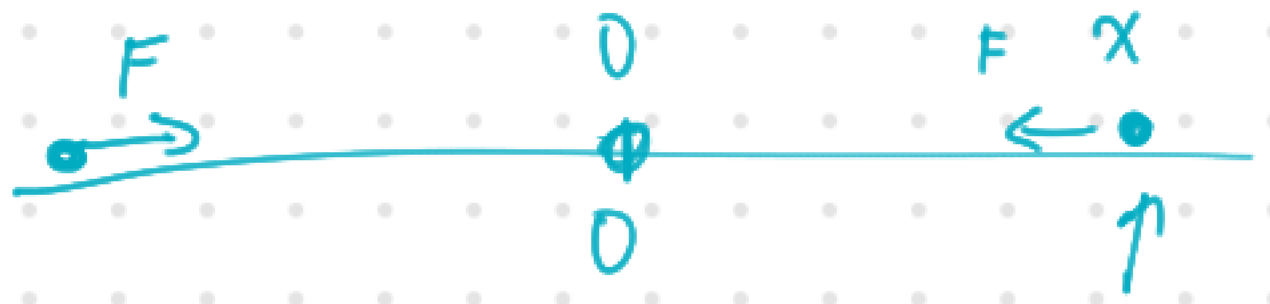
"Simple" harmonic motion

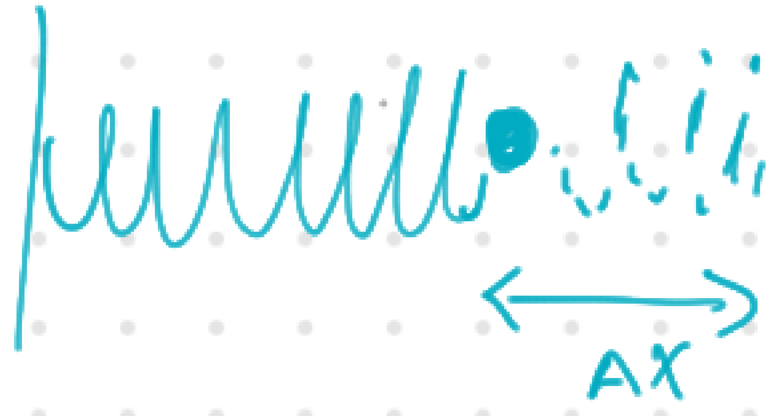


Condition / Definition

1. a always points to equilibrium position
   a opposes displacement
2. a is proportional to x

