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New insights in the theory of electron cooling

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Abstract

This report discusses the results of a calculation of the cooling force for electron cooling in magnetic fields of 0–4 kG. A computer simulation of the cooling force was found to noticeably differ from the theoretical formula for the absolutely magnetized electron beam. The second subject is the problem of electron cooling of intense ion beams. The nature of "electron heating" and the limits for ion beam intensity with electron cooling are discussed. © 2000 Elsevier Science B.V. All rights reserved.

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1. Main electron cooling features

The electron cooling method was suggested by Budker in the middle of the 1960s [1] on the basis of heat exchange in a two-component plasma: a hot ion and a cold electron beam are moving with the same average velocity. Experiments carried out at the first cooler ring NAP-M [2,3] showed that the magnetic field used to accompany the electron beam on the cooling section plays another significant role besides holding the electron beam together.

The magnetic field "magnetizes" the transverse electron motion, and as a result, the stored particles interact with a cool Larmor circle, and not with a hot free electron [4,5]. The effective temperature of the Larmor circle is only 1 K when the free electrons have temperature over 2000 K! A temperature of 1 K for the longitudinal motion of the stored particles was obtained in a 65 MeV proton beam. The discovered class of phenomena aroused so much interest that the authors specifically called the process "fast electron cooling". The main results on this magnetization cooling were obtained at the "MOSOL" facility with very intensive electron beams and magnetic fields of up to 4 kG [6,7]. These experiments gave a lot of information about the cooling force for intensive electron beams. It is interesting that in these measurements, the value of the cooling force was limited by intrabeam scattering in the intensive electron beam. The magnetic field plays a positive role in suppression of intrabeam scattering in the electron beam too. The experiments show that for high magnetic field it is possible to have higher current in cooling electron beam and more intensive cooling.

The analytical theory for calculation of the cooling force for intensive beams does not yet exist. For

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computer simulation of the cooling process in a real cooler we use the phenomenological description of the force (see below). The last comparisons with results of cooling at the SIS cooler show good agreements with this model [15].

But there are problems to cool intensive ion beams. Some effects, which are limiting the ion currents are observed at the interaction of the ion beam with the electron beam in the electron cooling system. Fast beam loss from intensive ion beams just after injection are observed at the CELSIUS facility if the electron beam is present. This effect was called "electron heating" in a report by Reistad et al. [8]. Similar effects have been observed at the Indiana [9] and COSY [10] rings.

In theory, the interaction of an ion beam with a low-temperature electron beam should result in cooling of all types of motion in the ion beam. But in an electron cooling system, the ion beam comes into the electron beam and then goes out of it after some time. A perturbation in the ion beam creates ion–electron oscillations, which begin when the perturbed part of the ion beam enters the electron beam. These oscillations are discontinued at the moment when the part of the ion beam leaves the electron beam. If the period of electron oscillations in the space charge field of the ion beam is smaller than the time of flight in the cooling section τ very large phase shifts of these oscillations provoke the development of instability.

This phenomenon is similar to beam–beam effects in colliders. The limit on the beam–beam tune shift $\Delta\nu$ is a limit on the phase shift of the electron oscillations in the space charge field of the opposite beam. If this phase advance is too large, the normal beam distribution is destroyed by beam–beam interaction.

2. Cooling force

In the absence of any magnetic field, the force, which affects a proton moving with velocity $\boldsymbol{V}$ in a perfectly cold electron gas is expressed by the well-known formula [1]

$$\boldsymbol{F} = -\frac{4\pi e^4 n_e \ln(\rho_{\max}/\rho_{\min})}{m}\frac{\boldsymbol{V}}{|V|^3} \tag{1}$$

where e, m are the electron charge and mass, n_e is the electron beam density, $\rho_{\max} = V/(\omega_e + 1/\tau)$ is the maximum impact parameter, $\omega_e = \sqrt{4\pi e^2 n_e/m}$ is the electron beam plasma frequency, τ is the time of flight through the cooling section (all in the beam reference system), $\rho_{\min} = e^2/(mV^2)$ is the minimum impact parameter. For a non-zero temperature electron gas, this equation must be folded with the velocity distribution of the electrons.

