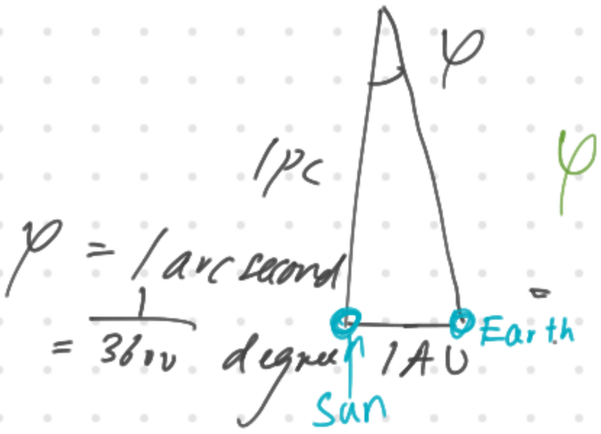
# Basic terms

**Parsec (pc)**: The distance at which 1 AU subtends an angle of 1 arcsecond.

**AU**: The mean distance between the Earth and the Sun (or the Earth's orbit).

**light year (ly)**: The distance which light travels in vacuum in 1 year.




$\varphi = 1 \text{ arcsecond} = \frac{1}{3600} \text{ degree}$

$\varphi = \frac{1AU}{1pc}$

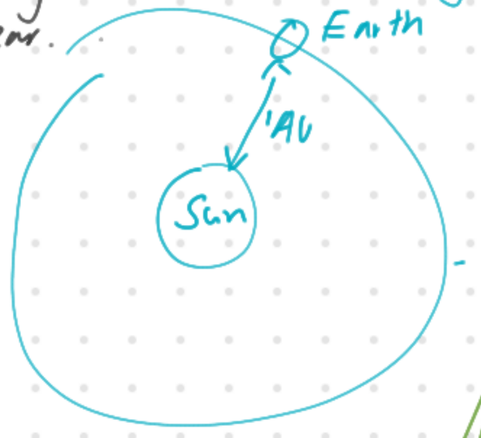
$\frac{\pi}{180}\left(\frac{1}{3600}\right) = \frac{1AU}{1pc}$

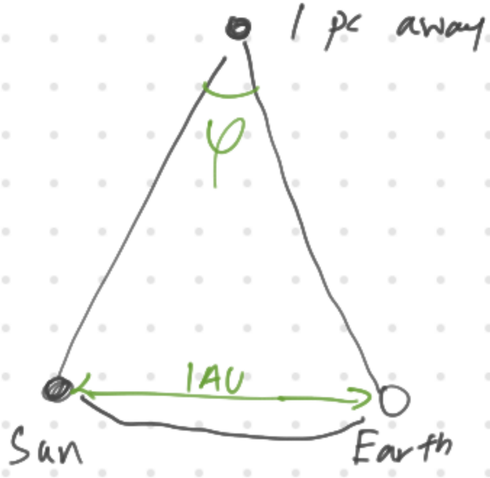
$1pc = 3.09 \cdot 10^{16} m$

$\approx 3.26 ly$

1 degree = 60 arcmin

1 arcmin = 60 arcsec




# Apparent magnitude (Hippacros scale)

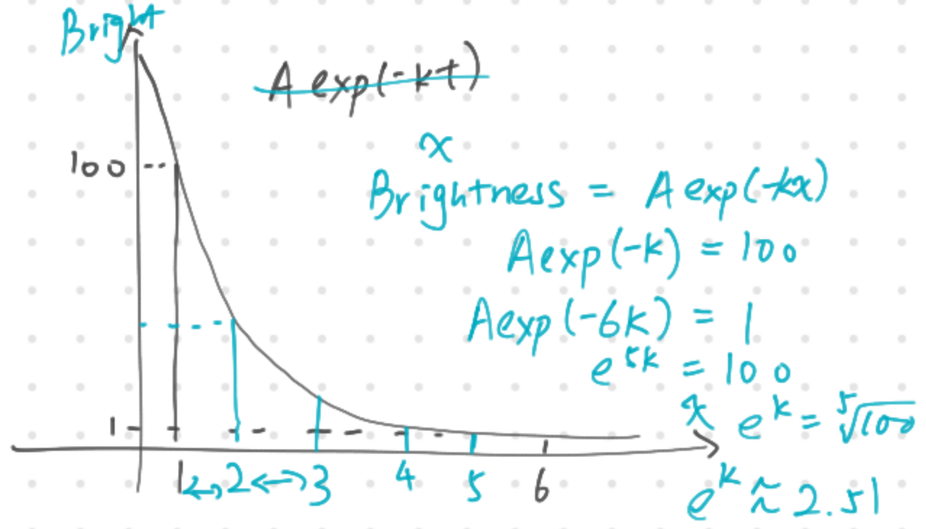

Stars in the sky have magnitudes 1–6 (assigned to by Hippacros)

bright → dim

Magnitude 1 is 100× brighter than 6

But brightness / intensity $= \frac{P}{A} = \frac{P}{4\pi r^2}$

↳ distance dependent




~~$A\exp(-kt)$~~

Brightness $= A\exp(-kx)$

$A\exp(-k) = 100$

$A\exp(-6k) = 1$

$e^{5k} = 100$

$e^k = \sqrt[5]{100}$

$e^k \approx 2.51$

To fix this, we introduce the Absolute magnitude (M)

