ELECTRON COOLING: STATUS AND PERSPECTIVES

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ABSTRACT

The electron cooling method became now conventional instrument of accelerator technique. Storage rings with electron cooling - the so-called coolers, permit generate very dense and high intensive beams of charged heavy particles - protons, antiprotons, ions. 9 such coolers - one for antiprotons and 8 for ions are operating effectively now in laboratories for high energy and nuclear physics. The review of the present understanding and of related problems is given in this report.

1. INTRODUCTION

The electron cooling method went through a long way of development, from the first Budker's idea [1], based on the model of two component electron – ion plasma, to very rich physics of a magnetized electron beam with "the flattened distribution" of electron velocities. It is a method of introduction of an effective "friction" in a particle beam that makes it possible to reduce the beam phase space volume (emittance) and, correspondingly, to compress the beam size and to decrease its particle momentum (energy) spread. As a result, an intensive, dense and "cold" particle beam can be generated. To produce such a friction, an "absorber" has to be brought into the beam.

In principle, one can use the mechanism of ionization losses in any material transparent for fast particles. Unfortunately, nature does not present us with such a gift, and ionization ("muon") cooling does not work for barions because of a short life time of the beam, limited by particle losses in strong interaction with absorber nucleons.

Budker's idea had two remarkable points:

1. to use "free" electrons, i.e. electron beam as an absorber;

2. to equalize average velocities of heavy particles and electrons, what gives the maximal value of the cross section of electromagnetic interaction.

Later these points were supplemented with a fortunate proposal to use homogeneous (solenoidal) magnetic field to form and transport the cooling electron beam [2], what gives additional advantages in cooling efficiency. It was discovered and understood later – in experimental studies of electron cooling [3].

The electron cooling method was successfully tested and developed in experimental studies done in the 1970-th at Budker Institute of Nuclear Physics [3-5], CERN [6] and Fermilab [7]. It is used now in 9 coolers (see [8]).

The development of the method is progressing in several directions that have a common goal of improving electron cooling efficiency and expanding the parameter area where the method can be applied. They are:

– generation of electron beams with the maximally achievable intensity and the lowest electron temperature possible: the guns with adiabatic optics, adiabatic acceleration, photocathode guns;

- elaboration of electron cooling systems with the neutralized space charge of an electron beam;
- extension of electron energy to the level in tens MeV;
- development of high effective recuperators of electron beam energy.

Today one can point out three main directions of the method application:
- storing, forming and keeping circulating antiproton beams in high energy physics, during the interaction with an internal target or slow extraction;
- the same for ions in nuclear atomic physics;
- storing and preliminary forming of heavy particle beams for following acceleration in a cascade of machines up to TeV-th energies (SSC and LHC projects).

The method can be used also for positron cooling, which permits to generate intensive flow of antihydrogen atoms[9]. The physics of cold intense beam of heavy particles – "beam crystallization"[10,11] – is also very interesting subject of the research. Numerical examples are given through at the report for some parameters, typical for middle energy coolers.

Unfortunately, the limited size of the report does not admit of any full review of the problem at all. Thereupon, one had to choose the mostly interesting, in author's opinion, points in the present status of electron cooling – some for a brief discussion, others – only to formulate. One can recommend a few review reports: [8,10-17].

2. THE PHYSICS OF ELECTRON COOLING

2.1. The uniform Maxwellian plasma relaxation

Two component plasma with a density n_e, consisting of heavy particles (mass M, the charge Ze) and electrons (m, e), each with uniform Maxwellian distribution and different initial temperatures T_p, T_e. Its relaxation process obeys Landau equation [18], which gives the instantaneous value of relaxation (cooling) time, or decrement

$$\tau_{plasma} = \frac{3Mm}{8\sqrt{2\pi}qL_c}\cdot\left(\frac{T_p}{M}+\frac{T_e}{m}\right)^{3/2} = \frac{3Mm}{8\sqrt{2\pi}qL_c}\times\begin{cases}\Delta_p^3, & \Delta_p \gg \Delta_e\ ,\\ \Delta_e^3, & \Delta_p \ll \Delta_e\ .\end{cases} \tag{1}$$

$$q = n_e Z^2 e^4,\ T_p = M\Delta_p^2,\ T_e = m\Delta_e^2\ ,$$

and the equilibrium condition:

$$T_p = T_e,\ \Delta_p = \sqrt{m/M}\cdot\Delta_e\ , \tag{2}$$

L_c is Coulomb logarithm. The decrement does depend on particle velocity for high magnitude of the last one. This relaxation process is "a competition" of friction force and diffusion, which appear, when a heavy particle penetrates an electron cloud.

