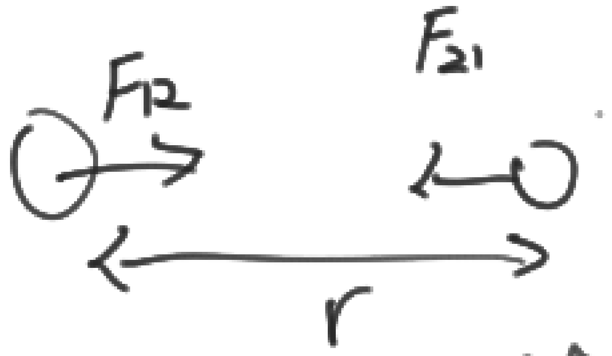
G field & E field

G field
due to
mass



$$|F_{12}| = |F_{21}| = \frac{Gm_1m_2}{r^2}$$

Energy:

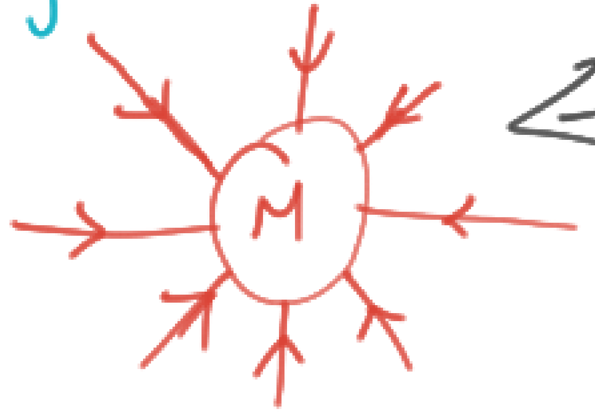
$$F \cdot r = \frac{Gm_1m_2}{r}$$

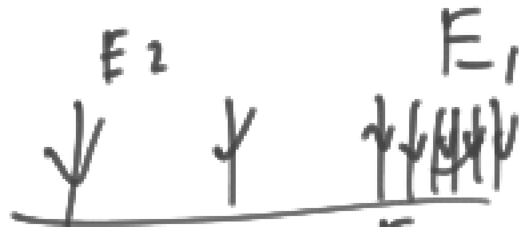
Definition of g-field:

$$g = \frac{F_g}{m} = \frac{GM}{r^2}$$

unit: NKg^{-1}

Gravitational force exerted per kilogram of the object's mass





$E_1 > E_2$

Electrostatic force

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

are the forces exerted on both particles equal?

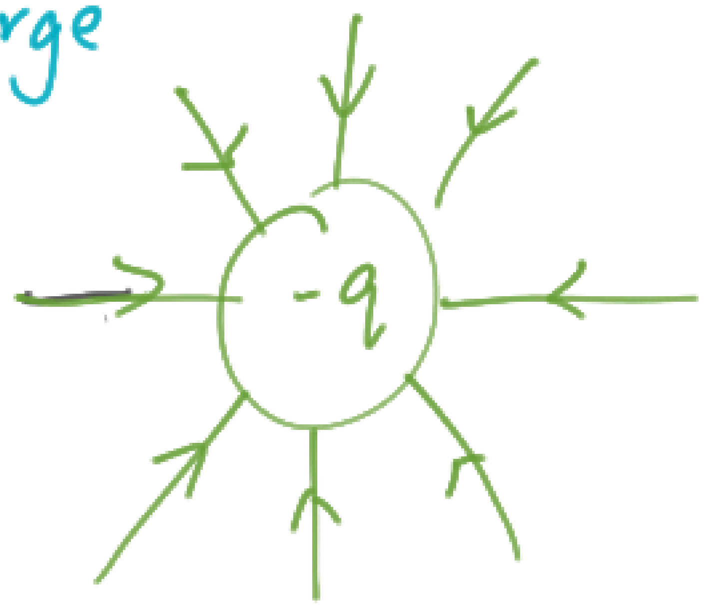
E field:

$$\frac{F}{q} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}$$

Electric force exerted on a positive test charge per unit charge

$$\vec{F} = q\vec{E}$$

$$\vec{F} = -k\vec{E}$$



\# line density = field strength

Gravitational potential energy

$\boxed{U = -\frac{GmM}{r}} \leftarrow \int \frac{GmM}{r^2}\,dr$

$r \to \infty \quad U \to 0 \quad (Max)$

$\frac{GmM}{r^2} = F$

$W = F \cdot s$

$(+) \xleftrightarrow{1m} (-)$

$V_i = 0 \quad V_f = \frac{1}{4\pi\varepsilon_0}\frac{-(q\,q)}{1}$

$-\Delta U = \int F \cdot dr$

Electrical potential energy

$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \leftarrow \int \boxed{\frac{1}{4\pi\varepsilon_0}}\boxed{\frac{q_1 q_2}{r^2}}\,dr$

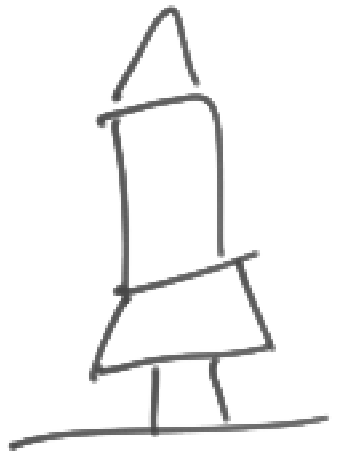
$(+)(-) \to (-) \qquad (+)(+) \to (+) \qquad (-)(-) \to (+)$

Lower U_E is always more stable.