A calculation for infinite magnetic field, in which the electrons can move only along the magnetic field lines, was made in [4,5] and the results can be written as

$$F_{||} = -\frac{2\pi e^4 n_e \ln(\rho_{\max}/\rho_{\min})}{m}\frac{3V_\perp^2}{|V|^5}\cdot V_{||} \tag{2}$$

$$F_\perp = -\frac{2\pi e^4 n_e \ln(\rho_{\max}/\rho_{\min})}{m}\frac{V_\perp^2 - 2V_{||}^2}{|V|^5}\cdot V_\perp \tag{3}$$

where $V_{||}$, $V_\perp$ are the vector components of $\boldsymbol{V}$ along and perpendicular to the magnetic field, respectively.

As it is easy to see, according to these formulae, the ions that have only longitudinal velocity, i.e. are moving along the magnetic field of the cooler ($V_\perp = 0$), would not be cooled any more. The reason is that this calculation uses an approximation of small change of the electron position during the time of interaction.

The cooling rate is proportional to the energy loss in the electron gas $\mathrm{d}E/\mathrm{d}t = \boldsymbol{F}\cdot\boldsymbol{V}$. The component of the friction force which is perpendicular to the velocity has the same effect as that of a weak magnetic field, which is oriented perpendicular to the velocity. There is an equivalent small coupling of the oscillations in different directions where energy of oscillation is transferred from one direction to the other, but the sum energy is constant. An increase of the transverse oscillation energy corresponds to a decrease of the longitudinal oscillation energy. If there is no magnetic field the displacement of the electrons is symmetric around the line of motion of the ion, and the perpendicular component of the fricion force is zero.

The negative value of the transverse force for $V_\perp < \sqrt{2}V_{||}$ corresponds to a friction force, which exists transverse to the ion velocity for the magnetized electron beam. The loss rate $\boldsymbol{F}\cdot\boldsymbol{V} \sim -(3V_{\perp}^2V_{\|}^2 + (V_{\perp}^2 - 2V_{\|}^2)V_{\perp}^2) \sim -V_{\perp}^2V^2 \sim \sin(\theta)^2$ is negative for any angle θ, corresponding to loss of energy and cooling of the total sum of energy of oscillations.

A useful practical equation for the cooling force is a result of fitting to the experimental data [12]

$$\boldsymbol{F} = -\frac{4e^4n_e}{m}\frac{\boldsymbol{V}}{(\sqrt{V^2 + V_{\mathrm{effe}}^2})^3} \times \ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_{\mathrm{L}}}{\rho_{\min} + \rho_{\mathrm{L}}}\right) \tag{4}$$

where $V_{\mathrm{effe}} = \sqrt{V_{\|\mathrm{e}}^2 + \Delta V_{\perp\mathrm{e}}^2}$ is not the electron velocity V_e, but an effective electron velocity, consisting of the longitudinal electron velocity $V_{\|}$ and the velocity component, which is due to transverse magnetic and electric fields $\Delta V_{\perp e}$, $\rho_{\mathrm{L}} = mV_e/eH$ is the Larmor radius of the transverse motion electrons with velocity V_e in the magnetic field in the cooler H. The logarithm in Eq. (4) is written as it is in order to make it possible to use this equation for a very wide region of parameters. For example, in an intensive electron beam with $\omega_e \gg 1/\tau$ and a low proton velocity, $\rho_{\min}$ becomes larger than $\rho_{\max}$ with the definitions given above, and $\ln(\rho_{\max}/\rho_{\min}) = \ln((V/c)^3/\sqrt{4\pi n_e r_e^3})$ becomes negative, so it cannot be used to calculate the friction force.

But $\ln(\rho_{\max} + \rho_{\min})/\rho_{\min} \approx (V/c)^3/(\sqrt{4\pi n_e r_e^3})$ transfers Eq. (4) for an ideal zero-temperature electron beam ($V_{\mathrm{effe}} = 0$) into a very nice equation for the friction force on ions with small velocity V:

$$\boldsymbol{F} = -2e^2n_e^{2/3}\frac{\boldsymbol{V}}{c\sqrt{\pi n_e^{1/3} r_e}} \tag{5}$$

as we really see at MOSOL experiments. The friction force increases linearly up to the velocity $V \approx c\sqrt{\pi n_e^{1/3} r_e}$, and then drops down as we can see from Eq. (4).