2.2. The flattened distribution

The peculiarity of an electron beam is (see Sec. 3) a sharply nonuniform velocity distribution, called "flattened" [3,10,17]:

$$f_{Flat}(\bar{v})\ d^3v = \frac{m^{3/2}}{(2\pi)^{3/2}T_\perp\sqrt{T_\parallel}}\cdot exp\left\{-\frac{mv_\perp^2}{2T_\perp}-\frac{mv_\parallel^2}{2T_\parallel}\right\}\cdot 2\pi v_\perp\ dv_\perp dv_\parallel\ , \tag{3}$$

$$v_\perp^2 = v_x^2+v_z^2,\ m < v_\perp^2 > \equiv m\Delta_\perp^2 = T_\perp \gg T_\parallel\ .$$

The flattened distribution has three different regions of particle velocities V:

high	("H")	$V > \Delta_\perp$.	
low	("L")	$\Delta_\parallel < V < \Delta_\perp$.	(4)
superlow	("S")	$V < \Delta_\parallel$.	

The friction force and the diffusion power for the particle in the electron beam with the flattened distribution and in the absence of any focusing magnetic field somewhat differs from these for free electrons (see [14,17,19]).

2.3. The magnetized electron beam

In a longitudinal magnetic field B electron trajectories are spirals with a radius (the transversal Larmor radius) and a "step"

$$\rho_\perp = mv_\perp c/eB, \ \lambda_\parallel = 2\pi m v_\parallel c/eB \ll \rho_\perp \ , \tag{5}$$

and the peculiarity of the distribution plays a crucial role. The magnetization defines also electron beam temperature formation, decreases the beam angular spread and limits its space charge influence (see Sec. 3).

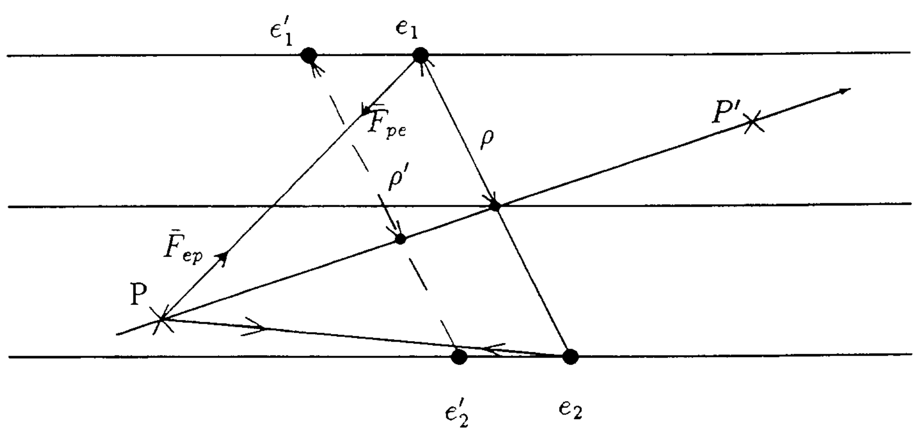

Fig.1. The schematics of collision of the particle with a magnetized electron: PP' – the particle trajectory, $e_1e_1'(\Delta\rho > 0)$ and $e_2e_2'(\Delta\rho < 0)$ – electron trajectories.

In the case of infinitely strong magnetization ($\rho_\perp \to 0$) the electron-particle interaction looks like the attraction ($Z > 0$) of an amber bead strung on a knitting needle, by a moving charged object (Fig. 1). When "the bead" can not move absolutely, the particle scattering is elastic, without any energy loss and, consequently, no friction force appears in this case. A free displacement of "the bead" along "the needle" leads to some energy transfer from the particle to the bead, so a friction force appears. In terms of a force action and a momentum transfer it means that the impact parameter alters during the collision – it increases or decreases depending on the mutual disposition of the electron and the particle (see Fig.1). Therefore, the attraction force also alters during collision, and it is weaker or stronger after the electron displacement. The total result for the whole electron ensemble is that the particle experiences the action of a decelerating (friction!) force. The calculations give (see [14,20,19]:

$$\mathbf{F}_{\perp} = -\frac{2\pi q L_c}{mV^3} \cdot \frac{V_{\perp}^2 - 2V_{\parallel}^2}{V^2} \cdot \mathbf{V}_{\perp} \ , \tag{6}$$

$$\mathbf{F}_{\parallel} = -\frac{2\pi q L_c}{mV^3} \cdot \frac{3V_{\perp}^2}{V^2} \cdot \mathbf{V}_{\parallel} \ , \quad L_c = ln\frac{\rho_{max}}{\rho_{min}} \ ,$$

$\mathbf{V}$ - the vector of the particle velocity.

