Experimentally, it was found that :

① $P \propto T$ (pressure law)
for constant V

② $P \propto \frac{1}{V}$ (Boyle's law)
for constant T

③ $V \propto T$ (Charles' law)
for constant P (冷縮熱脹)

Temperature in KELVINS!

(what if measured in °C)

Combining gives us,

$$\frac{PV}{T} = \text{constant}$$

naturally we know $P \propto n$ ← no of moles

$$\therefore \frac{PV}{T} = n \cdot \text{constant}$$

No of mole

No of molecule

$$\Rightarrow PV = nRT = Nk_BT$$

↑ universal gas constant

$$\frac{R}{N_A} = K_B$$

↑ Avogadro's constant

$$PV = nRT$$

$$P = \left(\frac{nR}{V}\right) T \qquad P \propto T$$

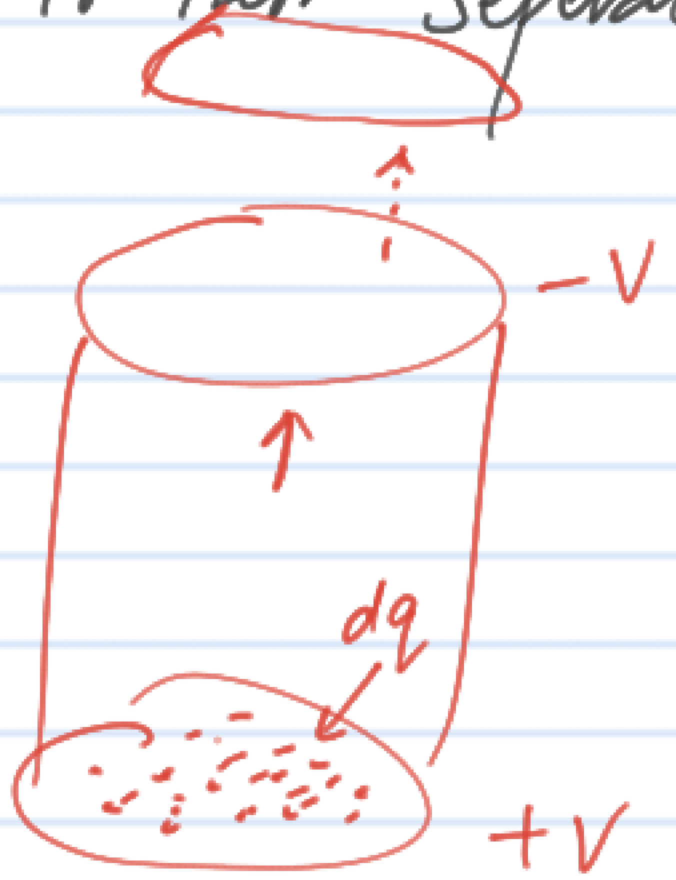
↑ const

molecule = (mol) • N_A

Kinetic theory

Assumptions:

1. Molecules are identical
2. Collisions between walls are elastic
3. Molecules do not interact w/ each other (no potential energy; only KE)
4. Size of molecules are negligible to their separation (low density)
5. Newtonian mechanics apply



Consider a cube, we now try to find F on one wall

Δp when hitting wall:

