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ELECTRON COOLING OF POSITRONS

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This article deals with the relaxation of a positron beam in electron gas moving in a magnetic field. A situation is considered where the electron longitudinal temperature is lower than the transverse one. Expressions for the friction force and scattering coefficients of positron momenta have been found, with plasma shielding taken into account. In a strong magnetic field the effective cross section of the cooling process of the longitudinal motion of positrons is shown to be determined by their longitudinal velocity relative to the electron gas. The similarity of frequencies of Larmor rotation of electrons and positrons sharply increases in its turn the cooling intensity of the transverse degree of freedom of positrons. These circumstances can be used for multiple acceleration of high-temperature positron beam cooling.

1. Introduction

The electron cooling method [1], which has shown good performance in high energy physics as a method of accumulation of intense cold beams of charged particles, has been developed theoretically and experimentally [2–6]. However, so far the studies have dealt with cooling heavy particles, mainly protons and antiprotons. Electron cooling of positrons has not been considered yet, because this problem is to some extent of rather exotic nature. Nevertheless it arises, for example, when there exists a problem of obtaining antihydrogen [6,7] and utilizing electron cooling as a means of accumulating an intense cold beam of both antiprotons and positrons.

In this paper we present a theory of cooling hot positrons which move in electron plasma in a homogeneous magnetic field (the presence of strong magnetic fields in the region of electron cooling is a necessary condition for this process to be effective). The questions of how this process can be employed for cooling the positron beam and for obtaining antihydrogen and how efficient it might be are discussed in ref. [7].

2. The kinetic equation for the interaction of positrons and electron plasma in a magnetic field

Electron–positron collisions in a strong magnetic field are to be considered qualitatively before calculating positron deceleration and diffusion in an electron plasma.

It is known that under the Coulomb interaction the momentum and energy exchange of colliding particles is logarithmically diverges in the range of large impact parameters and should cease at a certain macroscopic parameter, after which the interaction turns out to be effectively attenuated. In the absence of the magnetic field this parameter is the dynamic-screening radius $\rho_d = V/\omega_e$ and, hence, the maximum collision time proves to be roughly equal to the period of Langmuir oscillations.

If the plasma is in a magnetic field where the Larmor rotation frequency Ω is considerably higher than the plasma-oscillation frequency ω_e, the major contribution to the collision integral can be made by the region of impact parameters p such that $p > U/\Omega$ (U the a relative velocity of Larmor circles). In this case,

the collisions are slow relative to the Larmor rotation, with many cycles involved. Besides, if the condition $\rho_d \gg \rho$ (ρ is the Larmor radius of a positron) is satisfied, the intensity of energy exchange of a positron and an electron is only determined by the relative velocity of their Larmor circles. It is worth noting that in this region of impact parameters (the region of adiabatic collisions) the Larmor degree of freedom of electrons is effectively excluded from interaction kinetics and dynamics and is not involved in momentum and energy exchange when heavy particles are cooled. When positrons are cooled, this peculiarity is kept to the longitudinal motion alone. For the transverse motion, the situation is different since the frequencies of positron and electron Larmor rotation are equal (with an accuracy up to the energy dependence of the cyclotron frequency) and the interaction of transverse degrees of freedom is of the resonant nature. Thus, the effect of increasing positron cooling decrement must be observed in a strong magnetic field, at a small spread in relative velocities of Larmor circles of electrons and positrons ($\Delta V_{\parallel} \ll \Delta V_{\perp}$).

If the radius of dynamic screening appears to be less than the Larmor radius of a positron, the intensity of electron–positron energy exchange becomes independent of the relative velocity of Larmor circles. This is associated with the fact that the increase of collision time, as U reduces, is exactly compensated for by the ρ/ρ_d decrease of the interaction intensity. As a result, the rate of changing squared longitudinal and transverse momenta of the positron turns out to be proportional to $\rho_d/\rho U$.

Now we proceed to the calculation of the cooling process of positrons in a electron plasma in a magnetic field. An electron beam reference system is used here (with nonrelativistic motion of electrons and positrons). We will use the assumption that physical quantities, which evolve according to exact equations, are averaged over initial microscopic conditions with the probability distribution $D(\Gamma, 0)$. Consider the evolution of the positron density f in six-dimensional phase space, which, according to the motion laws, satisfies the Liouville equation:

$$\frac{\partial}{\partial t} f + \frac{\partial}{\partial \boldsymbol{r}} \boldsymbol{v} f + \frac{\partial}{\partial \boldsymbol{P}} \boldsymbol{F} f = 0.$$

Averaging this equation we obtain

$$\frac{\partial}{\partial t} \bar{f} + \frac{\partial}{\partial \boldsymbol{r}} \boldsymbol{v} \bar{f} + \frac{\partial}{\partial \boldsymbol{P}} \boldsymbol{F}_{\mathrm{r}} \bar{f} + \frac{\partial}{\partial \boldsymbol{P}} \bar{\boldsymbol{F}}_{\mathrm{s}} \bar{f} = -\frac{\partial}{\partial \boldsymbol{P}} \overline{\delta \boldsymbol{F}_{\mathrm{s}} \delta f}. \tag{1}$$

