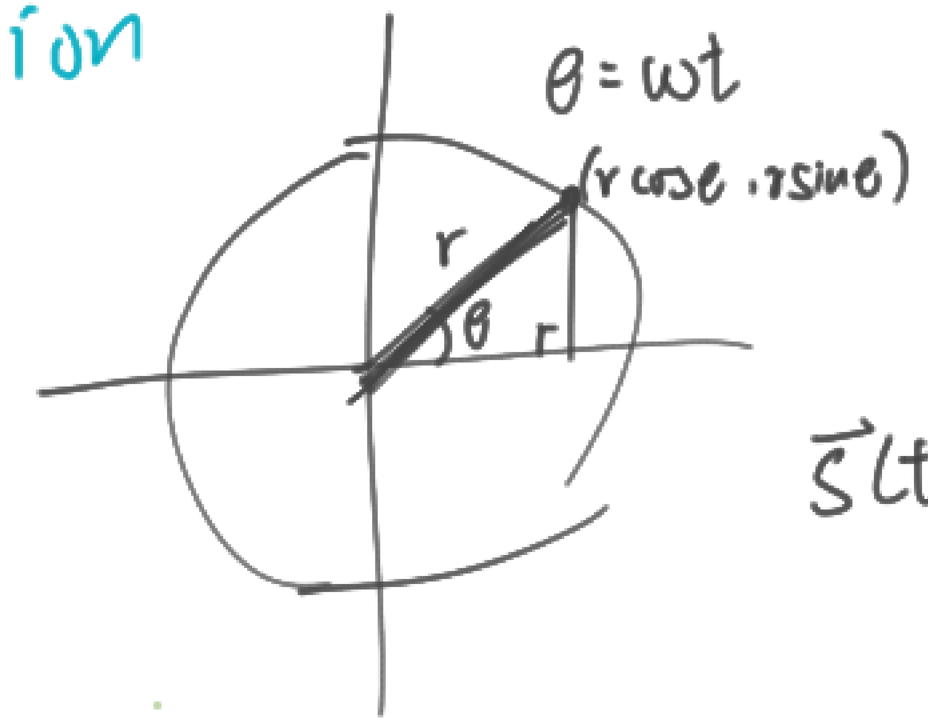
Uniform circular motion

① Theory

- what is radian
- Properties of S, v, a



$\vec{S}(t)$: circular (duh)

one of of expressing $\vec{s}(t)$

$$\vec{s}(t) = \underline{r\cos(\omega t)}\,\hat{i} + \underline{r\sin(\omega t)}\,\hat{j} + (\vec{s_0})$$

is this a circle?

$$\vec{v}(t) = \frac{d\vec{s}}{dt} \qquad |\vec{v}|^2 = (r\omega)^2\sin^2(\omega t) + (r\omega)^2\cos^2(\omega t) = (r\omega)^2$$

$$= -r\omega\sin(\omega t)\,\hat{i} + r\omega\cos(\omega t)\,\hat{j}$$

Other useful form: $\boxed{v = r\omega}$

Properties:
- Direction: $\perp$ to displacement $(\vec{v}\circ\vec{s} = 0)$
- Magnitude: Constant

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

$$= -r\omega^2\cos(\omega t)\,\hat{i} - r\omega^2\sin(\omega t)\,\hat{j}$$

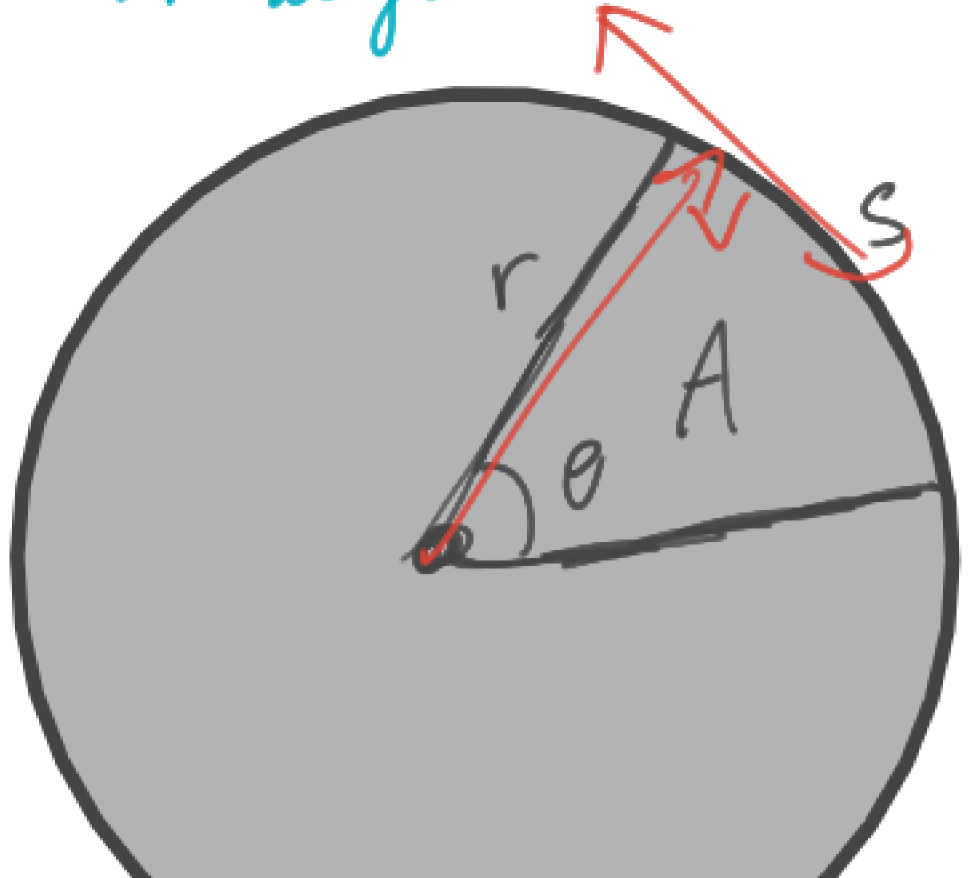
$$\boxed{\begin{aligned} a &= r\omega^2 \\ a &= \frac{v^2}{r} \end{aligned}}$$

Properties:
- Direction: Toward centre
- Magnitude: $r\omega^2$

Radian

Measurement for angles (alternative for deg)

- Defined as the ratio between the arc length of the sector and its radius
- One radian is the angle within a sector when the arc length of the sector equals to its radius

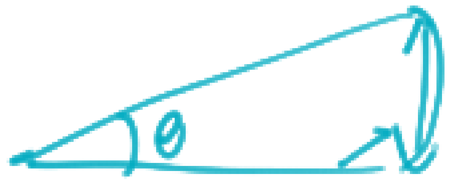


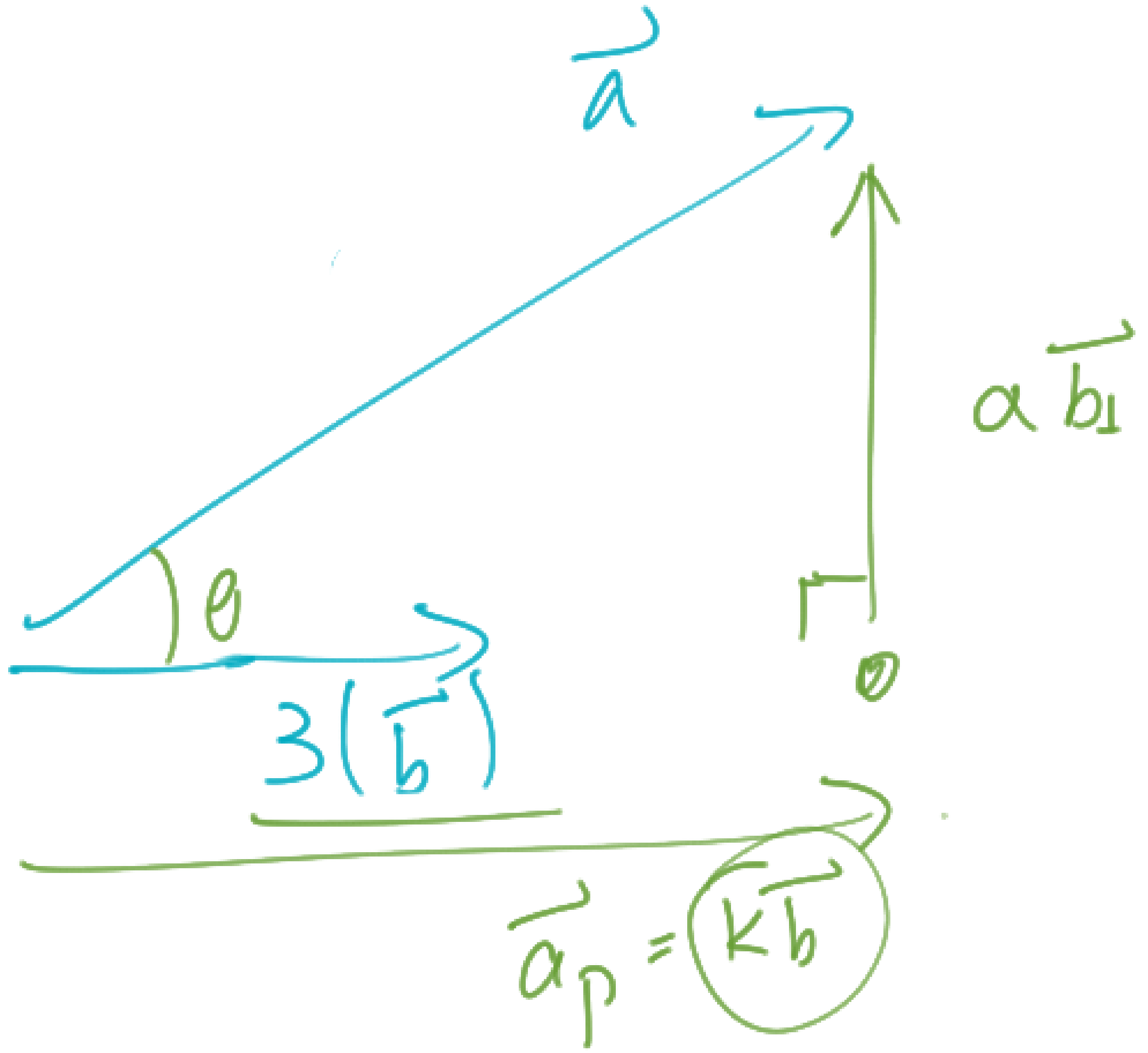
$S = r\theta$ (radian)