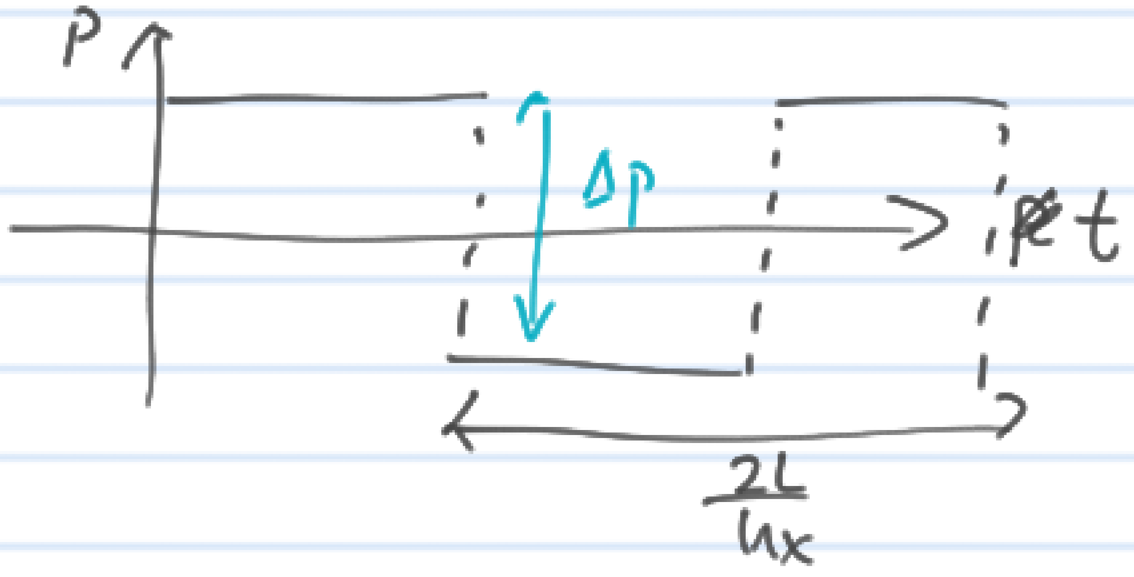
$$\Delta p = -2mv$$

$$\bar{F} = \frac{\Delta p}{\Delta t}$$

$$\Delta t = \frac{2L}{v}$$

$$\bar{F} = \frac{2mv \cdot v}{2L} = \frac{mv^2}{L}$$

$$\frac{2mu_x}{2L/u_x} = \frac{mu_x^2}{L}$$



$$P = \frac{F}{A} = \frac{m.v^2}{L^3}$$

Consider our n- gas molecule system.

$$P = \frac{nmv^2 \to \text{velocity}}{L^3 \to \text{Volume}}$$

$$PV = nm\,\bar{c}_x^2$$

$$\bar{c}^2 = u_x^2 + u_y^2 + u_z^2$$

these are equal

direction of motion doesn't matter for a lot of particles

$$PV = \frac{1}{3} Nm\bar{c}^2$$

or $P = \frac{1}{3}\rho\bar{c}^2$

$PV = \frac{1}{3} Nm\bar{c}^2$ ← p?

$\rho = \frac{M}{V}$

$C_{rms} = 500\ ms^{-1}$

$\Sigma m = 1\,kg$

$V = 0.5\,m^3$

$PV = \frac{1}{3} nmc^2$

$P(0.5) = \frac{1}{3}(1)(500^2)$

$P = 1.6\,atm$

Molecular KE.

all KE

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\bar{c}^2$$

$$\sum_i^n KE_i = \frac{1}{2}Nm\bar{c}^2 = \frac{3}{2}\left(\frac{1}{3}Nm\bar{c}^2\right) = \frac{3}{2}PV$$

$$k_B = \frac{R}{N_A}$$

$$N\left(\frac{1}{2}m\bar{c}^2\right) = \text{Total } KE = 3/2\,PV = 3/2\,nRT = \frac{3}{2}Nk_BT$$

consider only 1 molecule $E = \frac{3}{2}k_BT$ → Energy of a gas molecule is only dependent on temperature

Molecular speed ($\bar{c}$)

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} k_B T$$

$$\bar{c}^2 = \frac{3k_BT}{m} \Rightarrow \bar{c} = \sqrt{\frac{3k_BT}{m}}$$

$$\bar{c} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3RT}{M_R}}$$

Relative molecular mass (in kg)

typical gases have $\bar{c}$ at 500-1500 ms^{-1} at room temperature (298K)

1. **Figure 1** and **Figure 2** show apparatus used in an experiment to confirm the distribution of atom speeds in a gas at a particular temperature.