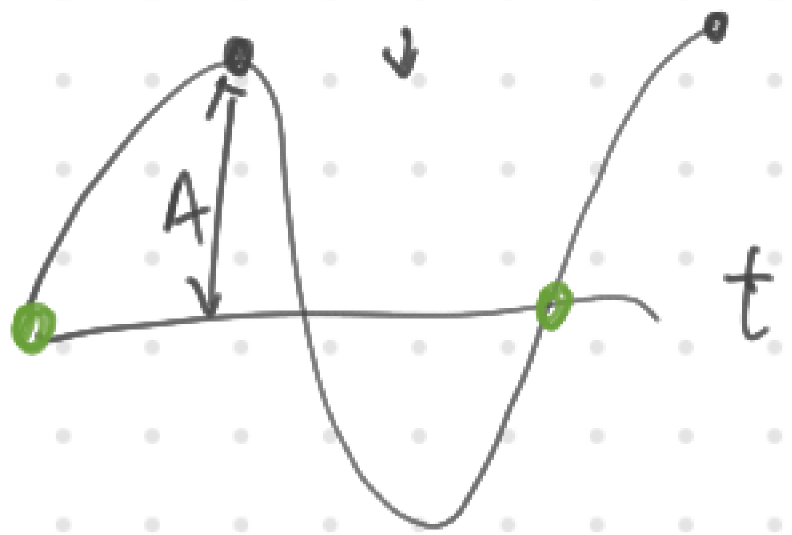
↳ $a = -\omega^2 x$

Hooke's law



$F = -k\Delta x$

$a = \frac{F}{m} = -\left(\frac{k}{m}\right)\Delta x$ → ω^2

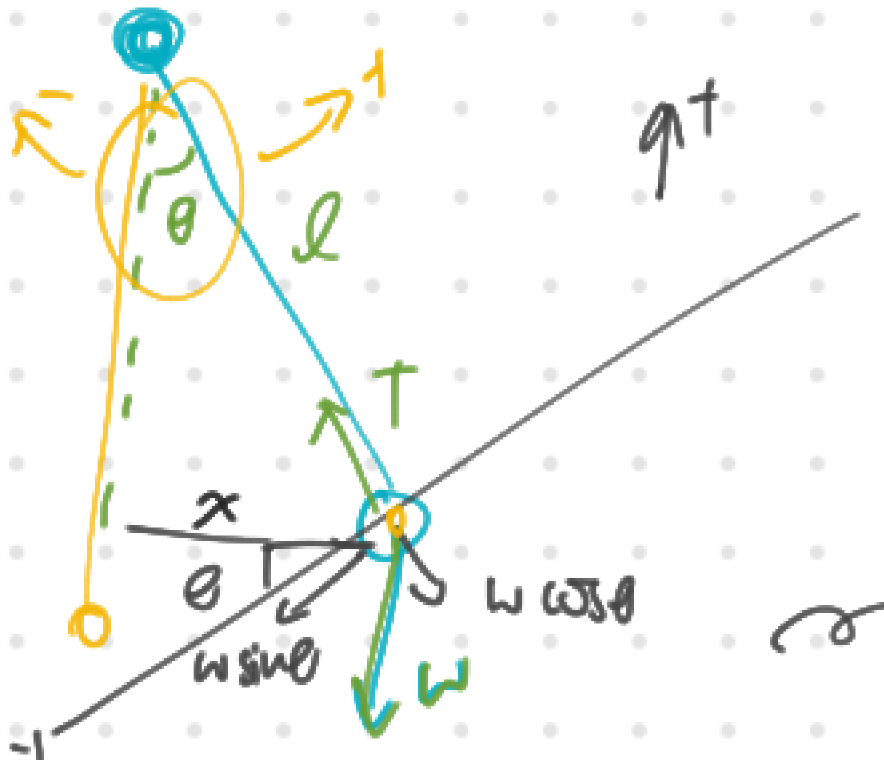


$x(t) = A\cos(\omega t + \phi)$

ω = Angular, how many angle s^{-1}

$\omega = 2\pi f$ ← how many complete wave / s

Simple pendulum



$\therefore m\alpha = -mg\sin\theta$

$\theta < 10°$

$\boxed{\sin\theta \approx \theta}$

$\alpha \approx -g\sin\theta \approx -g\frac{x}{l}$

$\alpha = -\frac{g}{l}x$

$-g\frac{x}{l} \approx -\omega^2 x$

$\omega^2 = \frac{4\pi^2}{T^2} = \frac{g}{l}$

$T^2 = 4\pi^2\frac{l}{g}$

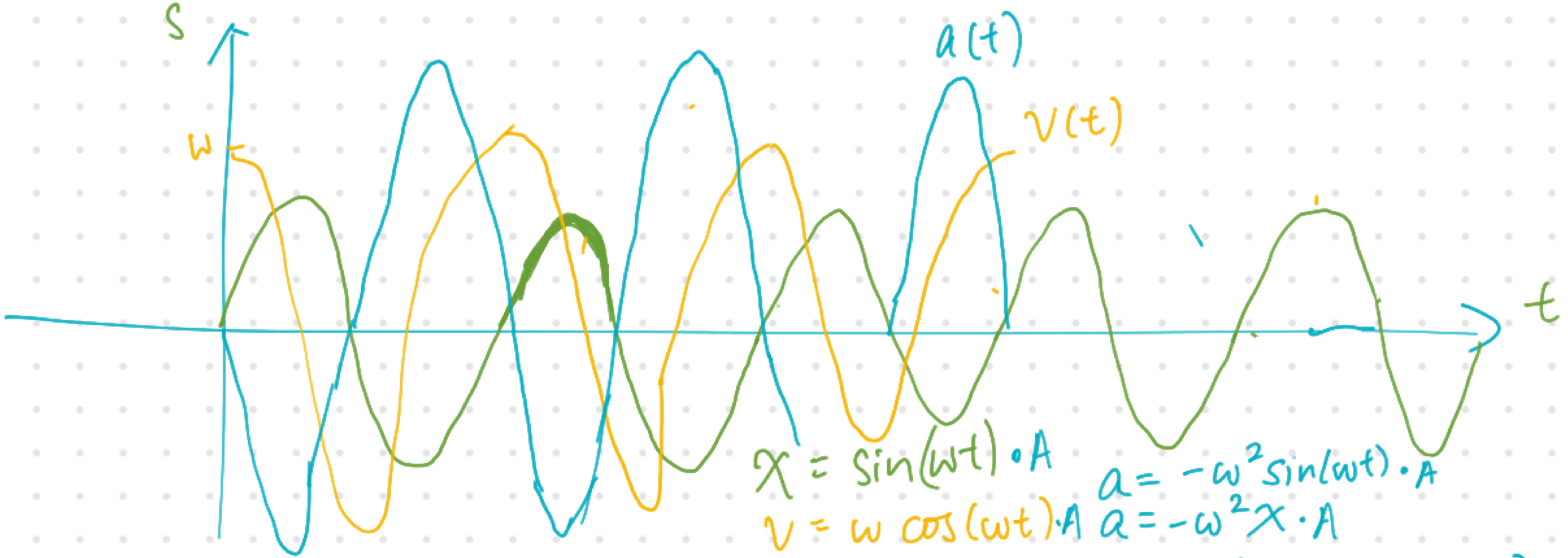
$T = 2\pi\sqrt{\frac{l}{g}}$

T ↑ i.e. slower

$T \propto \frac{1}{\sqrt{g}}$ $\quad T\sqrt{g} = k$

$T_1\sqrt{g_1} = T_2\sqrt{g_2}$

$T_m = T_e\sqrt{6}$



$x = \sin(\omega t) \cdot A$

$v = \omega \cos(\omega t) \cdot A$

$a = -\omega^2 \sin(\omega t) \cdot A$

$a = -\omega^2 x \cdot A$

Find v & a. Sketch them

Amplitude (A)

$x_{max} = A$

$v_{max} = \omega A$

$a_{max} = \omega^2 A$

when is v largest (in terms of x)

