Answers

1. Heat (Warm-up question)
   (a) The heater was switched on for 10 minutes, and the initial and final temperatures were recorded. Find the specific heat capacity for the liquid.

   $$P_h t = mc\Delta T$$
   $$c = \frac{P_h t}{m\Delta T}$$
   $$c = 1750\mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$$

   (b) Remove the lid.

   (c) Estimate the rate of decrease in the mass of the fluid.

   $$P_h t = ml_v$$
   $$\frac{\Delta m}{\Delta t} = \frac{P_h}{l_v}$$
   $$\frac{\Delta m}{\Delta t} = 0.4\mathrm{g\,s^{-1}}$$

   Rate of decrease is $\frac{\Delta m}{\Delta t}$.

2. Conservation of linear momentum (2D)
   (a) The two vehicles stick together after the collision. Find the resulting velocity of the two cars. Consider the x-axis first.

   $$m_c u_c = (m_c + m_t)v_x$$
   $$v_x = \frac{m_c}{m_c + m_t}u_c$$
   $$v_x = 4.20\mathrm{m\,s^{-1}}$$

   We consider the similar for the y-axis.

   $$m_t u_t = (m_c + m_t)v_y$$
   $$v_y = \frac{m_t}{m_c + m_t}u_t$$
   $$v_y = 5.60\mathrm{m\,s^{-1}}$$

   Final calculation for our velocity.

   $$\vec{v} = 4.20\hat{i} + 5.60\hat{j}$$
   $$|\vec{v}| = 7.00\mathrm{m\,s^{-1}}$$
   $$\theta = \arctan(\frac{5.6}{4.2}) = 53.1\,\mathrm{deg}$$

   (b) Determine the amount of kinetic energy lost during the collision.

   $$\Delta KE = KE_f - KE_i$$
   $$\Delta KE = \frac{1}{2}(m_c + m_t)(v)^2 - \frac{1}{2}m_c u_c^2 - \frac{1}{2}m_t u_t^2$$
   $$\Delta KE = -109200\mathrm{J} \text{ or } -109000\mathrm{J}$$

   (c) Reasons for loss of kinetic energy:
   - Energy is used to deform the cars
   - Some energy is released as sound during the crash
   - (Any one; Reasonable answers accepted)
   - **DO NOT ACCEPT**: Gravity as an external force is acting on both objects.

3. (a) Find the angular speed of the object in terms of rotation distance r, r_0, m and k

$$k(r - r_0) = mr\omega^2$$
$$\omega^2 = \frac{k}{m}\left(\frac{r - r_0}{r}\right)$$
$$\omega = \sqrt{\frac{k}{m}\left(1 - \frac{r_0}{r}\right)}$$

(b) For the object to actually undergo uniform circular motion, ω must have a real solution. Hence the term inside the bracket must be positive.

$$1 - \frac{r_0}{r} \geq 0$$
$$\frac{r_0}{r} \leq 1$$
$$r \geq r_0$$

Our condition is hence that rotation radius r must be larger than the natural length of the spring r_0. *Note: k is a positive constant*

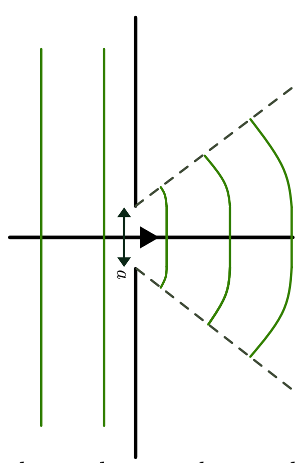
(c) When $r_0 = 0$, the bracket term vanishes, so we retrieve $\omega = \frac{k}{m}$, so there is no dependence on the rotation radius.

(d) Virial Theorem
In this situation we have $F \propto r^1$

$$K = \frac{1+1}{2}U$$
$$U = \frac{1}{2}mv^2$$
$$U = \frac{1}{2}m(r\omega)^2$$
$$U = \frac{1}{2}mr^2 \cdot \frac{k}{m}\left(1 - \frac{0}{r}\right)$$
$$U = \frac{1}{2}kr^2$$

4. Wave motion I

(a) Sketch diffraction pattern



I'm too lazy to redraw this but make sure the wavelength is equal on both sides.