The introduction of ρ_{L} helps to describe the friction force for the case of not too high magnetic field in the cooler and small proton velocity, when $\rho_{\mathrm{L}} > \rho_{\max}$. In this condition there is a region of velocity with $\rho_{\mathrm{L}} > \rho_{\min}$, and the cooling force can be written with $\ln((\rho_{\max} + \rho_{\mathrm{L}})/\rho_{\mathrm{L}}) \approx \tau\omega_{\mathrm{L}} V/V_e$ and so $F \sim 1/(VV_e)$. This is close to NAP-M results [12] in which we really find that the friction force drops down only as $\sim 1/V$ and not as $\sim 1/V^2$. Measurements made in these experiments show that the cooling decrement and the friction force decreased with increasing transverse Larmor motion of the electrons as $1/V_e$. In the model of perfect magnetisation the friction force should not be sensitive to the Larmor motion of the electrons. The author thinks that at NAP-M parameters the radius of the Larmor rotation was close to $\rho_{\max} = V\tau = \theta l_{\mathrm{cooler}} \approx 0.01$ cm by the small cooler length $l_{\mathrm{cooler}} = 1$ m and the small angular spread in the beam $\theta = 10^{-4}$. In this regime of parameters an increase of V_e results in a decrease of the cooling rate $\lambda \sim 1/V^2/V_e$, close to Eq. (4).

For calculation of the cooling force in real magnetic fields of 1–4000 G, a computer code was written. After entering the electron beam the proton moves from the initial to the final position with constant velocity during all the time of interaction in the cooler, $\boldsymbol{x}_p = \boldsymbol{x}_0 p + \boldsymbol{V}t$. The motion of the electron was calculated with the Runge–Kutta procedure. The change of the proton–electron interaction force was calculated and integrated over all the time of interaction in the cooler τ as

$$\delta p = \int\left(\frac{e^2(\boldsymbol{x}_p - \boldsymbol{x}_{0e})}{|\boldsymbol{x}_p - \boldsymbol{x}_{0e}|^3} - \frac{e^2(\boldsymbol{x}_p - \boldsymbol{x}_e(t))}{|\boldsymbol{x}_p - \boldsymbol{x}_e(t)|^3}\right)n_e\,\mathrm{d}v\,\mathrm{d}t, \tag{6}$$

where $n_e\,\mathrm{d}v$ is the number of electrons in the small fraction of the integration volume $\mathrm{d}v$.

The first term, the force if electrons did not move, should be zero after integration over all electrons $n_e\,\mathrm{d}v$, and is used for improving the digital accuracy of the calculation integral.

If we calculate it for not too large initial distance ρ, the time of interaction is really limited to $\tau_i = \rho/V \ll \tau$ and the shift of electrons is $\mathrm{d}x_e \approx e^2\tau_i^2/(m\rho^2) \approx e^2/(mV^2) = \rho_{\min}$. The δp from this region ρ is easy to see: $\delta p \approx e^2\,\mathrm{d}x_e/\rho^3\tau_i$ and the friction force from this region $\rho_{\min} < \rho < \rho_{\max}$ is equal to:

$$F = \int\frac{\mathrm{d}p}{\tau_i}n_e 4\pi\rho^2\,\mathrm{d}\rho = \frac{4\pi e^4 n_e}{mV^2}\ln(\rho_{\max}/\rho_{\min}). \tag{7}$$

For large ρ, when $\rho > \rho_{\max}$, the time of interaction is limited to the time of flight through the cooling section τ, and $dx_e \approx e^2\tau^2/(m\rho^2) \approx \rho_{min}(\rho_{max}/\rho)^2$; this decreases the region of the friction force and helps to stop the integration when $\rho \gg \rho_{max}$ without loss of accuracy.

The calculations were made on computer for the longitudinal and transverse components of the friction force, and results are shown in Figs. 1 and 2. The curves $B = 0$ show the results of calculation by Eq. (1). The curves marked $B =$ *infinite* show the results of calculation of the longitudinal component (Eq. (2)) in Fig. 1 and transverse component (Eq. (3)) in Fig. 2. The line marked *fit* shows the results of calculation with Eq. (4) for a magnetic field of 4 kG.

As it is easy to see, the numerical calculation of the longitudinal force for a proton moving along the magnetic field (Fig. 1, $\theta = 0$) is asymptotically close to that, which is calculated by Eq. (4), and not zero as predicted by the pure magnetized model. In the transverse direction, the value of the friction force for a proton, which is moving perpendicularly to the field ($\theta = \pi/2$), decreases with an increasing magnetic field more than a factor two (two is the ratio between Eqs. (1) and (3) for $V_{||} = 0$ and is again close to the estimation made with Eq. (4)).

