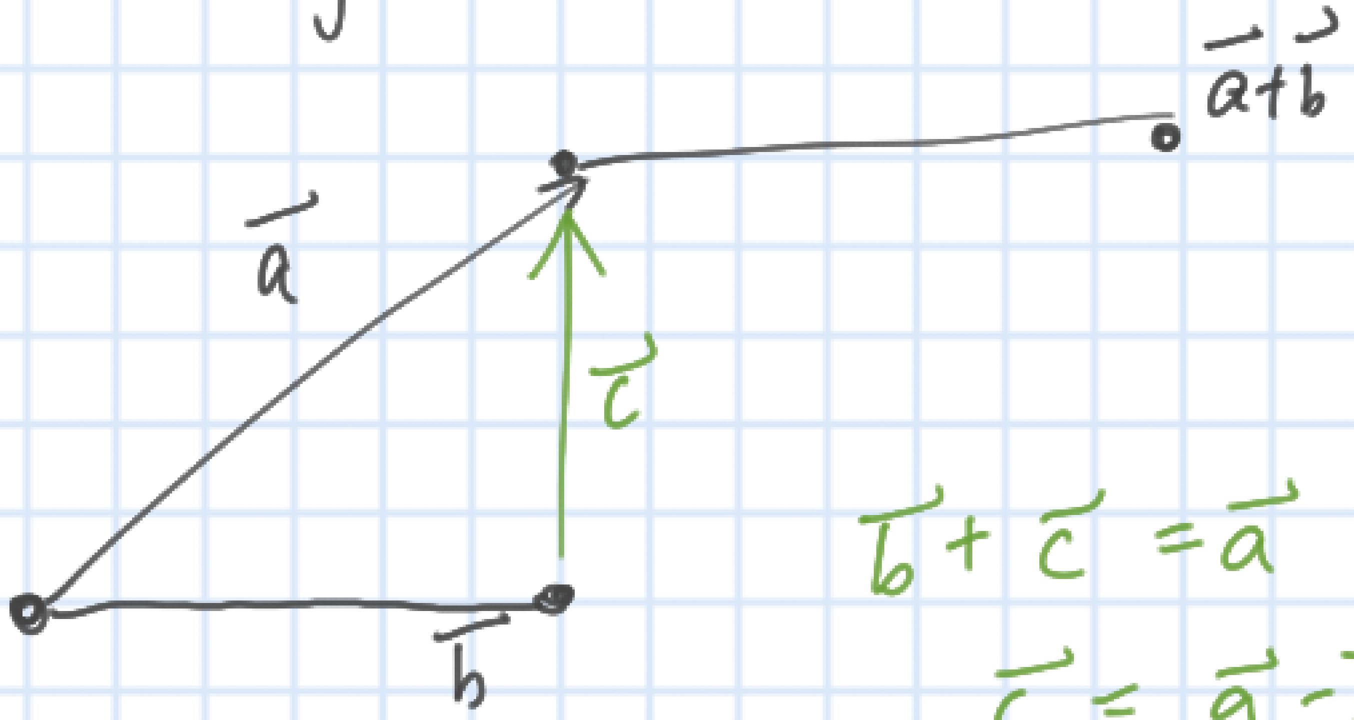
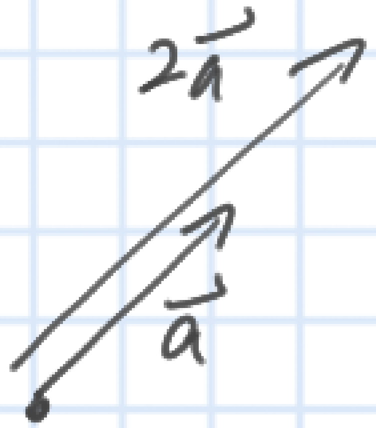
Vector algebra