(b) Diffraction grating with offset
Although the ruler has an offset, the precise first order maximum distance can still be found by taking the average of the two measurements. (Very useful experimentally!)

$$
\begin{aligned}
y &= \frac{1}{2}(y_1 + y_2) = 0.605\text{m} \\
d &= \frac{1 \cdot 10^{-3}}{500} = 2\mu\text{m} \\
d \sin\theta &= (1)\lambda \\
\lambda &= d \sin\left(\arctan\frac{y}{D}\right) \\
\lambda &= 579\text{nm}
\end{aligned}
$$

The angle θ is not small ($\approx 16\,\text{deg}$), the small angle approximation may not be used.

(c) Sound diffraction grating?! Maximum order is found as

$$
\begin{aligned}
\sin\theta &= \frac{m\lambda}{d} \leq 1 \\
m &\leq d/\lambda
\end{aligned}
$$

Substituting even the smallest value for λ, we see

$$m \leq 2 \cdot 10^{-4} << 1$$

The student cannot observe any maximums (except the central maximum) in the diffraction grating, hence it is impossible to deduce the wavelength of the source with this method.

5. Thin film interference (One of the harder questions)

(a) The wave will undergo a π phase change on boundary A.

(b) After the second reflected wave arrives at A, there is no longer any difference between the two paths. Thus we only account for the extra phase change for the reflected wave from B, as well as the π phase change on A as discussed above.

$$
\begin{aligned}
\delta\phi &= \phi_b - \phi_a \\
\delta\phi &= k(2d)n_p - \pi \\
\delta\phi &= \pi\left(\frac{4dn_p}{\lambda} - 1\right)
\end{aligned}
$$

(c) To block out purple light, we want it to *constructively interfere* for the reflected waves.

$$
\begin{aligned}
\delta\phi &= 2m\pi \quad (m \in \mathbb{Z}) \\
\pi\left(\frac{4dn_p}{\lambda} - 1\right) &= 2m\pi \\
\frac{4dn_p}{\lambda} &= 2m + 1 \\
d &= (2m+1)\frac{\lambda}{4n_p} \\
d &= \frac{\lambda}{4n_p} \\
d &= 61.8\text{nm}
\end{aligned}
$$

For the smallest thickness we substitute $\lambda = 0$

(d) Same thing, except this time we aim for destructive interference of the reflected waves.

$$\begin{aligned}
\delta\phi &= (2m+1)\pi \quad (m \in \mathbb{Z}) \\
\pi\left(\frac{4dn_p}{\lambda} - 1\right) &= (2m+1)\pi \\
\frac{4dn_p}{\lambda} &= 2m \\
d &= \frac{m\lambda}{2n_p} \\
d &= \frac{\lambda}{2n_p} \\
d &= 124\text{nm}
\end{aligned}$$

For the smallest thickness we substitute $\lambda = 1$

6. Oil drop in electric field.

(a) Obviously the charge is negative.

$$\begin{aligned}
mg &= qE \\
\frac{q}{m} &= \frac{g}{E} \\
\frac{q}{m} &= \frac{gd}{V} \\
\frac{q}{m} &= 0.0446 \ \mathrm{C\,kg^{-1}}
\end{aligned}$$

(b) No, a magnetic field can only exert a force on a charged and moving oil droplet;
Magnetic fields cannot exert a force on a stationary oil droplet to stop it from moving under the effect of gravity.

7. Circuits

(a) In the case where the thermistor's resistance is very small, the resistor can prevent the current of the circuit from being too high, which may damage the circuit's components.

(b) Potential divider circuit

$$\begin{aligned}
\frac{V_T}{V_0} &= \frac{IR_T}{I(R_T + R_0)} \\
\frac{V_T}{V_0}(R_T + R_0) &= R_T \\
\frac{V_T}{V_0}R_0 &= \left(1 - \frac{V_T}{V_0}\right)R_T \\
R_T &= 2000\Omega
\end{aligned}$$

(c) Finding constants β and C

$$\begin{aligned}
\frac{R_1}{R_2} &= \exp\left[\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \\
\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right) &= \ln\frac{R_1}{R_2} \\
\beta &= 5040\text{K}
\end{aligned}$$

To find C we substitute β into one of the two measured values.

$$C \exp\left(\frac{\beta}{T_1}\right) = R_1$$
$$C = 8.91 \cdot 10^{-5} \quad \Omega$$

(d) After the circuit is turned on, current flows through and thus heats up the circuit components. Since the thermistor's resistance decreases as temperature increases, the net resistance of the circuit decreases over time, which will increase the current passing through.

8. Radiation

(a) Experiment...