One should point out important differences between the force (6) and that one for free electrons: $\mathbf{F}_{\parallel} \rightarrow 0$ when $V_{\perp} \rightarrow 0$ and $\mathbf{F}_{\perp}$ changes its sign and becomes positive, when $V_{\perp} < 2V_{\parallel}$, that is cooling turns in heating.

The Coulomb logarithm L_c deserves a special discussion. So far as the electron transverse motion is "frozen", the response of the electron cloud to electric field of the particle occurs only through a longitudinal displacement of electrons, therefore Debay shielding radius is equal to

$$R_D = \frac{U_M}{\omega_{pe}} = \begin{cases} V/\omega_{pe} \sim 0.05cm, & V > \Delta_{\parallel}, \\ \Delta_{\parallel}/\omega_{pe} \sim 5 \cdot 10^{-4}cm, & V < \Delta_{\parallel}, \end{cases} \tag{7}$$

$$U_M = <| \mathbf{V} - \mathbf{\Delta}_{\parallel} |> \ ,$$

ω_{pe} - electron plasma frequency. On the other hand, Debay shielding needs some amount of electrons, therefore, it can not be less than

$$R_D > R_e \equiv (k/n_e)^{1/3}, \ k \gg Z \ . \tag{8}$$

This condition can be violated for the superlow particle velocity and Debay radius should be taken equal to R_e . It is, certainly, a very rough estimation, and a more accurate analysis of collective effects at superlow velocities is an open question yet.

Thus, the maximal impact parameter for a magnetized electron beam is the subject of a complicated choice:

$$\rho_{max} = min \begin{cases} a & \sim 1.5cm, \\ max\{R_D, R_e\} & \sim 0.05 - 0.1cm, \\ U_M \cdot \tau & \sim 0.5 - 6 \cdot 10^{-2}cm. \end{cases} \tag{9}$$

Here a is beam radius, τ - time of flight of the particle trough the cooling system.

A finite magnetic field brings into existence three regions of impact parameter magnitudes (Fig.2), which are defined by the beam parameters - electron Larmor radius (5) and longitudinal velocity spread $\Delta_{\parallel}$ and the particle velocity.

Small impact parameters - the particle interacts with an electron without any influence of magnetic field, if the collision time is less than the period of Larmor (cyclotron) revolution of an electron - "the fast collisions" ("F"):

$$\tau_{col} \sim \rho/U_M \leq \omega_B^{-1} \equiv mc/eB \ . \tag{10}$$

It defines the scale of small impact parameters:

$$\rho_{min} \leq \rho \leq \rho_F,$$

$$\rho_{min} = \frac{Ze^2}{mU^2} = \frac{Ze^2}{m} \cdot \begin{cases} V^{-2}, & V > \Delta_{\|}, \\ \Delta_{\perp}^{-2}, & V < \Delta_{\|}, \end{cases} \tag{11}$$

$$U = <| \mathbf{V} - \mathbf{\Delta}_{\perp} |>, \quad \rho_F = U_M/\omega_B.$$

The friction force and diffusion are described here with formulae for flattened distribution [14] with $\rho_{max} = \rho_F$.

Intermediate impact parameters $\rho_F \leq \rho \leq < \rho_{\perp} >$. In such collisions the particle and the electron travel together during a few periods of a cyclotron revolution while the electron rotates "around" the particle, colliding with it N times:

$$1 \leq N \leq \frac{\tau_{col}}{2\pi} \cdot \omega_B \leq \frac{\Delta_{\perp}}{U_M} \sim \begin{cases} \Delta_{\perp}/V, & V > \Delta_{\|}, \\ \Delta_{\perp}/\Delta_{\|}, & V < \Delta_{\|}. \end{cases} \tag{12}$$

The friction force and diffusion in these "adiabatic collisions" ("A") are amplified N times in comparison with a free electron beam [10,14,19].

At **large impact parameters** $\rho_{\perp} \leq \rho \leq \rho_{max}$ (see (5), (9)) the electron looks like a small "Larmor circle" and the collision has the same character as for strongly magnetized electrons (6).

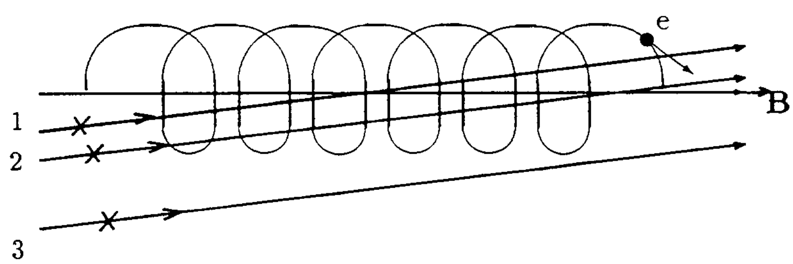

Fig.2. Three kinds of particle-electron collisions: fast (1), adiabatic (2) and magnetized (3).