$\frac{-k}{4\pi\varepsilon_0}\frac{1}{r \to 0} \to -\infty \qquad \frac{k}{4\pi\varepsilon_0}\frac{1}{r \to \infty} \to 0 \qquad \frac{k}{4\pi\varepsilon_0}\frac{1}{r \to \infty} \to 0$

Useful in energy calculations!

Example: we are firing a rocket from Earth, find V_{min} required s.t. the rocket escapes the earth's gravitational field.



GPE ↑ r ↑

r ↑ F ↓ $F \propto \frac{1}{r^2}$

$V = \boxed{-\frac{GmM}{r}}$ r ↑ U ↑ F ↓

$V = -\frac{GmM}{0}$ $U = \frac{-GmM}{1}$

$-GmM$ ↑ larger

KE → GPE

ΔV

$\frac{1}{2}mv^2 = \left(0 - \left(-\frac{GmM}{Re}\right)\right) - \infty$

$\frac{1}{2}mv^2 = \frac{GmM}{Re}$ $v = \sqrt{\frac{2GM}{Re}}$

Gravitational potential

$\Phi = \frac{U}{m}\ (J\,kg^{-1})$

$\Delta \Phi = 3\ J/kg$

Electrical potential

$V = \frac{U}{q}\ (V / JC^{-1})$

$500 = \frac{\Delta U}{q_1}$

$\Delta V = V_f - V_i \qquad U = -\frac{GmM}{r}$

$E = \frac{\Delta V}{d}$

$F_e = mg$

$qE = mg \quad E = \frac{mg}{q} \quad V = \frac{mgd}{q}$

Q9.

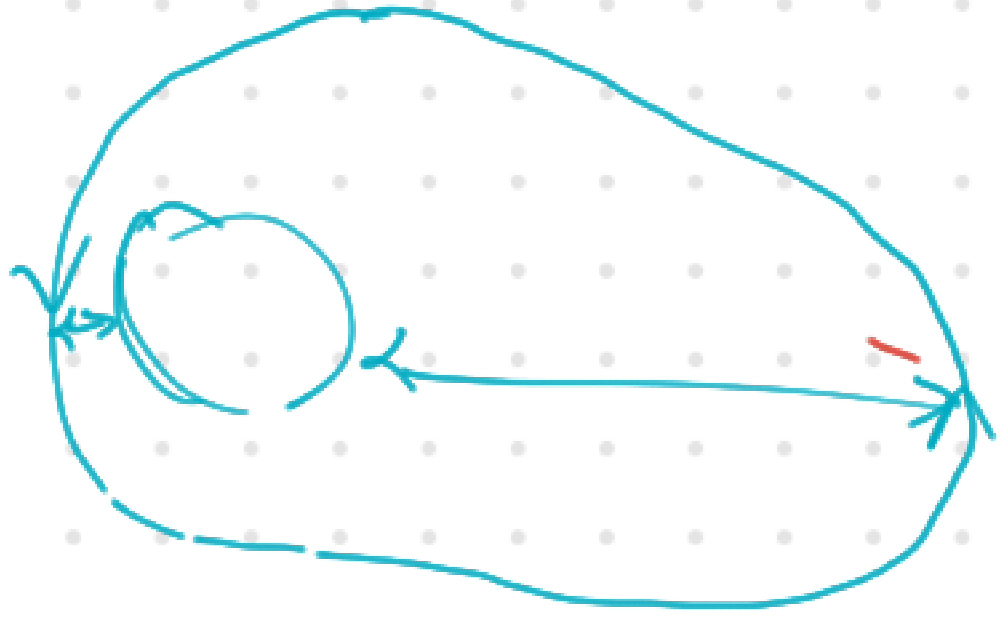
Astronomers observing stars at the centre of our galaxy have suggested that many of th are orbiting a supermassive black hole. The mass of this black hole is 9.2×10^{36} kg.

The star S0-2 is in a highly elliptical orbit around the position of the black hole.

At its point of closest approach, S0-2 is at a distance of 1.8×10^{13} m from the centre of t black hole.

At the most distant point of its orbit, S0-2 is 2.7×10^{14} m from the black hole.