Here $\bar{f}$ is the density of the probability distribution of positrons or the single-particle distribution function; $\boldsymbol{F}_{\mathrm{r}}$ is the regular part of the force acting from external fields; $\boldsymbol{F}_{\mathrm{s}}$ is the force caused by the positron–electron interaction; $\delta f = f - \bar{f}$, $\delta \boldsymbol{F}_{\mathrm{s}} = \boldsymbol{F}_{\mathrm{s}} - \bar{\boldsymbol{F}}_{\mathrm{s}}$ (the vinculum means averaging over the initial velocities and coordinates of electrons). The expression for the right-hand side of eq. (1) can be derived if one assumes that the force $\boldsymbol{F}_{\mathrm{s}}$ is small enough and its influence on the positron motion can be taken into account using the perturbation theory. Then, in first approximation, we have

$$\delta f = -\int_0^t \frac{\partial}{\partial \boldsymbol{P}_{t-\tau}} \{ \delta \boldsymbol{F}_{\mathrm{s}}(t-\tau) \bar{f}(t-\tau) \} \, \mathrm{d}\tau,$$

and the expression for the right-hand side of eq. (1) is of the form

$$\frac{\partial}{\partial P_i} \left[\int_0^t \frac{\partial P_k(t)}{\partial P_j(t-\tau)} \overline{\delta F_{\mathrm{s}}^i(t) \delta F_{\mathrm{s}}^j(t-\tau)} \frac{\partial}{\partial P_k(t)} \bar{f}(t-\tau) \, \mathrm{d}\tau \right].$$

If the single-particle distribution function $\bar{f}$ depends only on conservative integrals of unperturbed motion, the collision integral proves to be equal to

$$\mathrm{st}\, \bar{f} \equiv -\frac{\partial}{\partial \boldsymbol{P}} \overline{\delta \boldsymbol{F}_{\mathrm{s}} \cdot \delta f} = \frac{\partial}{\partial P_i} D_{ij} \frac{\partial}{\partial P_j} \bar{f},$$

where

$$D_{ij} = \int_0^t \frac{\partial P_i(t)}{\partial P_k(t-\tau)} \overline{\delta F_{\mathrm{s}}^j(t) \delta F_{\mathrm{s}}^k(t-\tau)} \, \mathrm{d}\tau.$$

For the motion in a magnetic field the following relations hold:

$$\begin{aligned}
&D_{zz} = \int_0^t \overline{\delta F_s^z(t)\delta F_s^z(t-\tau)}\mathrm{d}\tau, \quad D_{yz} = \int_0^t \overline{\delta F_s^y(t)\delta F_s^z(t-\tau)}\mathrm{d}\tau,\\
&D_{xz} = \int_0^t \overline{\delta F_s^x(t)\delta F_s^z(t-\tau)}\mathrm{d}\tau,\\
&D_{zy} = \int_0^t \left[\cos\tilde{\omega}\tau\overline{\delta F_s^z(t)\delta F_s^y(t-\tau)} + \sin\tilde{\omega}\tau\overline{\delta F_s^z(t)\delta F_s^x(t-\tau)}\right]\mathrm{d}\tau,\\
&D_{yy} = \int_0^t \left[\cos\tilde{\omega}\tau\overline{\delta F_s^y(t)\delta F_s^y(t-\tau)} + \sin\tilde{\omega}\tau\overline{\delta F_s^y(t)\delta F_s^x(t-\tau)}\right]\mathrm{d}\tau,\\
&D_{xy} = \int_0^t \left[\cos\tilde{\omega}\tau\overline{\delta F_s^x(t)\delta F_s^y(t-\tau)} + \sin\tilde{\omega}\tau\overline{\delta F_s^x(t)\delta F_s^x(t-\tau)}\right]\mathrm{d}\tau,\\
&D_{zx} = \int_0^t \left[\cos\tilde{\omega}\tau\overline{\delta F_s^z(t)\delta F_s^x(t-\tau)} - \sin\tilde{\omega}\tau\overline{\delta F_s^z(t)\delta F_s^y(t-\tau)}\right]\mathrm{d}\tau,\\
&D_{yx} = \int_0^t \left[\cos\tilde{\omega}\tau\overline{\delta F_s^y(t)\delta F_s^x(t-\tau)} - \sin\tilde{\omega}\tau\overline{\delta F_s^y(t)\delta F_s^y(t-\tau)}\right]\mathrm{d}\tau,\\
&D_{xx} = \int_0^t \left[\cos\tilde{\omega}\tau\overline{\delta F_s^x(t)\delta F_s^x(t-\tau)} - \sin\tilde{\omega}\tau\overline{\delta F_s^x(t)\delta F_s^y(t-\tau)}\right]\mathrm{d}\tau,
\end{aligned} \tag{2}$$

where $\tilde{\omega} = ZeB/E$ is the Larmor rotation frequency of a particle with charge Ze and energy E. Here, as follows from the kinetic equation, the rate of change of motion integrals P_z^2 and $P_x^2 + P_y^2 \equiv P_\perp^2$ of "test" particles interacting with the electron plasma in a magnetic field, is determined by the following relations:

$$\begin{aligned}
\frac{\mathrm{d}P_z^2}{\mathrm{d}t} &= 2(\boldsymbol{F}_{\mathrm{fr}} + \boldsymbol{F}_{\mathrm{fl}})\boldsymbol{P}_z + d_{zz},\\
\frac{\mathrm{d}P_\perp^2}{\mathrm{d}t} &= 2(\boldsymbol{F}_{\mathrm{fr}} + \boldsymbol{F}_{\mathrm{fl}})\boldsymbol{P}_\perp + d_\perp .
\end{aligned} \tag{3}$$