$\vec{b}+\vec{c}=\vec{a}$

$\vec{c}=\vec{a}-\vec{b}$

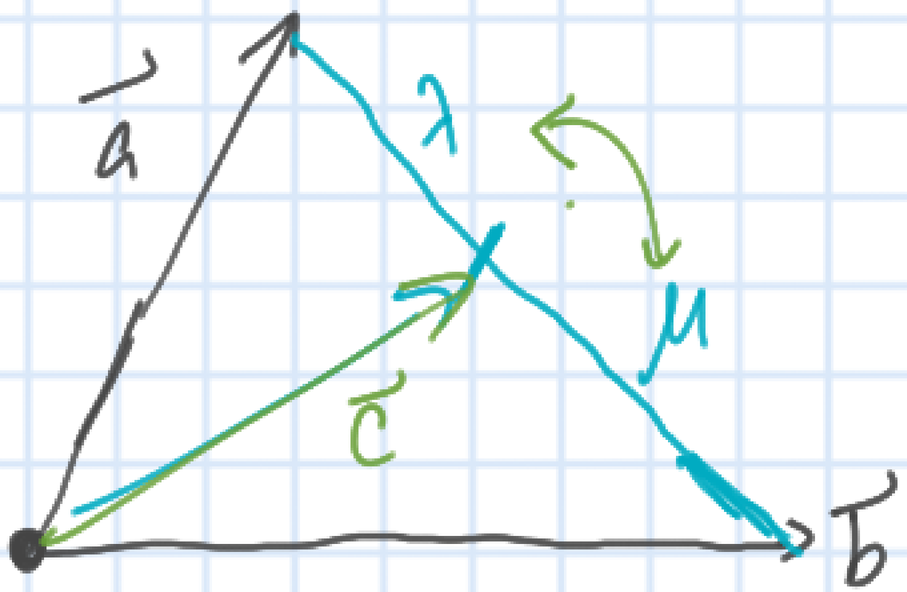


$$|\vec{a}|=\sqrt{\sum a_i^2}$$

if $\vec{a}=k\vec{b}$, then $\vec{a}\parallel\vec{b}$

Q1: Ratio formula

$$\frac{1}{\lambda+\mu}\left(\mu\vec{a}+\lambda\vec{b}\right)$$

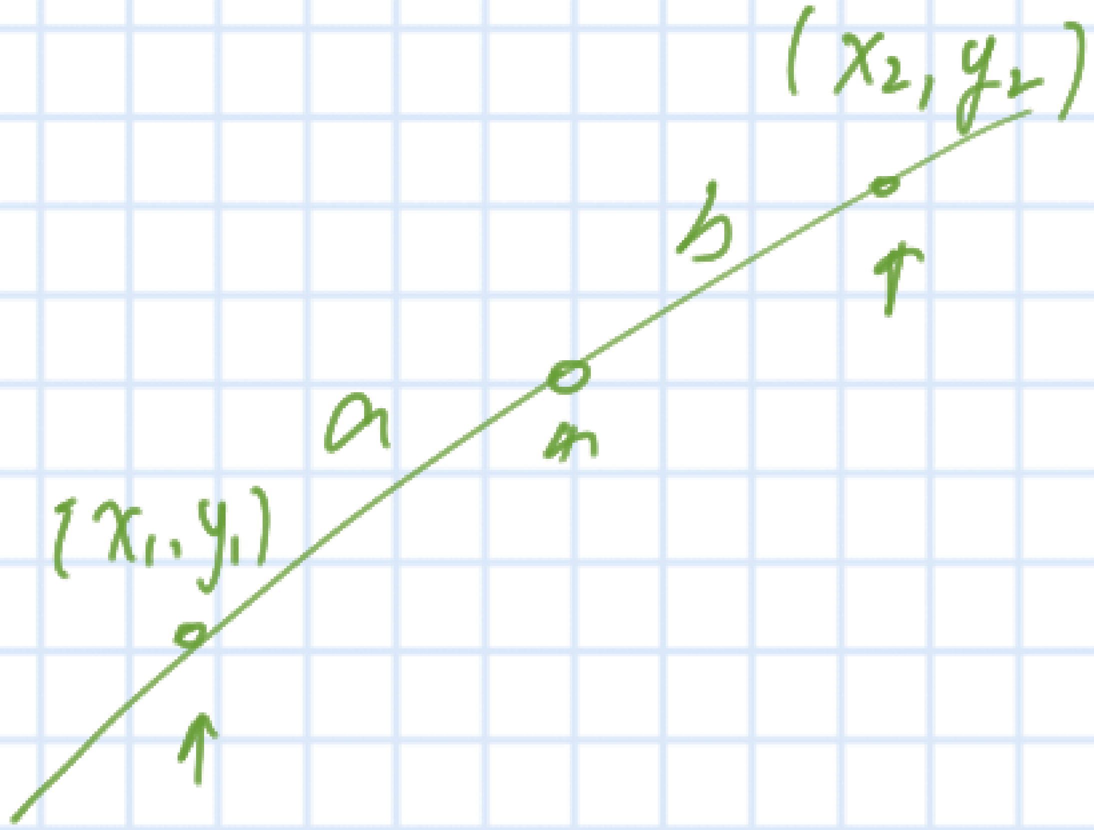


find $\vec{c}$ in terms of λ μ $\vec{a}$ $\vec{b}$

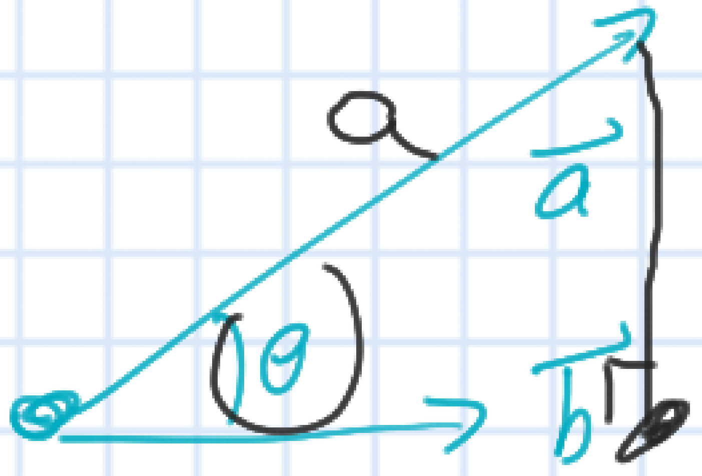
$$\vec{c}=\vec{a}+\frac{\lambda}{\lambda+\mu}(\vec{b}-\vec{a})$$

$$\vec{c}=\frac{\lambda+\mu}{\lambda+\mu}\vec{a}-\frac{\lambda}{\lambda+\mu}\vec{a}+\frac{\lambda}{\lambda+\mu}\vec{b}$$

$$\vec{c}=\frac{1}{\lambda+\mu}\left(\mu\vec{a}+\lambda\vec{b}\right)$$



Scalar product



$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta \Rightarrow \vec{a} \circ \vec{b} = 0$ when $\theta = \frac{\pi}{2}$

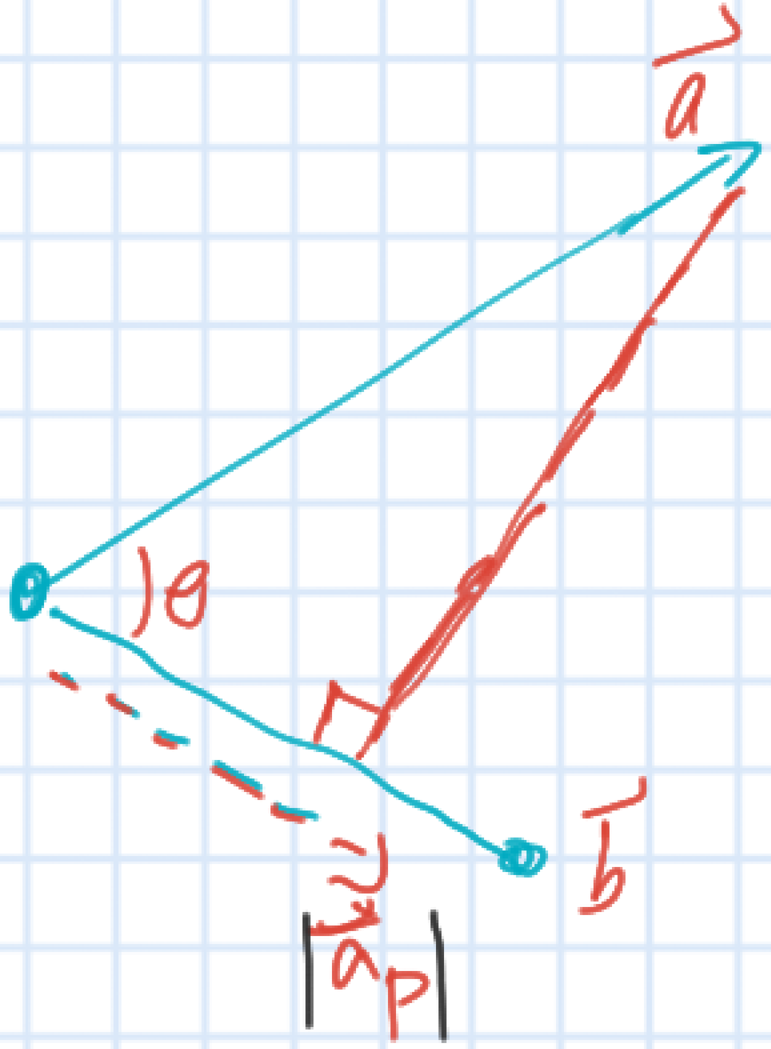
(given $|\vec{a}| \neq 0$ $|\vec{b}| \neq 0$)