(i) Show that the change in gravitational potential between the closest and most distant points in this orbit is about 3×10^{13} J kg^{-1}.



$$U = \frac{-GMm}{r}$$

$$\Phi = \frac{U}{m} \rightarrow \Delta\Phi = \frac{\boxed{\Delta U}}{m}$$

$$\Phi = -\frac{GM}{r}$$

$$\Delta\Phi = \Phi_f - \Phi_i$$

$$= -\frac{GM}{r_f} - \left(-\frac{GM}{r_i}\right)$$

$$= -\frac{GM}{r_f} + \frac{GM}{r_1}$$

$$\approx 3.18 \cdot 10^{13} \text{ J kg}^{-1}$$

(ii) At its point of closest approach, the star is travelling at a speed of 8.1×10^6 m s^{-1}.

Calculate the speed of S0-2 at the furthest point in its orbit using the change in gravitational potential.

mass of S0-2 = 2.4×10^{31} kg

V_0

(3)

Conservation of energy!

$$\Delta KE = -\Delta V$$

$$\frac{1}{2}\cancel{m}(v_f^2 - v_0^2) = -\cancel{m}\Delta\Phi$$

$$v_0^2 - v_f^2 = 2\Delta\Phi$$

$$v_f = 1.42 \cdot 10^6 \text{ ms}^{-1}$$

Speed = ...

Q2.

grav field strength

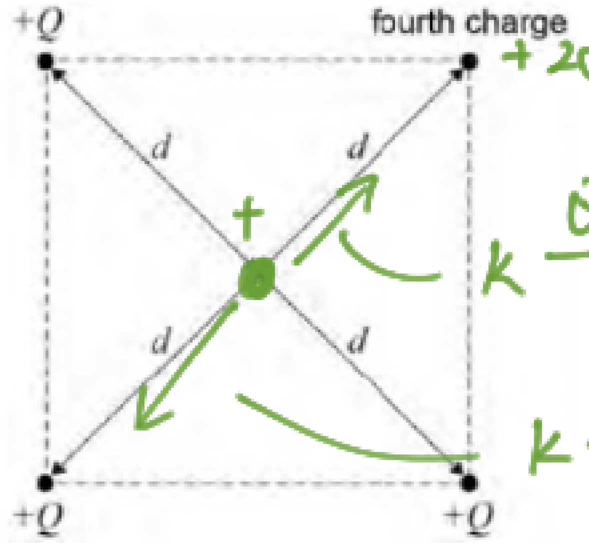
The acceleration of free fall at the surface of the Earth is 9.81 m s^{-2}.
The mass of the Earth is M and the diameter of the Earth is D.

Which of the following gives the acceleration of free fall, in m s^{-2}, at the surface of a planet with diameter $\frac{D}{2}$ and mass $\frac{M}{9}$?

- A $\frac{9.81 \times 2}{9}$
- B $\frac{9.81 \times 4}{9}$
- C $\frac{9.81 \times 2}{3}$
- D $\frac{9.81 \times 9}{4}$

$$a = \frac{GM}{r^2} = \frac{F_g}{m}$$

8. Four positive charges are fixed at the corners of a square as shown.



$k\frac{Qq}{r^2}$

$k\frac{2Qq}{r^2}$ larger

The total potential at the centre of the square, a distance d from each charge, is $\frac{5Q}{4\pi\varepsilon_0 d}$

Three of the charges have a charge of +Q

What is the magnitude of the fourth charge?

A $-\frac{7Q}{4}$

B Q

C $\sqrt{2}Q$

D $2Q$

(Total 1 mark)

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{U}{q}$

$= \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}_{U} \cdot \underbrace{\frac{1}{q_2}}_{\frac{1}{q}}$

$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$

neg

$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{d} + \cancel{\frac{Q}{d} - \frac{Q}{d}} + \frac{q'}{d}\right)$

$= \frac{Q}{4\pi\varepsilon_0 d} + \frac{1}{4\pi\varepsilon_0}\frac{q'}{d} = \frac{5Q}{4\pi\varepsilon_0 d}$