The expressions (2) are true for particles with any charge, mass and nonrelativistic motion energy. Further, $\boldsymbol{F}_{\mathrm{fr}}$, $\boldsymbol{F}_{\mathrm{fl}}$, d_{zz} and $d_\perp$ will be referred to as friction force, fluctuation force and diffusion coefficients, respectively. These quantities give a fairly complete understanding of the rate of relaxation of the positrons to a certain steady state. Its properties will be discussed below. In order to derive explicit expressions for friction force and diffusion coefficients, we assume the following as far as the properties of a single-component electron plasma are concerned.

(1) There is no correlation in the arrangement of electrons in the phase space at the initial moment.
(2) The single-particle electron distribution function depends only on P_z and $P_\perp^2$.
(3) $e^2 n^{1/3} < mv^2/2$, where $mv^2/2$ is the characteristic kinetic energy of the relative motion of electrons (in the magnetization region: electron circles) and $e^2 n^{1/3}$ are the characteristic fluctuations of their potential energy.

From the first two assumptions it follows that the friction force $\boldsymbol{F}_{\mathrm{fr}}$ coincides with the force at which the "test" particle acts on itself via electromagnetic fields induced in the electron plasma (note that it arises in the second order with respect to the interaction constant ($\sim e^4$), while $\delta\boldsymbol{F}_s$ corresponds to the force at which plasma electrons, unperturbed by the "test" particle, act on the latter). The contribution of friction-force fluctuations to the diffusion tensor is of a higher order of smallness with respect to the interaction constant e. The third assumption allows to use perturbation theory to calculate the fields created by plasma electrons. Thus the problem of finding the friction force, fluctuation force and diffusion coefficients reduces to finding the fields induced by the "test" particle in a single-component electron plasma; on the basis of assumptions (1)–(3) one can obtain the fields created by electrons using the substitution $Ze \to e$ and $\boldsymbol{r}(t) \to \boldsymbol{r}_a(t)$, where $\boldsymbol{r}(t)$ is an unperturbed, "test"-particle trajectory in a magnetic field and $\boldsymbol{r}_a(t)$ is an electron motion trajectory.

3. Friction and diffusion in the weak-interaction approximation

In the nonrelativistic weak-interaction approximation with the electron–electron collisions being neglected, there is a known connection between the Fourier components of the density of a free charge in plasma and the electric-field vector induced by it:

$$E_{k,\omega} = -4\pi i \rho_{k,\omega} k / k^2 \epsilon_l(\omega, k), \tag{4}$$

where $\epsilon_l = k_i k_j \epsilon_{ij}/k^2$ is the dielectric constant of a single-component electron plasma in a magnetic field:

$$\epsilon_l = 1 + \omega_e^2 \int_0^\infty \left[\frac{k_\parallel^2}{k^2} + \frac{k_\perp^2 \sin \Omega\tau}{k^2 \Omega\tau} \right] \left\langle J_0\left(2k_\perp r_\perp^a \sin\frac{\Omega\tau}{2} \right) e^{i(\omega - k_\parallel v_{e\parallel})\tau} \tau \, d\tau \right\rangle .$$

Here $k_\parallel$ and $k_\perp$ are the longitudinal and transverse components of the wave vector k respectively, relative to the magnetic field; Ω is the Larmor rotation frequency of electrons, $\omega_e = \sqrt{4\pi e^2 n/m}$ is the plasma frequency (e, m and n are charge, mass and density of electrons, respectively); the French quotes denote averaging over the distribution of the longitudinal and transverse electron velocities $v_{e\parallel}$, $v_{e\perp} = r_\perp^a \Omega$. Eq. (4) is written in the "nearly longitudinal" field approximation which is satisfied if $\omega/k \ll c/\sqrt{\epsilon}$. The applicability limit of describing the behaviour of a plasma by means of the collisionless dielectric constant will be discussed further.

With eq. (4) taken into account, the friction force acting on the "test" particle with charge Ze and the force correlation tensor $\langle \delta F_s^i(t) \delta F_s^j(t-\tau) \rangle$, which appears in the expression for the diffusion tensor D_{ij}, are as follows:

$$F_{fr} = ZeE[r(t)] = -Ze \int ik \langle \phi_k(t) \rangle e^{ikr(t)} d^3k \left\langle \delta F_s^i(t) \delta F_s^j(t-\tau) \right\rangle$$

$$= (2\pi)^3 Z^2 e^2 n \int \langle \phi_{ka}(t) \phi_{ka}(t-\tau) \rangle k_i k_j e^{ik[r(t) - r(t-\tau)]} d^3k, \tag{5}$$

where

$$\langle \phi_k(t) \rangle = \frac{Ze}{2\pi^2 k^2} \int_c d\omega \frac{L(\omega, k)}{\epsilon_l(\omega, k)} e^{-i\omega t} \tag{6}$$

is the spatial Fourier image of the potential induced by the "test" particle in the electron plasma;

$$\phi_{ka}(t) = \frac{e}{2\pi^2 k^2} \int_c d\omega \frac{L_a(\omega, k)}{\epsilon_l(\omega, k)} e^{-i\omega t} \tag{7}$$

is the same for a particular electron;

$$L(\omega, k) = \frac{1}{2\pi} \int_0^\infty e^{-ikr(t) + i\omega\tau} d\tau,$$

the integration contour lying in the complex plane above all the peculiarities of the integrals (6) and (7).

