Single-pass Simulation Studies of High-Energy Electron Cooling – Review and Future Directions

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Motivation

- Electron-ion colliders (EIC) are a high priority for the future of the worldwide nuclear physics community
- Relativistic, strongly-magnetized electron cooling may be essential for EIC, but never demonstrated
  - *key element of the MEIC conceptual design*
  - *also relevant to eRHIC, LHeC, …*
- Electron cooling at γ~100 requires different thinking
  - *friction force scales like $1/\gamma^2$ (Lorentz contraction, time dilation)*
    - challenging to achieve the required dynamical friction force
    - not all of the processes that reduce the friction force have been quantified in this regime → technical risk
  - *normalized interaction time is reduced to order unity*
    - $t=t\omega_{pe}$ >> 1 for nonrelativistic coolers
    - $t=t\omega_{pe}$ ~ 1 (in the beam frame), for γ~100
      - *violates the assumptions of introductory beam & plasma textbooks*
      - *as a result, our physical intuition can be wrong*

Context and Caveats

- Lessons learned over 8 years of work

  D.L. Bruhwiler, "Simulating single-pass dynamics for relativistic electron cooling," in *ICFA Beam Dynamics Newsletter* **65**, "Beam Cooling II," eds. Y. Zhang & W. Chou (Dec., 2014).

  - *we focus on work involving the author and closely related work*
    - there is an extensive worldwide literature on friction & stopping power
- We consider the microphysics of dynamical friction
  - *detailed simulations of a single pass for ions through the cooler*
  - *the parameter space is large & the simulations are demanding*
- Parametric and semi-analytic models are necessary
  - *accurate parametric models enable rapid conceptual design*
  - *codes like BETACOOL and MOCAC enable long-time studies*
    - semi-analytic models, electron & ion distributions, equilibration
  - *simulating single-pass physics helps to improve these models*
- Diffusive kicks must be suppressed for single-pass studies
  - *diffusive effects exceed friction in a single pass*
    - quiet start with correlated electron-positron pairs
    - use more macroparticles than there are physical electrons
  - *friction wins over millions of turns, but not in a single pass*

Outline

- Early simulations of magnetized cooling:

A.V. Fedotov, D.L. Bruhwiler, A. Sidorin, D. Abell, I. Ben-Zvi, R. Busby, J. Cary & V.N. Litvinenko, "Numerical study of the magnetized friction force," PRSTAB **9**, 074401 (2006).

A.V. Fedotov, I. Ben-Zvi, D.L. Bruhwiler, V.N. Litvinenko and A.O. Sidorin, "High-energy electron cooling in a collider," New Journal of Physics **8**, 283 (2006).

- Extensive studies of unmagnetized cooling:

G.I. Bell, D.L. Bruhwiler, A. Fedotov, A. Sobol, R. Busby, P. Stoltz, D.T. Abell, P. Messmer, I. Ben-Zvi and V.N. Litvinenko, "Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam", J. Comp. Phys. **227**, p. 8714 (2008).

A.V. Sobol, D.L. Bruhwiler, G.I. Bell, A. Fedotov and V.N. Litvinenko, "Numerical calculation of dynamical friction in electron cooling systems, including magnetic field perturbations and finite time effects," New Journal of Physics **12**, 093038 (2010).

G.I. Bell, I.V. Pogorelov, B.T. Schwartz, Y. Zhang and H. Zhang, "Single Pass Electron Cooling Simulations for MEIC," *Proc. Part. Accel. Conf.*, TUPHO02 (Pasadena, 2013).

