Assignment 1: BLAS and LAPACK

2022 Fall EECS205002 Linear Algebra

Due: 2020/10/19

The 2021 Turing award goes to Jack Dongarra for his contrbution in high-performance computing. One of his important works is to design and implement high-quality mathematical software, which has been widely used in all kinds of machines. In this assignment, we are going to learn two most important packages: BLAS (Basic Linear Algebra Subprograms) and LAPack (Linear Algebra Package).

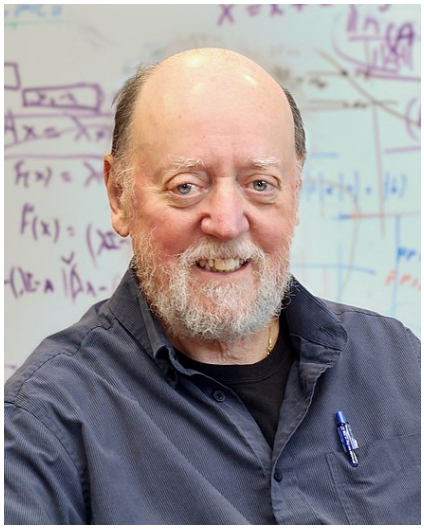

Figure 1: Jack Dongarra, the 2021 Turing Award winner.

Basic Linear Algebra Subprograms (BLAS) is a set of low-level routines for performing common linear algebra operations. It has three levels:

- Level 1: vector operations, such as vector addition, scalar multiplication, and dot products.
- Level 2: matrix-vector operations, such as matrix-vector multiplication.
- Level 3: matrix-matrix operations, such as matrix-matrix multiplication.

Their implementations are usually provided by CPU vendors, such as Intel or AMD. Most HPC chips are also designed to optimize the BLAS performance.

LAPACK is a higher level Linear Algebra library built upon BLAS. It provides routines for solving linear systems, linear least squares, eigenvalue problems, and singular value decomposition. LAPACK is the successor to LINPACK (solving linear systems and linear least sqaure problems) and EISPACK (solving eigenvalue problems). LINPACK is one of the most important benchmarks in the evaluation of supercomputers. The performance optimization is usually made based on the properties of problems. For instance, for different types of

matrices, such as general, banded, symmetric, or Hermitian, the used subroutines will be different.

1 Assignments

1. (10%) Run `numpy.show_config()` and `scipy.show_config()` to see which BLAS and LAPACK libraries are used in your python.

2. (20%) Answer the following questions. You can lookup the this link (`https://netlib.org/blas/#_blas_routines`) and (`https://netlib.org/lapack/explore-html/`). BUT DO NOT copy the explanation directly. Use your own word to explain them.

   (a) (8%) What are the namming convention, starting character S, D, C, Z, for BLAS and LAPACK?

   (b) (12%) What are the meaning of `SSPR2`, `ZGERC`, `DGBSVX`, and `CHEEVR`?

3. (30%) Find the subroutine to solve a linear system $Ax = b$, where A, b are in double precision. Try different matrix sizes and measure/plot their running time. How to verify the correctness of the solution?

4. (20%) The reason why BLAS/LAPACK can be so effective on modern CPUs is that they utilize block matrix operation. (See textbook 1.6.) How to use block matrix operation to solve a linear system $Ax = b$? Derive the formula by partitioning A into a 2×2 block matrix. Explain why a good matrix-matrix multiplication subroutine can accelerate the computation of solving $Ax = b$.

5. (20%) What is the Strassen Matrix-Matrix Multiplication algorithm? Why it is faster than the conventional matrix-matrix multiplication algorithm in theory? Why people usually do not use it in practice for high performance computation?

2 Submission

1. Write a report in PDF file that includes the answers of question (1), (2), (3), (4), and (5).

2. The code of (4).

3. Zip them and submit the zip file to the eeclass.