at eqm position

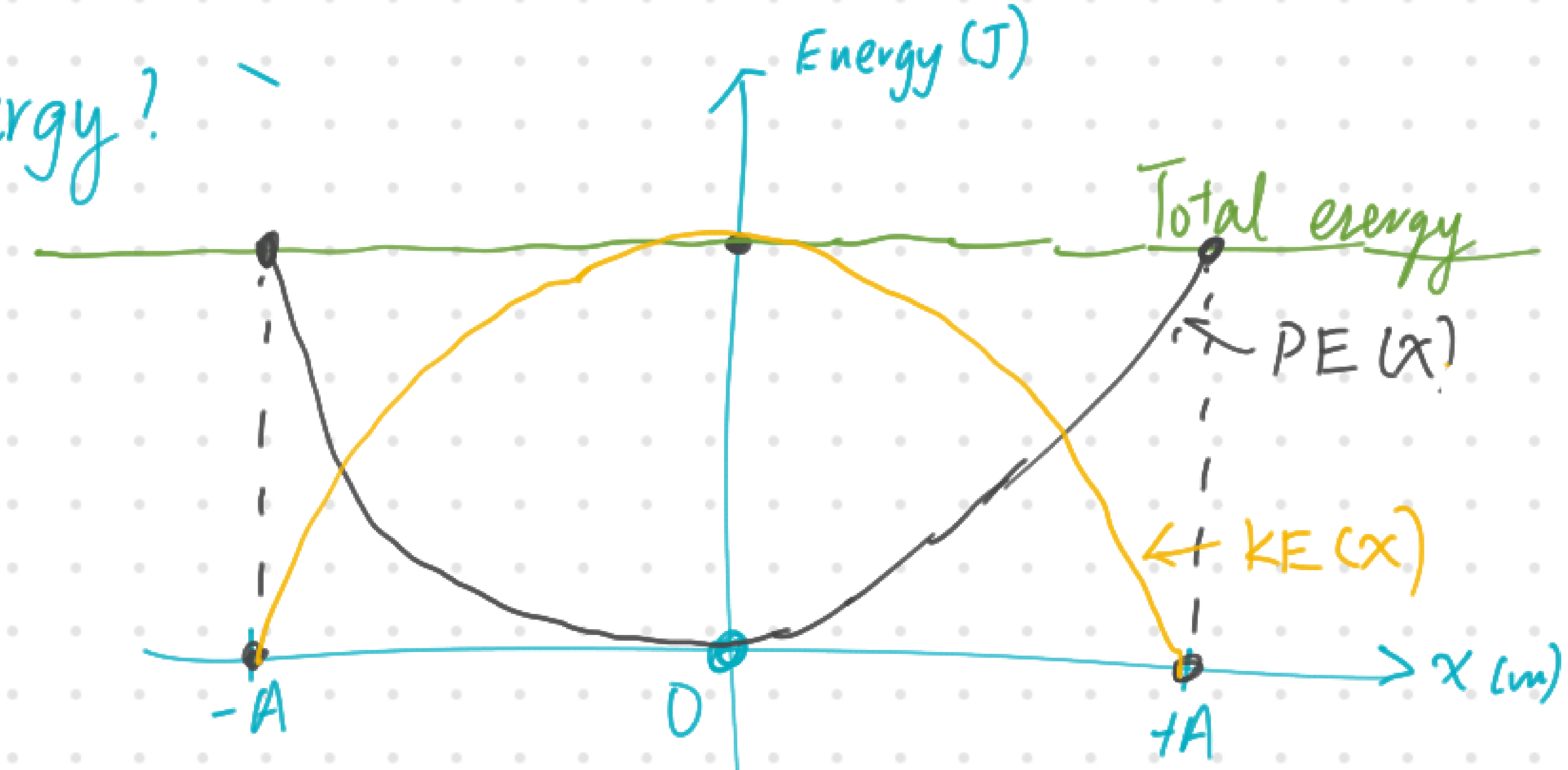
When is a largest

|a| is largest when x is at its max / min pt

Period (T)

frequency (f)

Energy?



Mechanical Energy

$KE + PE = \text{constant}$

$$E = \frac{1}{2} m v_{max}^2$$
$$= \frac{1}{2} k x_{max}^2$$

$KE = \frac{1}{2} m v^2$

$PE = ?\ \frac{1}{2} k x^2$

$KE = E - \frac{1}{2} k x^2$

Assuming an ideal spring mass system.

$W = Fx$

$\frac{dW}{dx} = F$

$$\int dW = \int +kx\,dx$$
$$W = \frac{1}{2} k x^2 + C$$

Q7.(a) (i) Name the two types of potential energy involved when a mass–spring system performs vertical simple harmonic oscillations.

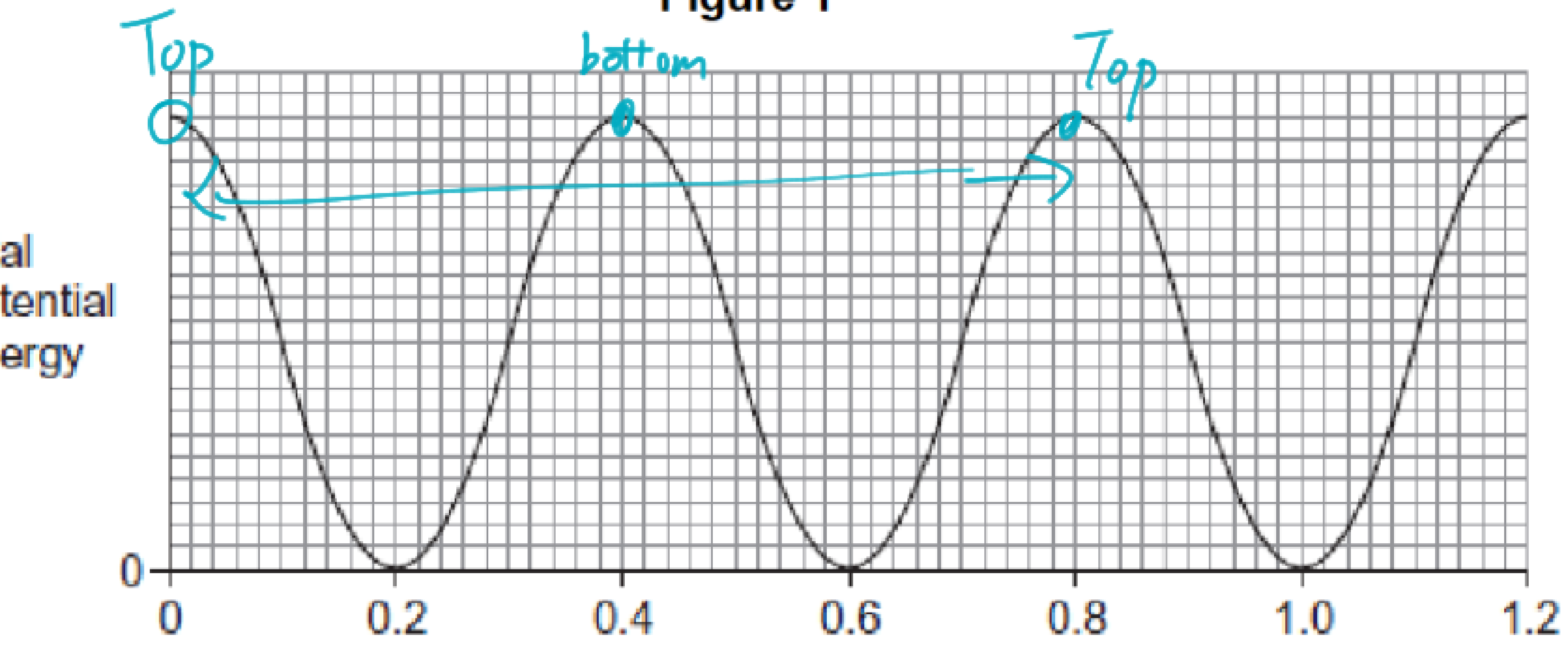
1. Gravitational PE
2. Elastic potential energy

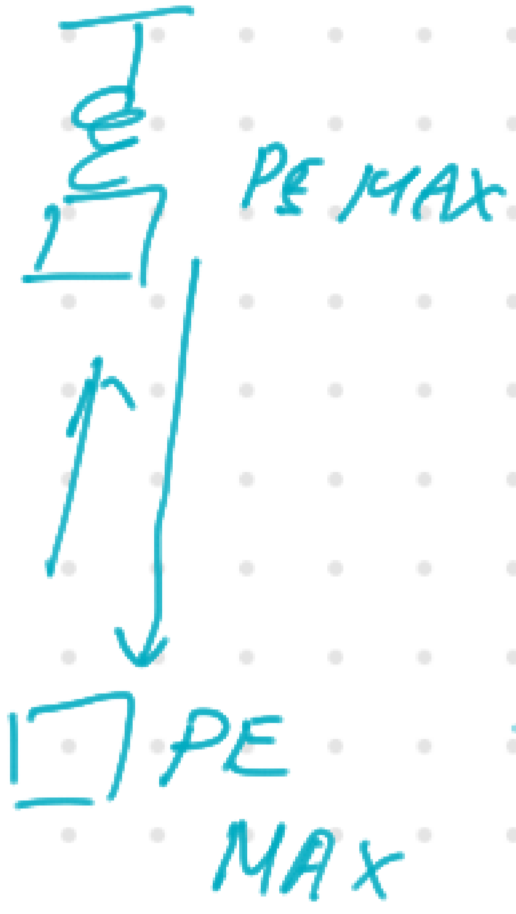
(ii) Describe the energy changes which take place during one complete oscillation of a vertical mass-spring system, starting when the mass is at its lowest point.