- Ideas for revisiting relativistic magnetized cooling
  - *finite time effects & many other difficulties must be quantified*

As of 2000, there were competing models

Ya. S. Derbenev and A.N. Skrinsky, "The Effect of an Accompanying Magnetic Field on Electron Cooling," Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, "Magnetization effects in electron cooling," Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

I. Meshkov, "Electron Cooling; Status and Perspectives," Phys. Part. Nucl. **25** (1994), 631.

$$\mathrm{F}_{\|}^{A} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\left[\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\left(\frac{V_{\perp}}{V_{ion}}\right)^2\right)+\frac{2}{3}\right]\frac{V_{\|}}{V_{ion}^3}$$

$$\mathrm{F}_{\perp}^{A} = -\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\frac{\left(0.5V_{\perp}^2-V_{\|}^2\right)}{V_{ion}^2}\frac{V_{\perp}}{V_{ion}^3}$$

$$r_L = V_{rms,e,\perp}/\Omega_L\left(B_{\|}\right)$$
$$\rho_{\min}^{A} = \max\left(r_L,\rho_{\min}\right)$$
$$\rho_{\max}^{A} = \min\left(r_{beam},\rho_{\max}\right)$$
$$\rho_{\max} = V_{rel}/\max\left(\omega_{pe},1/\tau\right)$$
$$V_{rel} = \max\left(V_{ion},V_{e,rms,\|}\right)$$
$$V_{ion}^2 = V_{\|}^2 + V_{\perp}^2$$

V.V. Parkhomchuk, "New insights in the theory of electron cooling," Nucl. Instr. Meth. in Phys. Res. **A 441** (2000), p. 9.

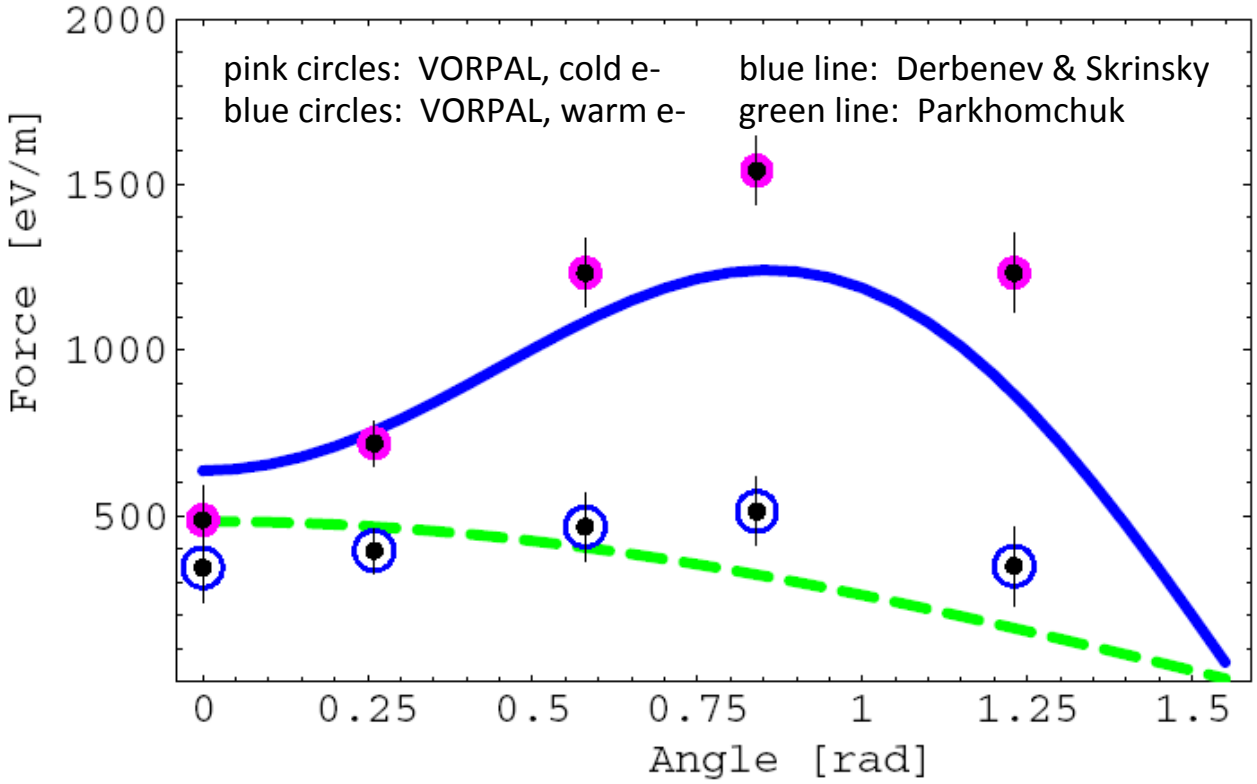
$$\mathbf{F} = -\frac{1}{\pi}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{\max}+\rho_{\min}+r_L}{\rho_{\min}+r_L}\right)\frac{\mathbf{V}_{ion}}{\left(V_{ion}^2+V_{eff}^2\right)^{3/2}}$$