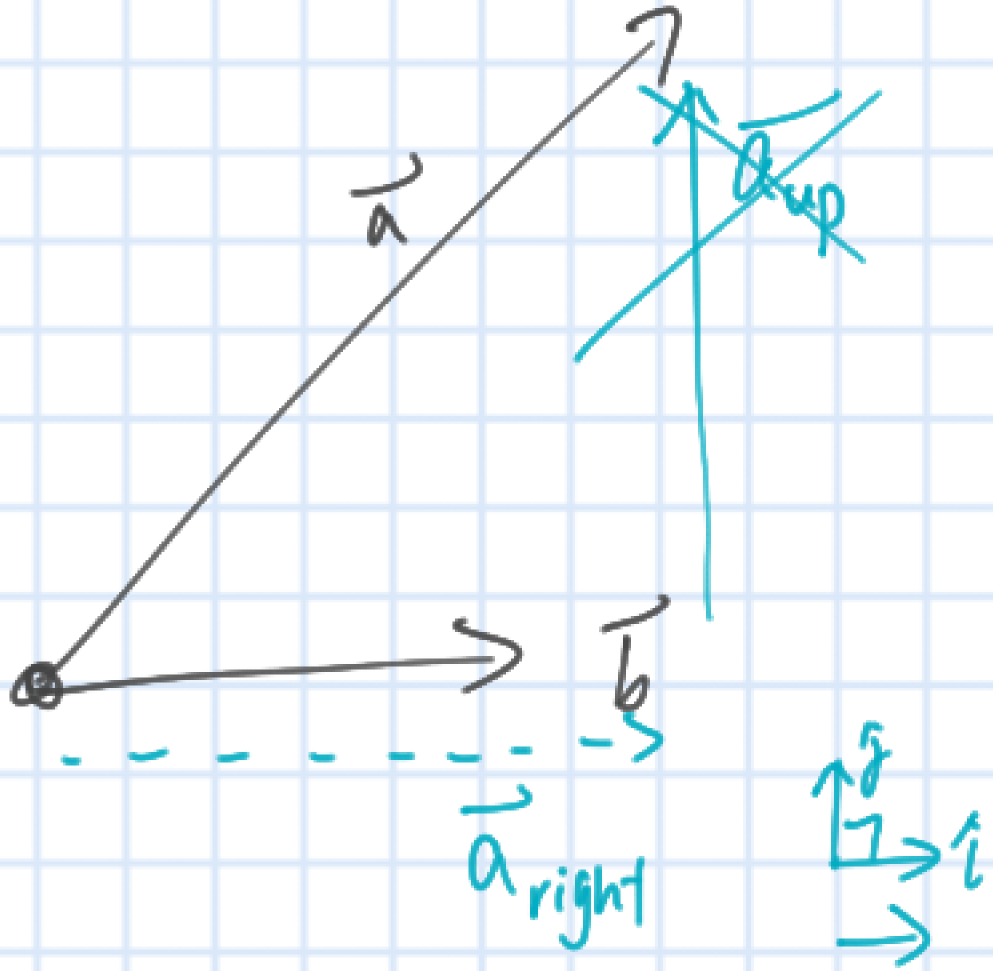
\# Commutative

$\vec{a} \cdot \vec{b} = \vec{b} \circ \vec{a}$

\# Distrubutive

$\vec{a} \circ (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \circ \vec{c}$





$\vec{a} \cdot \vec{b} = |\vec{a}| \underbrace{|\vec{b}| \cos\theta}_{|\vec{a_p}|}$

$\vec{a}_p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$ or $\frac{\vec{a} \circ \vec{b}}{|\vec{b}|} \hat{b}$

$|\vec{a_p}| = \frac{\vec{a} \circ \vec{b}}{|\vec{b}|}$

Magnitude of projection

find projection

$(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \circ (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$\hat{i} \cdot \hat{i} = 1$ $\quad \hat{i} \cdot \hat{j} = 0$

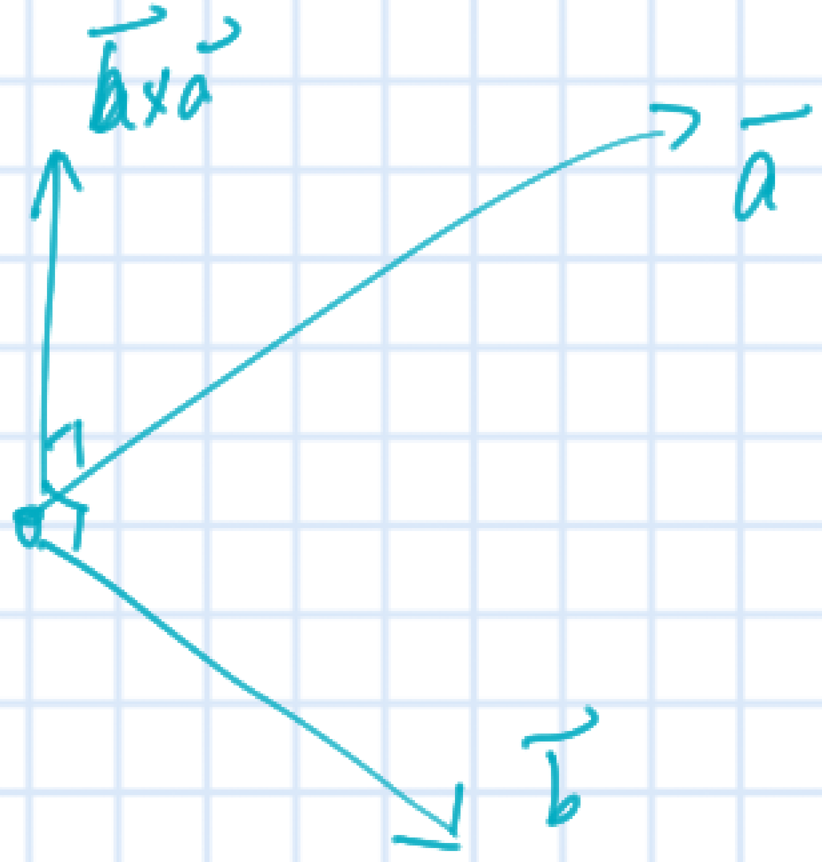
$\hat{j} \cdot \hat{j} = 1$ $\quad \hat{j} \cdot \hat{k} = 0$