$A = ??$

$$r\frac{d\theta}{dt} = \frac{ds}{dt}$$

$$r\omega = v$$



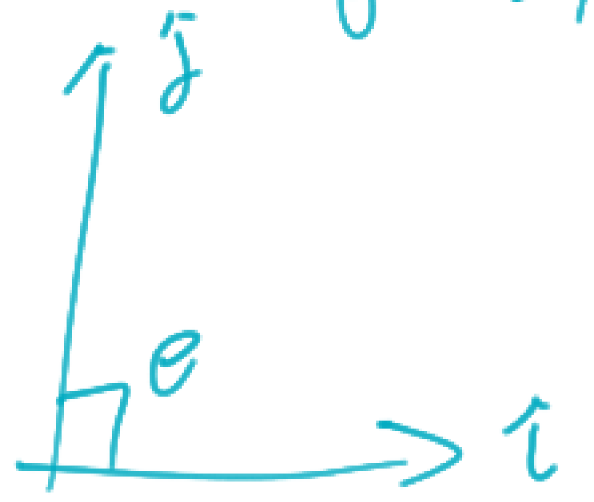


$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2} / \frac{3\pi}{2}$



$$\hat{i} \cdot \hat{j} = 0$$

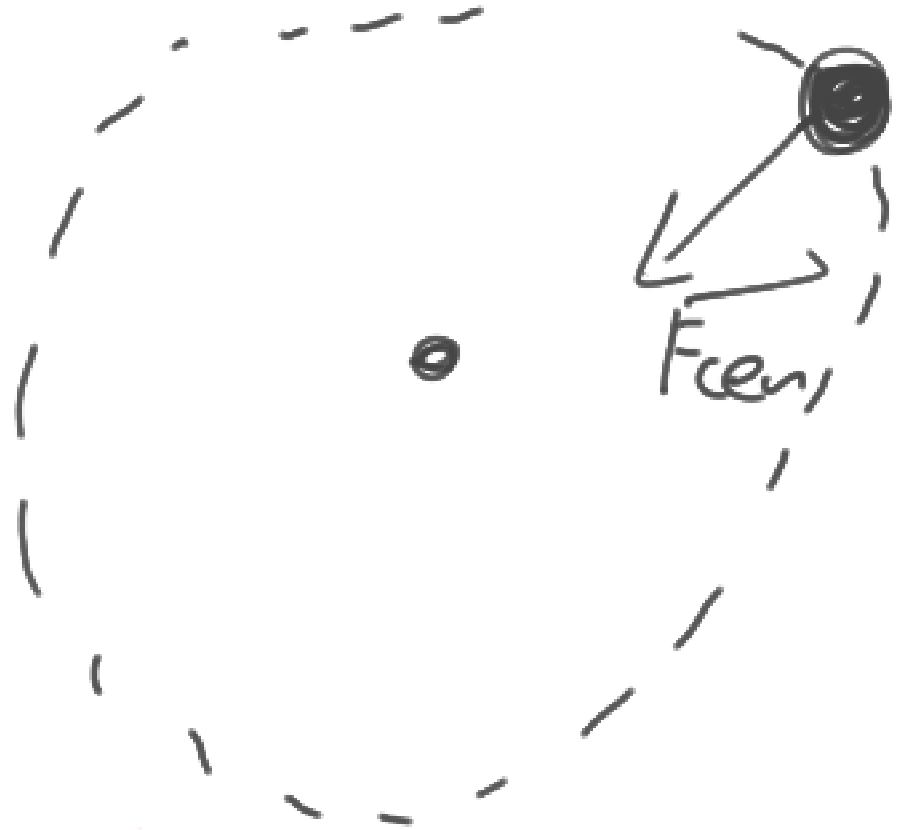
$$\vec{a} = x_1\hat{i} + y_1\hat{j} \qquad \vec{a} \cdot \vec{b} = x_1x_2 + y_1y_2$$

$$\vec{b} = x_2\hat{i} + y_2\hat{j}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \end{pmatrix}^T \cdot \begin{pmatrix} b_1 & b_2 & \dots \end{pmatrix}^T = (\vec{a} \cdot \vec{b})$$

if $\vec{a} \neq 0$, by $\vec{F} = m\vec{a}$

$\vec{F}_{net} = m\vec{a}$



Centripetal force is NOT a force, it is a requirement.

Centripetal force is only provided by other forces

e.g friction Tension Normal reaction

$\vec{F}_{net} = \vec{F}_{cen}$

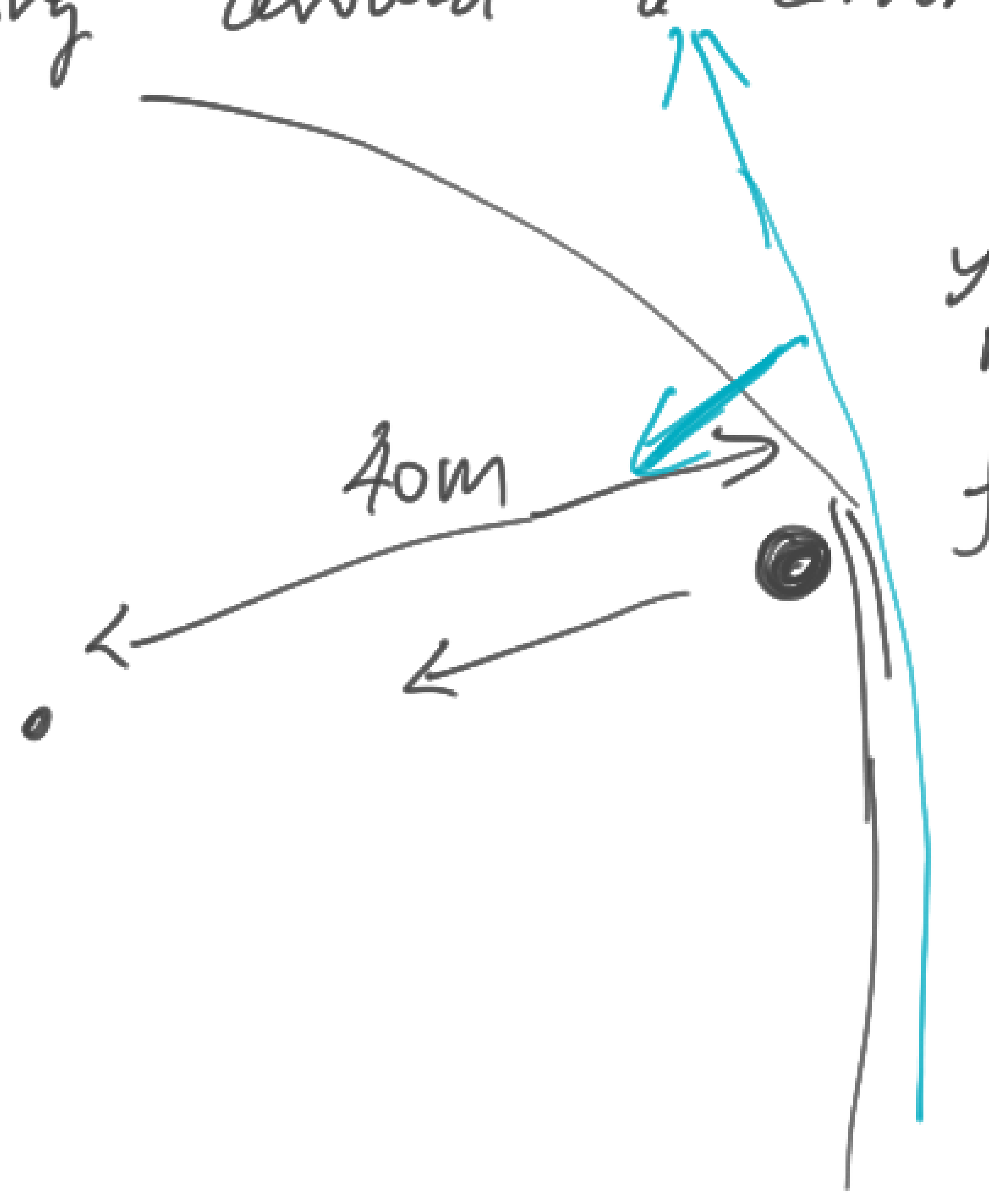
How to approach

① Label all forces

② Find the component responsible for a_{cm}

③ Solve equation (might need to set up another in ⊥ direction)

Car turning around a corner



$\mu = 0.4$

$m = 1000kg$

$f_{max} = \mu \cdot F_N$

① what is $|V_{max}|$?