$$r_L = V_{rms,e,\perp}/\Omega_L\left(B_{\|}\right)$$

$$\rho_{\min} = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^2 \qquad \rho_{\max} = V_{ion}/\max\left(\omega_{pe},1/\tau\right)$$

$$V_{eff}^2 = V_{e,rms,\|}^2 + \Delta V_{\perp e}^2$$

VORPAL modeling of binary collisions clarified differences in formulae for magnetized friction

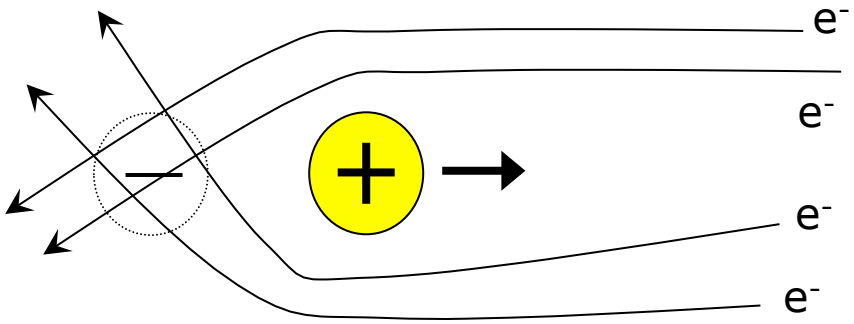


A.V. Fedotov, D.L. Bruhwiler, A.O. Sidorin, D.T. Abell, I. Ben-Zvi, R. Busby, J.R. Cary & V.N. Litvinenko, "Numerical study of the magnetized friction force," Phys. Rev. ST/AB **9**, 074401 (2006).

- D&S asymptotics are accurate for ideal solenoid, cold electrons – not warm
- Parkhomchuk formula often works for typical parameters, but not always
- 3D quad. of D&S with e- dist. works better (modified r_{min}, ideal solenoid)
- In general, direct simulation is required

Dynamical Friction/Diffusion is Long in the Tooth

- Case of isotropic plasma, with no external fields, was first explained 65 years ago
  - *S. Chandrasekhar, Principles of Stellar Dynamics (U. Chicago Press, 1942).*
  - *B.A. Trubnikov, Rev. Plasma Physics 1 (1965), p. 105.*
  - *NRL Plasma Formulary, ed. J.D. Huba (2000).*



- Physics can be understood in two different ways
  - *Binary collisions (integrate over ensemble of e-/ion collisions)*
  - *Dielectric plasma response (ion scatters off of plasma waves)*