$\hat{k} \cdot \hat{k} = 1$ $\quad \hat{i} \cdot \hat{k} = 0$

$\to a_1b_1 + a_2b_2 + a_3b_3$

Cross (vector product)

Cross product is only defined on $\mathbb{R}^3$ (3D space)



$(\vec{b}\times\vec{a}) \perp \vec{a}$ & $\vec{b}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

→ cofactor expansion // calculator.

NOT COMMUTATIVE

$\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$

$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

(c.f. determinant)

Distrubutive

$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

What if :

$\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$

Can you guess what is $\vec{a}\cdot(\vec{b}-\vec{c})$

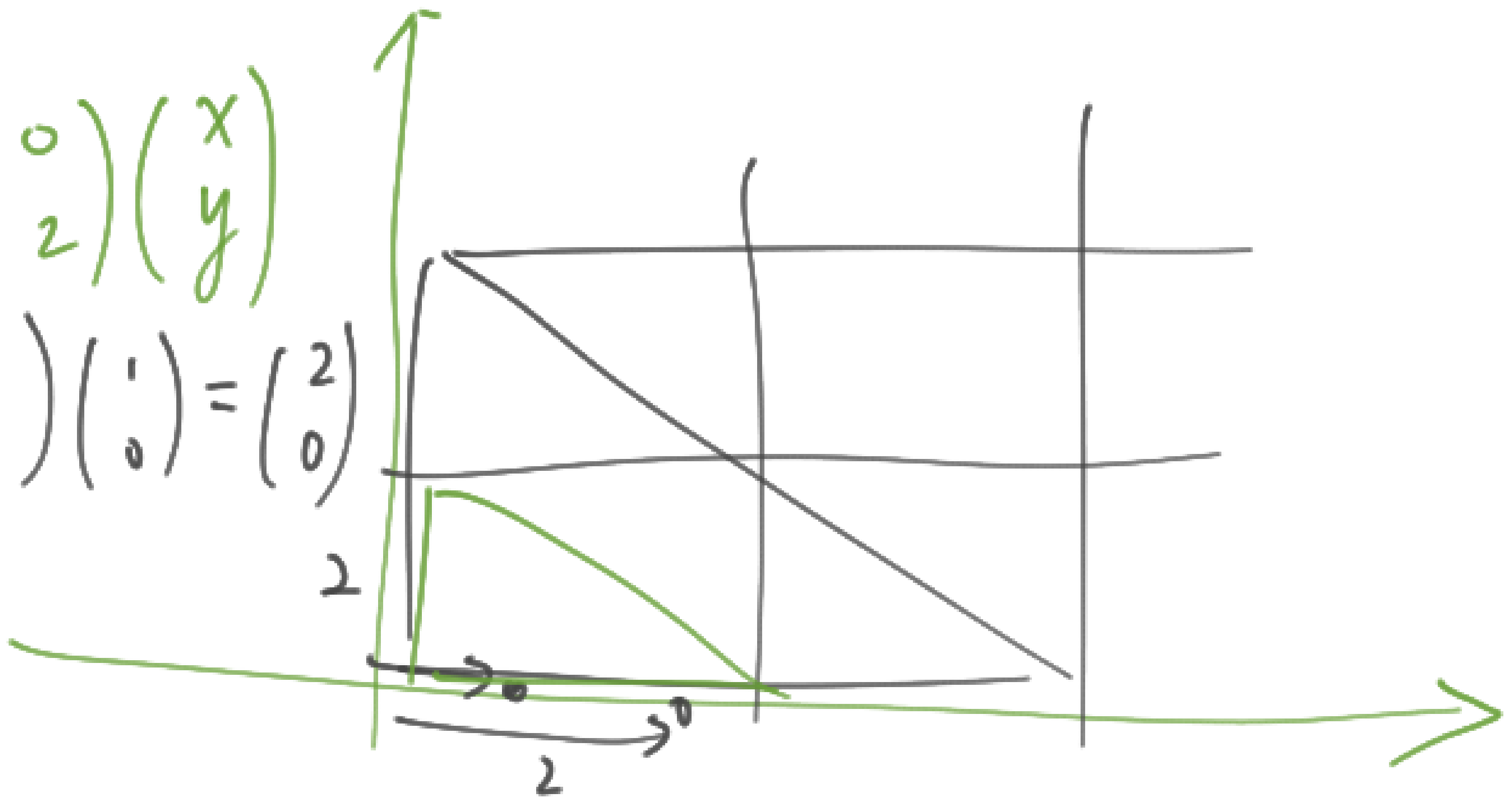
$\vec{a}\times\vec{b} - \vec{a}\times\vec{c} = 0$

$\vec{a}\times(\vec{b}-\vec{c}) = 0$

$\vec{b}-\vec{c} \,//\, \vec{a}$ → $|\vec{a}||\vec{b}-\vec{c}|(\pm 1)$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