Figure 1

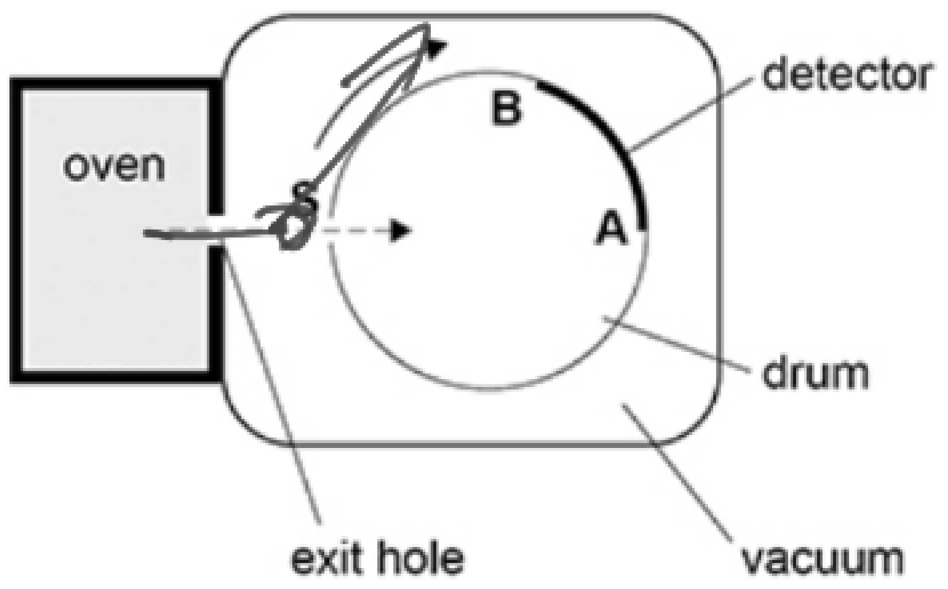
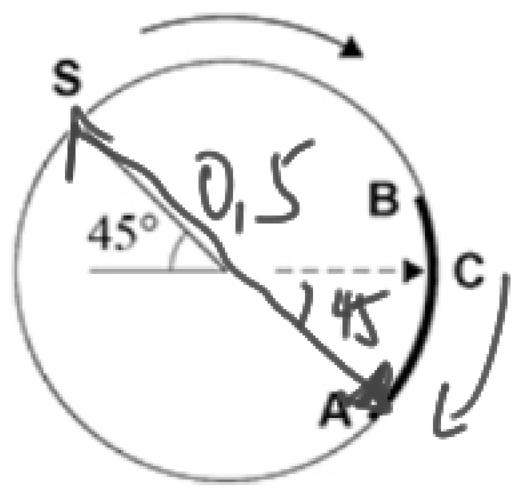


Figure 2



The oven contains an ideal gas kept at a constant temperature. Atoms of the gas emerge from the oven and some pass through the narrow slit **S** in a rapidly rotating drum. The drum is in a vacuum.

(a) Explain why the drum must be in a vacuum. (1 M)

To prevent the gas molecules from colliding with other particles which may deviate its path and stop it from getting into the drum

One atom leaves the oven, enters the drum through **S** and travels in a straight line across the drum.

In the time taken for the atom to move from **S** to the detector **AB**, the drum rotates through 45°.

The atom hits the detector at point **C**, as shown in **Figure 2**.

drum diameter = distance from **S** to **A** = 0.500 m

drum rotational speed = 120 revolutions per second = 120 Hz

(b) Show that the atom is moving at a speed of about 500 m s^{-1}.

$$\Delta\theta = \omega \Delta t \Leftrightarrow s = vt$$

$$\frac{\pi}{4} = (2\pi f)\Delta t$$

$$\Delta t = 1.04 \text{ ms}$$

$$AS = v\Delta t$$

$$v = 480 \text{ ms}^{-1}$$

(c) The speed of the atom in part (b) is equal to c_{rms}, the root mean square speed of the atoms of the gas in the oven.

The molar mass of the gas is 0.209 kg mol^{-1}. $\longrightarrow$ 1 mol of gas = 0,209 kg

Calculate the temperature of the gas in the oven.

$$E = \frac{1}{2} m \overline{c_{rms}^2} = \frac{3}{2} k_B T \qquad \frac{R}{N_A} = k_B$$

mass of one molecule

$$T = \frac{1}{3} \overline{c_{rms}^2} \frac{m}{k_B} \cdot \frac{N_A}{N_A} = \frac{\overline{c^2}}{3} \frac{0.209}{R}$$

$T = 1930K$

(d) The oven temperature is kept constant during the experiment but the pressure in the oven decreases as atoms leave through the exit hole.

Explain, using the kinetic theory, why the pressure decreases.