It can be distinctly seen from the discussion above, that the choice of the impact parameter magnitude depends primarily on particle velocity (for the given parameters of an electron beam). Hence the three regions of particle velocities (4) exist as before. At a **high velocity** the friction force has only two parts - the fast one and the magnetized one: no adiabatic collisions exist here, because $N < 1$, if $V > \Delta_{\perp}$.

Low and superlow velocities of the particle admit the existence of all three regions of impact parameters, and the friction force contains here three parts - fast, adiabatic and magnetized ones. However, the magnetized collisions **at superlow velocity** demand some additional analysis, because $\Delta_{\|}$ can not be neglected as before. The simplified estimation of a momentum, transferred to the particle in a collision with an electron, gives [19]

$$\mathbf{F} \sim -\mathbf{V} \cdot \frac{2\pi q}{\Delta_{\|}^3} \cdot L_{MS}, \; L_{MS} = ln \frac{\rho_{max}}{< \rho_{\perp} >}, \tag{13}$$

which only slightly differs from the result of the exact calculation [14].

Thus, the friction force in the magnetized electron beam is

$$\mathbf{F}_\perp = -\frac{2\pi q}{m}\cdot \mathbf{V}_\perp \cdot \begin{cases} \frac{1}{V^3}\left(2L_{FH} + \frac{V_\perp^2 - 2V_\parallel^2}{V^2}\cdot L_{MH}\right), & "H"; \\ \frac{L_{FL}}{\Delta_\perp^3} + \frac{L_{AL}}{\Delta_\perp^2 V} + \frac{V_\perp^2 - 2V_\parallel^2}{V^2}\cdot \frac{L_{ML}}{V^3}, & "L"; \\ \frac{L_{FS}}{\Delta_\perp^3} + \frac{L_{AS}}{\Delta_\perp^2 \Delta_\parallel} + \frac{L_{MS}}{\Delta_\parallel^3}, & "S"; \end{cases} \tag{14}$$

$$\mathbf{F}_\parallel = -\frac{2\pi q}{m}\cdot \mathbf{V}_\parallel \cdot \begin{cases} \frac{1}{V^3}\left(2L_{FH} + 3L_{MH}\cdot \frac{V_\perp^2}{V^2}\right), & "H"; \\ \frac{2L_{FL}}{\Delta_\perp^2 V_\parallel} + \frac{2L_{AL}}{\Delta_\perp V V_\parallel} + \frac{V_\perp^2}{V^2}\cdot \frac{3L_{ML}}{V^3}, & "L"; \\ \sqrt{\frac{2}{\pi}}\frac{L_{FS}}{\Delta_\perp^2 \Delta_\parallel} + \frac{2L_{AS}}{\Delta_\perp V V_\parallel} + \frac{3L_{MS}}{\Delta_\parallel^3}, & "S". \end{cases}$$

The Coulomb logarithms are gathered below by groups for each velocity region:

High particle velocity

$$L_{MH} = ln[(\rho_{max})_H \,/ < \rho_\perp >] \sim ln[(V\omega_B)\,/\omega_{pe}\Delta_\perp] \sim 3;$$

$$L_{FH} = ln[\rho_{FH}/\,(\rho_{min})_H] \sim ln[mV^3/\omega_B Ze^2] \sim 10 - lnZ.$$

Low particle velocity

$$L_{ML} = ln[(\rho_{max})_L \,/ < \rho_\perp >] = ln\frac{min\left\{V/\omega_{pe}, (\not{k}n_e)^{+1/3}\right\}}{< \rho_\perp >} \sim 3 \div 1;$$

$$L_{AL} = ln[< \rho_\perp > /\rho_{FL}] = ln[\Delta_\perp/\,(\Delta_\perp \to \Delta_\parallel)] \sim 0 \to 4;$$

$$L_{FL} = ln[\rho_{FL} \,/\, (\rho_{min})_L] = ln[mV^3 \,/\, Ze^2\omega_B] \sim 5 - lnZ$$

Superlow particle velocity

$$L_{MS} = ln[(\rho_{max})_S \,/ < \rho_\perp >] = ln[(\not{k}n_e)^{\;1/3} \,/ < \rho_\perp >] \sim 2;$$

$$L_{AS} = ln[< \rho_\perp > /\rho_{FS}] = ln[\Delta_\perp/\Delta_\parallel] \sim 4;$$

$$L_{FS} = ln[\rho_{FS}/\,(\rho_{min})_S] = ln[m\Delta_\parallel\Delta_\perp^2/Ze^2\omega_B] \sim 5 - lnZ.$$

This Coulomb logarithm "Zoo" shows that the friction force is nonlinear very significantly, because the logarithms themselves alter during the cooling process.