$F_N = 1000g \approx 10000N$

$f_{max} = 400g = 4000N$

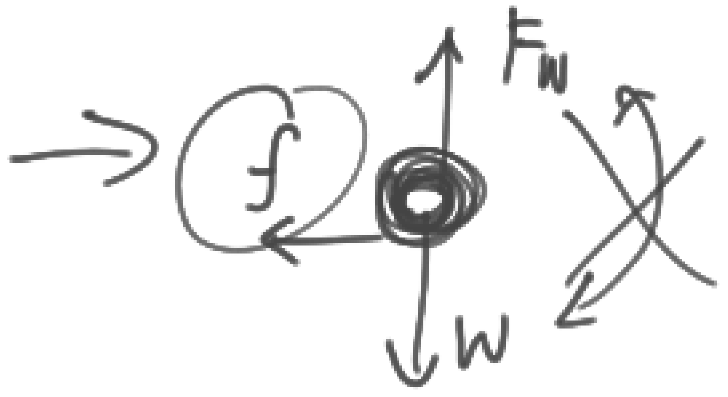
$F_{cen} = \frac{mv^2}{r}$

$f_{max} = \mu \cdot F_N = \frac{mv^2}{r}$

$\mu mg = \frac{mv^2}{r}$

$v^2 = \mu g r$

$v = 12.5\ ms^{-1}$



② what is $|F_{cen}|$ at $|V_{max}|$

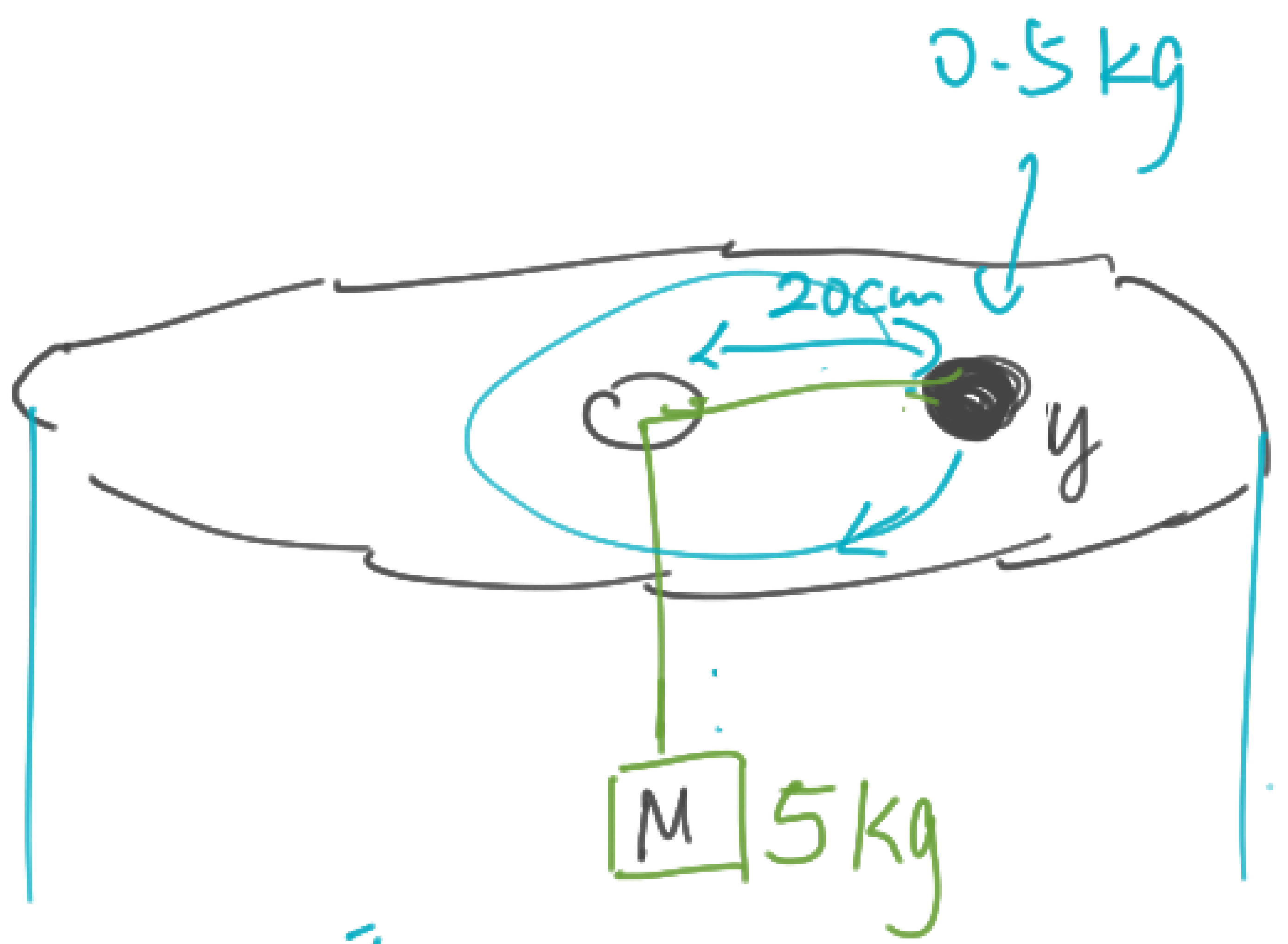
$f_{max} \approx 4000\ N$

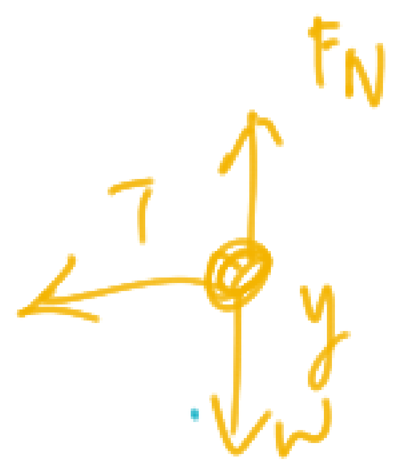
③ oily road... how does it affect V_{max}

More slippery → μ decrease

$v^2 \propto \mu$

table





Anywhere on an ideal string T is equal and pt toward centre

NO FRICTION

① What is ω of y if mass M doesn't fall down

$T = Mg = m_y r \omega^2$

$\omega = \sqrt{\frac{M}{m_y}\frac{g}{r}}$

$= 22.36 \text{ rad s}^{-1}$

③ The string breaks, find the velocity of y

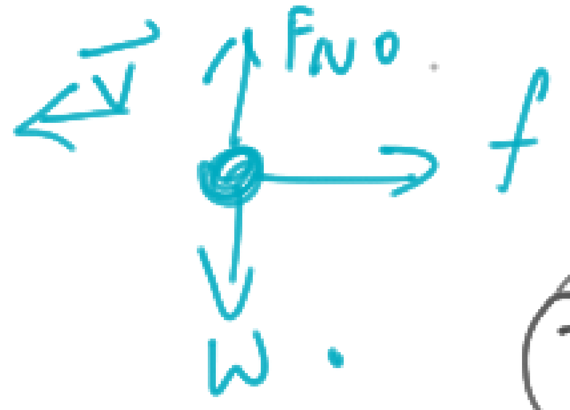
$v = \omega \cdot r$

$v = 4.47 \text{ ms}^{-1}$

④ Suppose $f = 10N$ when y is moving, find distance d moved

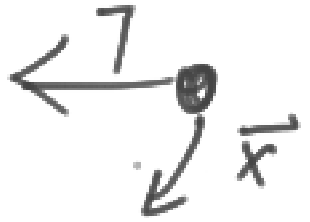
$a = \frac{f}{m}$ $\quad v^2 = u^2 + 2as$

$a = 20 \text{ ms}^{-2}$ $\quad s = 0.5m$



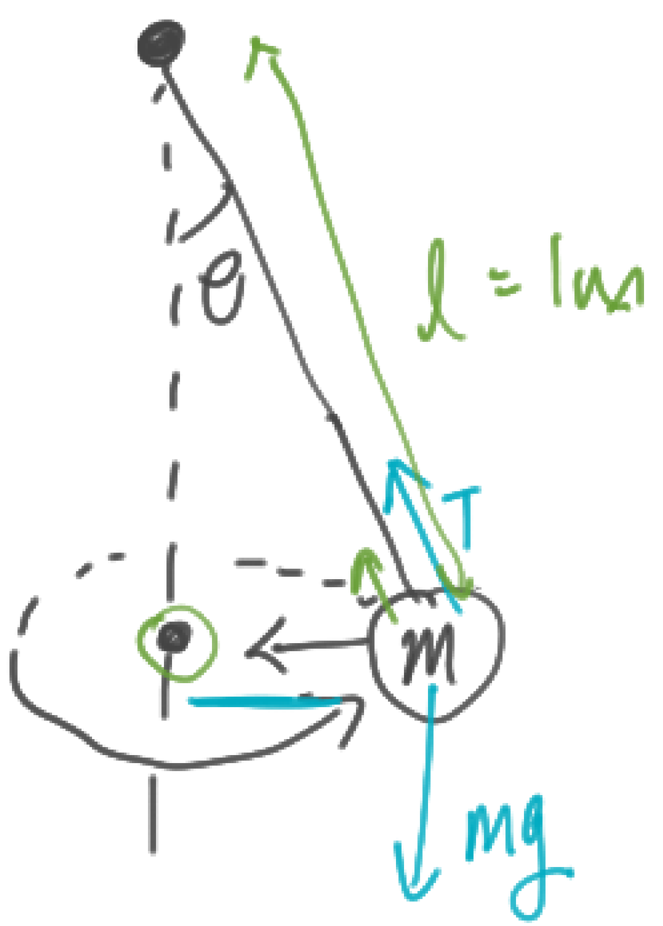
⑤ W.D by string on the mass?