$m - M = 5\log\left(\frac{d}{10}\right)$ in pc

Smaller = brighter

Since the scale is logarithmic, each 1 difference in absolute magnitude is $100^{1/5} = 2.51\times$ brighter

$x = 2 \Rightarrow A\exp(-2k)$

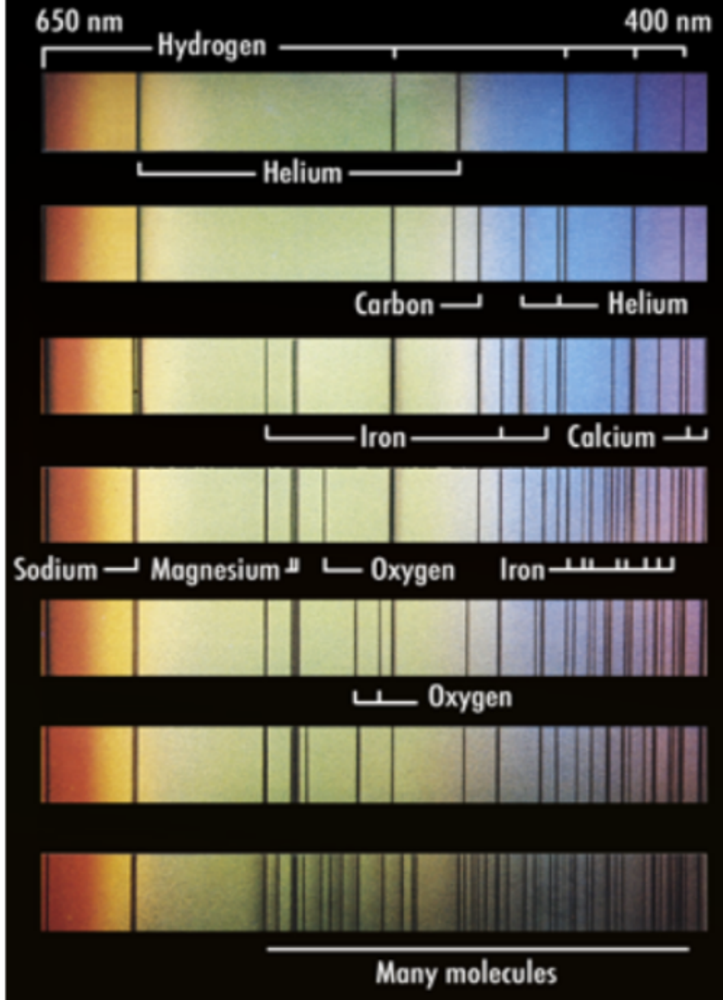
$\frac{Ae^{-k}}{Ae^{-2k}} = e^k$

$L\ dB = 10\left(\log\frac{I}{I_0}\right)$

Absolute magnitude is a measure of the brightness of the star at 10 pc away

Absolute magnitude: Vega → 0.03 Sirius → 1.4 Sun → 4.83

## STELLAR CLASSIFICATION (MAIN-SEQUENCE)




| CLASS | COLOR | TEMP. (C°) | HYDROGEN (Balmer) | SIZE(SOLAR R) | % OF ALL |
|---|---|---|---|---|---|
| O | | >30,000 | WEAK | >6.6 | ~0.00003% |
| B | | 9,700 - 30,000 | MEDIUM | 1.8 - 6.6 | 0.13% |
| A | | 7,200 - 9,700 | STRONG | 1.4 - 1.8 | 0.6% |
| F | | 5,700 - 7,200 | MEDIUM | 1.1 - 1.4 | 3% |
| G | | 4,900 - 5,700 | WEAK | .9 - 1.1 | 7.6% |
| K | | 3,400 - 4,900 | VERY WEAK | .7 - .9 | 12.1% |
| M | | 2,100 - 3,400 | VERY WEAK | <.7 | 76.5% |

Appear when the star is hot enough (but not too hot) to energise $e^-$ in $H^+$ atoms to the $n=2$ state

Too hot, $e^-$ in $H^+$ are ionised / excited to $n=1, \gamma$ states

**7.** The table shows some properties of the four brightest stars in the constellation Canis Minor.

| Name | Apparent magnitude | Absolute magnitude | Spectral class |
|---|---|---|---|
| Gamma A | 4.46 | –0.50 | K |
| Gomeisa | 2.89 | –0.70 | B |
| HD 66141 | 4.39 | –0.13 | K |
| Procyon | 0.34 | 2.65 | F |

(a) Discuss, with reference to the Hipparcos scale, why many star maps show only two stars in the constellation Canis Minor.