The Formulae (14) lead to one important conclusion: the friction force for a magnetized electron beam differs essentially from that for a nonmagnetized one: when a particle velocity diminishes under the cooling action, the force grows up as V^2, and this growth does not stop at $V \sim \Delta_\perp$, as for a nonmagnetized beam, but continues in the region of

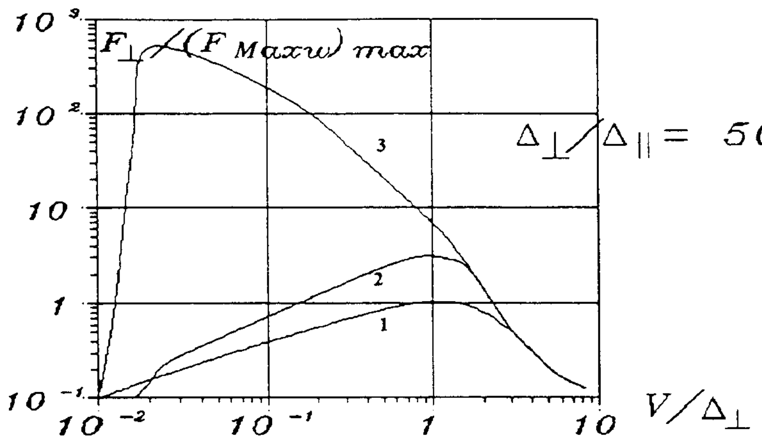

Fig.3. The dependence of the transversal friction force on particle velocity: 1 – the uniform Maxwellian plasma, 2 – the flatened distribution at $B = 0$, 3 – the magnetized electron beam.

a low velocity due to "adiabatic" and "magnetized" terms (Fig.3). The force reaches its maximum near $V \sim \Delta_{||}$ and then drops linearly with V at $V < \Delta_{||}$.

One can formulate at last the criterion of magnetization of an electron beam:

$$< \rho_\perp > < (\rho_{max})_S \sim 3n_e^{-1/3} , \tag{15}$$

or

$$B > \frac{m\Delta_\perp c}{3en_e^{-1/3}} .$$

The diffusion is important only in the region of a superlow velocity, because it defines the equilibrium state namely here. The estimations similar to (13), give for its power

$$Q \equiv \frac{M}{2} \cdot \frac{d < V^2 >}{dt} \sim \frac{4\pi q}{M\Delta_{||}} \cdot L_{MS} . \tag{16}$$

It differs insignificantly with results of exact calculations [14]. The equalizing of the power of the friction force (13) and diffusion (16) gives equilibrium state in a magnetized electron beam:

$$\sqrt{\mathbf{V}^2_{equi}} \approx \sqrt{\frac{m}{M}} \cdot \Delta_{||} . \tag{17}$$

The law of particle velocity behavior one can find with (14), neglecting with angular dependence of the friction force and the term with L_{AL}:

$$\sqrt{\mathbf{V}^2_\perp (t)} = \begin{cases} V^\circ_\perp (1 - t/\tau_\perp)^{1/3}, & "H"; \\ \left[\left((V^\circ_\perp)^3 + \alpha\Delta^3_\perp\right) e^{-t/\tau_\perp} - \alpha\Delta^3_\perp\right]^{1/3}, & "L"; \\ \left[(\mathbf{V}^2_\perp)_{equi} + \left((V^\circ_\perp)^2 - (\mathbf{V}^2_\perp)_{equi}\right) \cdot e^{-t/\tau_\perp}\right]^{1/2}, & "S"; \end{cases} \tag{18}$$

$$\alpha = L_{ML}/L_{FL};$$

where characteristic times $\tau_\perp$ are

$$\tau_\perp \approx \frac{mM}{2\pi q} \times \begin{cases} \left(V_\perp^o\right)^3/3\left(2L_{FH}+L_{MH}\right), & "H"; \\ \Delta_\perp^3/3L_{FL}, & "L"; \\ \Delta_\parallel^3/L_{MS}, & "S". \end{cases} \tag{19}$$

These $\tau_\perp$ can be treated as cooling time only in "H" and "S" cases. For "L" region it is some characteristic parameter, and time duration necessary to cool the particle from $V_\perp^o$ to $\Delta_\parallel$ is equal to

$$\left(\tau_{cool}\right)_L = \left(\tau_\perp\right)_L \cdot ln\frac{(V_\perp^o)^3+\alpha\Delta_\perp^3}{\Delta_\parallel^3+\alpha\Delta_\perp^3} \sim \frac{mM}{6\pi q L_{ML}}\cdot(V_\perp^o)^3, \quad "L". \tag{20}$$

This result demonstrates the effect of "The fast electron cooling", when τ_{cool} depends on particle velocity even at $V < \Delta_\perp$.