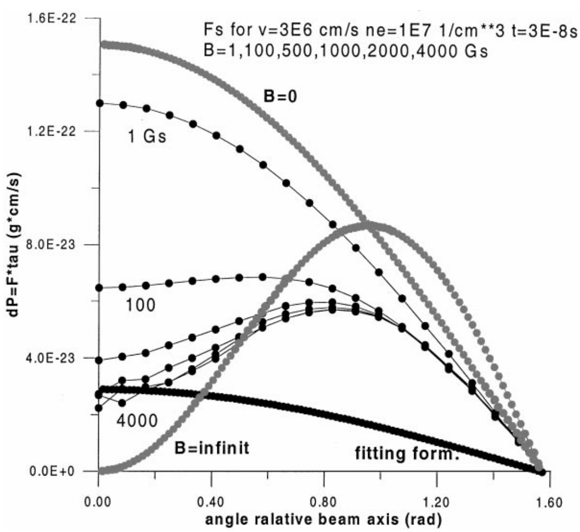

Fig. 1. The longitudinal component of the cooling force $F_{||}$ vs. angle of proton $\sin(\theta) = V_\perp/V$ for different values of the magnetic field.

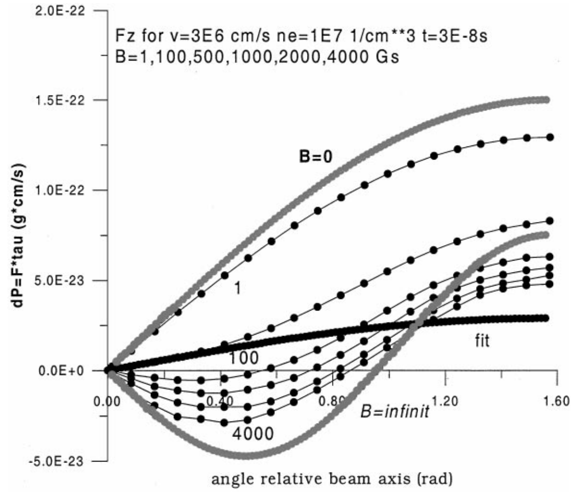

Fig. 2. The transverse component of the cooling force $F_\perp$ vs. angle of proton for different values of the magnetic field.

According to the computer simulation, the dependence of the angle of motion relative to the magnetic field θ on the energy loss is very weak; this is close to Eq. (4) where $(\boldsymbol{F} \cdot \boldsymbol{V})(\theta) = \text{const}$.

This calculation was made for a zero-temperature electron beam, and it appears as if an increase of the magnetic field decreases the cooling force. In real cooling experiments, the magnetic field suppresses the fast free motion of the electrons and an increase in the field helps to enhance cooling.

Fitting formula (4) can be recommended for use in computer simulations of cooling when V changes from a large value for the hot initial beam to a very small value near the equilibrium stage.

Let us demonstrate the use of the effective velocity V_{effe} in Eq. (4). If there are no other sources of transverse fields in the cooler except for the space charge of the electron beam, the value of the effective velocity is determined by the transverse drift in the magnetic field of the cooler H: $V_{effe} = c(2\pi e n_e a)/H$ (a is the electron beam radius). As it is easy to see from Eq. (4), an increase in the electron beam density, which is useful for cooling, is limited by the condition $V_{effe} \approx V$, and an optimum of the cooling beam density can be estimated as $n_e = \beta\gamma^2 H/(2\pi e \beta_\perp)$ (where $\beta_\perp$ is the beta function at the cooler, $a = \sqrt{\varepsilon\beta_\perp}$, $V = \beta\gamma c\sqrt{\varepsilon/\beta_\perp}$). For the electron beam current density it looks like (in practical units):

$$j \approx 16(\beta\gamma)^2 H(\mathrm{kG})/\beta_\perp(\mathrm{m})\ (\mathrm{A/cm}^2).$$

For the SIS cooler, as an example, (11 MeV/u, $\beta = 0.15$, $\beta_\perp = 10\,\mathrm{m}$, $H = 0.6\,\mathrm{kG}$) it gives $j \approx 0.02\ \mathrm{A/cm}^2$, very close to the really measured optimum current.