It is convenient to analyse the expressions for friction force and coefficients in two limiting cases: short ($\omega_e t \ll 1$) and long ($\omega_e t \gg 1$) electron–electron and positron–electron interaction, assuming the first and the second to be "switched on" simultaneously. Actually, the electron–electron interaction is "switched on" somewhat earlier, but this difference is insignificant in a stable electron beam, which allows theoretical consideration of a simpler situation. The first case corresponds to a relatively shorter section of electron cooling in a positron storage ring, while the second corresponds to a longer one.

3.1. Small interaction times (interaction without screening)

In this case, the expression for $\langle \phi_k(t) \rangle$ and $\phi_{ka}(t)$ will take the form:

$$\langle \phi_k(t) \rangle = -\frac{Ze\omega_e^2}{2\pi^2 k^2} \int_0^t \tau \, d\tau \chi(\tau) \, e^{-i k r(t-\tau)}, \tag{8}$$

$$\phi_{ka}(t) = \frac{e}{2\pi^2 k^2} e^{-i k r_a(t)}, \tag{9}$$

where

$$\chi(\tau) = \left(\frac{k_\parallel^2}{k^2} + \frac{k_\perp^2 \sin \Omega\tau}{k^2 \Omega\tau} \right) \left\langle J_0\left(2k_\perp r_\perp^a \sin\frac{\Omega\tau}{2}\right) e^{-i k_\parallel v_{e\parallel} \tau} \right\rangle.$$

Note that $\phi_{ka}(t)$ corresponds to the Coulomb potential of a free electron moving in a magnetic field, and $\langle \phi_k(t) \rangle$ corresponds to the potential which is created by an electron "cloud" induced by plasma charge at the time $t \ll \omega_e^{-1}$. Taking eqs. (8) and (9) into account, we obtain

$$\boldsymbol{F}_{fr} = (Ze\omega_e)^2 \int \frac{i\boldsymbol{k} \, d^3k}{2\pi^2 k^2} \int \tau \, d\tau \left\{ \left(\frac{k_\parallel^2}{k^2} + \frac{k_\perp^2 \sin \Omega\tau}{k^2 \Omega\tau} \right) \right.$$
$$\left. \times \left\langle J_0\left(2k_\perp r_\perp^a \sin\frac{\Omega\tau}{2}\right) e^{i k_\parallel \tau (v_\parallel - v_{e\parallel})} \right\rangle e^{i \boldsymbol{k}_\perp [\boldsymbol{r}_\perp(t) - \boldsymbol{r}_\perp(t-\tau)]\tau} \right\}, \tag{10}$$

$$\langle \delta F_s^i(t) \delta F_s^j(t-\tau) \rangle = \frac{2Z^2 e^4 n}{\pi} \int e^{i \boldsymbol{k}_\perp [\boldsymbol{r}_\perp(t) - \boldsymbol{r}_\perp(t-\tau)]}$$
$$\times \left\langle J_0\left(2k_\perp r_\perp^a \sin\frac{\Omega\tau}{2}\right) e^{i k_\parallel \tau (v_\parallel - v_{e\parallel})} \right\rangle \frac{k_i k_j}{k^4} d^3k, \tag{11}$$

$$\boldsymbol{F}_{fl} = \frac{m}{M} (Ze\omega_e)^2 \int \frac{i\boldsymbol{k} \, d^3k}{2\pi^2 k^2} \int \tau \, d\tau \left\{ \left(\frac{k_\parallel^2}{k^2} + \frac{k_\perp^2 \sin \Omega\tau}{k^2 \Omega\tau} \right) \right.$$
$$\left. \times \left\langle J_0\left(2k_\perp r_\perp^a \sin\frac{\Omega\tau}{2}\right) e^{i k_\parallel \tau (v_\parallel - v_{e\parallel})} \right\rangle e^{i \boldsymbol{k}_\perp [\boldsymbol{r}_\perp(t) - \boldsymbol{r}_\perp(t-\tau)]} \right\}. \tag{12}$$

Here M is the mass of the "test" particle and $v_\parallel$ is its velocity along the magnetic field, m and $v_{e\parallel}$ are the same for electrons, and $J_0(x)$ is a Bessel function of zero index.