Gamma A and HD66141 have very dim apparent magnitudes and are thus unlikely to observed when compared to the other 2 stars, which ~~have~~ ~~this~~ are bright in terms of the Hipparcos scale.

(e) Calculate the distance from the Earth to Procyon.
Give an appropriate unit for your answer.

$$m - M = 5 \log \frac{d}{10}$$

$$5 \log \frac{d}{10} = -2.31$$

$$d = 3.45 \text{ pc}$$

# Stars as black bodies

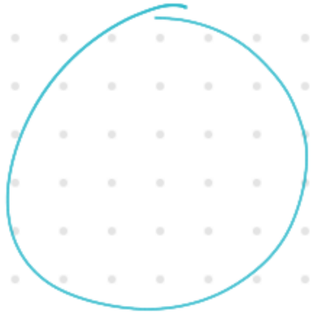
All stars are approximated as black bodies when estimating their power.

$$P = \sigma A T^4$$

Stefan Boltzmann constant

$T$: surface temp.

$A$: Surface area



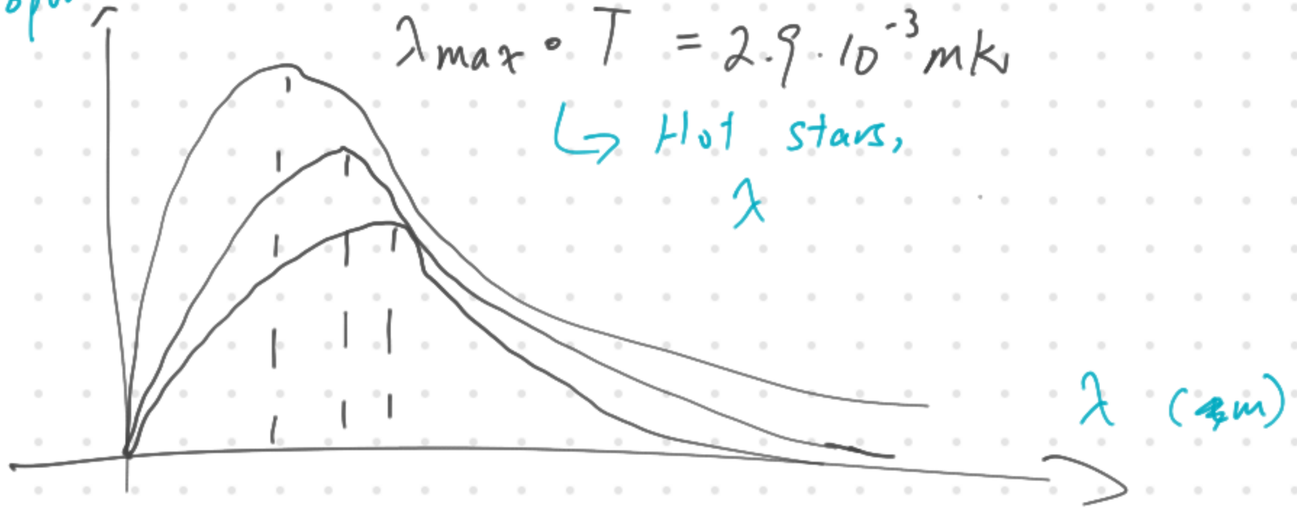
$$P = \sigma A (\Delta T)^4$$

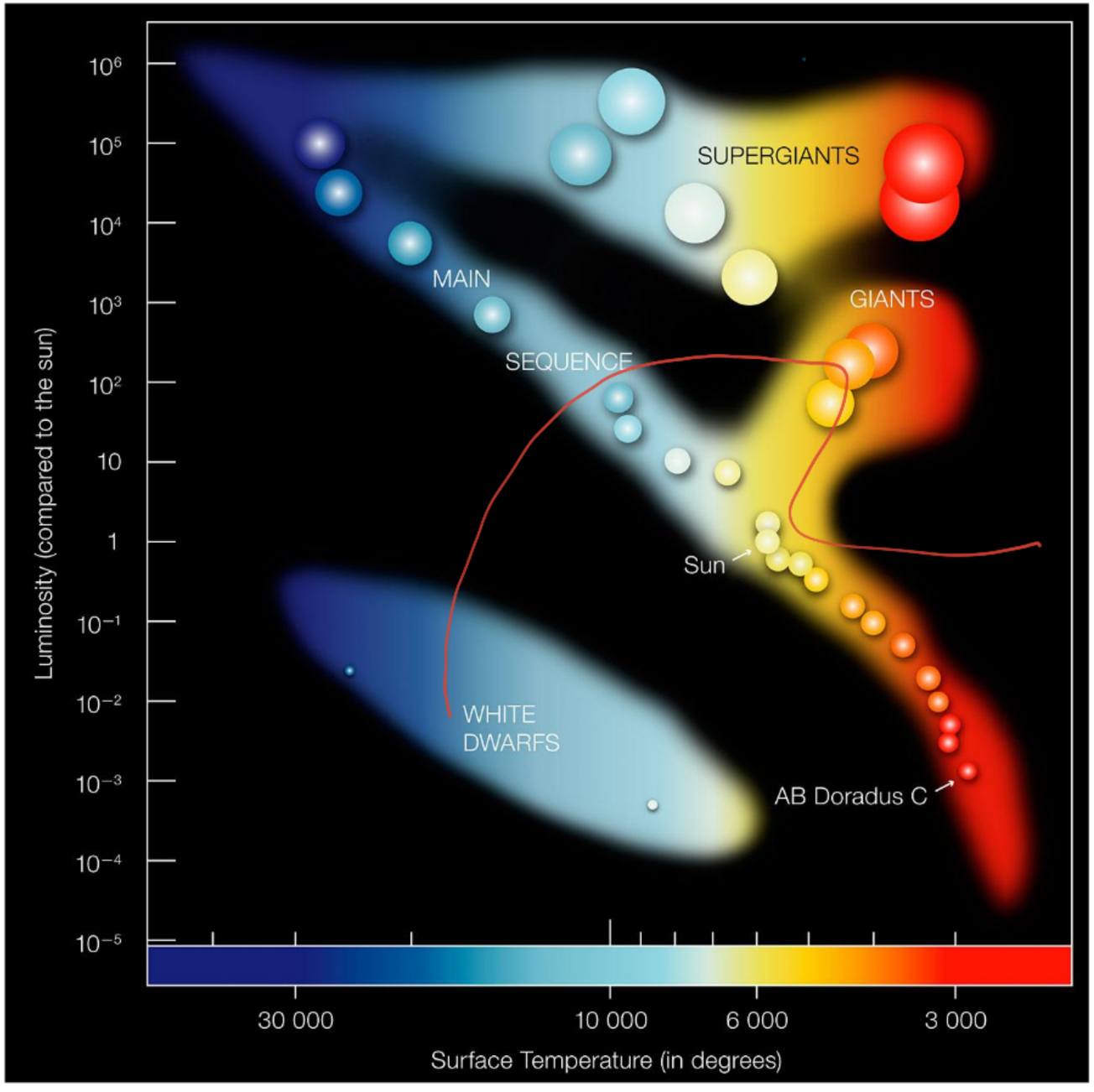
Stars also emit EM waves of different $\lambda$.

Proportion

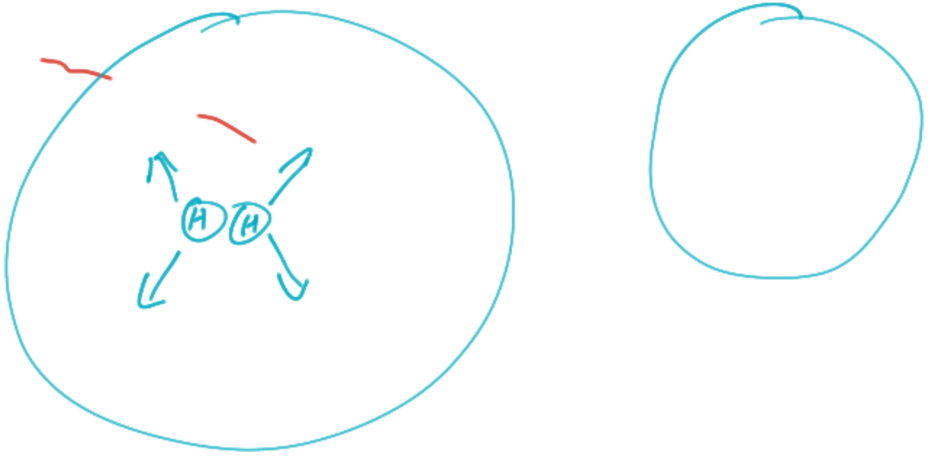
$$\lambda_{max} \cdot T = 2.9 \cdot 10^{-3}\ mK$$

