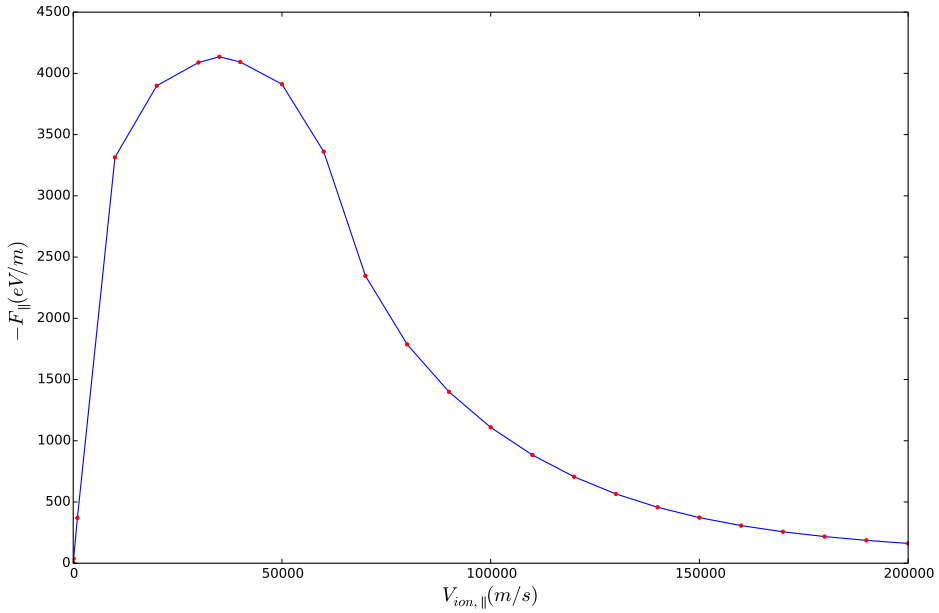
We have obtained some encouraging preliminary results from exploring our new reduced model for the longitudinal magnetized dynamical friction force. Figure .... shows the (minus) parallel friction force as a function of the ion velocity for the case of the ion moving parallel to the field lines (with zero transverse velocity) through a cold electron distribution in a strong magnetic field. In particular, we found a linear dependence on velocity at small ion velocity values, and we also saw a Z^2 dependence of the force on the ion charge in the few tests that we have done. We now describe the parameter regime in which we have been testing the model and the model itself in slightly more detail.



The parameter regime is that described in A.V. Fedotov, D.L.Bruhwiler, and A.O. Sidorin, Proc. of HB 2006, p. 210, WEAY04 (2006) and A.V. Fedotov et al., Phys. Rev. ST Accel. Beams 9, 074401 (2006). It was at one point considered for the RHIC upgrade, and is representative of the parameter regimes explored for the evolving EIC design. The simulations assume a locally-constant particle number density n_e = 2e15 m^{-3} in the beam frame, the (strong) magnetic field B = 5 T, and the electron longitudinal and transverse thermal velocity spread of 1.0e5 m/s and 4.2e5 m/s, respectively. Therefore, the "typical" gyration radius r_L is ~ 4.2e-8 m, and the expectation value of distance r_1 from the ion to the nearest electron at t = 0 is ~ 4.9e-6 m, or r_1 ~ 120 r_L, for this choice of parameters. The ion is a fully stripped gold ion (Z = 79). As regards some of the relevant timescales, the cooler traversal (i.e., interaction) time is T_{inter} = 4.0e-10 s, which is ~ 640 gyration periods, and ~ 0.16 of the plasma period. The calculation is done in the beam frame, where both the ion and electron velocities are non-relativistic.

The model we are developing takes advantage of the Larmor radius being small compared to the distance between the ion and the electrons, and follows the motion of only the guiding center of the electrons' trajectories. It is well known that in constant and uniform electric and magnetic fields (not parallel to each other) the guiding center of a charged particle trajectory undergoes a

drift in the configuration subspace orthogonal to **B** with constant velocity $\mathbf{v}_d$ = **E** x **B** / $|B^2|$ (in MKS). One would expect then the guiding center of an electron trajectory moving in a (weak) Coulomb field of the ion and a strong magnetic field to move in such a way that the projection of the motion onto a plane orthogonal to **B** is a superposition of a fast, small-radius gyration and a slow, large-radius precession of the guiding center along a circle centered on the ion. A formal canonical perturbation theory analysis that we performed (essentially using 1/B as the small parameter) confirms that this is the case at least in the first order in the perturbation. This means that, to the first order at least, if we model the electron motion by tracking the guiding center of its orbit, the impact parameter D of the electron during its interaction with the ion remains constant (although the precession frequency varies depending on the time-dependent distance from the ion). In other words, the longitudinal (along B) motion of each electron takes place in an effective 1D potential:

$$\ddot{z}(t) = -\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{z}{(D^2+z^2)^{3/2}}$$

where the z axis is aligned with the magnetic field lines, the ion is at the origin of the coordinate system, and the impact parameter D is, of course, different for different electrons.