3. Experimental limitation on the ion beam intensity

Injection of intensive proton beams into CELSIUS, COSY and the Indiana Cooler is associated with beam loss if the electron beam is present. Usually, just after cooling, the rest current was only a small fraction of a milliampere. All these cooler rings inject 40–50 MeV protons, and have a lifetime for a 2–5 mA proton beam of only a few seconds with cooling. Without electron cooling, the lifetime is 200–400 s, which is close to the estimate for multiple scattering on the residual gas in the vacuum chamber.

After injection and cooling of a very weak proton beam current in CELSIUS, the lifetime becomes limited only by single scattering on the residual gas, and is increased several times $\sim \ln(1/\theta_{\mathrm{maxaperture}})$. For example, the lifetime of the proton beam with < 0.1 mA current at CELSIUS is about 1000 s.

It was a popular idea that these effects are related to the bad cooling of particles with large amplitudes, for which the non-linear forces (for example, the electric field outside of the electron beam) do not permit to cool the particles. The oscillation amplitudes of these particles increase and they are lost from the beam. According to this hypothesis, cooling should only work for amplitudes, which are smaller than $A_\mathrm{c} = \sqrt{J_{\mathrm{cool}}/J_{\mathrm{inject}}}A_{\max} \approx \sqrt{(0.1\ \mathrm{mA})/(5\ \mathrm{mA})}A_{\max} = 0.14A_{\max}$ and the lifetime of cooled beams should be only $(A_\mathrm{c}/A_{\max})^2\tau_{\mathrm{single}} = 0.02\ln(1/\theta_{\max})\tau_{\mathrm{nocooling}} \sim 0.1\tau_{\mathrm{nocooling}} \sim 20$–$40$ s. But the good lifetime of cooled beams shows absolutely clearly that the whole aperture is free for cooling ($A_\mathrm{c}/A_{\max} \sim 1$).

It seems that coherent interaction of the intensive ion beams with the electron beam is a more realistic explanation of these phenomena. The experiments of Reistad et al. with detuned cooling, in which the electron beam energy was detuned away from the correct energy for cooling, have shown that the losses take place without cooling too. It means that the simple idea that after cooling the ion beam has very low momentum spread resulting in the development of coherent instability does not explain the observed beam loss, and only the presence of the electron beam on the proton beam orbit is sufficient to create the problem.

An analysis of the impedances connected with the electron beam was made by Burov in Ref. [14]. According to his analysis "normally, the coherent ion–electron interaction does not deteriorate the ion beam parameters". This is in contradiction to the results of the present report. The main reason for the mistake consists in the use of approximation of small perturbation, which is not relevant for the motion of high-frequency electrons. When the phase of the electron oscillations for the time of interaction becomes too large, it requires more careful calculation of the dynamic interaction.

4. Theory of the electron heating

Usually, to study coherent interactions between an ion beam and some object (for example a cavity) the model of two oscillators is very useful. One of the oscillators is the ion beam and the other one is the cavity. The beam passes the cavity periodically with the revolution frequency and excites fields acting back on the beam. The standard way to study this interaction is to use in equations only resonance terms, and to study the slowly changing amplitudes after averaging. If the time of amplitude increase is smaller than the Landau damping time in the ion beam, this system is unstable!

The electron cooling rate is proportional to the density of the electron beam n_e and has a factor Z^2/A, where Z is the charge and A the mass of the ions. When studying coherent cooling we should take a sample of the ion beam with volume V_s. Inside this sample we have $N_\mathrm{s} = n_\mathrm{i}V_\mathrm{s}$ ions. The formal cooling rate for this sample increases as N_s and for an ion beam of high intensity it becomes too large [11,13].

From this point of view, an electron beam has a resonance frequency but in a cooling system it interacts with an ion only once. In its next turn the ion beam interacts with a fresh electron beam. For the analysis of coherent motion in this system the matrix technique can be used.