For a longitudinal component the law of $V_\parallel(t)$ is the same as (18) for the "H" and "S" regions, but with different characteristic time (see (23)). For the "L" region the law is very intricate:

$$t(V_\parallel) = \tau_0\left\{\frac{V_\parallel^o - V_\parallel}{\Delta_\perp} - \kappa\cdot Arctg\left(\frac{V_\parallel^o - V_\parallel}{1+V_\parallel^o V_\parallel/(\kappa\Delta_\perp)^2}\cdot\frac{1}{\kappa\Delta_\perp}\right)\right\}, \tag{21}$$

$$\tau_0 = mM\Delta_\perp^3/4\pi q L_{FL}, \quad \kappa = \sqrt{3L_{ML}/2L_{FL}} \sim 1.$$

For typical parameters $\kappa \sim 1$, therefore the cooling time for a particle with a low initial velocity is about

$$\left(\tau_\parallel\right)_L \sim \frac{\tau_0}{3\kappa^2}\left(\frac{V_\parallel^o}{\Delta_\perp}\right)^3. \tag{22}$$

Then

$$\tau_\parallel \approx \frac{mM}{2\pi q} \times \begin{cases} \left(V_\parallel^o\right)^3/3\left(2L_{FH}+2L_{MH}\right), & "H"; \\ \left(V_\parallel^o\right)^3/9L_{ML}, & "L"; \\ \Delta_\parallel^3/L_{MS}, & "S". \end{cases} \tag{23}$$

The comparison of (20) and (23) shows, that for low velocities the longitudinal cooling time is about 3 times shorter than the transversal one.

The question of practical interest is the cooling time for a particle with high initial velocity. As could be seen from (19), (23), it slightly differs from the case of a non-magnetized electron beam (1). Besides, what is much more significant: the influence of magnetization on cooling time in the region of low velocities is also very modest. It's not very surprising, because, notwithstanding a very fast increase of friction force (and instant increment!) with V decreasing, the full cooling time is defined mostly by large magnitudes of the initial particle velocity $V \sim \Delta_\perp$.

Nevertheless, instant value of friction force plays crucial role when equilibrium state is concerned. It is very important, particularly, in consideration of particle beam interaction with internal target.

3. THE FINE EFFECTS IN ELECTRON COOLING

3.1. The additional friction force for a negatively charged particle

In a collision with the low impact parameters a negatively charged particle reflects magnetized electrons, while a positively charged one attracts them. The addition to the friction force for a negatively charged ion is equal to [19]

$$\Delta F_{\|} = -n_e V \cdot 2mV \cdot \pi\rho_{min}^2 \approx -\frac{2\pi q}{m} \times \begin{cases} 1/V^2 , & V > \Delta_{\|}; \\ V^2/\Delta_{\|}^4 , & V < \Delta_{\|}; \end{cases} \tag{24}$$

and the maximal friction force is nearly 2 times more powerful for negatively charged particle (see [19], Fig.7).

3.2. The additional diffusion for a positively charged particle[19]

The junction of electron and particle beams in a cooling device is a very fast process in the particle rest frame: an electron displaces in the junction section for one period of Larmor revolution in the direction transversal to the particle trajectory by the distance, which is much larger of $< \rho_{\perp} >$. Hence, the particle and the electron meet together "instantly". Then, the positively charged particle attracts the electron, which travels close enough to the particle, oscillating along the magnetic field and slowly rotating around the particle trajectory due to the drift in a transversal electric field of the particle and a longitudinal magnetic field of the cooler. The particle-electron interaction gives the additional diffusion power [19]:

$$Q \approx \frac{e^2}{M\tau_0} \cdot \left(\frac{Bq\tau^2}{c}\right)^{2/3} . \tag{25}$$

τ_0 – period of particle revolution in the storage ring, τ – time of flight through cooling section. As result,

$$\sqrt{(\mathbf{V}^2)_{equi}} \approx \sqrt{\frac{m}{M}} \cdot (1+\delta) \ , \delta = \frac{1}{\sqrt{2\pi\tau_0\Delta_{\|} L_{MS}}} \cdot \left(\frac{r_e c^2 \tau^4 \omega_B^2}{Z^2}\right)^{1/6} \sim 0.2 \cdot Z^{-1/3} . \tag{26}$$