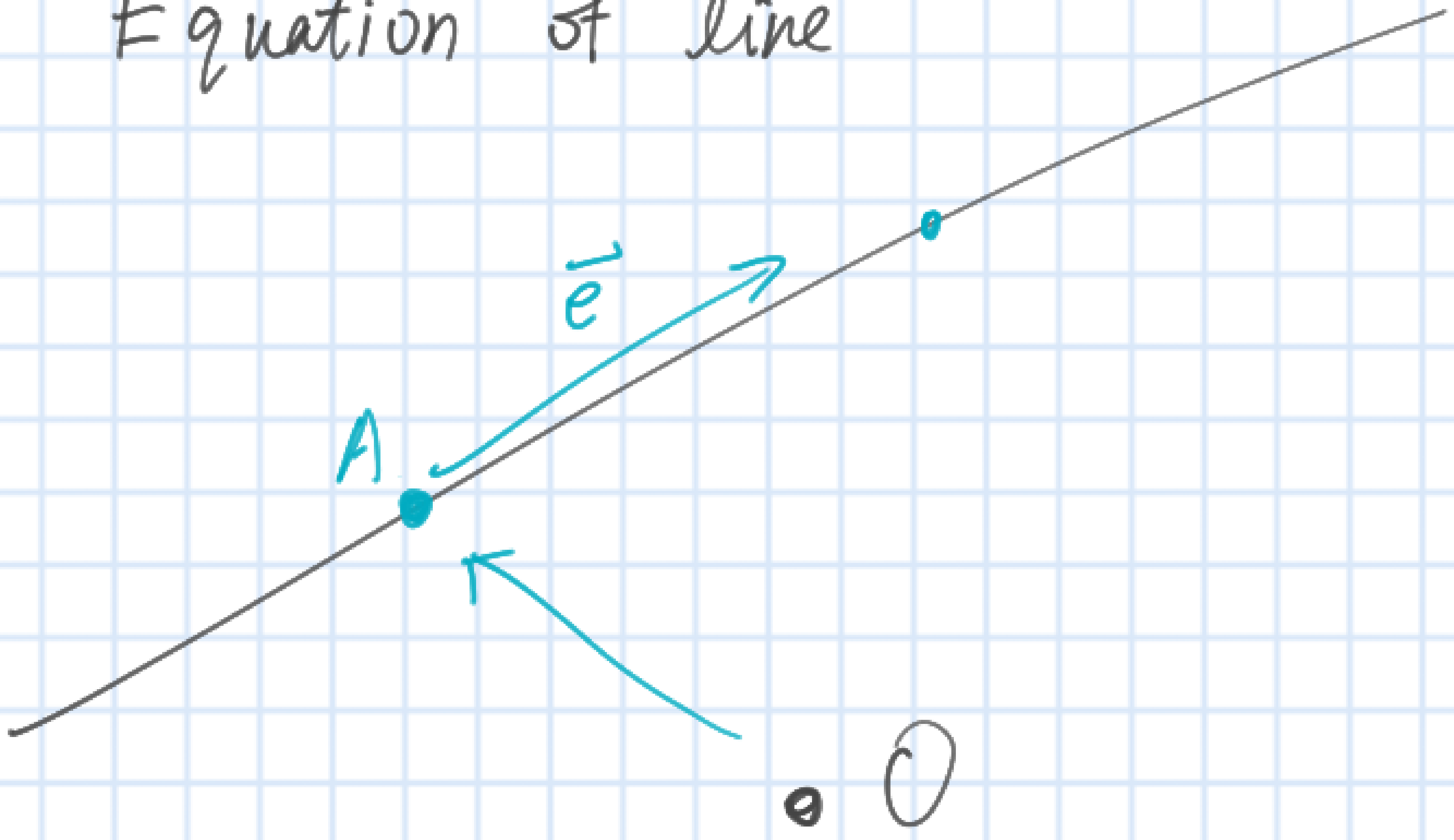
$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$