0J // if W.D ⇒ ΔKE ≠ 0 i.e Δv ≠ 0



$W = F x \cos\theta$

$dW = \boxed{\vec{F} \cdot d\vec{s}}$



① Label all forces

② Suppose $m = 0.3\,kg$, $\theta = 30°$,

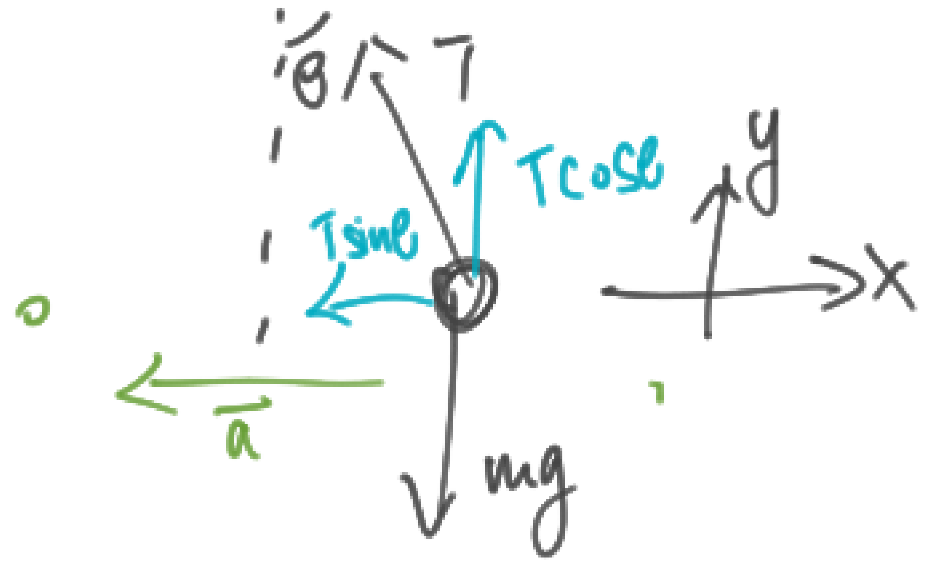
Find: 1. T 2. ω $r = 0.2m$

$T\cos\theta = mg$

$T = 3.40N$

$T\sin\theta = mr\omega^2$

$\omega = 5.32\ rad\,s^{-1}$



~~$l : 1m$~~ $v : 5ms^{-1}$ ($m : 0.3\,kg$)

find θ $r = 0.5m$

$T\cos\theta = mg$ ①

$T\sin\theta = ma = \frac{mv^2}{r}$

$T\sin\theta = \frac{mv^2}{r}$ ②

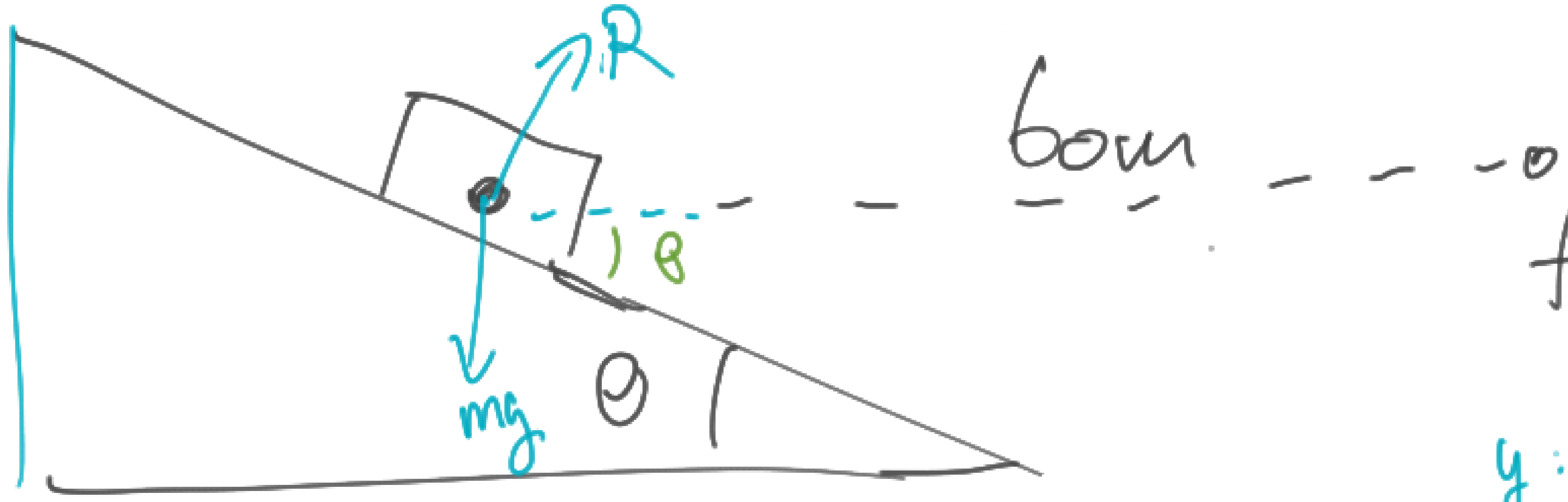
$\frac{②}{①}$ $\frac{T\sin\theta}{T\cos\theta} = \frac{\cancel{m}v^2}{r} \cdot \frac{1}{\cancel{m}g}$

$\tan\theta = \frac{v^2}{rg}$

Banked road

$m_{car} = 1200\,kg$

v of car $= 100\,kmh^{-1}$



If no friction is used in Fcen, find min θ required

$v = 27.7\,ms^{-1}$

$y:\ mg = R\cos\theta$

$x:\ F_x = ma$

$R\sin\theta = m\frac{v^2}{r}$

$\tan\theta = \frac{mv^2}{r} \cdot \frac{1}{mg}$

$\theta = \arctan\left(\frac{v^2}{rg}\right)$

$\theta = 52.66^\circ$

Explain the benefits of a banked road compared to a non-banked one

$\tan\theta = \frac{v^2}{rg}$ $\quad \theta = 0$

$v^2 \uparrow \quad \theta \uparrow$ $\quad \theta \geq 0$

$|v_{max}|$ on banked road is larger than the non-banked one

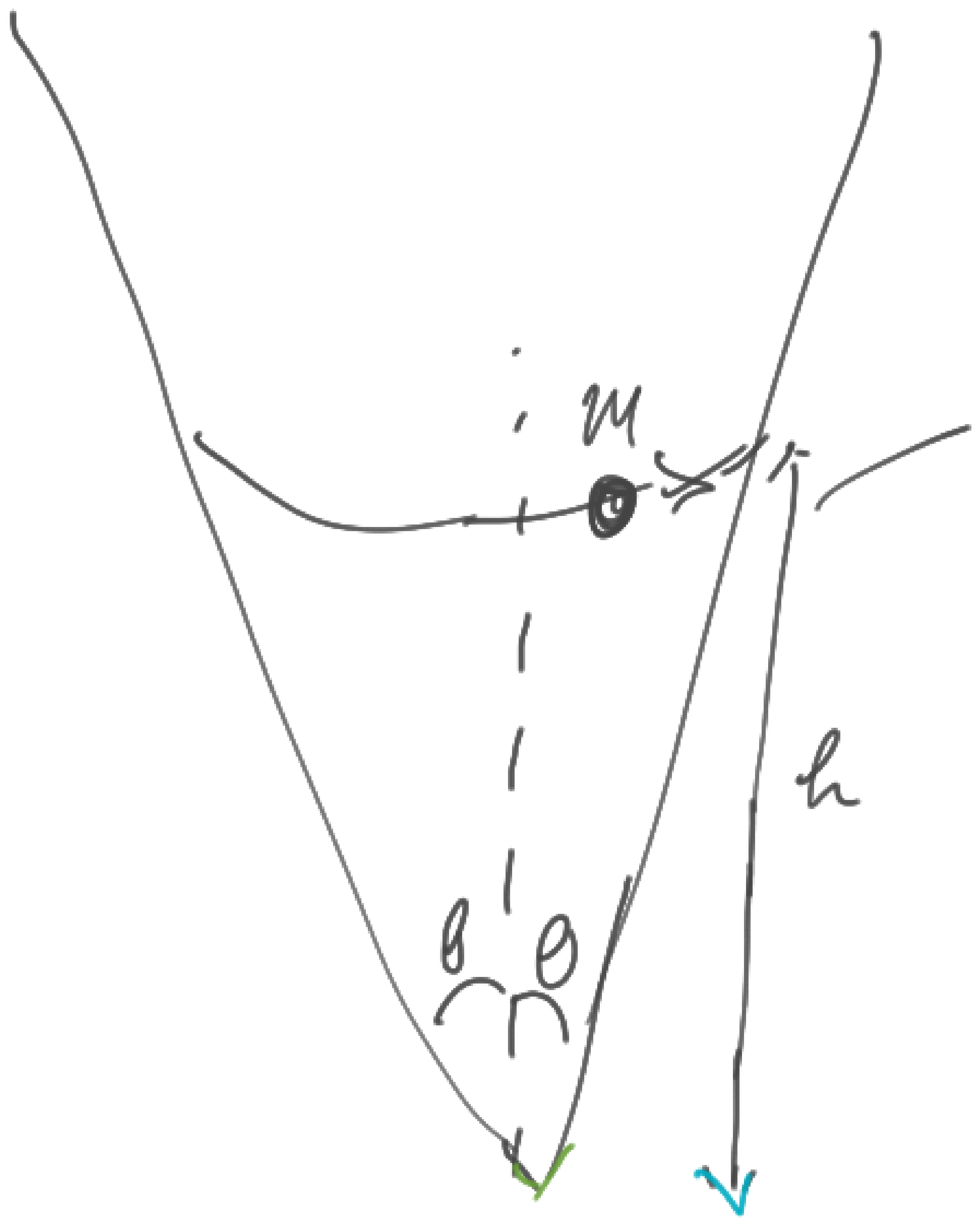
→ Allowing cars to take the corner at higher v