3.3. The Coulomb logarithm and multi-charged ions

It became "a tradition" in electron cooling numerical estimations to put the Coulomb logarithm in any formula of order 10-15. However, a more attentive analysis shows that one should be careful with such estimations. The influence of the Coulomb logarithm is the only possible way of explanation of the results, obtained recently in experiments at TSR [21,22]. It was found, that longitudinal friction force and its decrement decrease with ion charge Z. These results do not contradict the dependence of the Coulomb logarithm L_{FL} on ion charge in formulae (14). Our analysis shows, that TSR experiment results stricly follow the law (Fig.4)

$$\lambda_{\|} = \lambda_1 - \lambda_2 \cdot lnZ, \quad \lambda_{\|} = \left(\tau^{-1}\right) \cdot A / Z^2. \tag{27}$$

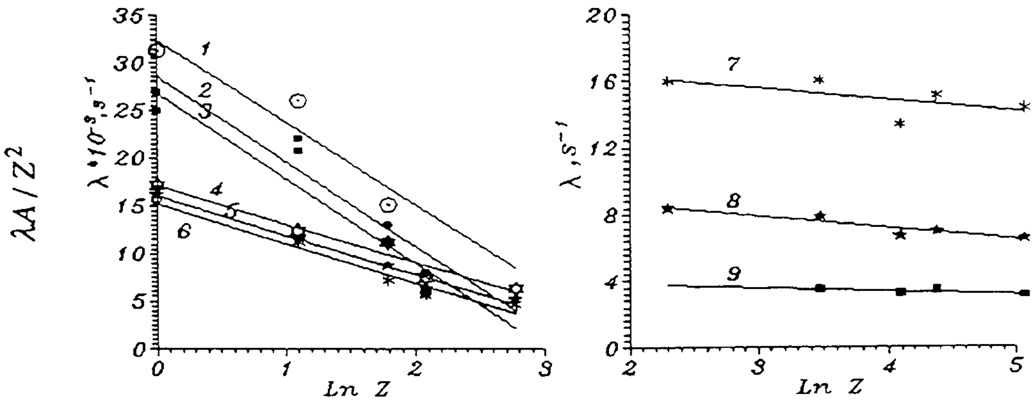

Fig.4. The dependence of longitudinal decrement of ion transverse velocity for different Z, obtained from the analysis of TSR-experiment [21].

4. THE COOLING ELECTRON BEAM

4.1. The electrostatic acceleration and the flattened distribution

The distribution (3) has a kinematical origin in a low intensity beam. Because distribution function is the solution of the collisionless Vlasov equation with full energy as the function argument, one can write (in Laboratory frame after acceleration):

$$f(v_{\parallel}) = A \cdot e^{-\varepsilon_c/T_c}, \quad \varepsilon_c \approx p_0 \cdot (p - p_0)/\gamma m \ , \tag{28}$$

$$p_0 = < p > = \beta\gamma mc \ ,$$

p, p_0 – electron momenta in Lab. frame, ε_c – electron kinetic energy at the cathode, T_c – cathode temperature. The transformation of (28) from the Lab. frame to the electron one and normalization by unit leads to

$$f(v_{\parallel}) = \frac{\beta\gamma mc}{T_c} \cdot e^{-\beta\gamma mcv_{\parallel}/T_c} \tag{29}$$

$$T_{\parallel} = m \int_{-\infty}^{\infty} v_{\parallel}^2 \cdot f(v_{\parallel}) \cdot dv_{\parallel} = 2T_c^2/\beta^2\gamma^2 mc^2 \ \rightarrow \ T_c^2/\varepsilon_0 \ , \quad \varepsilon_0 \ll mc^2 \ . \tag{30}$$

This distribution function is not an equilibrium one. Electron gas comes to thermoequilibrium after some relaxation time, that is very short (see Sec. 4.3).

Electron transverse momentum does not change during the acceleration, if optical aberrations of the gun can be neglected. Hence

$$T_{\perp} \sim T_c \ . \tag{31}$$

N^o	1	2	3	4	5	6	N^o	7	8	9
$V_{\perp}$, $10^5 cm/s$	6	7	8	6	7	8	$V_{\parallel}$, $10^7 cm/s$	2.4	3.4	4.8
$n_e \cdot 10^8 cm^{-3}$		0.72			3.42		–	–	–	–

4.2. The electron density fluctuations and longitudinal temperature

The result (29),(30) is correct for an electron beam of very low density, when the Coulomb interaction between electrons is negligible. The beams used in electron cooling are fairly dense and this interaction plays an important role. Because of a random character of distances between electrons, the local electric field in a beam is not equal to zero and such a field accelerates or decelerates electrons, i.e. transfers to them additional kinetic energy (temperature)[10,19]:

$$\Delta T \sim e^2/l_e \sim 10^{-4}eV \ , \quad l_e = n_e^{-1/3} \sim 1.5 \cdot 10^{-3}cm \ - \tag{32}$$

the average distance between electrons in the beam (all parameters are taken in the particle rest frame). The result of a strict analysis differs by the numerical coefficient of order of unit.