$PV = \frac{1}{3} N m \bar{c}^2 \qquad p = \frac{1}{3} \rho \bar{c}^2$

less atoms $\rightarrow$ N decreases. ($\bar{c}$ constant $\because T$ constant)

V is constant so $P \downarrow$

(e) The pressure of gas in the oven is initially 5.0×10^4 Pa.
The volume of the oven is 2.7×10^{-2} m^3.
During the experiment the pressure in the oven decreases to 4.5×10^4 Pa.

Calculate, in mol, the amount of gas that has emerged from the oven.

$P = \frac{RT}{V} n$

$\Rightarrow \Delta P = \frac{RT}{V} \Delta n$

$\Delta n = \frac{(\Delta P) V}{RT}$

$\Delta n = -8.41 \cdot 10^{-3}$ mol

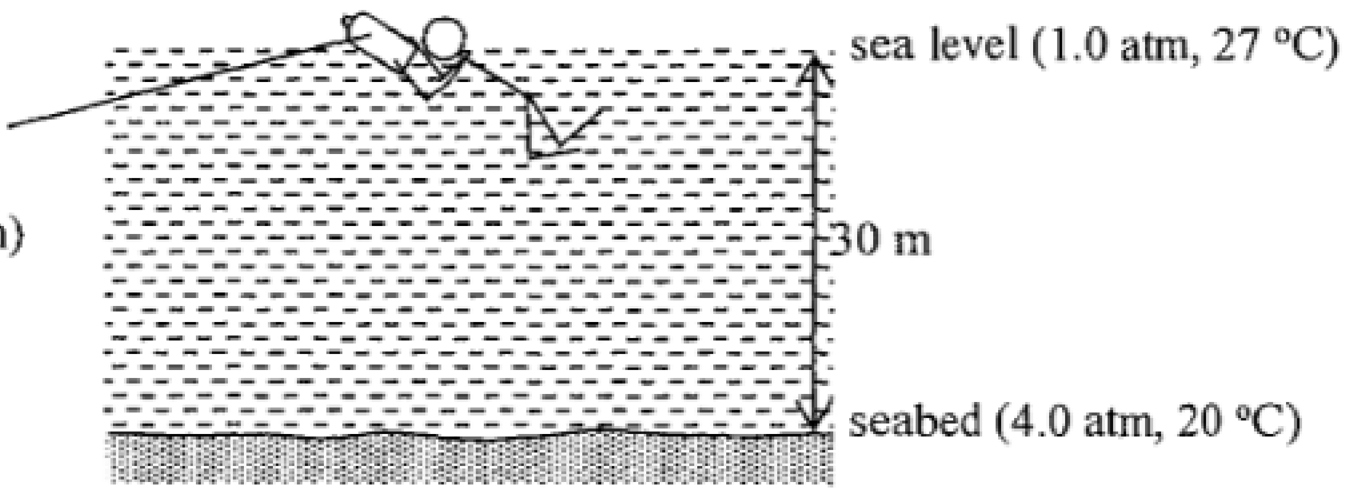


Figure 2.1

The metal cylinder of volume 0.012 m^3 contains compressed gas under a pressure of 18.5 atm is initially at sea level, where the pressure is 1.0 atm and the temperature is 27 °C. The diver then brings the cylinder to the seabed where the pressure is 4.0 atm and the temperature is 20 °C. Assume that the volume of the cylinder remains unchanged. Given: atmospheric pressure 1.0 atm = 1.01×10^5 Pa

*(b)(i) Show that at the seabed the pressure in the cylinder becomes 18.1 atm. (1 mark)

$P \propto T$ (constant V)

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$P_2 = \frac{T_2}{T_1} P_1 = \left(\frac{20+273}{27+273}\right)(18.5) = 18.1 \text{ atm}$$

(ii) Explain the pressure drop in the cylinder using the kinetic theory. (2 marks)

$T\downarrow \rightarrow P\downarrow$

When T decreases, avg molecular kinetic energy decrease → gas molecules moves slower

∴ Gas molecules collide with the walls of the container less vigorously and less frequently.