Before : $\square = 1$

After : $\square = 4 = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}$

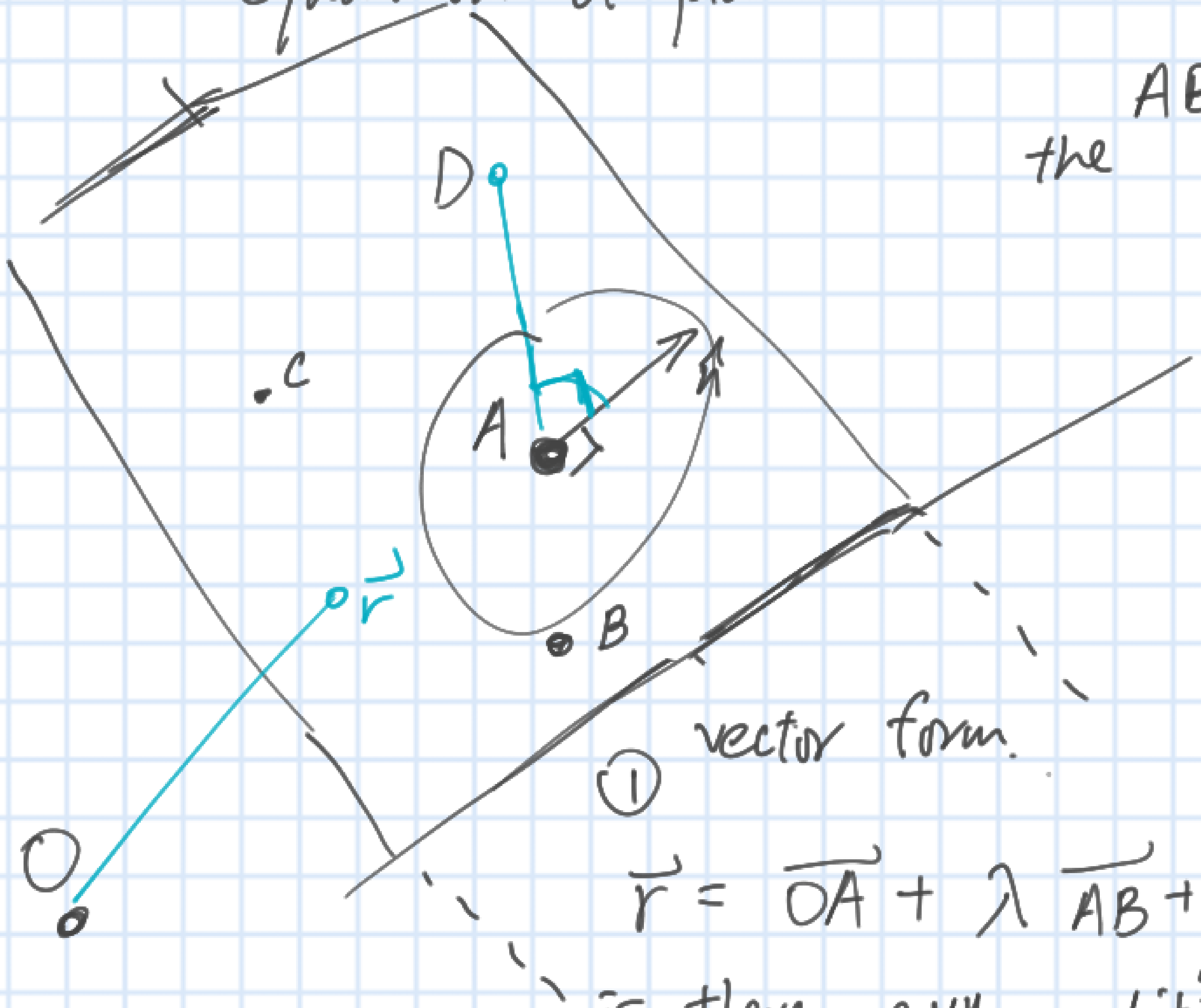
Equation of line



$\vec{r} = \vec{OA} + \vec{e} \cdot k$

$y = mx + c$

equation of plane



ABC all on the plane

① vector form

$\vec{r} = \vec{OA} + \lambda \vec{AB} + \mu \vec{AC}$

is there any condition to $\vec{AB}$ & $\vec{AC}$?

$\vec{AB}$ & $\vec{AC}$ are li

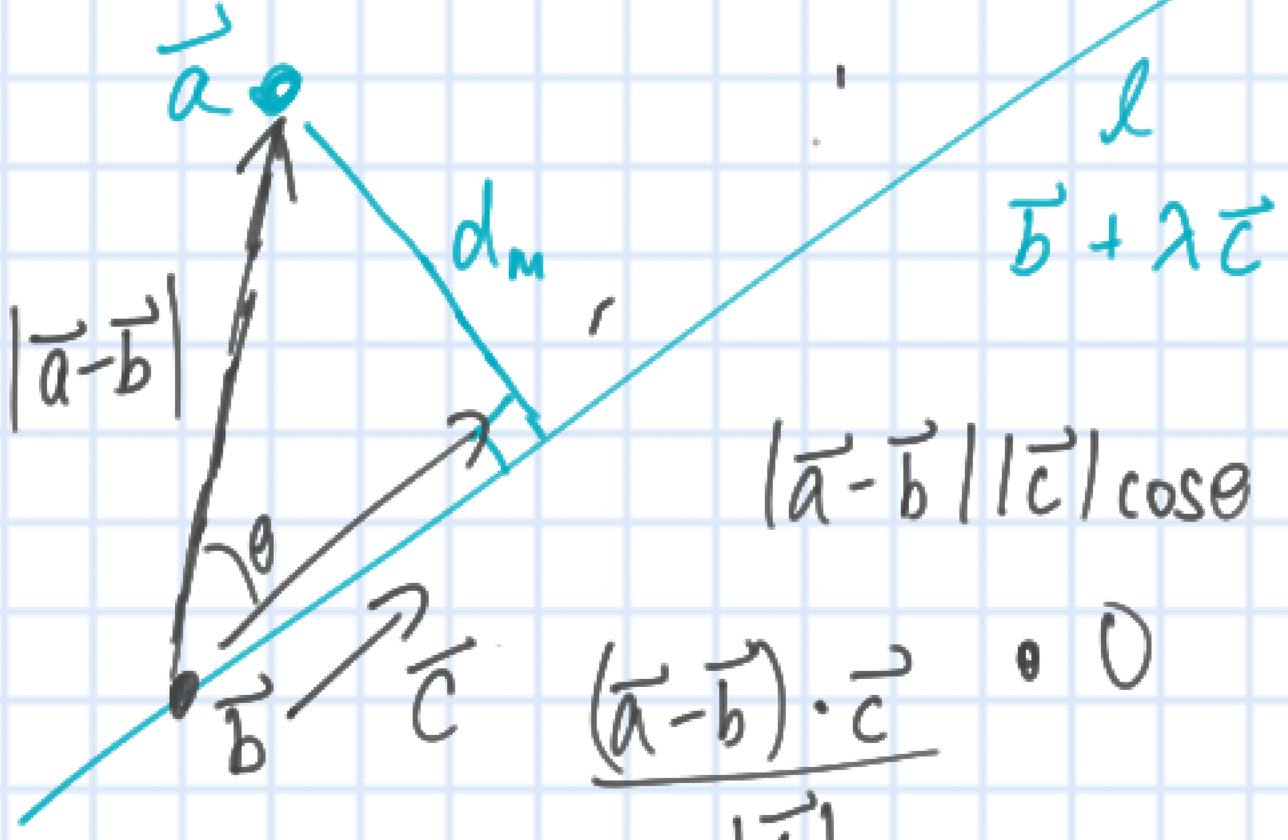
② Cartesian

$\vec{AR} \cdot \hat{n} = 0$ $\quad \vec{AR} \to \vec{OR} - \vec{OA} \to \vec{OA} = x_a \hat{i} + y_a \hat{j} + z_a \hat{k}$