Elastic potential energy
↳ Kinetic energy
↳ Gravitational potential energy
↳ Kinetic energy
↳ Elastic potential energy

igure 1 shows how the **total** potential energy due to the simple harmonic motion aries with time when a mass-spring system oscillates vertically.

Figure 1

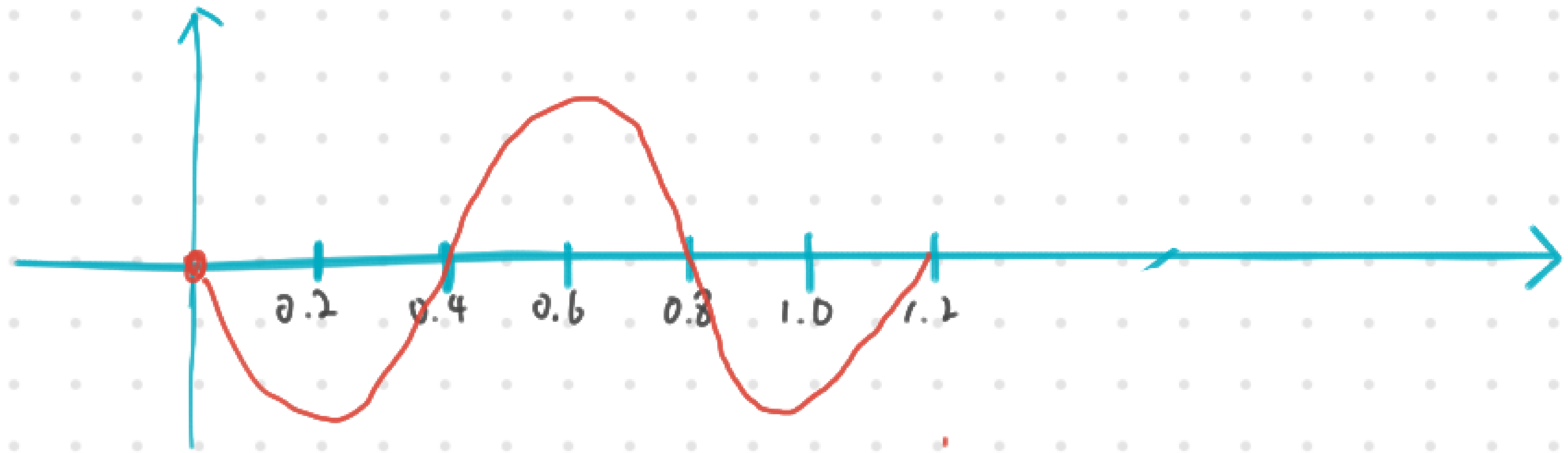


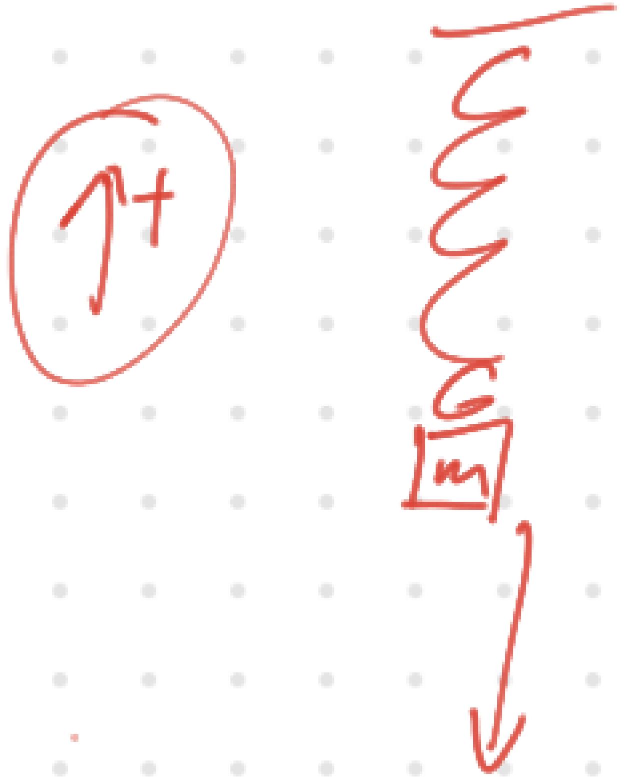


(i) State the time period of the simple harmonic oscillations that produces the energy–time graph shown in Figure 1, explaining how you arrive at your answer

0.8 s

(ii) Sketch v(t) in the following graph





(c) Given that m = 0.35kg, find the spring constant k with appropriate units

$T = 0.8 \quad m = 0.35$

as $T = 2\pi\sqrt{\frac{m}{k}}$

$k = 21.6 \ Nm^{-1}$

(ii) The maximum kinetic energy of the oscillating object is 2.0 × 10−2 J. Show that the amplitude of the oscillations of the object is about 40 mm.

