the clouds— they just zero out the clouds so that they're not a source of confusion to a later algorithm in the workflow. But ground features on shore would be just as confusing to an automated algorithm.

Maybe I've been doing the wrong thing by projecting out the clouds as a linear transformation in radiance. Maybe it should be in log(radiance) because absorbtion is multiplicative.

The very last line in my `loglikelihood` function for Naïve Bayes has a division that I think should be a subtraction.

Maybe we could do clustering in spectrum space to find the major components. Clusters close to what we expect for water and for clouds could then be labeled as such and we can define brightness as distance away from the bad clusters. (That's equivalent to centering the water distribution and then projecting out the clouds and calling the origin the bad cluster center.)

Other ideas???