Dynamic friction calculations assume long times

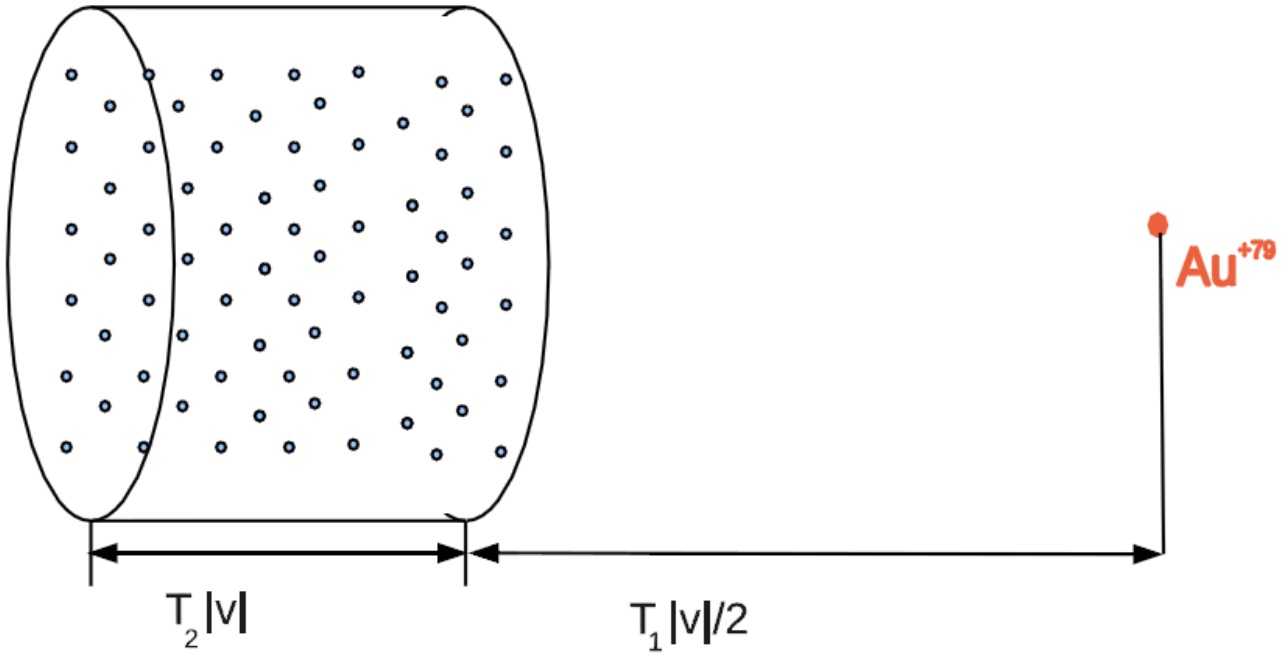


Figure 2. The choice of the volume that leads to the classic friction force formula. The frame of reference is chosen such that the ion is at rest.

- This scenario is not valid for short interaction times
  - *in reality, ion is immersed suddenly in an e- distribution*

Small impact parameter collisions are important

(for unmagnetized friction) Sobol *et al.*

- Impact parameters follow a modified Pareto distribution
  - *like income distrib.*
  - *small values are rare but significant*
- Uncertainties are intrinsically large
- The central limit theorem is not valid
  - *using ever more collisions to average away noise → artificially large result*