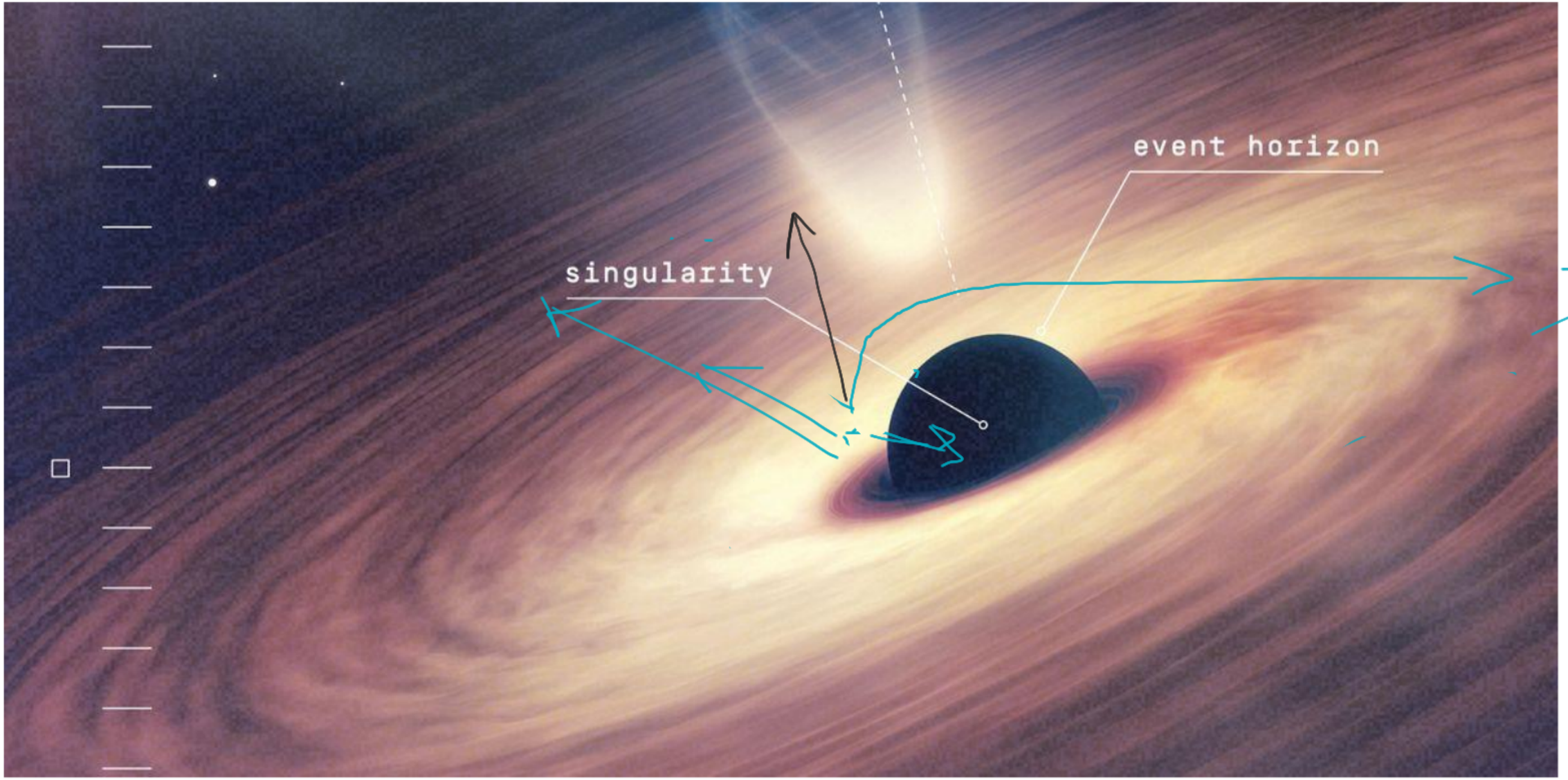
↳ Hot stars, $\lambda$

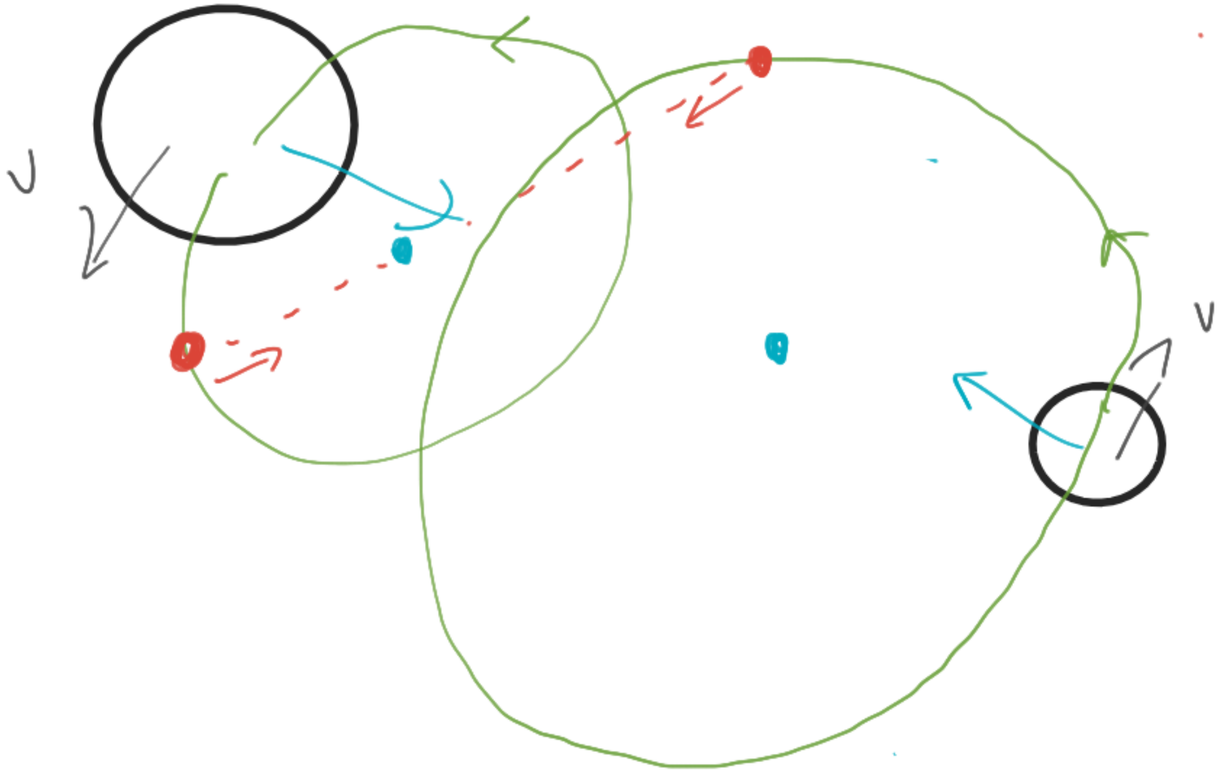





1. Protostar
2. Main sequence star
   - core $\sim 10^9$ k
   - # Fusion occurs
3. Giant phase → White Dwarf $< 1.4\ M_\odot$
   - (Supernova)
4. Neutron star // Black hole