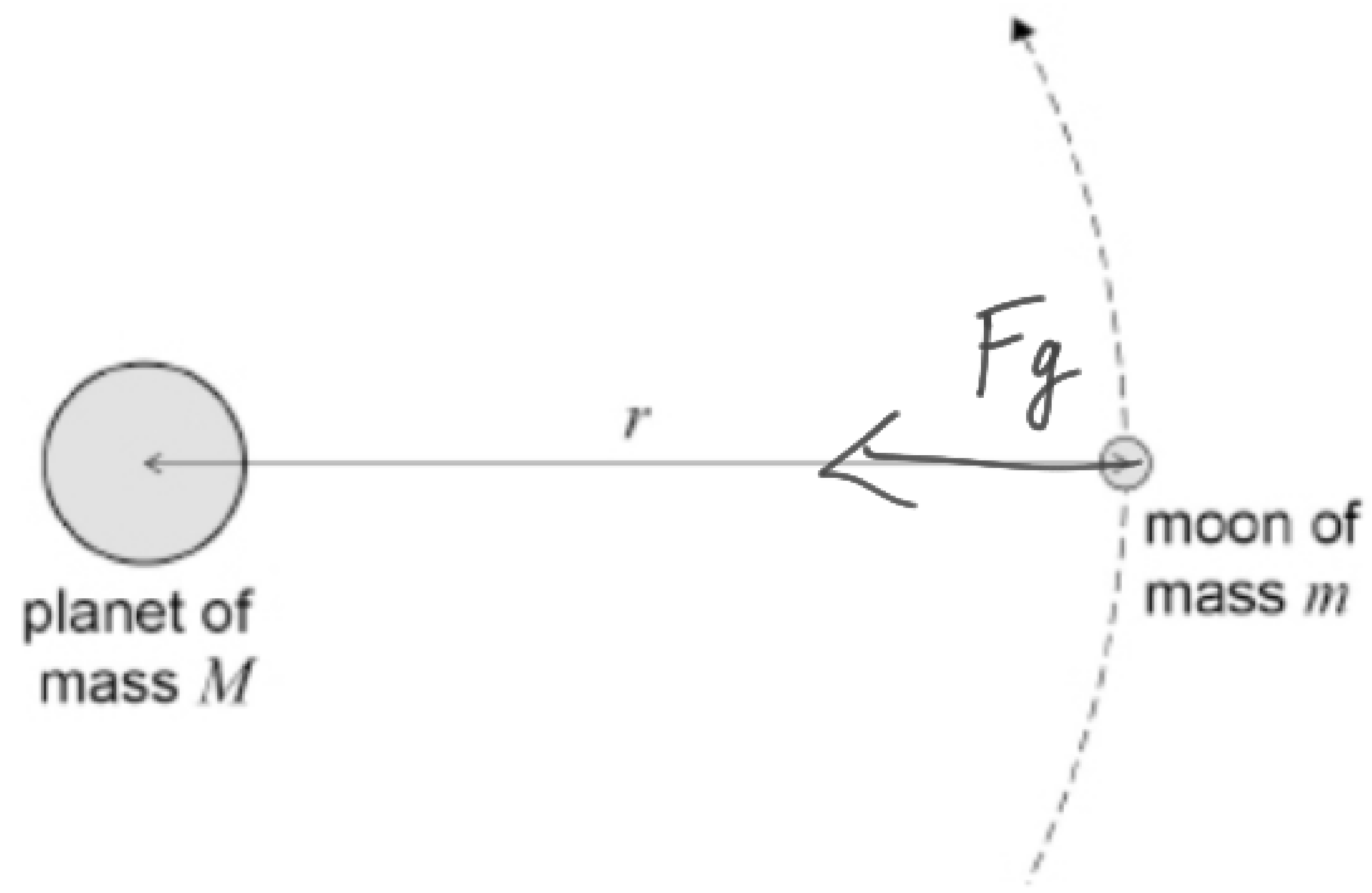
$\frac{q'}{4\pi\varepsilon_0 d} = \frac{4Q}{4\pi\varepsilon_0 d}$

$q' \leftrightarrow 4Q$

$\frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{d^2}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{d^2}\right)$

$|\vec{E}| = \frac{Q}{4\pi\varepsilon_0 d^2}$ NC^{-1} / Vm^{-1} $\quad E = \frac{V}{d}$

4. The figure shows a moon of mass *m* in a circular orbit of radius *r* around a planet of mass M where $m << M$.



The moon has an orbital period T.
T is related to r by

$$\boxed{T^2} = kr^3$$

where k is a constant for this planet.

(a) Show that $k = \frac{4\pi^2}{GM}$

$$\frac{GM}{r^2} = rw^2$$

Kepler's 3rd law

$$\frac{r}{w^2} = \frac{1}{GM}r^3 \rightarrow T^2 = \boxed{\frac{4\pi^2}{GM}}r^3 \quad T^2 \propto r^3 \rightarrow \boxed{\frac{T^2}{r^3}} = \frac{4\pi^2}{GM} = \text{Constant}$$

Table 1 gives data for two of the moons of the planet Uranus.

Table 1

Name	T / days	r / m
Miranda	1.41	1.29×10^8
Umbriel	4.14	**X**

(b) Calculate the orbital radius **X** of Umbriel.

$$\frac{1.41^2}{(1.29\cdot10^8)^3} = \frac{4.14^2}{x^3}$$

$$x^{\cancel{3}} = 2.65\cdot10^8 \text{ m}$$

(c) Calculate the mass of Uranus.

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$$M = \frac{4\pi^2 r^3}{GT^2}$$

mass = $8.60 \cdot 10^{25}$ kg

Table 2 gives data for three more moons of Uranus.

Table 2

Name	Mass / kg	Diameter / m
Ariel	1.27×10^{21}	1.16×10^{6}
Oberon	3.03×10^{21}	1.52×10^{6}
Titania	3.49×10^{21}	1.58×10^{6}

(d) Deduce which moon in **Table 2** has the greatest escape velocity for an object on its surface.
Assume the effect of Uranus is negligible.

Planet escape velocity, given r & M

$$\frac{1}{2}\cancel{m}v_{min}^2 = G\frac{\cancel{m}M}{r}$$

$$v_{min}^2 = \boxed{\frac{2GM}{r}}$$

Titania has largest V_{escape}

(3)

(e) A spring mechanism can project an object vertically to a maximum height of 1.0 m from the surface of the Earth.