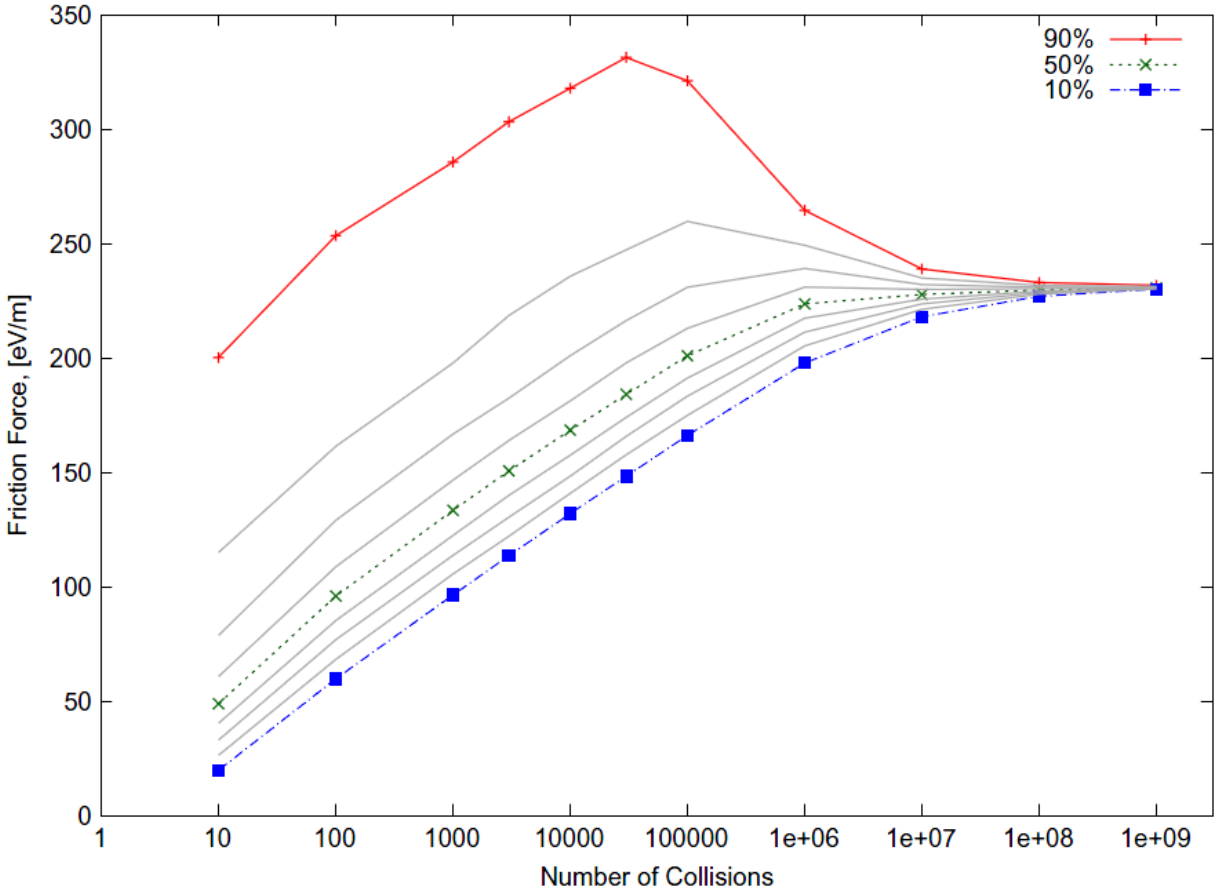


Figure 8. The friction force resulting from a sequence of collisions as an RV. The 10-quantiles (deciles) are plotted as a function of the number of collisions: thick lines are 10%-, 50%- and 90%-quantiles and thin lines are 20%-, 30%-, 40%-, 60%-, 70%- and 80%-quantiles. The 80% confidence interval is very wide unless the number of collisions is extremely large.

Modified Coulomb Log Captures Main Effects

- The following can be used to replace standard eq'n
  - *this effect is important; not yet in BETACOOL or MOCAC*

$$\Lambda = \frac{1}{2}\log\left[\left(\frac{\rho_{max}^2+\rho_\perp^2}{\rho_\perp^2+\rho_c^2}\right)\left(\frac{\rho_c^2+d^2}{\rho_{max}^2+d^2}\right)\right]$$

$$d = |\vec{v}_{\rm rel}|\tau/2 \quad \tau = s/(\gamma\beta c)$$

- r_c defines the minimum impact parameter that is statistically sampled at in a meaningful way
  - *value of Nc is chosen in adhoc manner*

$$\rho_c = \sqrt{\frac{N_c}{\langle|\vec{v}_{\rm rel}|\rangle\tau\pi n_e}}$$

$$N_c = 5$$

- standard Coulomb log is easily recovered
  - *in the limit that:* $\rho_c = 0 \quad d \gg \rho_{max} \gg \rho_\perp$