   - 1.4-3 $M_\odot$ // $> 3\ M_\odot$




event horizon
singularity



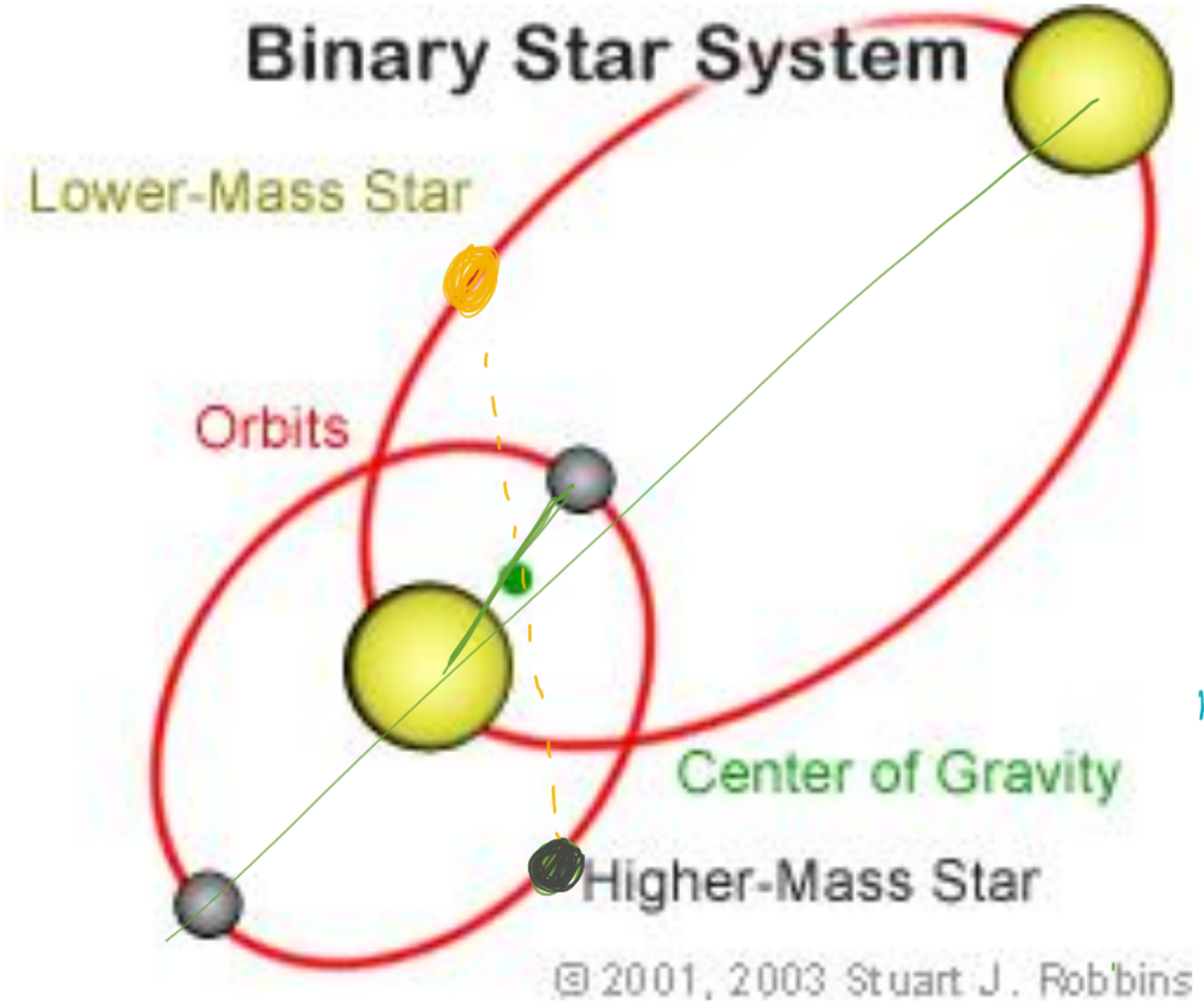

$$F = \frac{GmM}{r^2}.$$

$$a = \frac{F}{m}.$$

$$a_1 = \frac{F_g}{m_1} \qquad a_2 = \frac{F}{m_2}$$

$$a \propto \frac{1}{m}$$

(bigger star have more inertia)