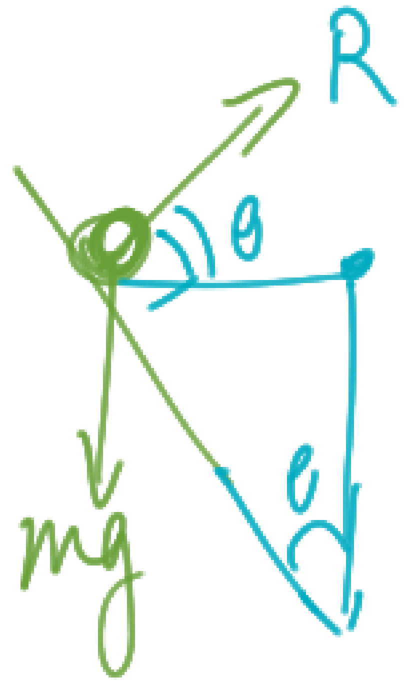
Curved cylinder

Assuming no friction

Given $\theta = 40^\circ$, $m = 0.1\,kg$, $h = 0.25\,m$

Find v_{min} required to sustain circular motion.





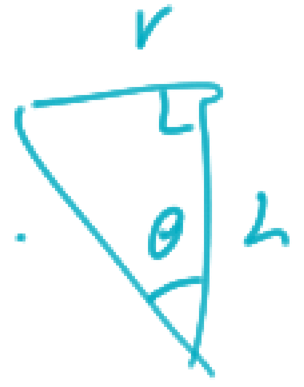
$y: \; mg = R\sin\theta$

$x: \; m\frac{v^2}{r} = R\cos\theta$

$$\frac{gr}{v^2} = \tan\theta$$

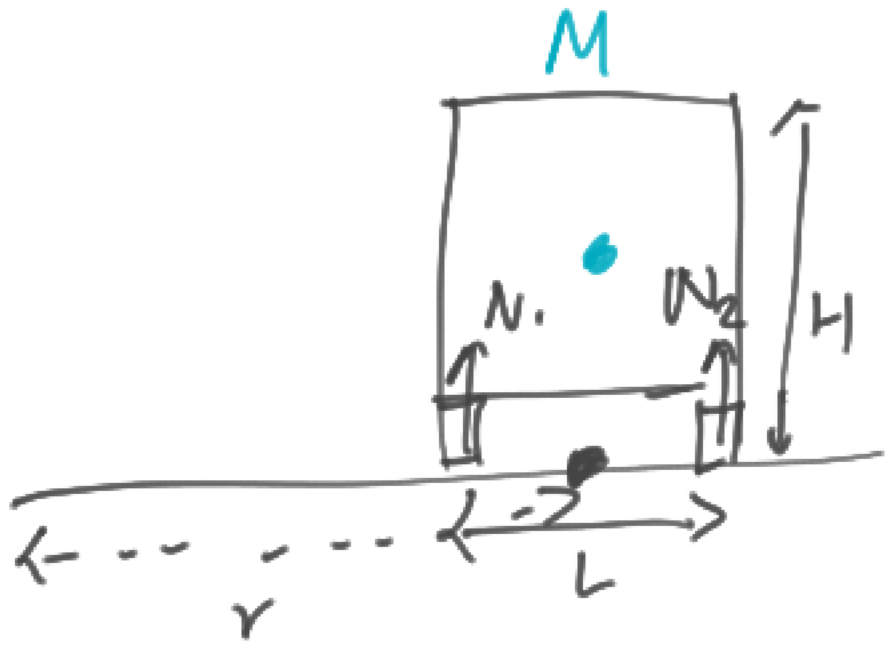
$$v^2 = \frac{gr}{\tan\theta}$$

$$v^2 = gh$$



$$\frac{r}{h} = \tan\theta$$

v required for sustain uniform circular motion



Determine N_1 N_2. for some μ

$$(N_1+N_2)\mu = \frac{Mv^2}{r}$$

$$\left(\frac{Mv^2}{r}\right)\left(\frac{H}{2}\right) + N_1\frac{L}{2} = N_2\frac{L}{2}$$

$$N_1+N_2 = Mg$$

$$N_2 = mg - N_1$$

$$\left(\frac{Mv^2}{r}\right)\left(\frac{H}{2}\right) + N_1 L = mg\frac{L}{2}$$

$$LN_1 = mg\frac{L}{2} - \frac{Mv^2}{r}\frac{H}{2}$$

$$N_1 = M\left(\frac{g}{2} - \frac{v^2H}{2Lr}\right)$$

$$N_1 = \frac{M}{2}\left(g - \frac{v^2H}{LR}\right)$$

$$N_2 = \frac{M}{2}\left(g + \frac{v^2H}{LR}\right)$$

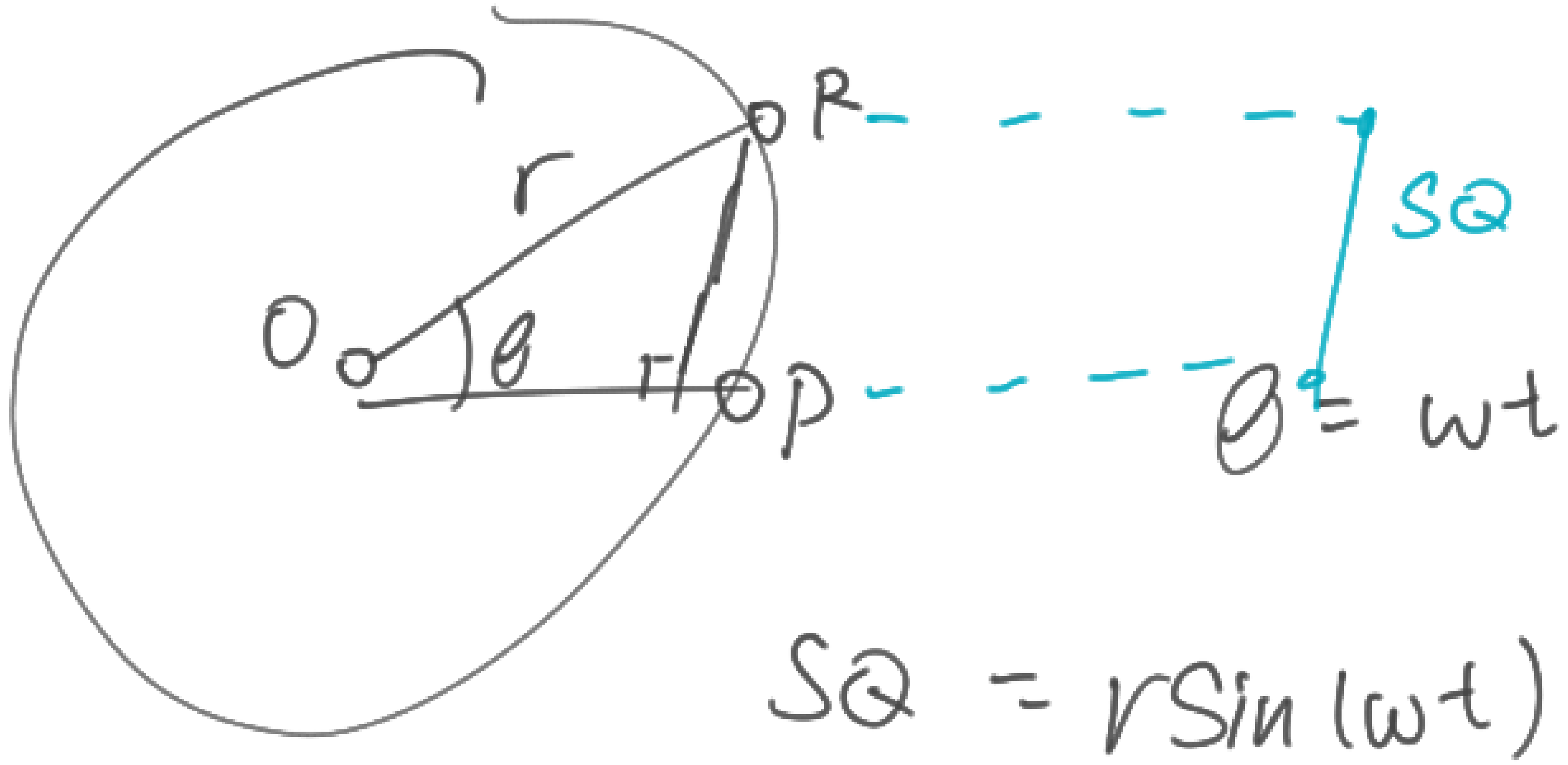
$$N_1 = 0, \quad \cancel{\frac{M}{2}}\left(g - \frac{v^2H}{LR}\right) = 0$$