Numerical Approaches for Electron Cooling Simulations

- Fast multipole method (FMM) and tree-based algorithms
  - *requires constant time step; inefficient for MD with a few close collisions*
    - not a problem for magnetized cooling (sometimes OK for unmagnetized)
  - *could be combined with semi-analytic treatment (see below)*
- 4th-order predictor-corrector "Hermite" algorithm
  - *taken from astrophysical dynamics community*
    - generalized to include solenoid field
- Semi-analytic binary collision model (Bell *et al.*)
  - *also MD approach; close connection to "Hermite" algorithm above*
    - accurately models arbitrarily strong Coulomb collisions
    - arbitrary external fields included via 2nd-order operator splitting
- df PIC (electrostatic)
  - *taken from plasma fusion community*
  - *cannot directly capture close Coulomb collisions*
    - successful demonstration in VORPAL for unmagnetized case
- Importance sampling method (Sobol *et al.*)
  - *simulate one parameter set at a time*
    - choose impact parameters to optimally sample the full range
    - integrate saved results over the full range of impact parameters
  - *orders of magnitude faster than an 'integrated' simulation*

Replacing SC solenoid with a conventional wiggler offers lower cost & technical risk

- Why look for alternatives to solenoid design?
  - *solenoid design & beam requirements for RHIC are challenging*
    - 80 m, 5 T, superconducting, field errors $<10^{-5}$
- Advantages of a wiggler
  - *like a solenoid, it provides focusing & suppresses recombination*
    - modest fields (~10 Gauss) effectively reduce recombination via 'wiggle' motion of electrons:

$$\rho_w = \frac{\Omega_{gyro}}{k_w^2 v_{beam}} \sim 1.4x10^{-3}\,\lambda_w^2[m]B_w[G]/\gamma$$

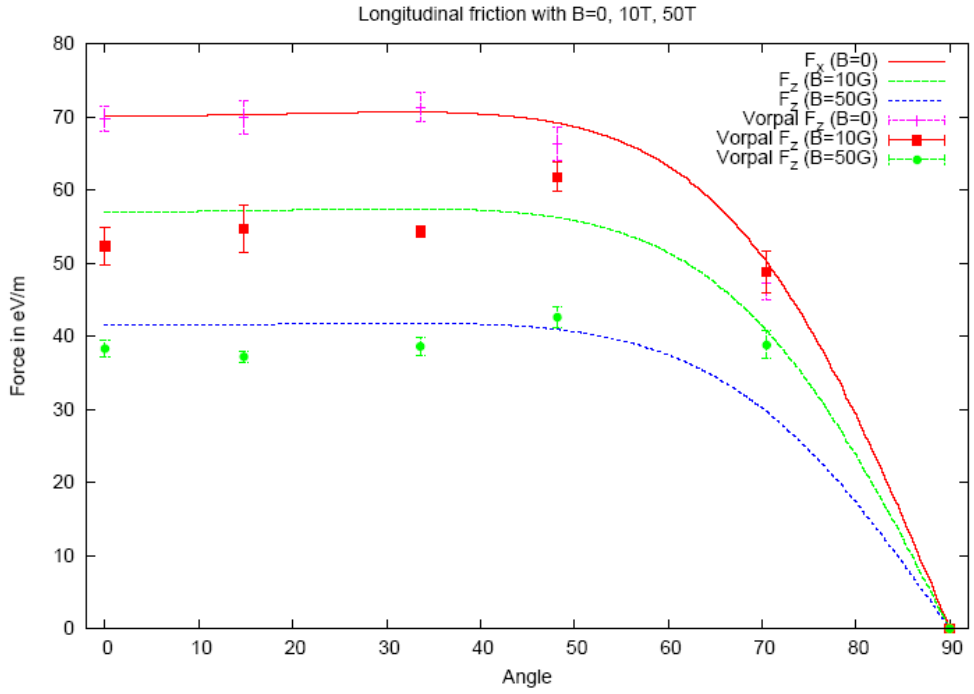
  - *e- bunch is easier: less charge and un-magnetized*
  - *lower construction costs; less technical risk*
- What's the effect of 'wiggle' motion on cooling?
  - *independent suggestion of V. Litvinenko & Ya. Derbenev*
    - increases r_{min} of Coulomb logarithm: $\rho_{\min} \rightarrow \rho_w$
  - *strong need for simulations*