$\frac{1}{2} m v_{max}^2 = 2 \cdot 10^{-2}$

$\frac{1}{2} m (\omega A)^2 = 2 \cdot 10^{-2}$

$m \omega^2 A^2 = 4 \cdot 10^{-2}$

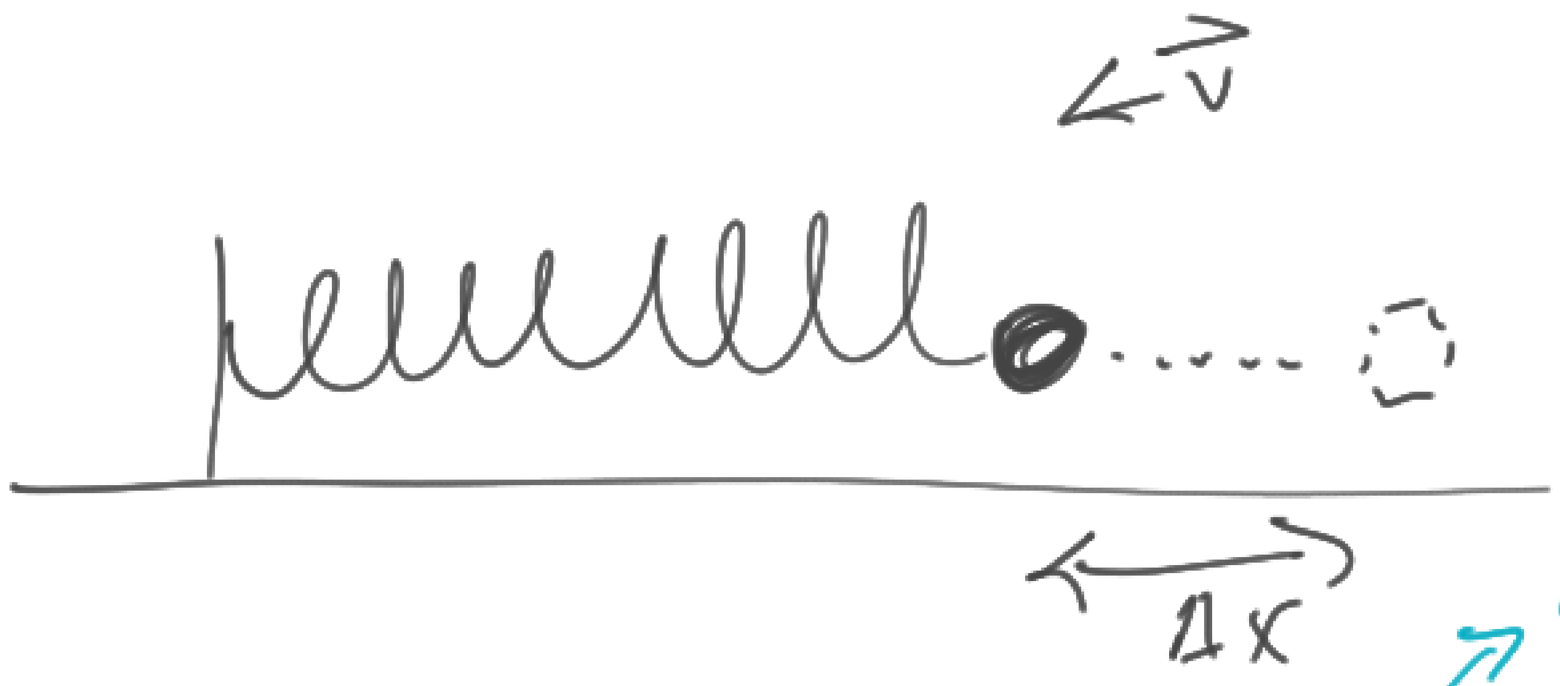
$A \simeq 0.043 m$

$A \simeq 40 mm$

$\frac{1}{2} m v_{max}^2 = \frac{1}{2} k x_{max}^2$

$\frac{1}{2} k x_{max}^2 = 2 \cdot 10^{-2} J$

DAMPED SHM & Driven SHM



drag coefficient

$$F_{net} = ma = -kx - \alpha v$$

$$\ddot{x} + \frac{\alpha}{m}\dot{x} + \frac{k}{m}x = 0,$$

Second order homogenous ODE.

$$x = Ae^{-\beta t}\cos\left(\sqrt{\omega^2-\beta^2}\,t + \phi\right)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \beta = \frac{\alpha}{2m}$$

① under damp

✓ oscillating

α ~~k~~ not very big

② Critically damped

α just right $\longrightarrow$ $\alpha^2 - 4mk = 0$

③ Over damped

α very big

Resonance!

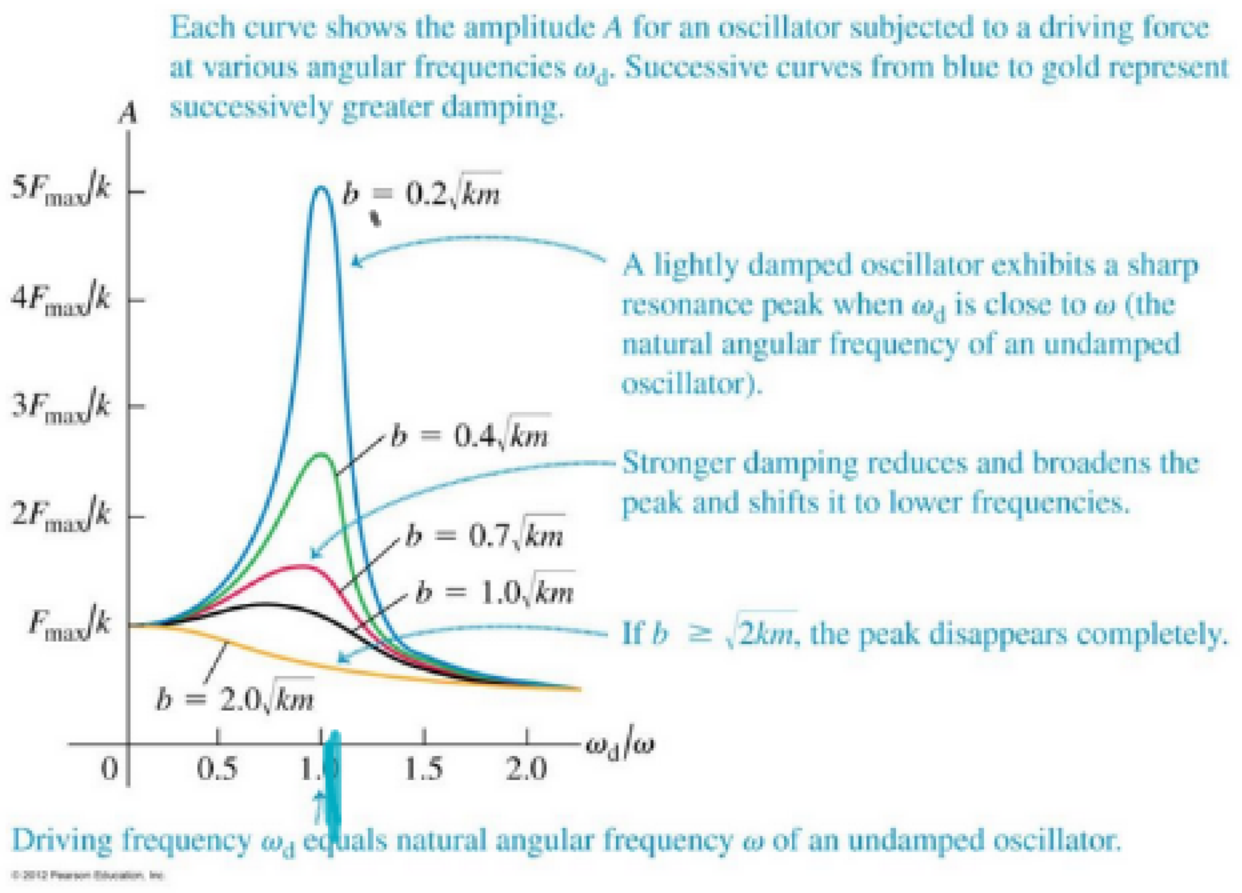
① Presence of oscillating driving force : $F_{ext} = F_0 \cos \omega t$