It is worth noting that the ratio of friction-to-fluctuation force turns out to be of the order M/m and, hence, the action of the latter may be neglected for heavy particles ($M \gg m$). As far as positrons are concerned, these forces in the unscreened interaction approximation are equal for them. Assuming that $M = m$ and $\tilde{\omega} = -\Omega$, and with the help of eqs. (10)–(12), we find

$$d_\parallel = \frac{4e^4 n}{\pi} \int \frac{k_\parallel^2 \, d^3k}{k^4} \int_0^t d\tau \left\langle J_0\left(2k_\perp r_\perp^a \sin\frac{\Omega\tau}{2}\right) e^{i k_\parallel \tau (v_\parallel - v_{e\parallel})} \right\rangle e^{i \boldsymbol{k}_\perp [\boldsymbol{r}_\perp(t) - \boldsymbol{r}_\perp(t-\tau)]}$$
$$\equiv 4\frac{e^4 n}{\pi} \int \frac{k_\parallel^2 \, d^3k}{k^4} \int_0^t g_k(\tau) \, d\tau,$$

$$d_\perp = \frac{4e^4 n}{\pi} \int \frac{k_\perp^2 \, d^3k}{k^4} \int_0^t g_k(\tau) \cos \Omega\tau \, d\tau,$$

$$2F_\parallel P_\parallel = 2(F_{fr\parallel} + F_{fl\parallel}) P_\parallel = 4F_{fr\parallel} P_\parallel = \frac{8e^4 n}{\pi} \int \frac{i k_\parallel v_\parallel \, d^3k}{k^2} \int_0^t g_k(\tau) \left(\frac{k_\parallel^2}{k^2} + \frac{k_\perp^2 \sin \Omega\tau}{k^2 \Omega\tau} \right) \tau \, d\tau,$$

$$2\boldsymbol{F}_\perp\boldsymbol{P}_\perp = \frac{8e^4n}{\pi}\int\frac{\mathrm{i}\boldsymbol{k}_\perp\boldsymbol{v}_\perp}{k^2}\mathrm{d}^3k\int g_k(\tau)\left(\frac{k_\parallel^2}{k^2}+\frac{k_\perp^2\sin\Omega\tau}{k^2\Omega\tau}\right)\tau\,\mathrm{d}\tau,$$

$$g_k(\tau) \equiv \left\langle J_0\left(2k_\perp r_\perp^{\mathrm{a}}\sin\frac{\Omega\tau}{2}\right)\mathrm{e}^{\mathrm{i}k_\parallel\tau(v_\parallel - v_{\mathrm{e}\parallel})}\right\rangle \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp[\boldsymbol{r}_\perp(t)-\boldsymbol{r}_\perp(t-\tau)]}.$$

Integrating the above expressions over ϕ and $k_\parallel$ we obtain:

$$d_\perp = 4\pi e^4 n\left\langle\int\mathrm{d}k_\perp\int_0^t[1+k_\perp|U|\tau]\cos\Omega\tau\cdot J(\tau)\,\mathrm{d}\tau\right\rangle,$$

$$d_\parallel = 4\pi e^4 n\left\langle\int\mathrm{d}k_\perp\int_0^t[1-k_\perp|U|\tau]J(\tau)\,\mathrm{d}\tau\right\rangle,$$

$$2F_\parallel P_\parallel = 2v_\parallel\frac{\partial d_\parallel}{\partial v_\parallel}+\frac{8\pi e^4 n v_\parallel}{\Omega}\left\langle\int k_\perp^2\,\mathrm{d}k_\perp\int_0^t\sin\Omega\tau\cdot J(\tau)U\tau\,\mathrm{d}\tau\right\rangle, \tag{13}$$

$$2\boldsymbol{F}_\perp\boldsymbol{P}_\perp = 8\pi e^4 n\int\mathrm{d}k_\perp\int_0^t\mathrm{d}\tau\left[\left(1+\frac{\sin\Omega\tau}{\Omega\tau}\right)-\left(1-\frac{\sin\Omega\tau}{\Omega\tau}\right)k_\perp|U|\tau\right]$$
$$\times\mathrm{e}^{-k_\perp|U|\tau}J_0\left(2k_\perp r_\perp^{\mathrm{a}}\sin\frac{\Omega\tau}{2}\right)\frac{\mathrm{d}}{\mathrm{d}\tau}J_0\left(2k_\perp\rho\sin\frac{\Omega\tau}{2}\right),$$

$$J \equiv J_0\left(2k_\perp r_\perp^{\mathrm{a}}\sin\frac{\Omega\tau}{2}\right)J_0\left(2k_\perp\rho\sin\frac{\Omega\tau}{2}\right)\mathrm{e}^{-k_\perp|U|\tau}.$$

Here ρ is the Larmor radius of the positron and $U = v_\parallel - v_{\mathrm{e}\parallel}$.

The entire region of impact parameters of electron–positron collisions is likely to be divided into a region of fast collisions, for which the cyclicity of positron and electron motion in a magnetic field is insignificant, and a region of slow collisions, when the particles manage to make a great deal of Larmor rotations during the collision. The integration interval $K_{\max} > k_\perp > \Omega/U$ corresponds to the first region, and $\Omega/U > k_\perp > 0$ to the second one.

In the fast-collision region the time integrals in eqs. (13) converge on the intervals $\tau \sim (k_\perp U)^{-1} \ll \Omega^{-1}$. In this case, substituting $\sin\Omega\tau \to \Omega\tau$ and returning to the integral representation of Bessel functions for zero index, we obtain expressions for friction force and diffusion coefficients which correspond to charged-particle collisions in the absence of a magnetic field [3].