$E_s = mgh$

Determine whether the same mechanism could project the same object vertically to a maximum height greater than 100 m when placed on the surface of Ariel.

$\Delta h \ll r_a$

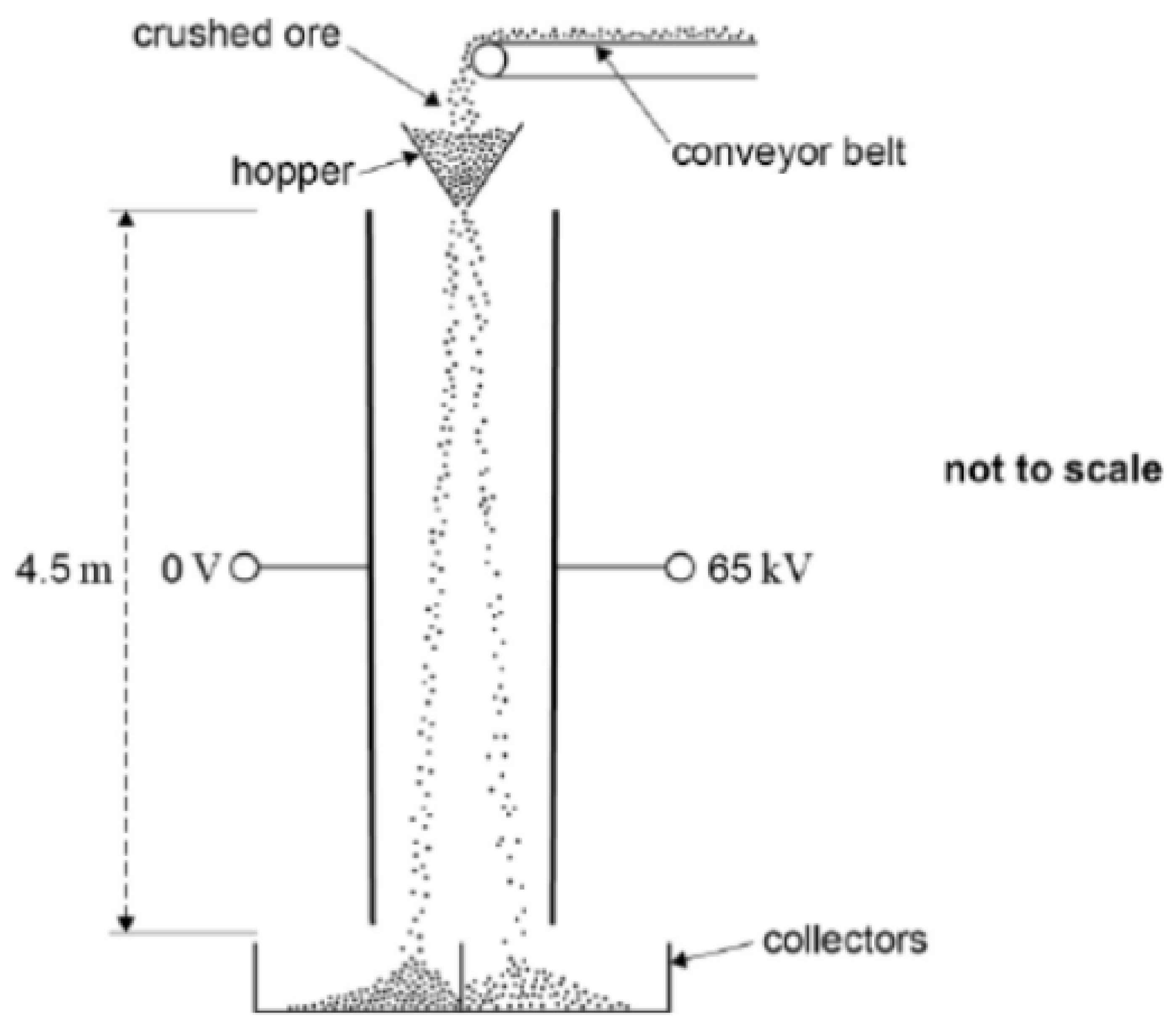
$$g_a = \frac{GM}{r^2}$$

$g_a = 0.25\ ms^{-2}$

$\cancel{m}g_a h_a = \cancel{m}g_e h_e$ $\rightarrow$ ~~9.8~~ 10

$h_a = 40\ m < 100\ m \therefore$ No!

10. The figure below shows a system that separates two minerals from the ore containing them using an electric field.



The crushed particles of the two different minerals gain opposite charges due to friction as they travel along the conveyor belt and through the hopper. When they leave the hopper they fall 4.5 metres between two parallel plates that are separated by 0.35 m.

(a) Assume that a particle has zero velocity when it leaves the hopper and enters the region between the plates.

Calculate the time taken for this particle to fall between the plates.

(b) A potential difference (pd) of 65 kV is applied between the plates.

Show that when a particle of specific charge 1.2×10^{-6} C kg^{-1} is between the plates its horizontal acceleration is about 0.2 m s^{-2}.