It is convenient to study the motion in a reference frame where the ion is at the coordinate system origin and at rest. (For the interaction time and strength in this parameter regime, one can assume that the ion does not appreciably move in response to the net force from the electrons, and only the net momentum change during the interaction time needs to be computed by adding up the contributions from individual electrons.) For the case of the ion moving with a (longitudinal) velocity V_i through a gas of cold electrons, one then initializes the electron distribution such that all electrons have the same initial velocity $-V_i$, with the ion having zero velocity. Depending on the given electron's energy $E = 1/2(dz/dt)^2 + U(z)$, the motion can be bounded (finite) or unbounded (infinite). Back-of-the-envelope estimates with Z = 79 and the electron longitudinal temperature and density for our test case indicate that a non-negligible fraction of the electrons can be moving on bound orbits in the effective 1D potential. The potential exhibits a weak nonlinearity, i.e., the period of oscillation is a monotonically increasing function of amplitude. For a given value of D, the shortest period is for the near-zero amplitudes, where the linearized oscillator equation yields

$$T_{lin} = \frac{2\pi}{c}\sqrt{\frac{D^3}{Zr_e}}$$

(where r_e is the classical electron radius); that is, T_{lin} ~ 4.9e-10 s, if we take the expected distance to the nearest electron for D, or $T_{lin} \sim 1.2 * T_{inter}$ in terms of the the interaction time. Although we can write the expressions for the individual trajectories in quadratures or in terms of elliptic functions (and perhaps use some asymptotic approximations of those), the Coulomb

force being a nonlinear function of coordinates, a closed-form analytic expression for the force on the ion averaged over the electron phase space distribution is not easy to produce. While we intend explore the analytic approach further, the oscillatory orbits at smaller values of E introduce a certain amount of phase mixing, the combined effect of which on the net force from the electrons on the ion we explored via simulation.

While a careful analysis of the statistics of effects of close electron-ion encounters is clearly important, initially we focus on computing the expectation value of the friction force. With this in mind, we use a kind of a deterministic quiet start technique for initializing the electron positions (as mentioned, all electrons have the same initial velocity $-V_i$ in this example). For each electron macroparticle, we specify the distance from the ion as well as the initial z position (fixing thereby the value of D), as follows. Assuming a locally uniform and isotropic electron number density, the expectation value of distance from the ion to the k^{th} nearest electron is $r_k = (3k / 4 \pi n_e)^{1/3}$. (This can be found by analogy with event occurrence times in a constant-rate Poisson process, the volume of a sphere centered on the ion playing the role of "time" in our case.) We place an equal number n of macroparticles of charge e/n and mass m_e/n at each r_k, up to some maximum number k_{max}. The z coordinates of particles in each n-tuple are chosen so as to match the lowest z-moments of the uniform (continuous) density distribution on a thin spherical shell of radius r_k up to the order n for n odd and n+1 for n even (by symmetry). We saw a reasonable degree of convergence in the n-tuple averaged result already in going from n = 4 to n = 5 (to within 4%); the z positions for n=5 are given by

$$z_0^{(k)} = 0, \quad \frac{z_{1,-1}^{(k)}}{r_k} = \pm\sqrt{\frac{5-\sqrt{11}}{12}}, \quad \frac{z_{2,-2}^{(k)}}{r_k} = \pm\sqrt{\frac{5+\sqrt{11}}{12}} \, .$$

Because the dynamics is effectively 1D, the azimuthal angle is irrelevant and is not specified or used. We plan to cross-verify in the near future the results produced with this quiet-start procedure with the results produced by randomly sampling the initial configuration space.

The initial conditions of the electrons are evolved (e.g.) with a 2^{nd} order symplectic integrator and the force on the ion is computed, electron-by-electron, as a function of time. Before computing the ensemble average of the longitudinal force on the ion, it is important to subtract, particle-by-particle, the longitudinal force on the ion that would come from the same electron moving on the unperturbed (by the ion), constant-velocity trajectory. This negates the purely numerical bulk force on the ion whose origin is in the finite extent of the distribution of macroparticles used in the simulation. Ensemble average then gives the total longitudinal dynamical friction force on the ion as a function of time, and the time-average over the interaction time is what is plotted (with reversed sign) on the vertical axis of the Fig. .... . The omitted half of the graph for negative ion velocities is obviously anti-symmetric with respect to $V_i = 0$.

The procedure described above yields the longitudinal friction force on the ion as a function of longitudinal ion velocity assuming zero longitudinal electron temperature. All that is needed to compute the corresponding force on the ion when the longitudinal electron temperature is non-zero, is to compute the convolution of the function in Fig. ... with the electron distribution density in the longitudinal velocity space. The convolution acting essentially as a smoothing filter, the peak friction force produced by a warm electron beam will be somewhat smaller than in the cold electron case, the exact value depending on the rms electron velocity spread and other details of the electron distribution.

We remark in passing that the procedure for generating the initial conditions described above becomes, as expected, numerically inefficient for large ion velocities, when the relevant initial conditions are not symmetrically distributed around the ion (as is the case for small velocities), but are concentrated in the direction of the ion's motion. [[For this reason, the high-velocity tail in Fig. ... underestimates somewhat the magnitude of the force]]. For small ion velocities, however, a convergence in the computed time-averaged friction force value was seen with as few as ~1000 nearest neighbor physical electrons. The procedure for generating initial conditions will be optimized for accuracy and performance in the near future.

We are presently working on the computation of the friction force for the case of transverse ion velocities.