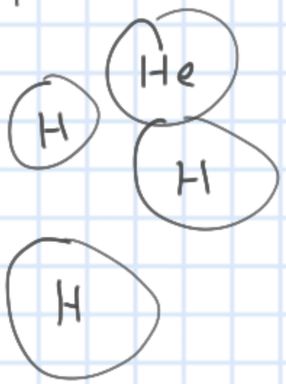

$$\vec{x}_{CM} = \frac{m_1 \vec{x}_1 + m_2 \vec{x}_2}{m_1 + m_2}$$

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{G m_1 m_2}{(r_1 + r_2)^2}$$


$$m_1 r_1 \omega^2 = F_g = m_2 r_2 \omega^2$$

$$\boxed{m_1 r_1 = m_2 r_2}$$

Proto star



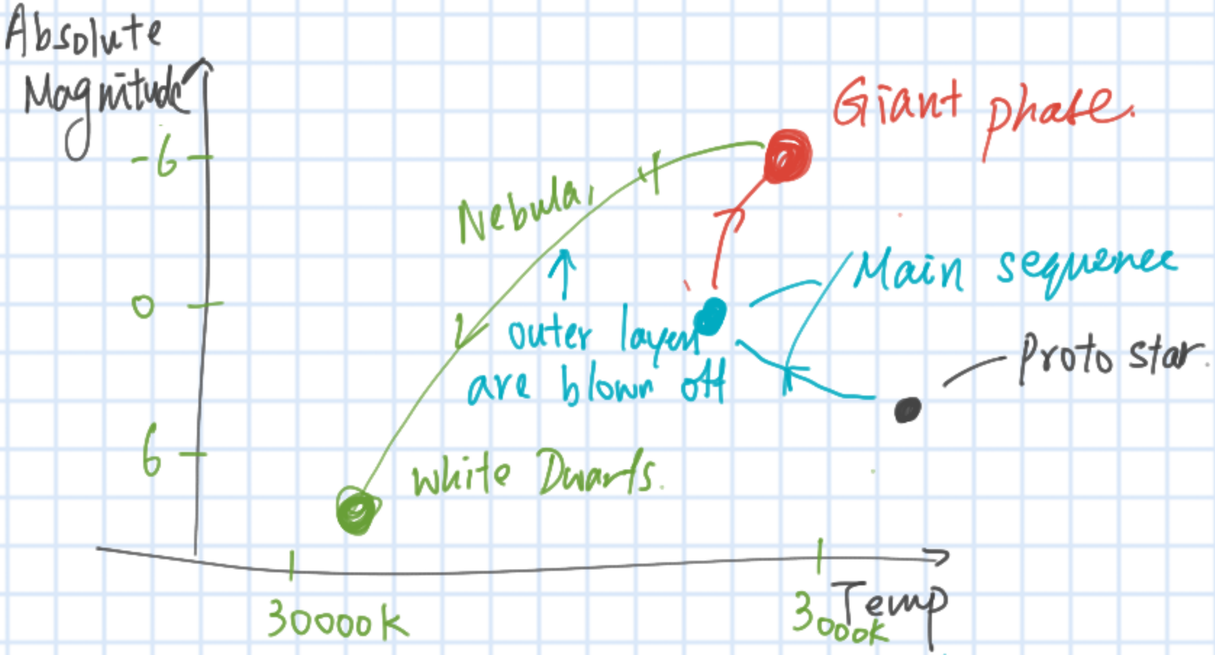

→ Not very hot

→ Not very bright

Main sequence star

— surface T ~5000k (dependent on star)

Core (very dense ~$10^8$k) — have fusion




Giant phase

- Core H used up.
- take up H of outer layer

→ Swell

→ Brighter. (More red)

- All fuel is progressively used up

White dwarf:

- Not very bright
- Small
- Very hot
- life time : Much longer.

(b) State and explain which star in the table above has the most prominent Hydrogen Balmer absorption lines.

Gomeisa since its a B class star.
B class stars are hot enough (but not too hot) to energise $e^-$ in H atoms to the $n=2$ state.

**7.** The table shows some properties of the four brightest stars in the constellation Canis Minor.

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(a) Discuss, with reference to the Hipparcos scale, why many star maps show only two stars in the constellation Canis Minor.

(c) Deduce which star, Gamma A or HD 66141, has the larger diameter.