② $f_{drive} \approx f_{natural}$

$f_{natural}$: The frequency at which the system oscillates on its own given an initial displacement

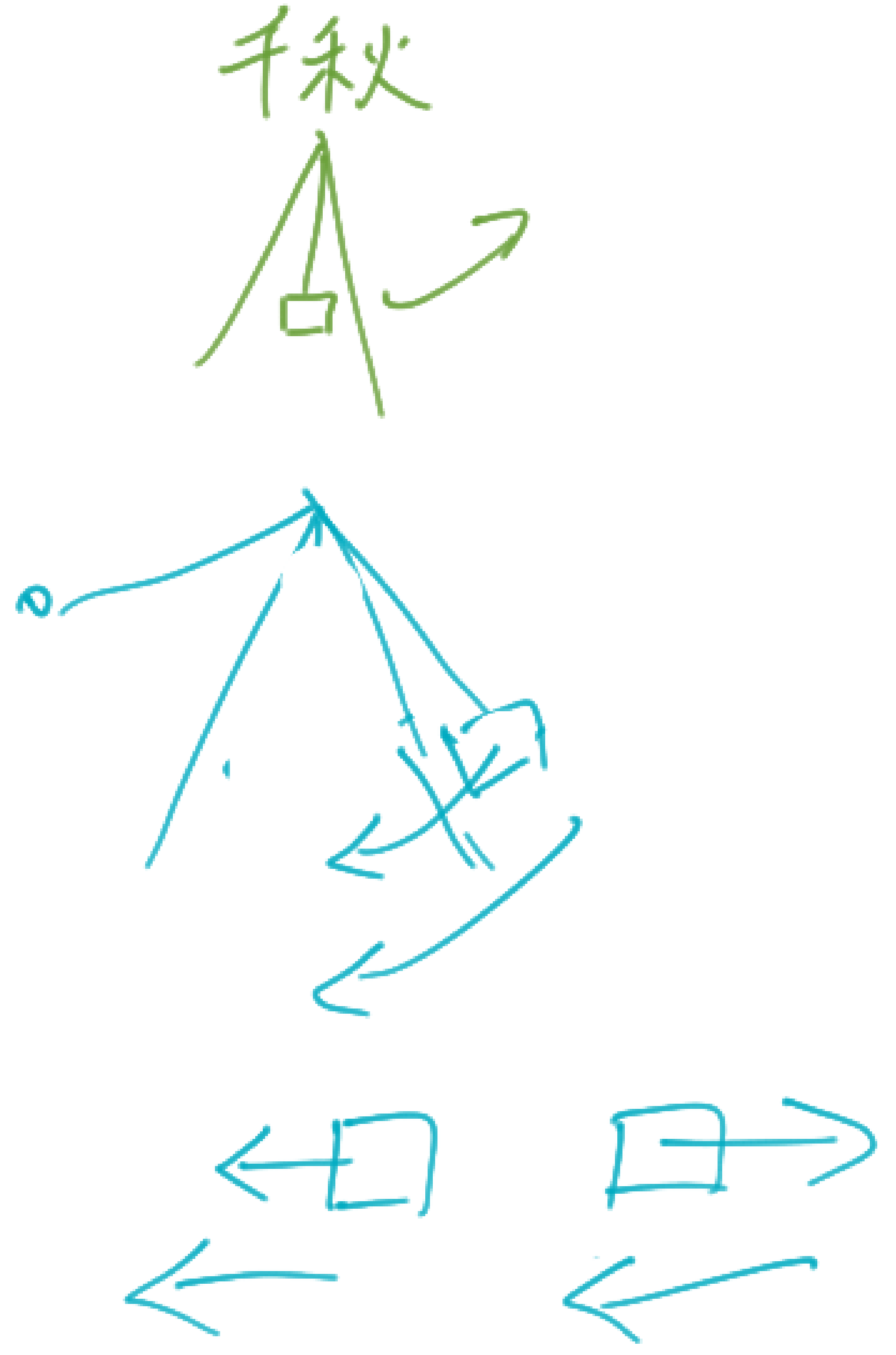
③ Effects at ~~reso~~ resonance

- Increased Amplitude
- Max energy transfer ($P \propto A^2$)



$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

A reaches maximum when $\omega_d \approx \sqrt{k/m}$, called **resonance**



$$\omega_c = \sqrt{\frac{k}{m}}$$

$$F = ma = -kx - \alpha v + F_0 \cos \omega t$$

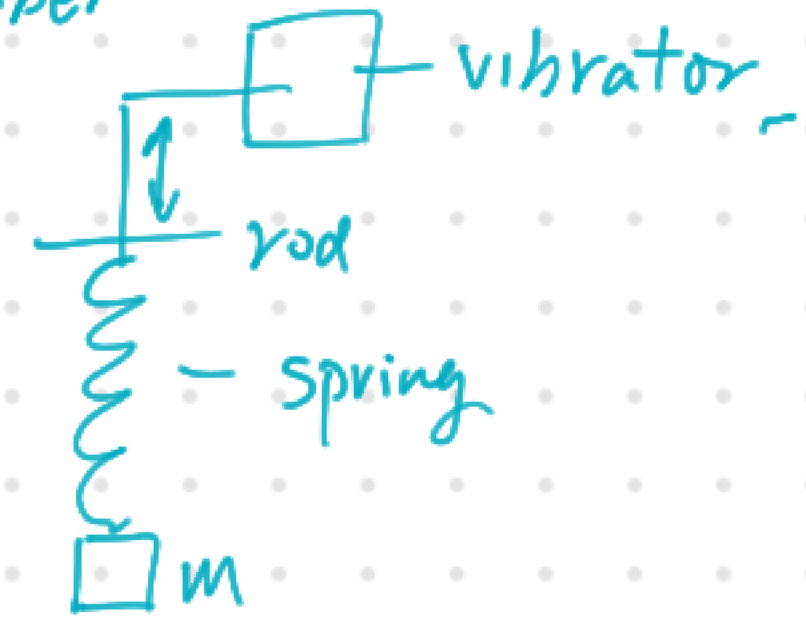
$$\ddot{x} + \frac{\alpha}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos \omega t$$

$$x = C_4 \cos(\omega' t) + C_3 \cos(\omega t)$$

$$x = A' \cos\left(\frac{\omega' + \omega}{2} t\right) \cos\left(\frac{\omega' - \omega}{2} t\right)$$

If an oscillating system is made to perform forced oscillations at a frequency close to its natural frequency, then resonance occurs.
Describe how you could demonstrate qualitatively the meanings of the terms:
forced osciallations,
natural frequency and
resonance

No need number



Forced oscillations.

