

Lesson 1 Homework: Simultaneous equations and equations of straight lines

1. Solve the following sets of simultaneous equations

$\begin{cases} 3x + 5y = 6 \\ x = 6 - 2y \end{cases}$	$\begin{cases} 5y - 3x = 3 \\ y = 9 - x \end{cases}$	$\begin{cases} x + y = 0 \\ 5y + 2 = 3x \end{cases}$
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2. This question examines another method of solving simultaneous equations. Consider the following set of simultaneous equations:

$$\begin{cases} 32x + 11y = 237 \rightarrow (1) \\ 16x + 55y = 465 \rightarrow (2) \end{cases}$$

- By multiplying equation (2) by 2 (i.e. $2 \times (2)$), obtain another equation in the form of $32x + ky = c \rightarrow (3)$, where k , c are numbers to be determined.
 - Now subtract (3) from (1), in this step, you should be able to eliminate x from the equation. In other words, you should obtain another equation in the form of $my = d$, where m and d are numbers. Solve for y and then x .
 - Try doing the entire process again, but instead multiply (1) by a suitable factor to eliminate y from the equation.
- [You may find using a calculator useful for this question]
3. A farm has 80 animals in total. We know there are two more cows than pigs; the sum of the number of pig and ducks is $\frac{5}{3}$ times the number of cows; set up 3 simultaneous linear equations and solve for the number of each animal (assuming the farm only has those 3 types of animals). You should be able to set up the following system.

$$\begin{cases} c + p + d = 80 \\ c = p + 2 \\ p + d = \frac{5}{3}c \end{cases}$$

Hint: Try to replace c with p and d first. You should then be able to set up 2 equations to solve for p and d .

4. Suppose Alfred has \$10 dollars to begin with, and his friend, Marco has \$50 dollars to start with. Alfred is a hard worker and earns \$20/week while Marco only earns \$10/week.
- Express the amount of money, y_a , Alfred will have after x weeks. Do the same for Marco, with y_m ?
 - Alfred is trying to be richer than Marco, how many weeks will it take for Alfred to have the same amount of money than Marco?
 - Find the rate of decrease of the wealth difference between Alfred and Marco. In other words, find the slope of $(y_a - y_m)$ against x
 - How much money must Marco make per week so Alfred will never become richer than him?

5. Alfred and John start off in two different locations, travelling towards each other. Initially, Alfred is at a distance $y = 80\text{m}$ away from John, who is standing at the origin. Alfred moves towards the origin (directly) at 5 m/s whilst John moves towards Alfred at 3m/s
- Plot John and Alfred's initial position on a graph.
 - Find where John and Alfred will be after 4 seconds.
 - Show that the slope of the two lines is equal to their speeds. Hence find the straight-line equations describing John and Alfred's positions.
 - When will John and Alfred meet? And where will they meet?
 - Kerry initially stood in the middle of Alfred and John, suppose Kerry will walk at 2 m/s , when must she start walking and towards which direction so all 3 of them would meet.
 - Considering the geometry of the two straight lines representing Alfred's and John's positions, is it possible for Kerry to meet *neither* of them, assuming all 3 people walk for indefinitely long.
6. These questions will be based on more algebraic proofs.
- Show that any straight line passing through the origin will have an equation in the form of $y = mx$, where m represents the line's slope.
 - Another way of expressing a straight line equation is in the form

$$ay + bx + c = 0$$

What is the condition on c so the line passes through the origin. Here, a, b, c , are constant numbers.