In the slow-collision region $k_\perp U < \Omega$ and under the condition $\Omega t \gg 1$ (otherwise the magnetic field exerts an inessential effect on the collision kinetics) the integral in eq. (13) can be averaged over the period of electron and positron Larmor rotation in the magnetic field. Note that at $\Omega t \gg 1$ averaging over the Larmor rotation can be performed after averaging over the time. The difference of the expressions obtained from eqs. (14) and (15) proves to be relatively small in the $k_\perp U \ll \Omega$ region. Integrating over the time and substituting the obtained values into eq. (3) we derive the following expressions, which define the rates of change of the squares of longitudinal and transverse momenta of the positron slowly colliding with electrons during its motion in the plasma (note that these are slow collisions with respect to the Larmor rotation):

$$\frac{\mathrm{d}P_\parallel^2}{\mathrm{d}t} = \left(1+2v_\parallel\frac{\partial}{\partial v_\parallel}\right)4\pi e^4 nt\left\langle\int_0^{\Omega/|U|}\mathrm{d}k_\perp\,\mathrm{e}^{-k_\perp|U|t}\sum_l J_l^2(k_\perp\rho)J_l^2(k_\perp r_\perp^{\mathrm{a}})\right\rangle, \tag{14}$$

$$\frac{\mathrm{d}P_\perp^2}{\mathrm{d}t} = 4\pi e^4 n\left\langle\int_0^{\Omega/|U|}\frac{\mathrm{d}k_\perp}{k_\perp|U|}\left[2-(2+k_\perp|U|t)\,\mathrm{e}^{-k_\perp|U|t}\right]\right.$$
$$\left.\times\sum_{l=0}^{\infty}(2l+1)\left[J_l^2(k_\perp\rho)J_{l+1}^2(k_\perp r_\perp^{\mathrm{a}})-J_l^2(k_\perp r_\perp^{\mathrm{a}})J_{l+1}^2(k_\perp\rho)\right]\right\rangle. \tag{15}$$

Our further analysis will be confined to the case when positron transverse velocities are considerably higher than their longitudinal velocities. In expression (15) the upper integration limit is assumed to be equal to ∞. The same can be done when integrating eq. (14) regardless of the relationship between positron longitudinal and transverse velocities, since the characteristic divergence interval is $k_\perp \sim 1/|U|t \ll \Omega/|U|$. The procedure of extending the integration in eqs. (14) and (15) to the region of infinitely small impact parameters is correct in the perturbation-theory approximation if expressions (16) and (17) thus obtained are such that changes in squares of transverse and longitudinal positron momenta for the characteristic times of formation or action of friction and diffusion ($\tau \sim \min(t, \rho/|U|)$ turn out to be relatively small. Using the approximate relation valid at $\rho \gg r_\perp$:

$$\sum_{l=-\infty}^{\infty} J_l^2(k_\perp \rho) J_l^2(k_\perp r_\perp) \simeq \begin{cases} 1, & k_\perp \rho \ll 1, \\ \dfrac{1}{\pi k_\perp \rho}, & k_\perp \rho \gg 1, \end{cases}$$

we obtain

$$\frac{dP_\parallel^2}{dt} = 4\pi e^4 n \begin{cases} \left(1 + 2v_\parallel \dfrac{\partial}{\partial v_\parallel}\right)\langle 1/|U|\rangle, & \rho \ll |U|t, \\ \dfrac{4t}{\pi\rho}\left(1 + 2v_\parallel \dfrac{\partial}{\partial v_\parallel}\right)\langle \ln(\rho/|U|t)\rangle, & \rho \gg |U|t. \end{cases} \tag{16}$$

Now consider eq. (15) for the rate of change of the square of the transverse positron momentum, which is conveniently rewritten as follows:

$$\frac{dP_\perp^2}{dt} = 4\pi e^4 n \int \frac{dk_\perp}{k_\perp^3 |U|}\left[2 - (2 + k_\perp |U| t)\, e^{-k_\perp |U| t}\right]\left(\frac{\partial}{\partial \rho^2} - \frac{\partial}{\partial (r_\perp^a)^2}\right)$$
$$\times 2\sum_l l^2 J_l^2(k_\perp r_\perp^a) J_l^2(k_\perp \rho).$$

Using the approximate relation

$$\sum_l l^2 J_l^2(k_\perp \rho) J_l^2(k_\perp r_\perp^a) = \begin{cases} k_\perp^4 (r_\perp^a)^2 \rho^2/8, & k_\perp \rho \ll 1, \\ k_\perp^2 r_\perp^a/2\pi\rho, & k_\perp \rho \gg 1, \end{cases}$$

we find

$$\frac{dP_\perp^2}{dt} = -4\pi e^2 n \begin{cases} \langle 1/|U|\rangle, & \rho/|U|t \ll 1, \\ \dfrac{4t}{\pi\rho}\langle \ln(\rho/|U|t)\rangle, & \rho/|U|t \gg 1. \end{cases} \tag{17}$$

In the inverse case of very small Larmor radii of positrons, they will increase, the rate of this process being obtained from eq. (17) by replacing $\rho \to r_\perp^a$ and by changing the sign. In this case the rate of change of squares of longitudinal positron momenta is also derived from the obtained eq. (16) by substitution $\rho \to r_\perp^a$.

So far we have assumed that Larmor frequencies of electrons and positrons are equal, but this is not true because their energies differ. This effect may appear to be significant even in the nonrelativistic case. Its influence on the kinetics of electron cooling can be neglected, i.e. $\Delta\Omega \ll k_m |U|$, where $\Delta\Omega$ is the characteristic spread in Larmor frequencies, k_m is the maximum impact parameter of the effective

Coulomb interaction: $k_{\mathrm{m}} = \max(1/|U|t,\ k_x)$, k_x is the characteristic divergence interval for the integrals (14) and (15).