$\hat{n} : n_x \hat{i} + n_y \hat{j} + n_z \hat{k}$

$(x - x_a) n_x + (y - y_a) n_y + (z - z_a) n_z = 0$

Dist from pt to line



$|\vec a-\vec b||\vec c|\cos\theta$

$$\frac{(\vec a-\vec b)\cdot\vec c}{|\vec c|}$$

$\vec a\times\vec b = |\vec a||\vec b|\sin\theta$

$(\vec a-\vec b)\times\vec c = |\vec a-\vec b||\vec c|\sin\theta$

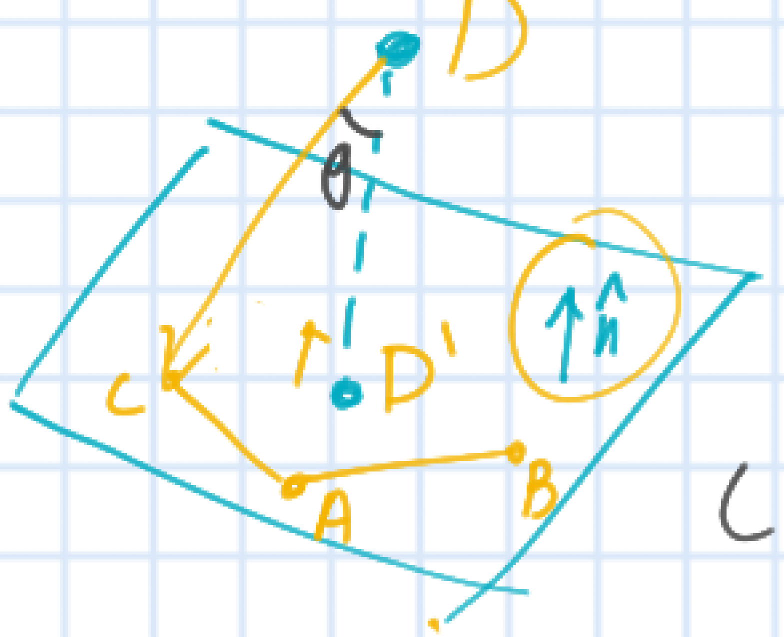
$(\vec d-\vec a)\cdot\hat n = 0$

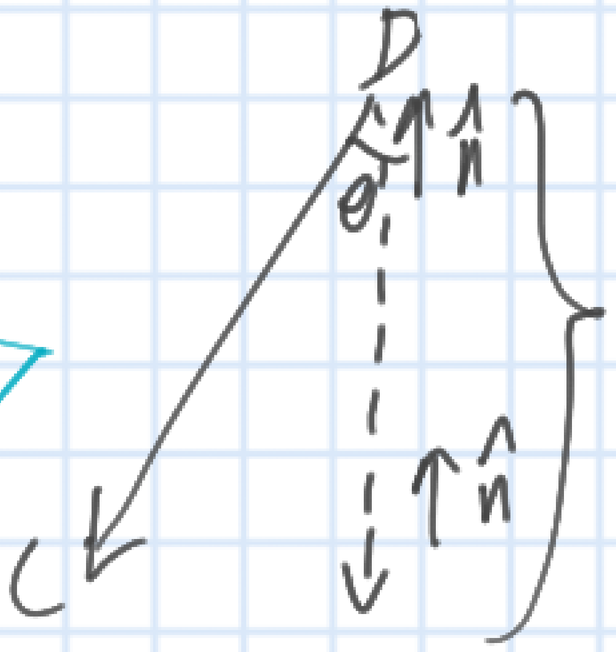
$\vec d\cdot\hat n = \vec a\cdot\hat n$

$(x\hat i + y\hat j + z\hat k)\cdot(n_x\hat i + n_y\hat j + n_z\hat k) = \vec a\cdot\hat n$

$n_x\cdot x + n_y\cdot y + n_z\cdot z \underbrace{- \vec a\cdot\hat n}_{d} = 0$

Pt to plane





$$\vec{DD'} = \frac{\vec{DC}\cdot\vec n}{|\vec n|^2}\vec n$$

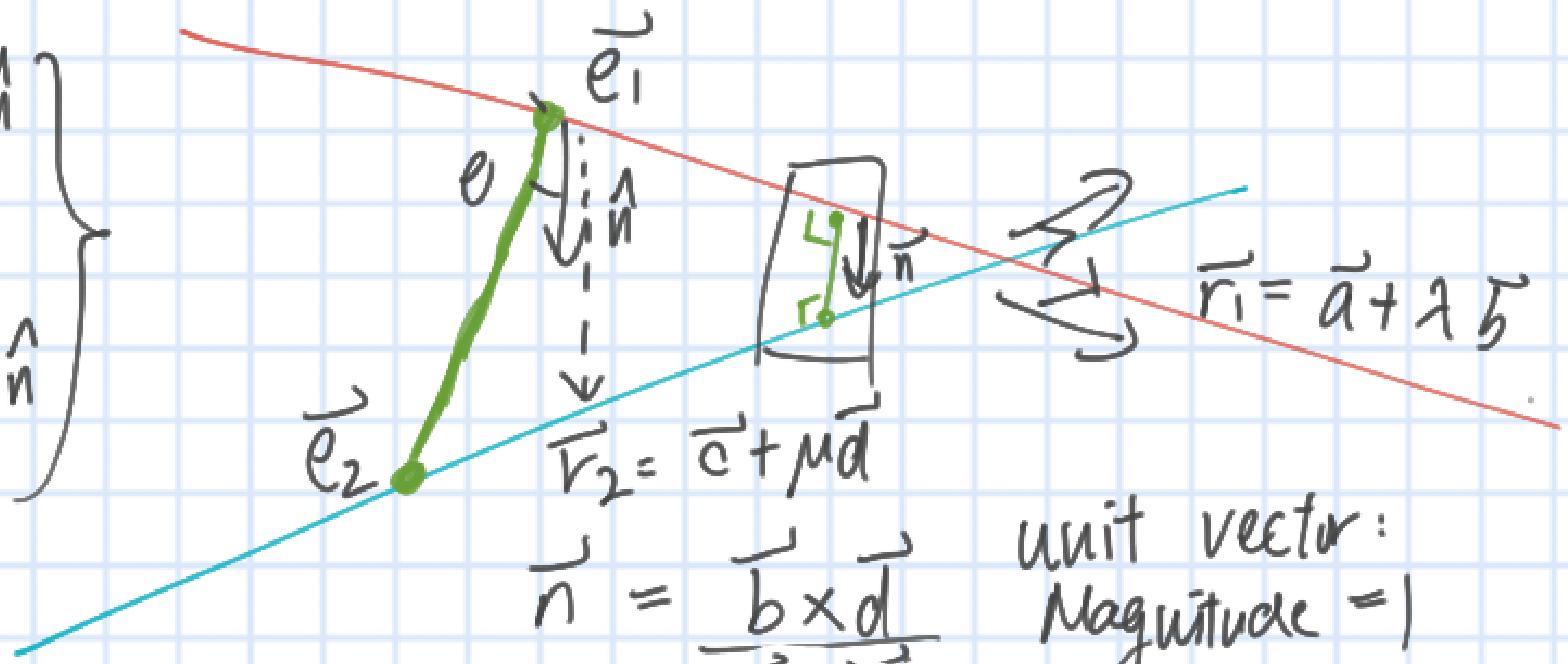
$$\cos\theta = \frac{|\vec{DD'}|}{|\vec{DC}|}$$