∵ They are in the same spectral class
∴ Similiar surface T.
∵ Gamma A ~~has~~ has a brighter absolute magnitude
→ Power output of ~~HD 66141~~ Gamma A is higher

$$P = \sigma A T^4 = \sigma T^4 4\pi\left(\frac{d}{2}\right)^2 = \pi\sigma T^4 d^2$$

→ $P \propto d^2$
So Gamma A is larger

**2.** The table summarises some information about four stars in the constellation Cassiopeia.

| Name | Colour | Apparent magnitude | Distance / ly |
|---|---|---|---|
| Caph | white | 2.3 | 55 |
| Ruchbah | blue/white | 2.7 | 99 |
| Schedar | orange | 2.2 | 228 |
| Tsih | blue | 2.2 | 610 |

1 pc = 3.26 ly

1,16

0,28 $M = m - 5 \log \frac{d}{10}$

-2,02

-4,16

(a) Which star has the highest surface temperature?
Tick (✓) **one** box.

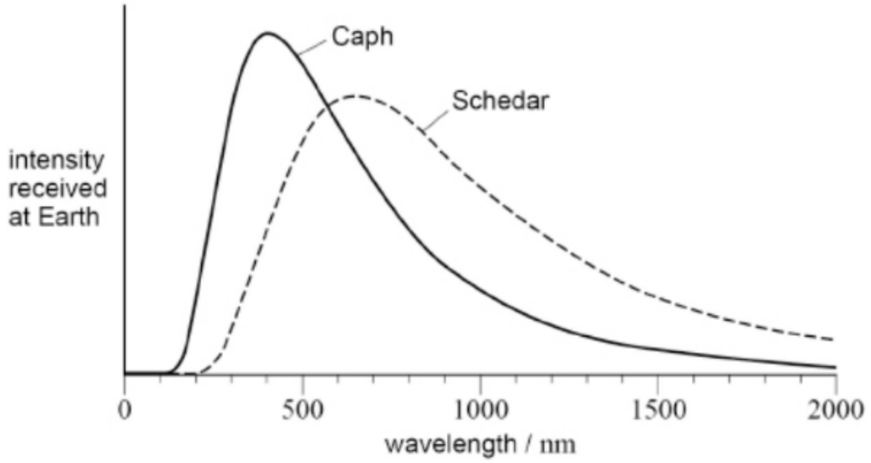
Caph ☐

Ruchbah ☐

Schedar ☐

Tsih ☑

(b) The graph below shows the intensity received at Earth from two of the stars, plotted against wavelength.
The effect of absorption by the Earth's atmosphere is not shown.




Discuss what information can be found from the graph about the temperature and colour of these stars.
Support your answer with suitable calculations.

Temperature:

Caph: $\lambda_{max} T = 2.9 \cdot 10^{-3}$ mK

$T_C = 7250$ K

Schedar: $\lambda_{max} T = 2.9 \cdot 10^{-3}$ mK

$T_S = 4400$ K

Colour

Caph B in class A ∴ its $T_C$ 7250k

⇒ Caph B white

Schedar B in class K

→ Schedar B orange

(c) State which star in the table above is dimmest on the absolute magnitude scale.

Caph

(1)

(d) Calculate the absolute magnitude of Schedar.

$1\,pc = 3.26\,ly$

$$m - M = 5 \log \frac{d}{10}$$

$$M = m - 5 \log \left( \frac{228/3.26}{10} \right)$$

$$M = -2.02$$

(e) Tsih has a mass over 15 times the mass of the Sun.
Tsih may eventually collapse to form a black hole.

Calculate the radius of the event horizon for a black hole with a mass 15 times that of the Sun.

$$c^2 = \frac{2GM}{R_S}$$

$$R_S = \frac{2GM}{c^2}$$

$$R_S \approx 4.4 \cdot 10^4\ m$$


$$v_{esc}^2 = \frac{2GM}{R}$$

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infinitely far away

$$\frac{1}{2}mv^2 = -\Delta U$$

$$\frac{1}{2}mv^2 = -\left( \frac{-GmM}{r} - \frac{GmM}{R_e} \right)$$

($r \to \infty$, $\frac{1}{r} \to 0$)

$$\frac{1}{2}mv_{esc}^2 = \frac{GmM}{R_e}$$

$$v_{esc}^2 = \frac{2GM}{R_e}$$

**12.** The table summarises some of the properties of four stars in the constellation Hercules.

| Star | Distance/pc | Spectral class | Apparent magnitude |
|---|---|---|---|
| Kornephoros | 43 | G | 2.8 |
| Rasalgethi | 110 | M | 3.0 |
| Rutilicus | 11 | G | 2.8 |
| Sarin | 23 | A | 3.1 |

(a) Define the parsec. You may use a diagram as part of your answer.

**(2)**

(b) Deduce which star is larger, Kornephoros or Rutilicus.

(3)

(c) One of the four stars has the peak in its black-body radiation curve at a wavelength of 1.0 μm.

Calculate the corresponding temperature for this curve.

(d) Explain which star produced the black-body radiation curve described in question (c).

(e) Which star has the brightest absolute magnitude?

Tick (✓) the correct box.

Kornephoros ☐

Rasalgethi ☐

Rutilicus ☐

Sarin ☐

**(1)**

(f) Determine the absolute magnitude of Sarin.