Outside the cooling section, the equation of motion for plasma oscillations of the electron beam can be written in the form (in the system of beams)

$$\frac{d^2x_e}{dt^2} = -\frac{4\pi e^2 n_e}{m}x_e = -\omega_e^2 x_e \tag{8}$$

and for the ion beam:

$$\frac{d^2x_i}{dt^2} = -\frac{4\pi e^2 n_i}{M}x_i = -\omega_i^2 x_i \tag{9}$$

where ω_e and ω_i are the plasma oscillation frequencies for the electron and ion beams, respectively. (The definition of plasma frequency depends on the form shape of oscillations in the ion cloud. For one-dimensional flat oscillations $\omega_e = c\sqrt{4\pi n_e r_e}$, but for the axially symmetric round beams $\omega_e = c\sqrt{(2\pi n_e r_e)}$.) In the cooling section, where the beams move along a common orbit, the electric fields of plasma oscillations act on both beams, and the equations can be written as:

$$\frac{d^2x_e}{dt^2} = -\frac{4\pi e^2(n_e x_e - n_i x_i)}{m} \tag{10}$$

$$\frac{d^2x_i}{dt^2} = -\frac{4\pi e^2(n_i x_i - n_e x_e)}{M}. \tag{11}$$

For the electric field $E = 4\pi e(n_e x_e - n_i x_i)$ it is easy to see that the oscillation equation has a simple form

$$\frac{d^2E}{dt^2} = -(\omega_e^2 + \omega_i^2)E. \tag{12}$$

It is an equation for plasma oscillations with a frequency $\omega_p = \sqrt{\omega_e^2 + \omega_i^2}$.

At the entrance of the cooling section the ion beam has some coordinates $x_i(0)$ and initial velocity $dx_i(0)/dt$ but the fresh electron beam has $x_e(0) = 0$, $dx_e(0)/dt = 0$.

The self-consistent solution of this equation can be written in the form

$$\begin{pmatrix} x_i \\ dx_i/dt \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} x_i \\ dx_i/dt \end{pmatrix}_0 \tag{13}$$

where the matrix elements are calculated by integration of equations along the cooling section using specific initial conditions. For the initial conditions $x_i(0) = 1$, $dx_i/dt(0) = 0$ we calculate elements A_{11} and A_{21} but for initial conditions $x_i = 0$, $dx_i/dt = 1$ we calculate matrix elements A_{12} and A_{22}

$$\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} = \begin{pmatrix} x_p(\tau) \\ dx_p/dt(\tau) \end{pmatrix} \tag{14}$$

where τ is the time of interaction in the cooling section.

We should remember that the determinant (det) of this matrix is not equal to 1. This system is not closed, and the electron beam can either absorb or increase the plasma oscillation energy of the ion beam. If det < 1 it means the loss of this energy and cooling, but for det > 1 the energy increases and we have heating.

Fig. 3 shows the variations of det versus the electron beam density written in units $\omega_e\tau = c\tau\sqrt{2\pi n_e r_e}$ for two cases of the proton beam density $n_i = 10^7$ and 10^8 cm^{-3} ($\tau = 4\times10^{-8}$ s).

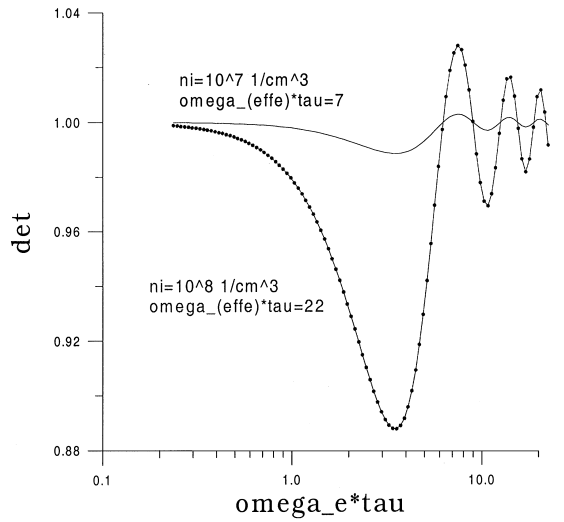

Fig. 3. Variation of matrix determinant vs. electron beam density for different densities of proton beam.

As it is easy to see from Fig. 3, for small electron beam density we have det < 1, which corresponds to fast cooling of plasma oscillations in the proton beam but for large density $\omega_e \tau > 2\pi$ we have det > 1 and fast heating of coherent oscillations. The reason is that the interaction is too strong, and at the moment of interruption, the electron cloud moves in the direction opposite to the ion cloud motion.

Fig. 4 shows the variation of the determinant of this matrix vs. the ion beam density written in units $\omega_{\mathrm{effe}}\tau = \sqrt{M/m}\omega_i \tau = c\tau\sqrt{2\pi n_i r_e}$ for two cases of electron beam density $n_e = 10^7$ and 10^8 cm^{-3}. For small ion beam density we can see that det ~ 1 but when the frequency of electron oscillations in the space charge of an ion beam becomes larger than $\omega_{\mathrm{effe}} > 2\pi/\tau$ we have a rapidly increasing det.