(3)

(c) Calculate the total horizontal deflection of the particle that occurs when falling between the plates.

horizontal deflection = ____________ m

(1)

(d) Explain why the time to fall vertically between the plates is independent of the mass of a particle.

___

___

___

___

(2)

(e) State and explain **two** reasons, why the horizontal acceleration of a particle is different for each particle.

(4)

(Total 12 marks)

5. A rocket carrying an artificial satellite is launched vertically from the Earth. When the rocket is at a certain height from the Earth's surface, it expels 2.60×10^3 kg of gas per second with a certain speed v towards the Earth's centre. As a result, an average thrust of 5.20×10^6 N is produced. Neglect air resistance.

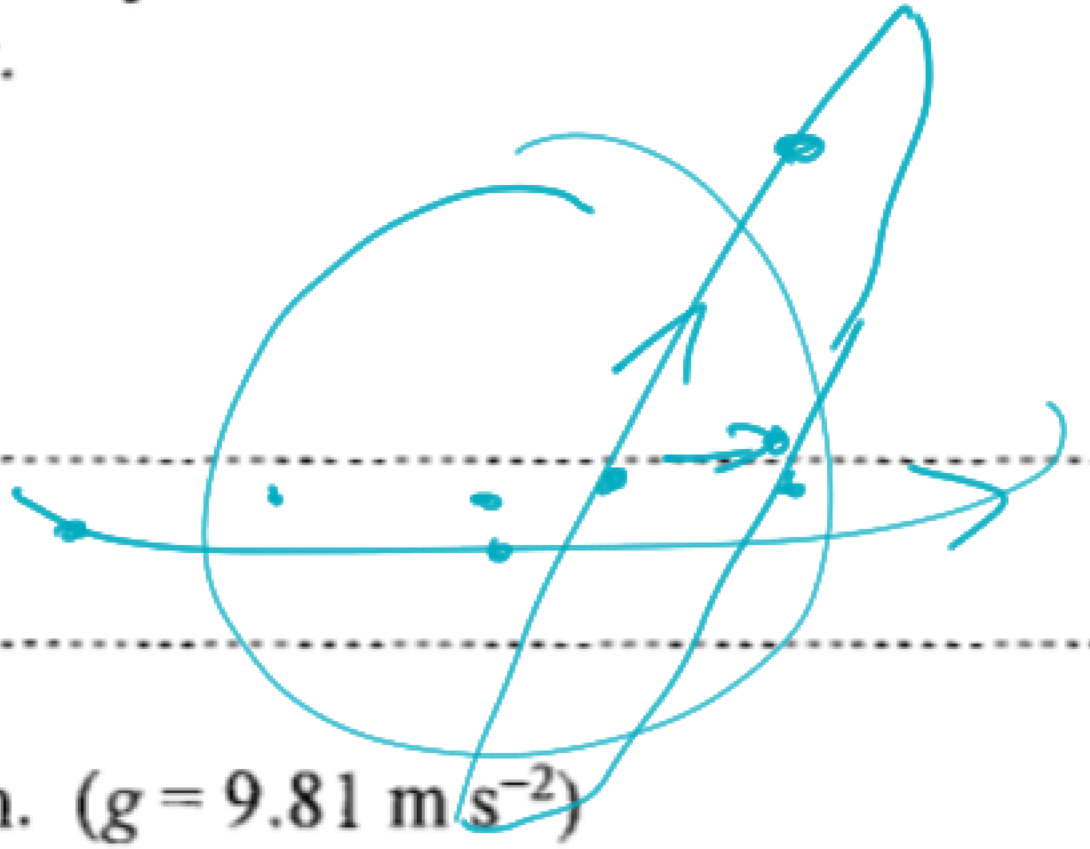
(a) (i) Assuming that the speed of the rocket is negligible, estimate v. (2 marks)

(ii) At a certain instant, the total mass of the rocket and the artificial satellite is 3.60×10^5 kg while the acceleration due to gravity at the rocket's position is 8.56 m s^{-2}. Estimate the acceleration a of the rocket at this position. (2 marks)