- Put the GM counter close to the source (3-5 cm away) and measure the count rate, then put a piece of paper between the GM counter and the source. There should be no significant changes to the count rate, this shows that the source does not emit α radiation.
- Put the 5mm aluminium sheet between the source and the GM counter, the count rate should decrease compared to its previous value, but is still higher than the background count rate. This shows that the source emits β radiation.
- Put the 25mm lead block between the source and the GM counter, the count rate should decrease again, (but is still slightly higher than background count rate) showing that γ radiation is emitted by the source.
- Measure the background count rate first for reference later.
- The GM counter should never be moved.

(b) Half life calculation

$$A_1 = A_0 \cdot 0.5^{\frac{t}{\tau}}$$
$$\tau = \frac{t}{\log_2\left(\frac{A_0}{A_1}\right)}$$
$$\tau = 5.27 \quad \text{yr}$$

9. Ideal gas
   (a) The temperature inside the sun's core is very high;
   The density of gas inside the sun's core is still relatively low.
   (b) Estimation of gas pressure.

$$
\begin{aligned}
PV &= \frac{1}{3}Nm\bar{c^2} \\
P &= \frac{1}{3}\rho\bar{c^2} \\
\frac{1}{2}m\bar{c^2} &= \frac{3}{2}k_bT \\
\rightarrow \bar{c^2} &= \frac{3k_bT}{m} \\
\rightarrow P &= \frac{k_b\rho T}{m} \\
P &= 1.86 \cdot 10^{16} \quad \text{Pa}
\end{aligned}
$$

   (c) $^{1}_{0}\text{n}$
   (d) Since $PV = nRT$, we have $PV/NT = const$. The subscript 1 denotes the conditions for hydrogen gas, while the subscript 2 denotes the conditions for helium gas.

$$
\begin{aligned}
\frac{P_1V_1}{n_1T_1} &= \frac{P_2V_2}{n_2T_2} \\
T_2 &= \frac{P_2}{P_1} \cdot \frac{V_2}{V_1} \cdot \frac{n_1}{n_2}T_1
\end{aligned}
$$

   From the question we have the following ratios. Note the mole ratio is due to the fusion reaction.

$$
\begin{aligned}
&\frac{P_2}{P_1} = 2000; \quad \frac{V_2}{V_1} = \frac{1}{600}; \quad \frac{n_1}{n_2} = 2 \\
&T_2 = 10^8 \quad \text{K}
\end{aligned}
$$

   (e) The sun's mass does not change during the fusion process, so there is no change in the gravitational force exerted on the Earth.

10. Electromagnetic induction
   (a) From X to Y
   (b) When the rod rolls down the rail, the loop experiences a change in magnetic flux. Hence, an induced current flows from X to Y and then through the rod. The rod experiences a magnetic force (due to its current), which opposes the weight of the rod and reduces the acceleration of the rod. At some velocity, the magnetic force on the rod is able to balance out the weight of the rod. Since the rod has no net force, it travels at constant velocity.
   (c) Magnetic flux through the loop

$$\Phi = Blz\cos\theta$$

   (d) Calculation of terminal velocity; We start off with Faraday's law.
   Note we denote: $\dot{z} \equiv \frac{\partial z}{\partial t}$

$$V = -\frac{\partial \Phi}{\partial t}$$
$$V = -Bl\cos\theta \cdot \frac{\partial z}{\partial t}$$
$$V = -Bl\dot{z}\cos\theta$$

   Then we obtain the current and the force on the rod.

$$I = -\frac{V}{R}$$
$$I = -\frac{Bl\dot{z}\cos\theta}{R}$$
$$F_{rod} = B|I|l = \frac{(Bl)^2\dot{z}\cos\theta}{R}$$

   Finally we balance out the components of the force on rod and gravity.

$$F_{rod}\cos\theta = mg\sin\theta$$
$$\frac{(Bl)^2\dot{z}\cos^2\theta}{R} = mg\sin\theta$$
$$\dot{z} = \frac{mgR}{(Bl)^2}\sec\theta\tan\theta$$