The rate of change of p experienced gas molecules ↓

→ By 3rd law, ~~P ∝ F~~ ~~impulse~~ force exerted on container walls ↓ → ∴ $P = \frac{F}{A}$, $P\downarrow$

*(c) The diver then inflates identical balloons each to a volume of 0.015 m^3 by using the cylinder of compressed gas at the seabed. Assume that the balloons are inflated slowly so that the temperature of the gas remains unchanged and the final pressure in the balloon equals that at the seabed.

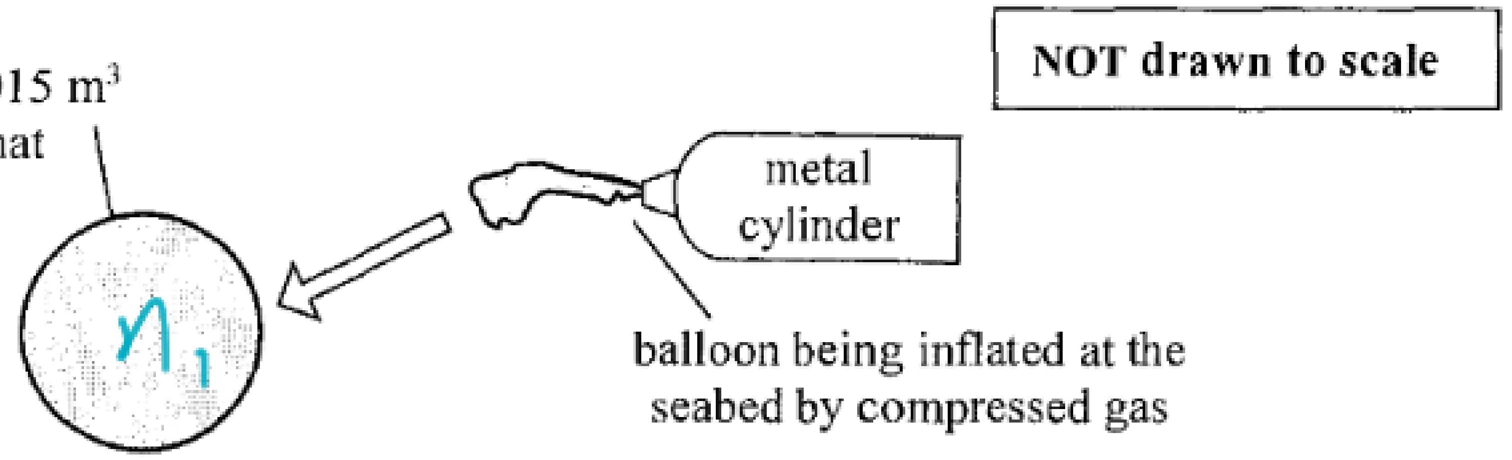


Figure 2.2

(i) Show that the gas pressure in the cylinder decreases by 5.0 atm after inflating one balloon. (2 marks)

$P_b V_b = n_1 RT$

$n_1 = 2.49$ mol

$(\Delta P)V = (\Delta n)RT$

$\Delta P = 5$ atm

(ii) Hence, find the total number of balloons that the diver can inflate completely. (2 marks)

Usable pressure $= 18.1 - 4$
$= 14.1$

∴ Total balloon $= \frac{14.1}{5}$
$= \cancel{2.67}\ 2.80$

2 balloons are filled COMPLETELY

(2)

10. (a) State what is meant by the internal energy of a gas.

(b) Absolute zero of temperature can be interpreted in terms of the ideal gas laws or the kinetic energy of particles in an ideal gas.

Describe these two interpretations of absolute zero of temperature.

(c) A mixture of argon atoms and helium atoms is in a cylinder enclosed with a piston. The mixture is at a temperature of 310 K.

Calculate the root mean square speed (c_{rms}) of the argon atoms in the mixture.

molar mass of argon = 4.0×10^{-2} kg mol^{-1}

(d) Compare the mean kinetic energy of the argon atoms and the helium atoms in the mixture.

The magnetic field strength required to provide a 1N/m force per unit length on a 1A current carrying wire, where the B field is perpendicular to the wire

(e) Explain, in terms of the kinetic theory model, why a pressure is exerted by the gas on the piston.

(f) The mixture of gases in the cylinder stays the same.

Explain, using the kinetic theory model, **two** changes that can be made independently to reduce the pressure exerted by the gas.