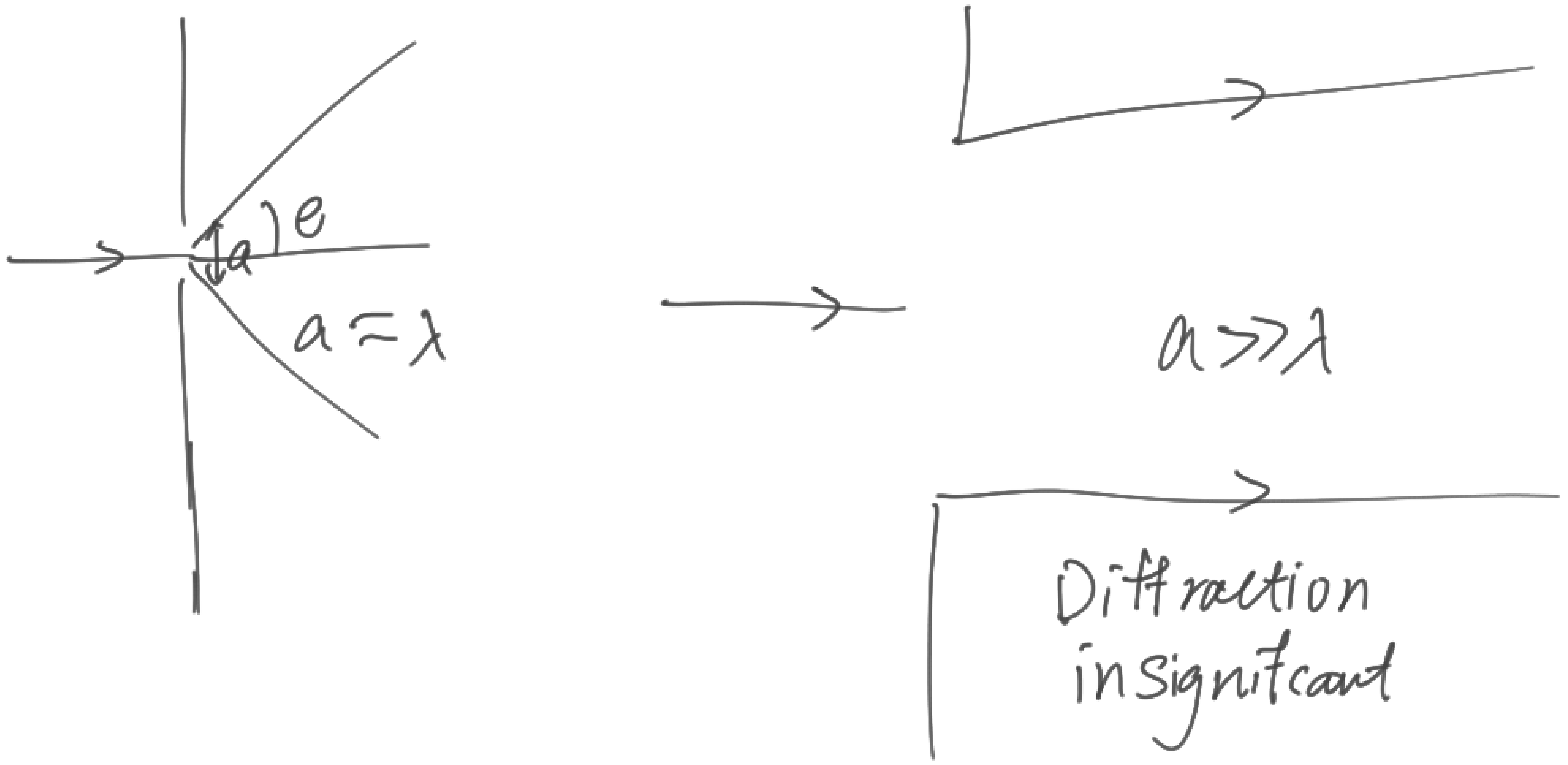
(iii) Suppose the rocket keeps expelling gas at the same rate for a few seconds. Would the rocket's acceleration increase, decrease or remain unchanged in that duration ? Explain. (2 marks) *(b) The artificial satellite is put in the geostationary orbit of radius r around the Earth. It appears to be always stationary above an observer at the equator.