Otherwise,

$$\frac{\mathrm{d}P_{\|}^2}{\mathrm{d}t} = 4\pi e^4 nt\left(1 + 2v_{\|}\frac{\partial}{\partial v_{\|}}\right)\left\langle \int \mathrm{d}k_{\perp} J_0^2(k_{\perp}\rho) J_0^2(k_{\perp} r_{\perp}^{\mathrm{a}})\, \mathrm{e}^{-k_{\perp}|U|t} \right\rangle$$

$$\simeq 4\pi e^4 n\left(1 + 2v_{\|}\frac{\partial}{\partial v_{\|}}\right)\begin{cases} 1/|U|\rangle, & |U|t \gg \rho \gg r_{\perp}^{\mathrm{a}}, \\ \dfrac{4t}{\pi\rho}\langle \ln(\rho/|U|t)\rangle, & \rho \gg |U|t \gg r_{\perp}, \\ \dfrac{4t}{\pi\rho}\langle \ln(\rho/|U|t)\rangle, & \rho \gg r_{\perp}^{\mathrm{a}} \gg |U|t; \end{cases}$$

$$\frac{\mathrm{d}P_{\perp}^2}{\mathrm{d}t} = -4\pi e^4 n\left\langle \int \mathrm{d}k_{\perp} \int_0^t [1 + k_{\perp}|U|\tau] J_0^2(k_{\perp} r_{\perp}) J_0^2(k_{\perp}\rho) \cos \Delta\Omega\tau\, \mathrm{e}^{-k_{\perp}|U|\tau}\, \mathrm{d}\tau \right\rangle$$

$$= -4\pi e^4 n\begin{cases} \langle 1/|U|\rangle, & \rho \ll |U|/\Delta\Omega, \\ 1/3\pi\rho\, \Delta\Omega, & \rho \gg |U|/\Delta\Omega \gg r_{\perp}^{\mathrm{a}}, \\ |U|/\pi^2\rho r_{\perp}^{\mathrm{a}}(\Delta\Omega)^2, & \rho \gg r_{\perp}^{\mathrm{a}} \gg |U|/\Delta\Omega. \end{cases}$$

3.2. Large interaction times (interaction under plasma screening)

This section will begin with the analysis of the friction force acting on a nonrelativistic positron moving in a single-component electron plasma. The spatial Fourier component of the Coulomb potential, induced by a positron, can be written as follows:

$$\phi_k(t) = \frac{e}{2\pi^2 k^2}\sum_l J_l^2(k_{\perp}\rho)\left\{ \frac{\mathrm{e}^{-\mathrm{i}(k_{\|}v_{\|} + l\Omega)t - \mathrm{i}\psi_l}}{\epsilon_l(k_{\|}v_{\|} + l\Omega,\ \boldsymbol{k})} + \sum_s \frac{\mathrm{e}^{-\mathrm{i}\omega_s t - \mathrm{i}\psi_l}}{(\omega_s - k_{\|}v_{\|} - l\Omega)\ \partial\epsilon_l/\partial\omega_s} \right\}, \tag{18}$$

where ω_s is the solution of the dispersion equation $\epsilon_l(\omega_s,\ \boldsymbol{k}) = 0$. Eq. (18) has been derived from eq. (6) by integrating over ω along the contour lying above all ω_s. The scalar products of the friction force and the longitudinal and transverse components of the positron momentum prove to be equal correspondingly to

$$\boldsymbol{F}_{\mathrm{fr}} \cdot \boldsymbol{P}_{\|} = \frac{me^2}{\pi}\int \frac{v_{\|}k_{\|}k_{\perp}\ \mathrm{d}k_{\|}\ \mathrm{d}k_{\perp}}{k^2}\sum_l J_l^2(k_{\perp}\rho)$$
$$\times \mathrm{Im}\left[\frac{1}{\epsilon_l(k_{\|}v_{\|} + l\Omega,\ \boldsymbol{k})} + \sum_s \frac{\mathrm{e}^{-\mathrm{i}(\omega_s - k_{\|}v_{\|} - l\Omega)t}}{(\omega_s - k_{\|}v_{\|} - l\Omega)\ \partial\epsilon_l/\partial\omega_s} \right], \tag{19}$$

$$\boldsymbol{F}_{\mathrm{fr}} \cdot \boldsymbol{P}_{\perp} = \frac{me^2}{\pi}\int \frac{\Omega k_{\perp}\ \mathrm{d}k_{\|}\ \mathrm{d}k_{\perp}}{k^2}\sum_l lJ_l^2(k_{\perp}\rho)$$
$$\times \mathrm{Im}\left[\frac{1}{\epsilon_l(k_{\|}v_{\|} + l\Omega,\ \boldsymbol{k})} + \sum_s \frac{\mathrm{e}^{-\mathrm{i}(\omega_s - k_{\|}v_{\|} - l\Omega)t}}{(\omega_s - k_{\|}v_{\|} - l\Omega)\ \partial\epsilon_l/\partial\omega_s} \right], \tag{20}$$

where $\mathrm{Im}[\cdots]$ is the imaginary part of the expression in square brackets. Due to the action of friction on a charged particle the obtained regular energy losses of it are different from those presented in ref. [8] by the presence of the second, time-dependent term. This distinction is due to the fact that in ref. [8] the expression is given for stationary energy losses without transients caused by the finiteness of time during which the plasma responds to an external impact. However, one can show that this difference becomes insignificant for times far shorter than the characteristic damping time of the most "long-lived" plasma oscillations. Indeed, ω_s defined as the solutions of the dispersion equation $\epsilon_l = 0$ are complex quantities; the entire region of wave vectors $\boldsymbol{k}$ can be divided into two:

$$\mathrm{Im}\,\omega_s \gtrsim |\omega_s|; \quad \mathrm{Im}\,\omega_s \ll |\omega_s|.$$