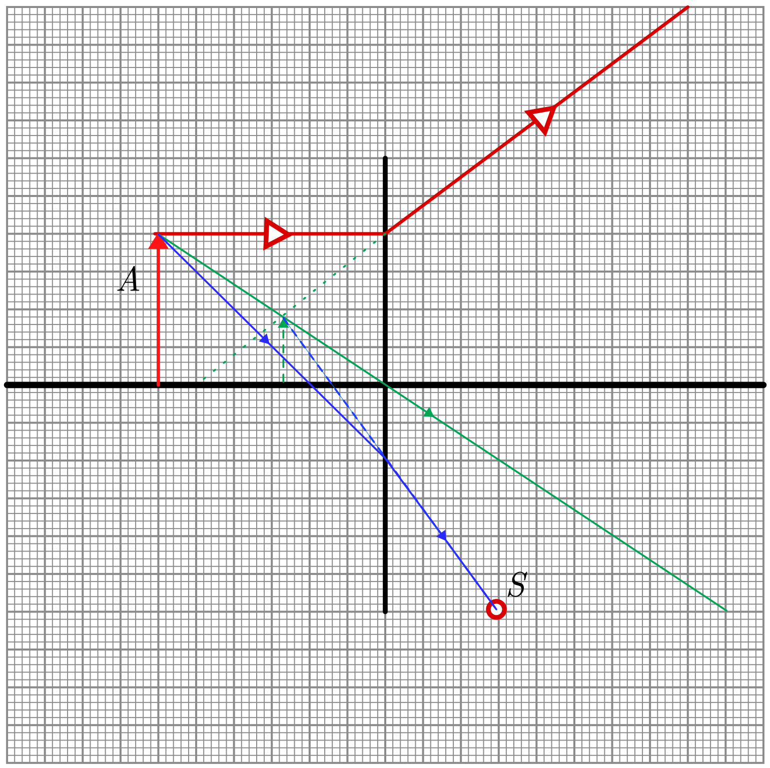
   (e) Current at terminal velocity

$$I = -\frac{Bl\dot{z}\cos\theta}{R}$$
$$I = -\frac{mg}{Bl}\tan\theta$$

   So we can see there is no R dependence.
   (f) Energy conservation; we first find the rate of loss of gravitational potential energy.

$$\frac{\partial U}{\partial t} = mg\frac{\partial h}{\partial t}$$
$$= mg\dot{z}\sin\theta$$
$$\frac{\partial U}{\partial t} = \left(\frac{mg}{Bl}\tan\theta\right)^2 R = I^2 R$$
$$\frac{\partial U}{\partial t} = P_{resistor}$$

11. Lens
   (a) The lens is concave because only a concave lens can diverge rays away from the optical axis.
   (b) See drawing



   (c) 23-27 cm
   (d) The lens formula states:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

   As object distance u increases, v must decrease. Since magnification is $m = \frac{v}{u}$, we see that m must decrease too.

12. Projectile motion
   (a) We start off with conservation of energy since at the peak N, there is no velocity in the vertical direction.

$$\begin{aligned}\frac{1}{2}mv_y^2 &= mgh\\ v_y &= \sqrt{2gh}\\ v_y &= 6.26 \quad \mathrm{m\,s^{-1}}\end{aligned}$$

Time of flight and x-direction velocity:

$$\begin{aligned}s_y &= v_y t_f - \frac{1}{2}gt_f^2\\ t_f &= 0.639\mathrm{s}\\ v_x &= \frac{s_x}{t_f}\\ v_x &= 11.3 \quad \mathrm{m\,s^{-1}}\end{aligned}$$

Direction of velocity:

$$\begin{aligned}\theta &= \arctan\frac{v_y}{v_x}\\ \theta &= 33.7 \quad \mathrm{deg}\end{aligned}$$

   (b) Time of flight (ϕ is launch angle):

$$\begin{aligned}s_y &= u\sin\phi t_f - \frac{1}{2}gt_f^2\\ t_f &= 1.83\mathrm{s}\\ s_x &= u\cos\phi t_f\\ s_x &= 11.3\mathrm{m}\end{aligned}$$

Maximum distance Mandy can cover:

$$\begin{aligned}x_{max} &= 6 + 3t_f = 11.5\mathrm{m}\\ x_{max} &> s_x\end{aligned}$$

Therefore Mandy can catch the ball.

   (c) By Newton's 3rd law, force by the ball on Mandy is equivalent to force by Mandy on the ball.

$$\begin{aligned}F_b &= \frac{\Delta p}{\Delta t}\\ F_b &= \frac{mv'}{\Delta t}\\ \frac{1}{2}mv'2 &= \frac{1}{2}mu^2 + mgh\\ v' &= 11.8 \quad \mathrm{m\,s^{-1}}\\ \therefore F_b &= (59m) \quad \mathrm{N}\\ &= 23.6 \quad \mathrm{N}\end{aligned}$$