$\vec{DC}\cdot\hat n = |\vec{DC}||\hat n|\cos\theta$

$$\underbrace{\frac{\vec{DC}\cdot\hat n}{|\vec{DC}||\hat n|} = \cos\theta}$$

$$\frac{\vec{DC}\cdot\hat n}{\cancel{|\vec{DC}|}|\hat n|} = \frac{|\vec{DD'}|}{\cancel{|\vec{DC}|}}$$

line to line

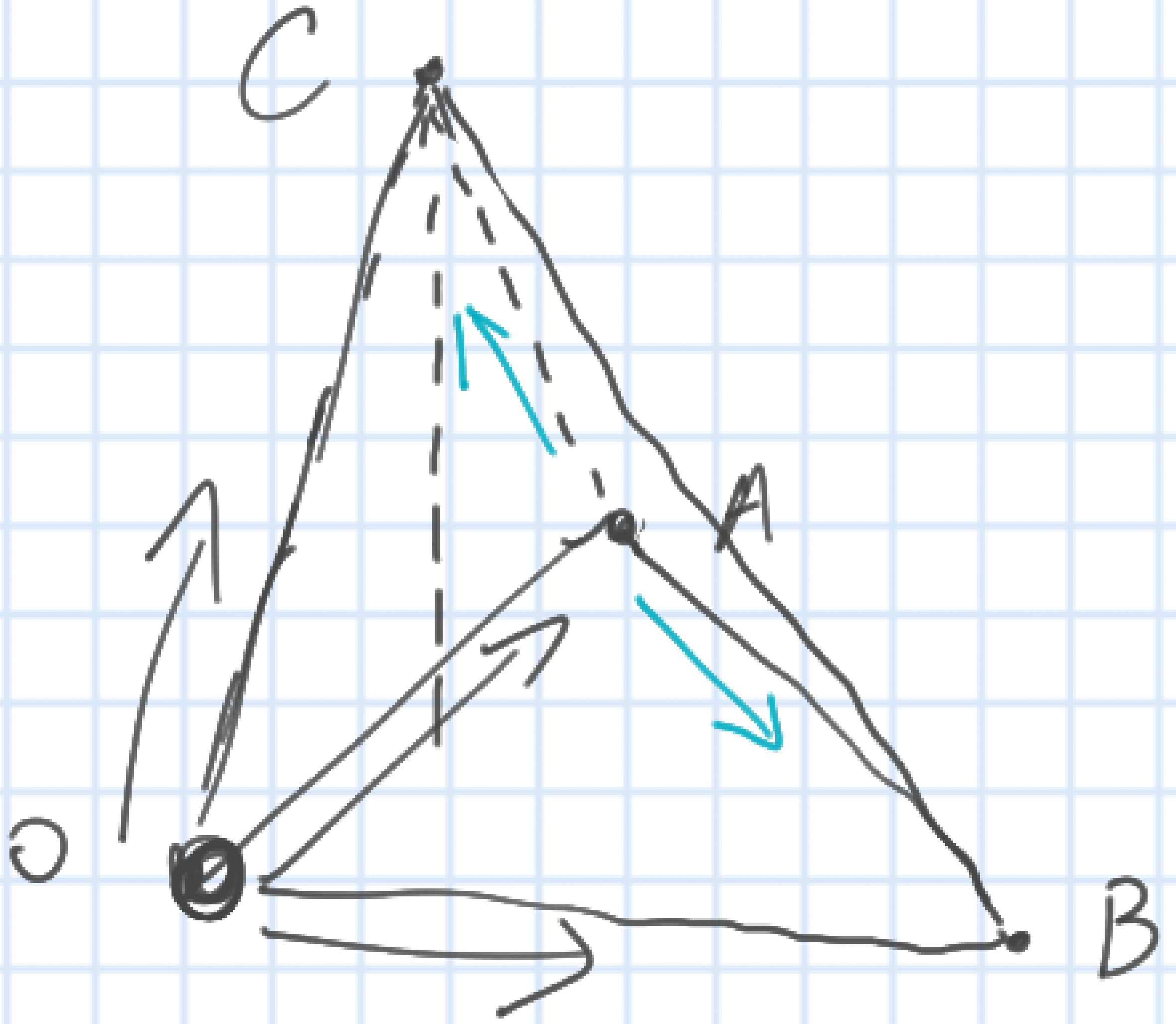


$\vec n = \vec b\times\vec d$

$$\hat n = \frac{\vec b\times\vec d}{|\vec b\times\vec d|}$$

unit vector: Magnitude = 1

$|(\vec e_1-\vec e_2)\cdot\hat n|$



Area ΔOAB =

$$\begin{vmatrix} 2 & 3 & 5 \\ 6 & 2 & 4 \\ 5 & 8 & 3 \end{vmatrix} \qquad \begin{vmatrix} 2 & 3 & 5 \\ 0 & -7 & -11 \\ 0 & \frac{1}{2} & -\frac{19}{2} \end{vmatrix}$$

$R_2 - R_1 \cdot 3$

$R_3 - R_1 \cdot \frac{5}{2}$

$$\begin{vmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{vmatrix}$$

determinant

$R_3 \leftarrow R_3 + \frac{1}{14} R_2$

$$\begin{vmatrix} 2 & 3 & 5 \\ 0 & -7 & -11 \\ 0 & 0 & -\frac{72}{7} \end{vmatrix}$$

$2 \cdot -\cancel{7} \cdot -\frac{72}{\cancel{7}}$

$= 144$

$$\underbrace{A = \left|\overrightarrow{AO} \cdot \left|\overrightarrow{AB} \times \overrightarrow{AC}\right|\right| \cdot \frac{1}{6}}$$

$\overrightarrow{AO} = a_1 \hat{i} + b_1 \hat{j} + c_1 \hat{k}$

$\overrightarrow{AB} = a_2 \hat{i} + b_2 \hat{j} + c_2 \hat{k}$

$\overrightarrow{AC} = a_3 \hat{i} + b_3 \hat{j} + c_3 \hat{k}$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \left(\frac{1}{6}\right)$$

10. Let O be the origin. The position vectors of P and Q are $-2\mathbf{i}-\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ respectively. Denote the circle passing through O, P and Q by C. Let R be a point lying on PQ such that OR is perpendicular to OQ.

(a) By considering the ratio of PR to RQ, find $\overrightarrow{OR}$. (3 marks)

(b) OR produced meets C at another point S. Find $\overrightarrow{OS}$. (3 marks)

(c) Let Π be the plane which contains C.

(i) Find a non-zero vector which is perpendicular to Π.

(ii) Let G be the centre of C. Denote the projection of point $A\,(-6, -22, 2)$ on Π by B. Describe the geometric relationship between O, B and G. Explain your answer.

(6 marks)

$\overline{OG} = \frac{1}{4}\overline{OB}$