VORPAL simulations support decision to use conventional wiggler for e- cooling of 100 GeV/n Au+79

- Culmination of years of work, beginning in 2002

G.I. Bell, D.L. Bruhwiler, A. Fedotov, A.V. Sobol, R. Busby, P. Stoltz, D.T. Abell, P. Messmer, I. Ben-Zvi and V.N. Litvinenko, "Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam", J. Comp. Phys. **227** (2008), p. 8714.

- Conventional wiggler could replace expensive solenoid
  - friction force is reduced only logarithmically



VORPAL implementations of binary collision algorithm and df PIC show good agreement when used carefully

G.I. Bell, I.V. Pogorelov, B.T. Schwartz, Y. Zhang and H. Zhang, "Single Pass Electron Cooling Simulations for MEIC," *NA-PAC*, TUPHO02 (2013).

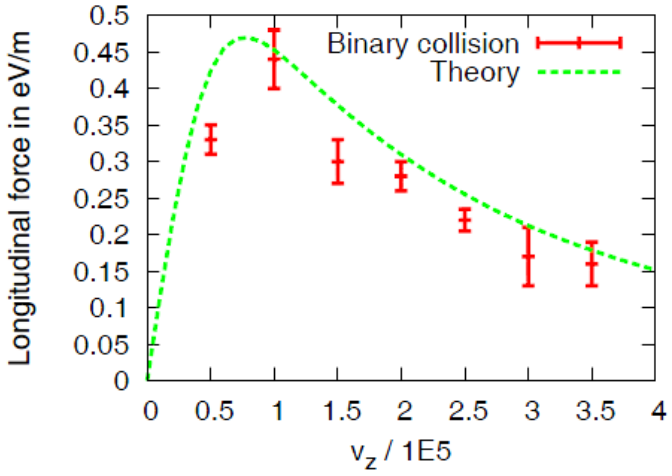


Figure 2: Theoretical longitudinal friction, eq. (2) versus that obtained from the binary collision model.

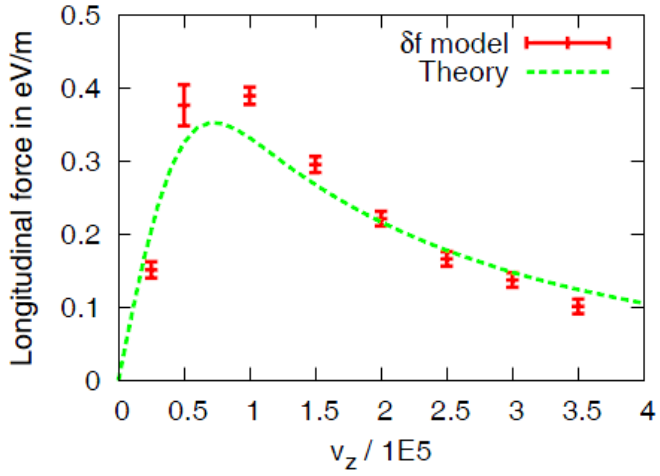


Figure 4: Theoretical longitudinal friction, eq. (2) versus that obtained from the δf model.