$$v^2 = \frac{LRg}{H}$$

$$v_{slip}^2 < v_{flip}^2$$

$$\mu g r < \frac{LRg}{H}$$

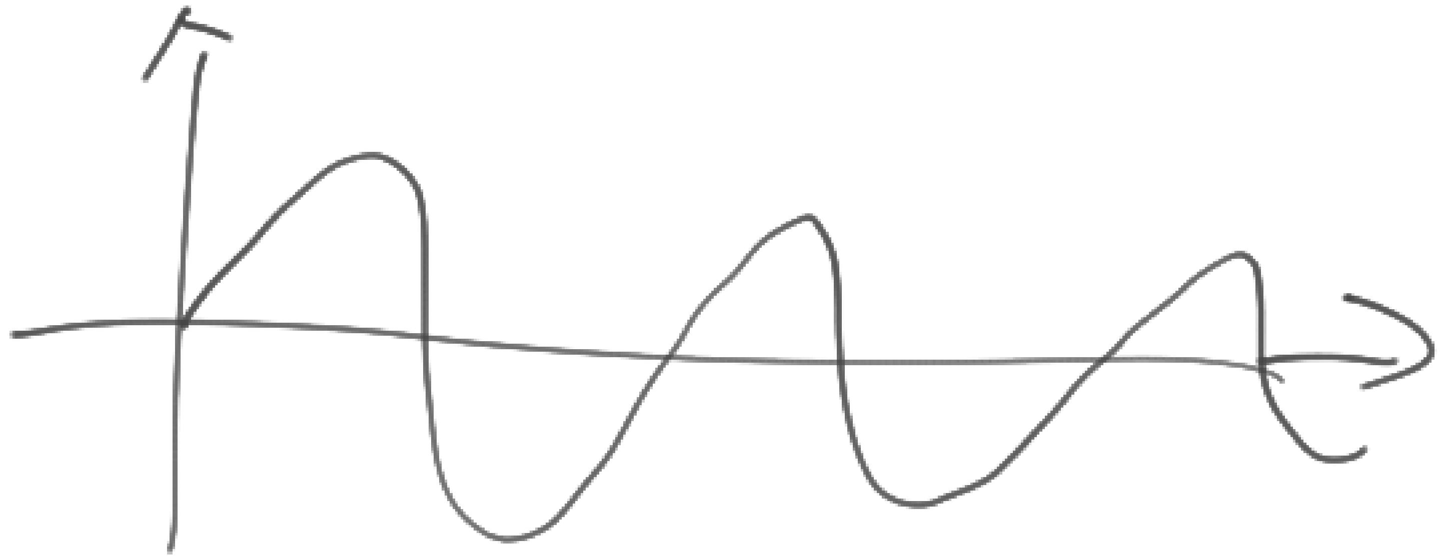
$$\frac{H}{2} < \mu$$



$$SQ = r\sin(\omega t) \longrightarrow -r\omega^2\sin(\omega t)$$

$$a = -\omega^2 x$$

SHM



$$\vec{F} = -k\vec{x}$$

$$a = \frac{F}{m} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

let $x = e^{\lambda t}$

$\vec{a} = \quad -k\vec{x} = \vec{F}$ k is positive ~~posi~~ constant

$\overset{Q}{\circ\rightarrow}$ $\quad$ O $\quad$ $\overset{\vec{a}}{\leftarrow}\circ$

$$\lambda^2 e^{\lambda t} + \frac{k}{m}e^{\lambda t} = 0$$

$$\cancel{e^{\lambda t}}\left(\lambda^2 + \frac{k}{m}\right) = 0$$

$$\lambda = \pm\sqrt{\frac{k}{m}}\,i$$

$x = Ae^{\omega t} + Be^{-\omega t} \longleftarrow$ let $\omega = \pm\sqrt{\frac{k}{m}}\,i = \lambda$

$$\omega^2 = -\frac{k}{m}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$Ae^{i\omega t} + Be^{-i\omega t}$$

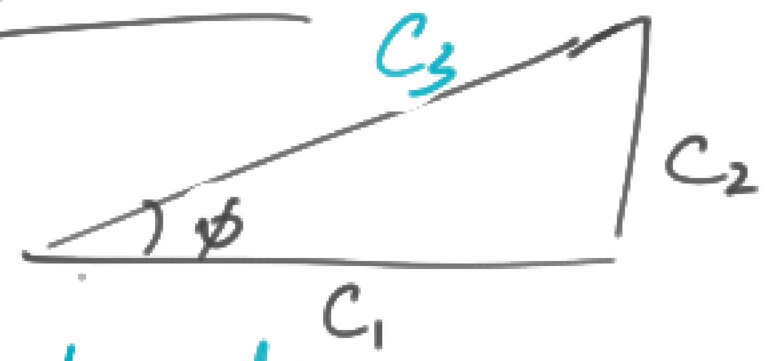
$$= A\cos\omega t + iA\sin\omega t + B\cos\omega t - iB\sin\omega t$$

$$= \underbrace{(A+B)}_{C_1}\cos\omega t + \underbrace{i(A-B)}_{C_2}\sin\omega t$$

C_1 & C_2 are new arbitrary constants.