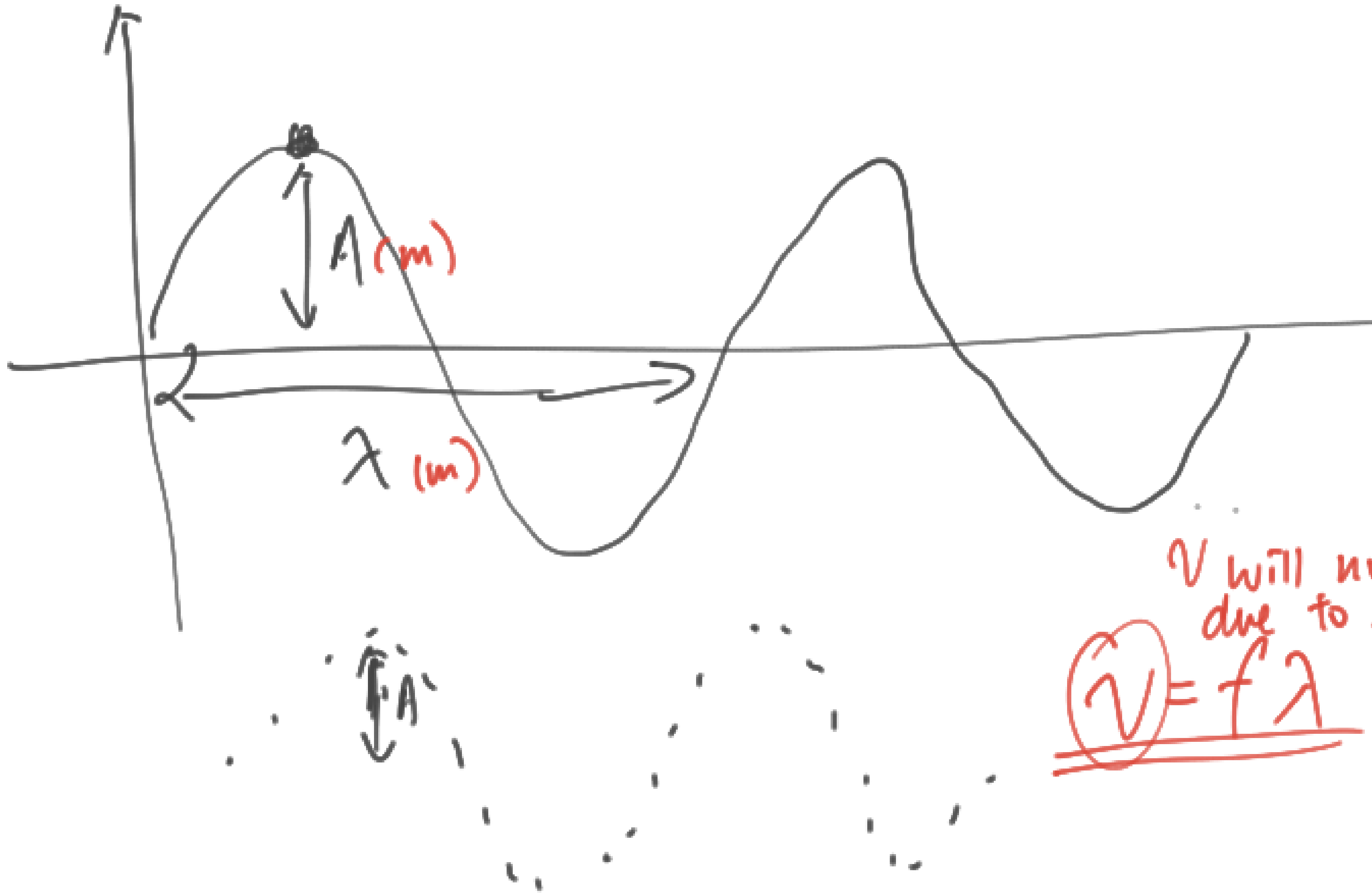
(i) State the period of this satellite. (1 mark)

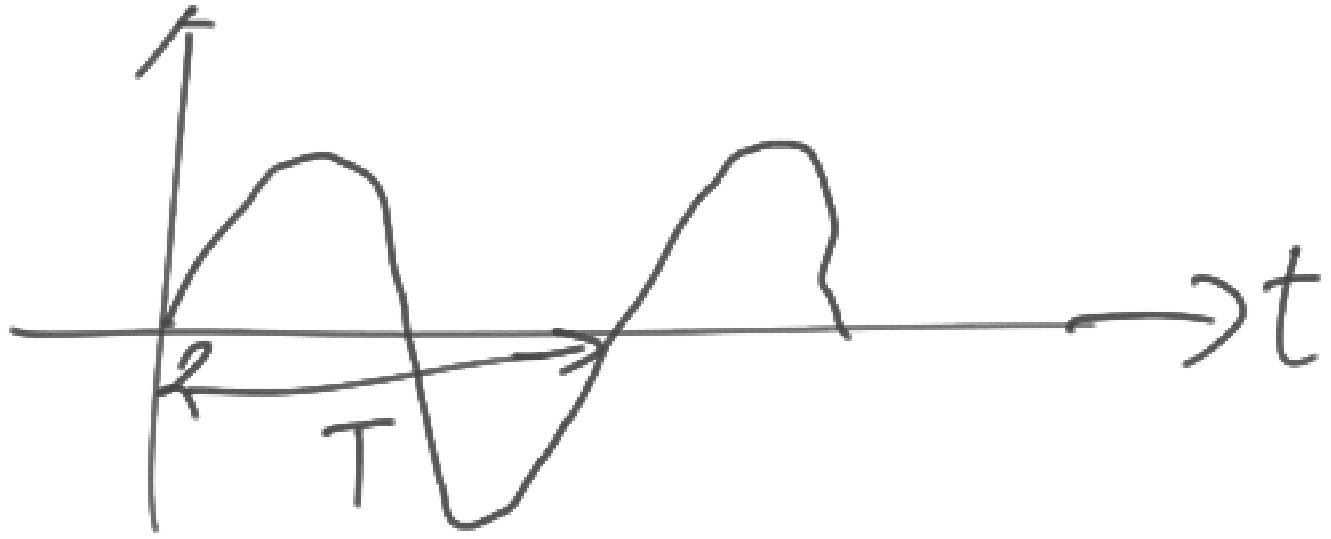
1 day



(ii) Show that r is approximately 42000 km. ($g = 9.81 \text{ m s}^{-2}$) (2 marks)
Given: radius of the Earth = 6.37×10^{6} m







v will not increase due to a change in f / λ

$v = f\lambda$

$f = \frac{1}{T}$ (Hz) (s^{-1})

↳ how many complete wave form per second