As it is easy to see from Figs. 3 and 4, the region for cooling is limited (det < 1) by conditions for the frequency of electron beam oscillations in the fields of the space charge of the electron and ion beams:

$$\omega_e = c\sqrt{2\pi n_e r_e} < \frac{2\pi}{\tau} \tag{15}$$

$$\omega_{\mathrm{effe}} = c\sqrt{2\pi n_i r_e} < \frac{2\pi}{\tau}. \tag{16}$$

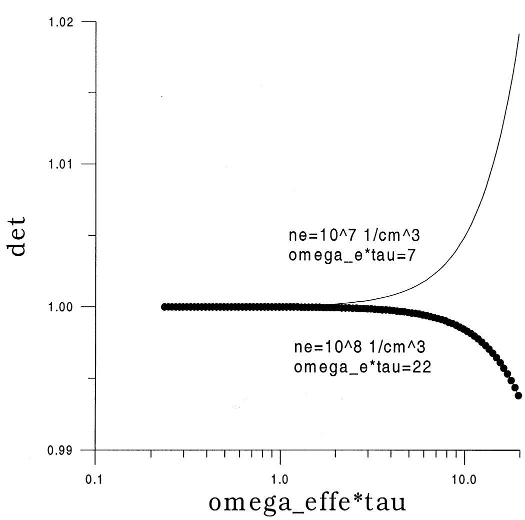

Fig. 4. Variation of matrix determinant vs. ion beam density for different densities of electron beam.

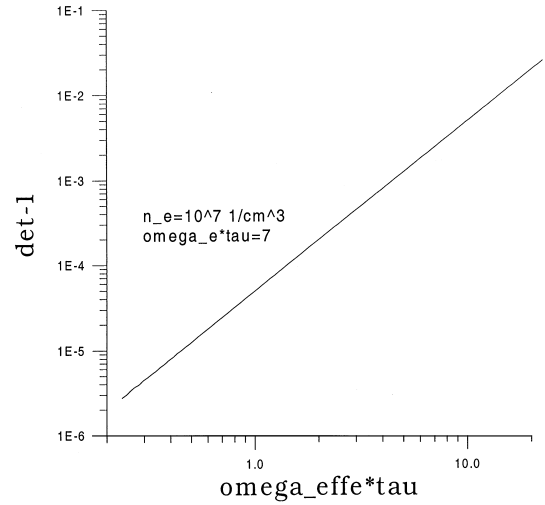

Fig. 5. Variation of heating rate vs. ion beam density for electron beam density 10^7 cm^{-3}, $\omega_e \tau = 7$.

These conditions put limits for the maximum density of the electron and ion beams in order to avoid heating.

For high electron current, $\omega_e \tau > 2\pi$, the problem of electron heating exists also for very small ion currents as it is easy to see from Fig. 5. When $\omega_e \tau > 2\pi$ the determinant is greater than 1 for any ion current. But for the beam emittance to grow, the heating rate ((det $-$ 1)f_0) must be higher than the cooling rate given by the Landau damping – the mixing of the ion position by the thermal motion. Fig. 5 shows that for low ion density the heating rate is very small. For example, at $\omega_{\mathrm{effe}}\tau \sim 1$, the heating time corresponds to about 10^5 beam turns and if we have a momentum spread sufficient for fast decay of fluctuations, the heating does not lead to emittance growth.

5. Longitudinal wake field in the cooler

Outside the cooling section, there is an ion beam fluctuation, which propagates along and opposite to the motion of the beam as a wave with phase velocity [16]:

$$V_{\mathrm{iw}} = c\sqrt{\pi a_i^2 n_i r_i (1 + 2\ln(Ap/a_i))} \tag{17}$$

where a_i is ion beam radius, $r_i = (Ze)^2/AM_pc^2$ is the classical radius of the ion, Ap is the aperture of the vacuum chamber, n_i is the density of the ion beam. If the velocity of the thermal motion of the ions after cooling is small ($V \ll V_{iw}$) we can see this wave in Schottky noise spectra as two peaks with frequency $n(V_0 \pm V_{iw})/R_0$ (n is the harmonic number, V_0 is the velocity of the beam, R_0 is the radius of the ring). But inside the electron beam the velocity of this electrostatic wave increases by many orders of magnitude:

$$V_{ew} = c\sqrt{\pi a_e^2 n_e r_e (1 + 2\ln(Ap/a_e))} \tag{18}$$

because the electron current $J_e = e\pi a_e^2 n_e V_0$ is very high and electrons are very light, $r_e = r_i 1836 A/Z^2$.