$$= C_1\cos\omega t + C_2\sin\omega t$$

Consider

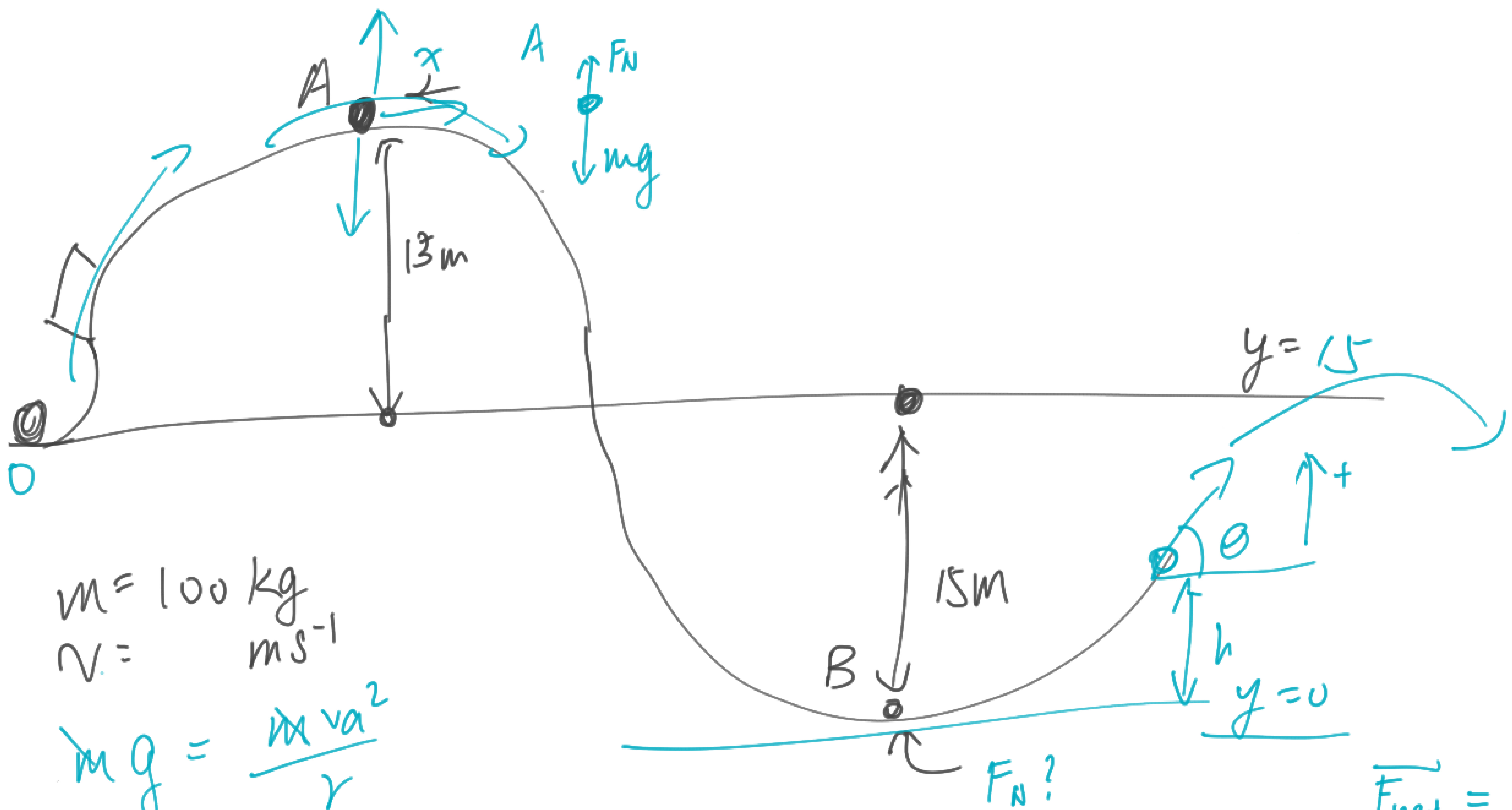


$$= C_3\cos\phi\cos\omega t + k\sin\phi\sin\omega t$$

$$= C_3(\cos\phi\cos\omega t + \sin\phi\sin\omega t)$$

$$x = C_3(\cos(\omega t - \phi))$$

compound angle formula



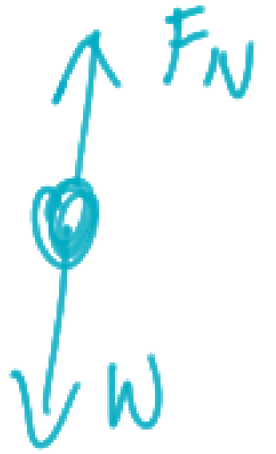
$m = 100\ kg$

$v = \quad ms^{-1}$

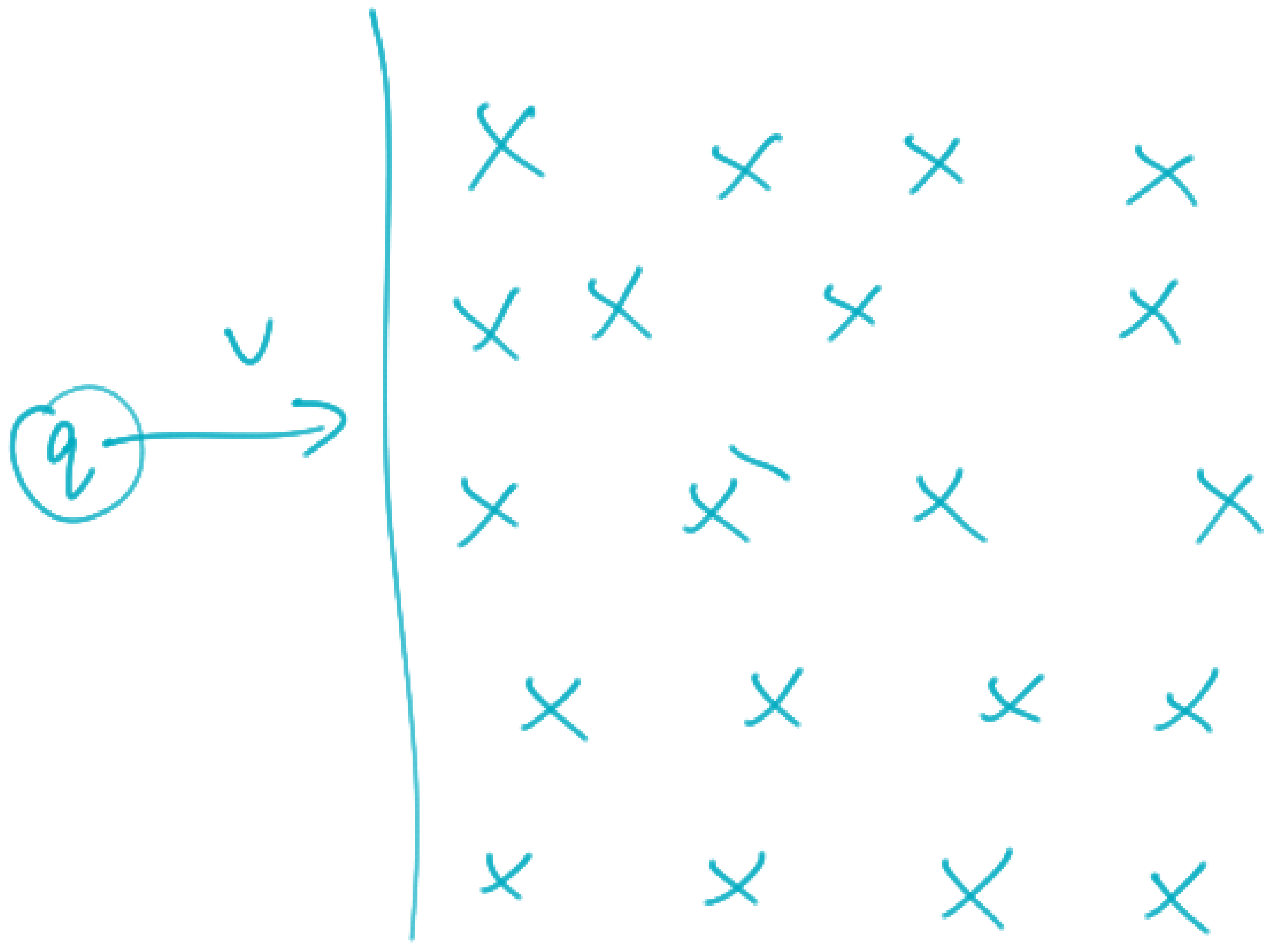
$$\cancel{m} g = \frac{\cancel{m} v_a^2}{r}$$

$$v_a = \sqrt{rg}$$
$$= 12.13\ ms^{-1}$$

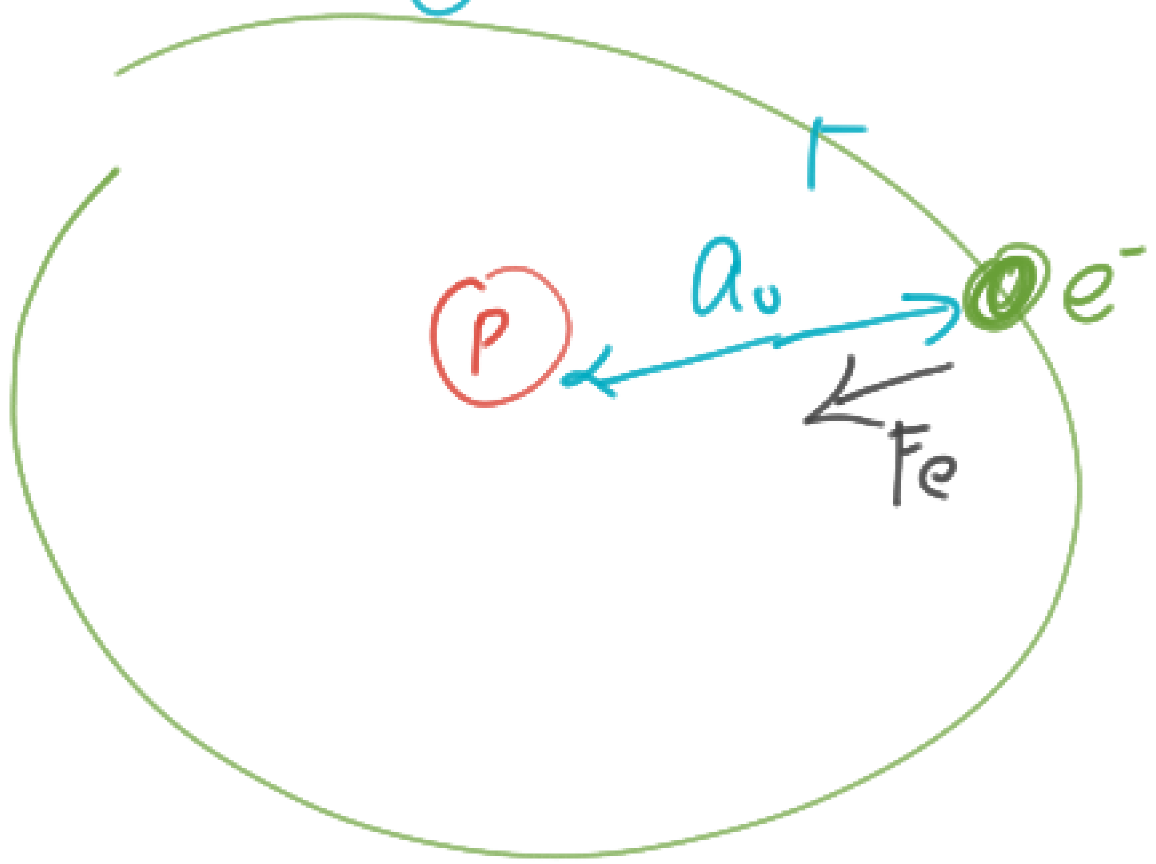
$$mgh + \frac{1}{2} m v_a^2 = \frac{1}{2} m v_b^2$$
$$v_b = 27.12\ ms^{-1}$$



$$\vec{F}_{net} = F_N - W = \frac{m v_b^2}{r}$$
$$F_N = \frac{m v_b^2}{r} + W$$
$$F_N = 5890\ N$$