Turn the vibrating f to some frequency f. The mass will oscillate with same f as the vibrator.

Natural frequency

Turn off vibrator. Give the mass an initial displacement. The frequency of the subsequent oscillating motion of the mass is the natural frequency.

Resonance

- Adjust f of ~~vibrator~~ vibrator to an f equal to natural frequency of spring
- Amplitude will greatly increase

$$E = hf$$

$$E^2 = (mc^2)^2 + (pc)^2$$

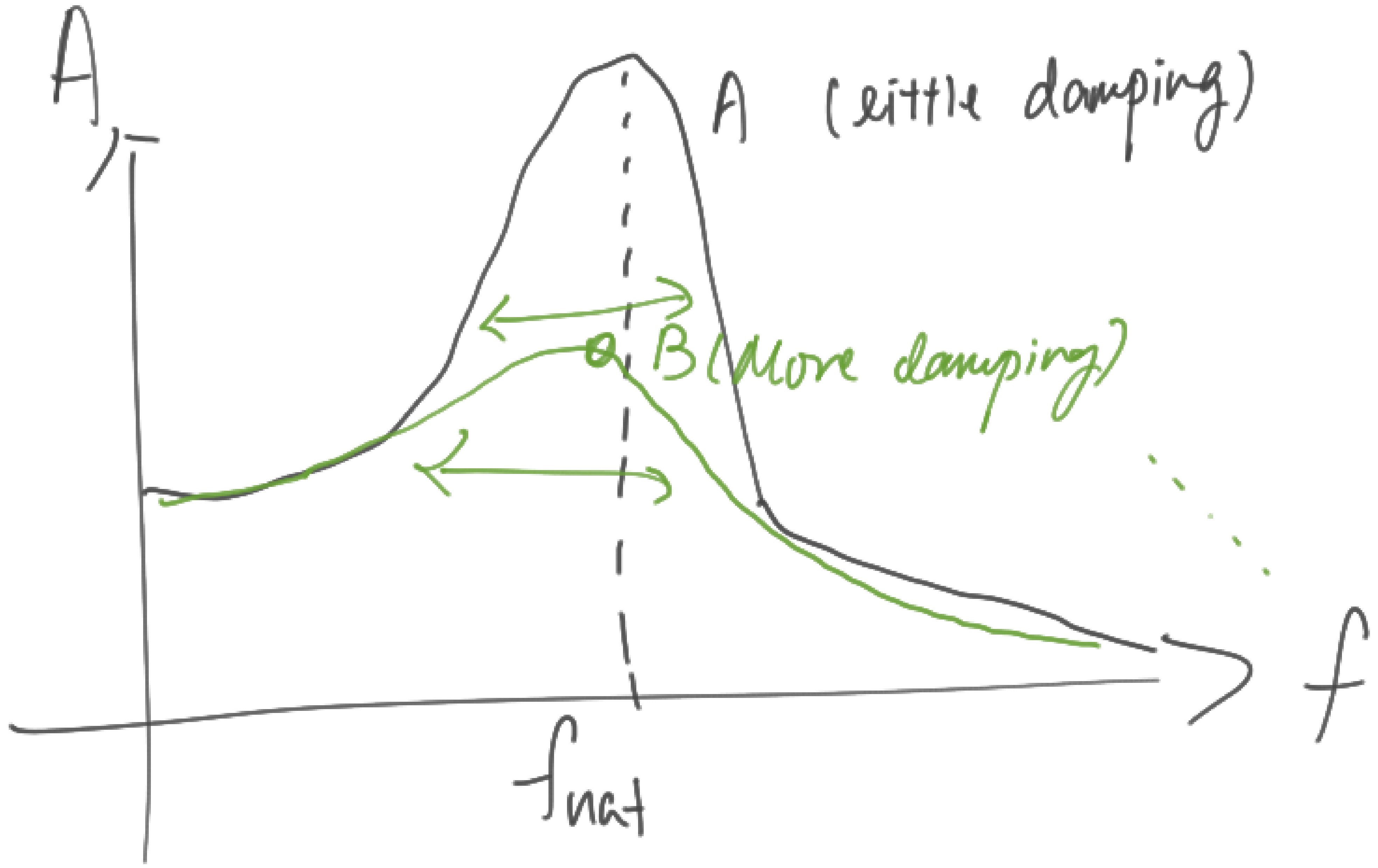
$$E^2 = (pc)^2$$

$$E = pc$$

$$p = \frac{E}{c} = \frac{hf}{c}$$

$$p = \frac{hc}{\lambda c}$$

$$\lambda = \frac{h}{p}$$

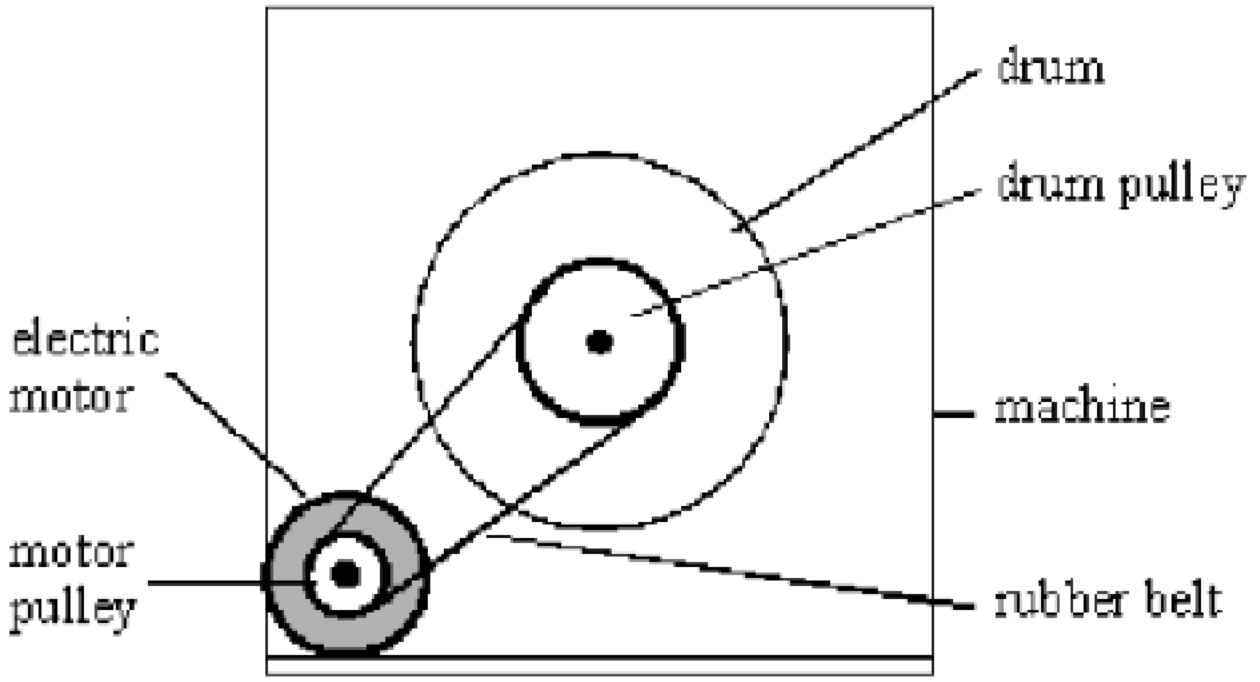


Reduce resonance:

① Apply more damping

↳ absorb more energy → Reduce amplitude of vibration at freq

Q9. An electric motor in a machine drives a rotating drum by means of a rubber belt attached to pulleys, one on the motor shaft and one on the drum shaft, as shown in the diagram below.



(b) When the motor rotates at a particular speed, it causes a flexible metal panel in the machine to vibrate loudly. Explain why this happens.

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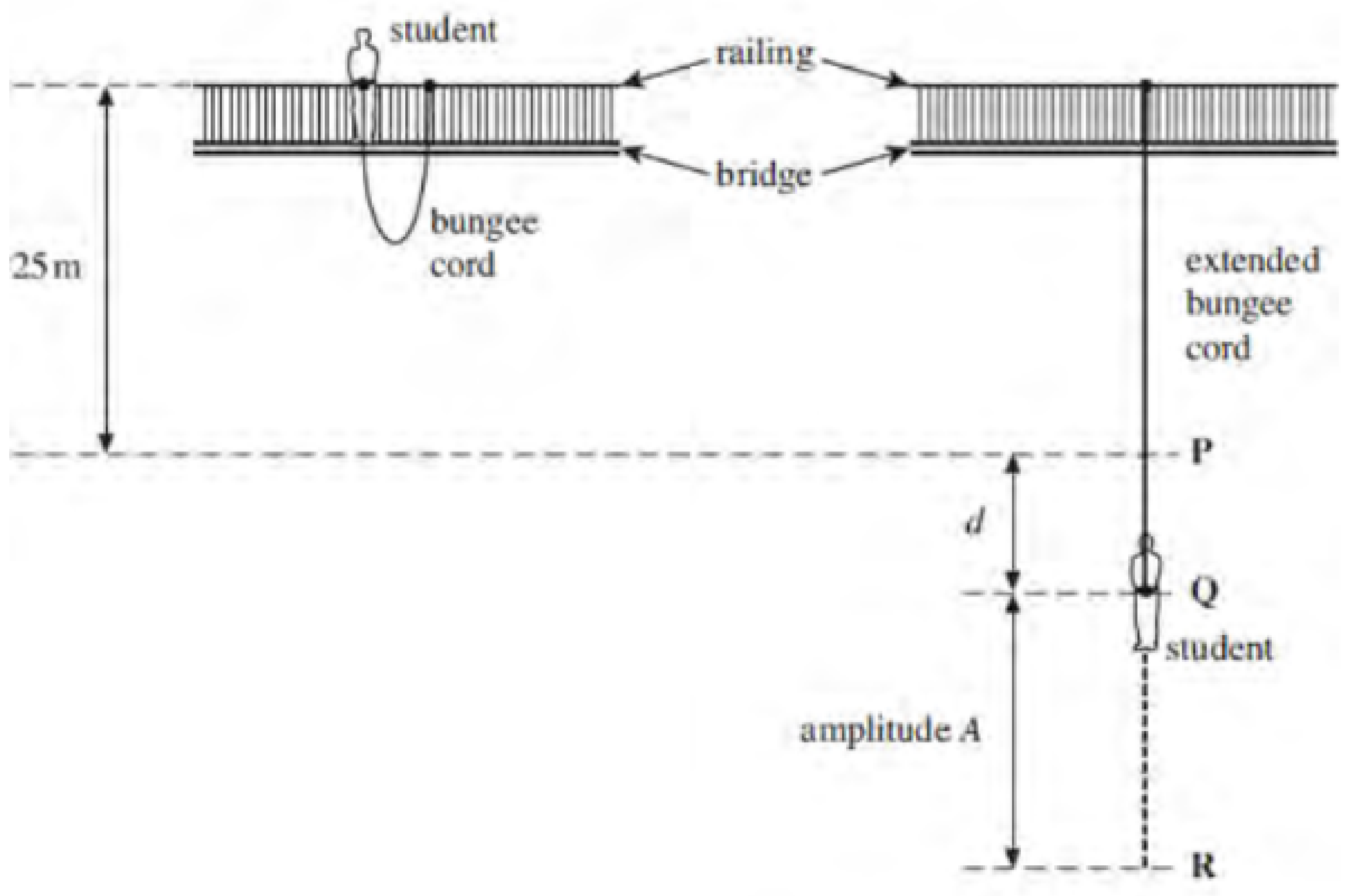
(2)
(Total 7 marks)

Q18. The two diagrams in the figure below show a student before and after she makes a bungee jump from a high bridge above a river. One end of the bungee cord, which is of unstretched length 25 m, is fixed to the top of a railing on the bridge. The other end of the cord is attached to the waist of the student, whose mass is 58 kg. After she jumps, the bungee cord goes into tension at point **P**. She comes to rest momentarily at point **R** and then oscillates about point **Q**, which is a distance *d* below **P**.

BEFORE **AFTER**



(a) (i) Assuming that the centre of mass of the student has fallen through a vertical distance of 25 m when she reaches point **P**, calculate her speed at **P**.
You may assume that air resistance is negligible.

answer = ... ms^{-1}

(2)

(ii) The bungee cord behaves like a spring of spring constant 54 Nm^{-1}.
Calculate the distance *d*, from **P** to **Q**, assuming the cord obeys Hooke's law.

(b) As the student moves below **P**, she begins to move with simple harmonic motion for part of an oscillation.

(i) If the arrangement can be assumed to act as a mass-spring system, calculate the time taken for one half of an oscillation.

answer = ... s

(2)

(ii) Use your answers from parts (a) and (b)(i) to show that the amplitude *A*, which is the distance from **Q** to **R**, is about 25 m.

(3)

(c) Explain why, when the student rises above point **P**, her motion is no longer simple harmonic.

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(2)