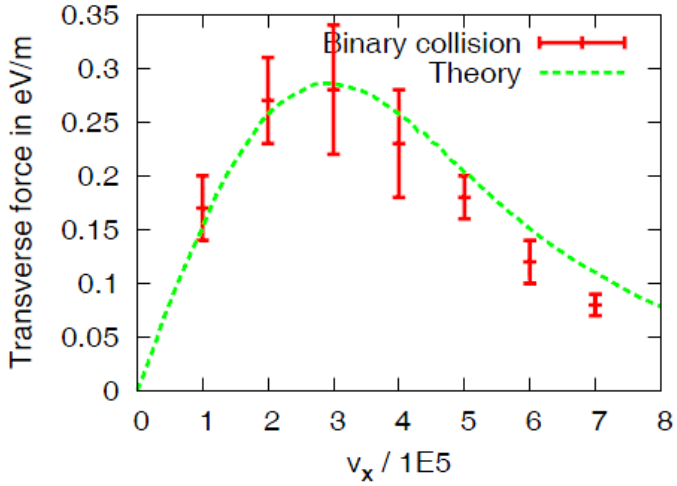


Figure 3: Theoretical transverse friction versus that obtained from the binary collision model.

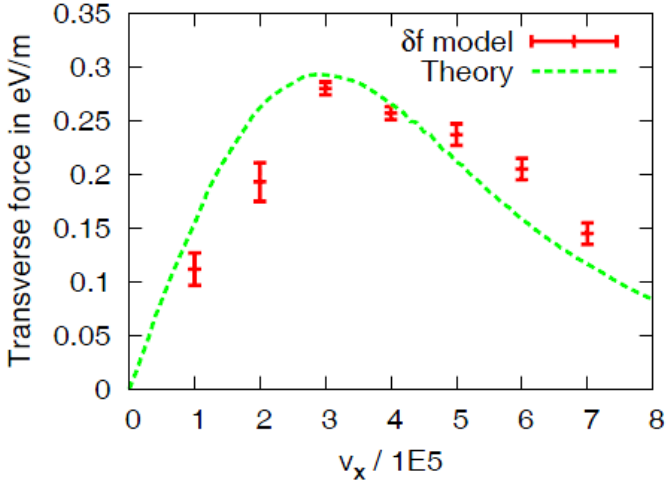


Figure 5: Theoretical transverse friction versus that obtained from the δf model.

What needs to be done next

- Use importance sampling method (ISM) [Sobol *et al.*]
  - *simulate one parameter set at a time*
    - choose impact parameters to optimally sample the full range
    - integrate saved results over the full range of impact parameters
  - *orders of magnitude faster than an 'integrated' simulation*
- Generalize ISM to the case of magnetized friction
  - *simulate one parameter set at a time*
    - can no longer use analytic binary collision algorithm
    - need to simulate ~10 magnetized electrons and one ion
- Problems to be studied
  - *magnetic field errors; space charge forces*
  - *coherent longitudinal features of simulated electron distrib.'s*
  - *weak to strong magnetization*
- Goals
  - *improve semi-analytic models in BETACOOL and MOCAC*
  - *find parametric equation(s) that is validated at high* g
  - *reduce technical risk for MEIC and other EIC designs*

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Most of the simulations discussed here were conducted using the parallel VORPAL framework (now known as VSim) [1,2,3], including the implementation of several algorithms (Hermite, BCC, δf-PIC, Vlasov/Poisson), and I acknowledge the important contributions of John Cary and all members of the VORPAL development team at Tech-X Corp.

[1] C. Nieter and J.R. Cary, "VORPAL: a versatile plasma simulation code," J. Comput. Phys.196, 448 (2004)

[2] G.I. Bell, D.L. Bruhwiler, A. Fedotov, A. Sobol, R.S. Busby, P. Stoltz, D.T. Abell, P. Messmer, I. Ben-Zvi and V. Litvinenko, "Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam," J. Comput. Phys. 227, 8714 (2008).

[3] The VSim website; http://www.txcorp.com/home/vsim/vsim-overview