v
q

Hydrogen atom



$p = +1.602 \cdot 10^{-19}$ C

$e^- = -1.602 \cdot 10^{-19}$ C

$a_0 = 5.30 \cdot 10^{-11}$ m

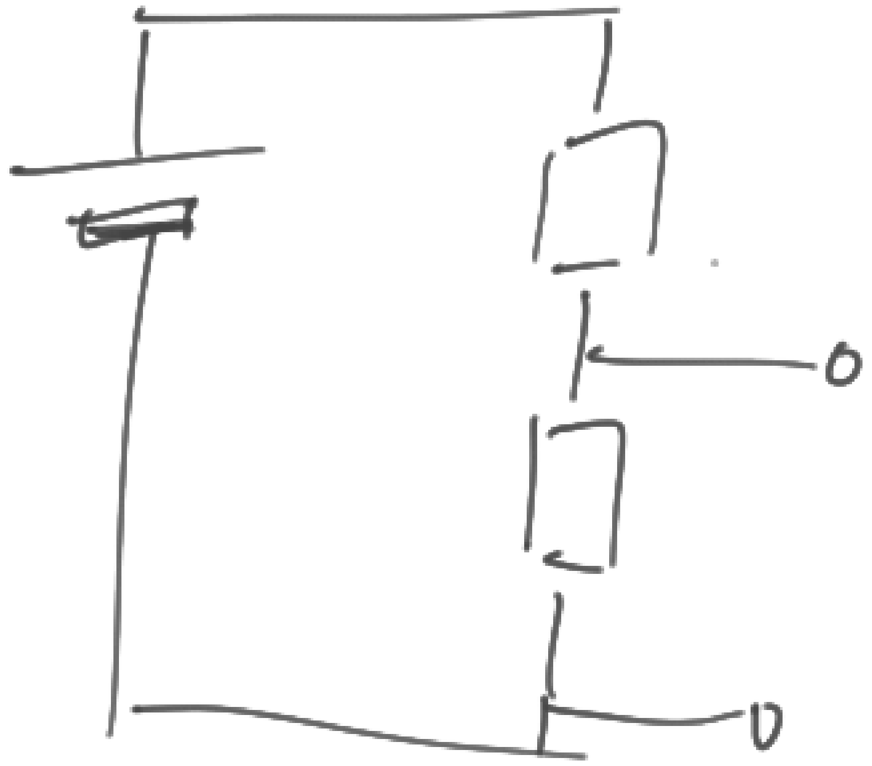
$m_e = 9.11 \cdot 10^{-31}$ kg

what is v of e^-

(Assume p is at rest)

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_e}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q}{\frac{4}{3}\pi R^3}\,\frac{4}{3}\pi r^3\,\frac{1}{\varepsilon_0}$$



$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{|q_1 q_2|}{4\pi\varepsilon_0\, r m}$$

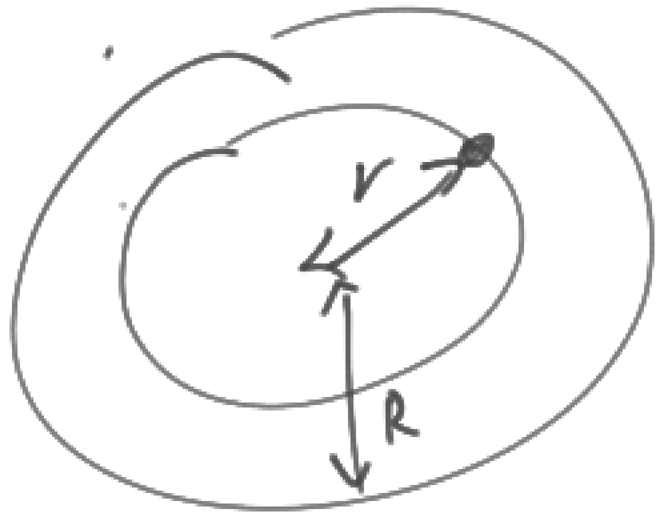
→ capacitor

③ - Potential divider

① - SHM (DAMPING + RESO)

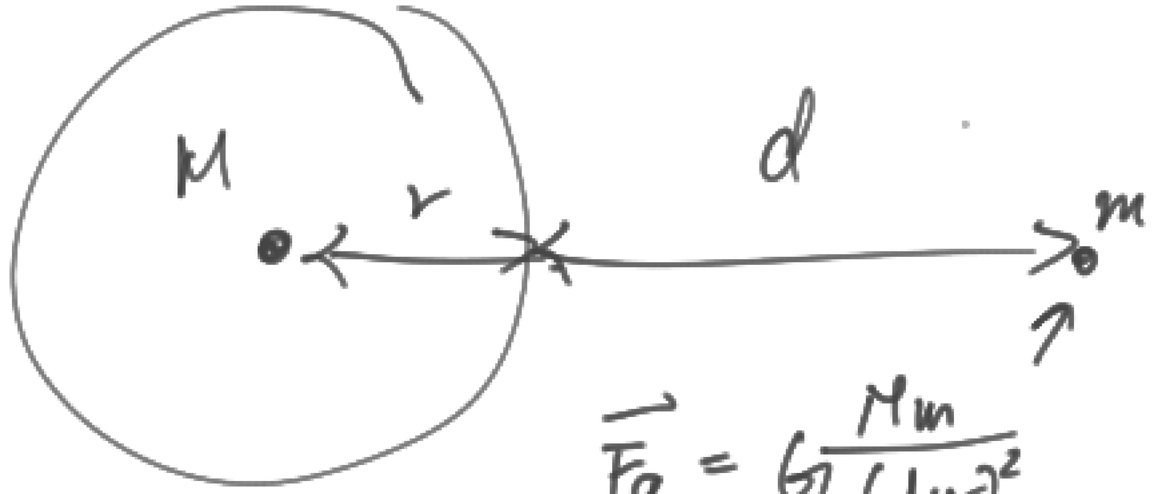
② - $\vec{E}$ + $\vec{G}$

④ - Wave (Double + diff)

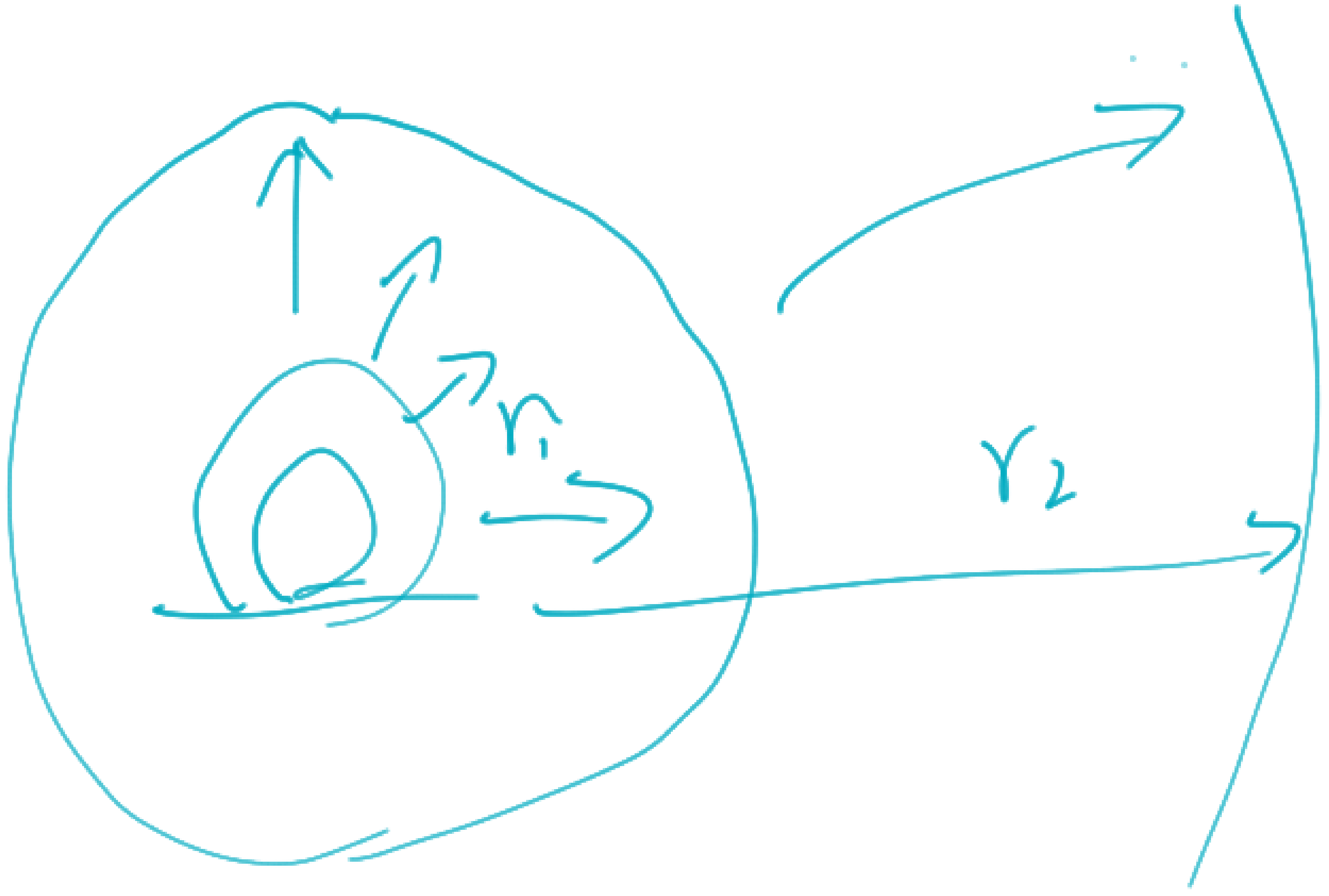


$$\vec{E}_r = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}.$$

$$\vec{G} = GM\frac{r}{R^3}.$$



$$\vec{F_g} = G\frac{Mm}{(d+r)^2}$$



$$B_1 = \frac{L}{4\pi r_1^2}$$

average light per unit area

$$B_2 = \frac{L}{4\pi r_2^2}$$

$$\frac{B_2}{B_1} = \frac{r_1^2}{r_2^2}$$

$$B_2 r_2^2 = B_1 r_1^2 = k$$

$$B \propto \frac{1}{r^2} \qquad B = \frac{k}{r^2}$$