Fig. 6 shows the initial distribution of the electric field of the ion beam fluctuation and the electric field after the time of motion in the cooler $\tau = l_{cooler}/V_0$. As it is easy to see, the electric field of the fluctuation is compensated by shifting electrons, and the electric field splits into two waves acting on ions, that are far away from the real fluctuation (the shift of position of action from the initial point is $V_{ew}\tau$).

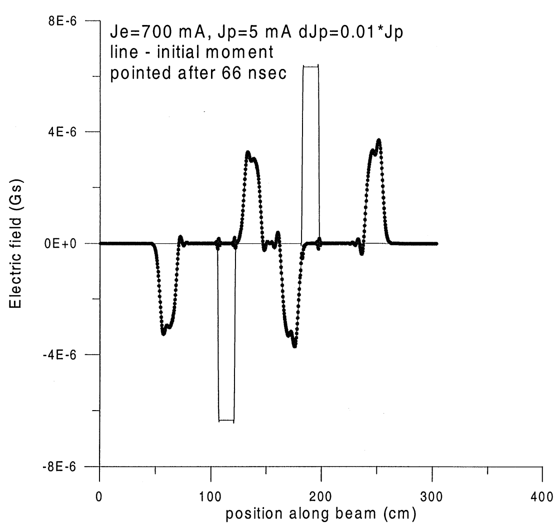

Fig. 6. The electric field distribution along the ion beam at the initial moment and after some time inside the cooler (66 ns). The rectangular lines show the electric field of the ion beam wave having $\Delta n_e = 0.01 n_e$ at points from 100 cm to 200 cm and propagating with the ion wave velocity. The pointed curve shows the electric field distribution after which waves have been excited in the electron beam.

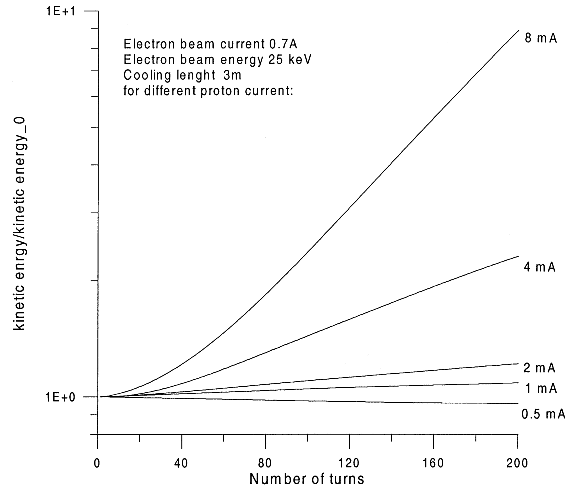

Fig. 7. Change of the kinetic energy of motion of ions inside a proton bunch after passing the cooling section for different proton beam currents.

This type of wake, when fields act on neighboring ions (but not on themselves) is well-known and produces instability if the increment of instability is larger than the Landau damping decrement. Fig. 7 shows the change of kinetic energy in a proton bunch for different currents while cooling with parameters close to those of CELSIUS. As it is easy to see, for currents less then 1 mA we have cooling but for high proton beam currents we have fast heating. As it is easy to see from this figure it is a hard problem for the Landau damping to suppress heating with time scale a few hundred turns in the ring.

6. Conclusions

A numerical simulation of the friction force for real magnetic fields shows that the analytical equation for the case of pure magnetization has a large discrepancy and for practical use it should be modified. An empirical equation shows better coincidence with a computer simulation. For calculation of the equilibrium of intensive ion beams it seems reasonable to use this equation for the friction force in a large range of parameters.

The electron heating results in very strong effect of the ion beam space charge field on the electron beam. For the phase shift of plasma oscillations of electrons at the time of flight through the cooling section corresponding to $\omega_e \tau > 1$ there is usually the development of electron heating instead of cooling. The picture of this phenomenon is similar to that of beam–beam interaction in colliders when the beam–beam tune shift is too high. It is possible to achieve an increase of the cooled beam intensity by decreasing this interaction by a good choice of the ion beam magnet system and by decreasing the cooler length.

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