4.3. Electron relaxation. "Fast" and "slow" acceleration

The relaxation of the distribution function (29) occurs due to electron-electron collisions and, in principle, is the same process of the Coulomb interaction, as the electron cooling. If the electron beam is magnetized, one can use for the estimation of the relaxation time formulae (23), the case "S", if to put $m = M$. Then for the longitudinal relaxation one has [24,19]

$$(\tau_{\|})_{e-e} \sim \frac{\Delta_{\|}^3}{2\pi n_e r_e^2 c^4 L_{MS}} \sim 2 \cdot 10^{-9} sec \ . \tag{33}$$

This relaxation works very fast and the electron beam comes from the gun into the cooling section (a distance of about 0.5 m) very well "maxwellized". The relaxation does not influence in a conventional electron gun because of a high acceleration rate. Therefore electrons do not redistribute their additional energy (32) and come to the gun exit, where after relaxation time electron energy becomes equal to

$$T_{\|} \sim \frac{T_c^2}{\varepsilon_0} + e^2 n_e^{1/3} \approx e^2 n_e^{1/3} \ . \tag{34}$$

One can write the criterion of slow acceleration, when relaxation "intermixes" electrons and maxwellizes electron velocity distribution in the gun:

$$\frac{1}{T_{\|}} \cdot \frac{dT_{\|}}{dt} < min \left\{ (\tau_{\|})_{e-e}^{-1}, \, \omega_{pe} \right\} \ . \tag{35}$$

In this case the instant value of the electron longitudinal temperature follows the law (30), and

$$\frac{1}{T_{\|}} \cdot \frac{dT_{\|}}{dt} = \frac{1}{\varepsilon} \cdot \frac{d\varepsilon}{dt} = \frac{v}{\varepsilon} \cdot eE \ , \tag{36}$$

where E is accelerating electric field in the gun. Then, the criterion has the form

$$E < 2.1\sqrt{j \cdot \sqrt{m\varepsilon_0/e^2}} \ . \tag{37}$$

The electric field of the Pierce's gun is about 3 times higher, the Pierce gun is too "fast" and forms the electron beam with a temperature, which follows the law (34), with the flattened distribution, generally speaking.

Numerical estimations show, that the effect of density fluctuations defines the longitudinal temperature of the electron beam. A special design of the gun is required to reach a lower temperature limit (30). Such a gun consists of two parts: a Pierce gun with "fast" acceleration and section of slow ("adiabatic") acceleration. One can show, that for given $T_{\|}$ the maximal beam current generated with such a gun does not exceed

$$I(T_{\|}) \approx \frac{mc^3}{e} \cdot \left(\frac{\pi a^2}{r_e^2}\right)^{9/8} \cdot \left(\frac{T_{\|}\varepsilon_0}{m^2c^4}\right)^{3/8} , \quad \frac{T_c^2}{\varepsilon_0} \leq T_{\|} \leq e^2 n_e^{1/3} , \tag{38}$$

and the current corresponding $T_{min} = T_c^2/\varepsilon_0$ does not depend on energy ε_0 :

$$I(T_{min}) = \frac{mc^3}{e} \cdot \left(\frac{a}{r_e}\right)^{3/4} \cdot \left(\frac{T_c}{mc^2}\right)^{9/4} \approx 40mA \times a_{cm}^{3/4} \ \ at \ \ T_c = 0.1 \ eV \ . \tag{39}$$

To generate the beam with T_{min} the gun at $\varepsilon_0 = 30keV$ has to have about 5 mm "the Pierce gap" and 0.7 m long slow section. First experimental achievements [23] look promising.

The conservation of flattened distribution during beam transportation is limited with electron-electron collisions, which lead to the so called transversal – longitudinal relaxation. It equalizes the longitudinal temperature to the transversal one. A longitudinal magnetic field suppresses essentially the transversal – longitudinal relaxation, if [24] $\rho_\perp \leq l_e$.

4.4. The magnetized intense electron beam in a drift chamber

The space charge of an intense electron beam, travelling in a longitudinal magnetic field, creates two effects: electron drift in the plane, transversal to the beam axis and electron energy variation across the beam. Both are essential for electron cooling process. Therefore the neutralization of the space charge is a problem of great practical interest [25].

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