The first corresponds to the energy exchange of a charged particle and electron plasma through collisions with separate electrons. To calculate the energy losses one may use eqs. (19) and (20) excluding the second term if

$$t \gtrsim 1/\mathrm{Im}\,\omega_s.$$

The second "transmittance" region corresponds to the interaction of positrons and "long-lived" plasma oscillations. In this case, assuming $\mathrm{Im}\,\epsilon_l = 0$, $\mathrm{Im}\,\omega_s = 0$ and

$$\mathrm{Im}\left[\epsilon_l^{-1}\right] = -\pi \sum_s \frac{\delta(k_\parallel v_\parallel + l\Omega - \omega_s)}{\partial\epsilon_l/\partial\omega_s},$$

we find

$$2\boldsymbol{F}_{\mathrm{fr}} \cdot \boldsymbol{P}_\parallel = -\frac{2me^2}{\pi} \int \frac{k_\parallel v_\parallel k_\perp \,\mathrm{d}k_\perp \,\mathrm{d}k_\parallel}{k^2} \sum_l J_l^2(k_\perp \rho) \sum_s \frac{\sin(\omega_s - k_\parallel v_\parallel - l\Omega)t}{(\omega_s - k_\parallel v_\parallel - l\Omega)\,\partial\epsilon_l/\partial\omega_s}, \tag{21}$$

$$2\boldsymbol{F}_{\mathrm{fr}} \cdot \boldsymbol{P}_\perp = -\frac{2me^2}{\pi} \int \frac{\Omega k_\perp \,\mathrm{d}k_\perp \,\mathrm{d}k_\parallel}{k^2} \sum_l lJ_l^2(k_\perp \rho) \sum_s \frac{\sin(\omega_s - k_\parallel v_\parallel - l\Omega)t}{(\omega_s - k_\parallel v_\parallel - l\Omega)\,\partial\epsilon_l/\partial\omega_s}. \tag{22}$$

Note that the electron plasma in question is stable in hydrodynamic approximation relative to the Coulomb interaction and, therefore, at $\mathrm{Im}\,\epsilon_l \to 0$, $\mathrm{Im}\,\omega_s \sim \mathrm{Im}\,\epsilon_l/(\partial\epsilon_l/\partial\omega_s) \to 0$. At $\omega_l/\Omega \ll 1$, $\omega_l t \ll 1$, $\Omega t \gg 1$ and $k_\perp r_\perp^{\mathrm{a}} \ll 1$ these expressions coincide with those derived in section 3.1. At $t \gg 1/(\omega_s - l\Omega)$ we again return to the expression for stationary energy losses, i.e. to eqs. (19) and (20) excluding the second term, since in this case

$$\frac{\sin(\omega_s - k_\parallel v_\parallel - l\Omega)t}{(\omega_s - k_\parallel v_\parallel - l\Omega)\partial\epsilon_l/\partial\omega_s} = \frac{\pi\delta(\omega_s - k_\parallel v_\parallel - l\Omega)}{\partial\epsilon_l/\partial\omega_s}.$$

Our further analysis of energy losses of the longitudinal and transverse motion under the action of friction will be concerned with the magnetization region of electron–positron interaction in a rather strong magnetic field ($T_\perp \omega_{\mathrm{e}}^2 \ll m v_\parallel^2 \Omega^2$). Here (as in the case $\omega_{\mathrm{e}} t \ll 1$) the major contribution to the collision integral will be made by the region of impact parameters $k_\perp r_\perp^{\mathrm{a}} \ll 1$. If longitudinal positron velocities are considerably higher than the characteristic spread of longitudinal velocities in the electron plasma, positrons will interact mainly via the excitation of plasma waves, the spectrum of which is defined from the equation

$$1 = \frac{\omega_{\mathrm{e}}^2}{k^2}\left(\frac{k_\parallel^2}{\omega^2} + \frac{k_\perp^2}{\omega^2 - \Omega^2}\right).$$

Substituting the solutions of this equation, $\omega^2 = \omega_{\mathrm{e}}^2 k_\parallel^2/k^2$ and $\omega^2 = \Omega^2 + \omega_{\mathrm{e}}^2 k_\perp^2/k^2$, into eqs. (21) and (22) and assuming

$$\frac{\sin(\omega_s - k_\parallel v_\parallel - l\Omega)t}{\omega_s - k_\parallel v_\parallel - l\Omega} = \pi\delta(\omega_s - k_\parallel v_\parallel - l\Omega), \quad t \to \infty,$$

we obtain

$$2\boldsymbol{F}_{\mathrm{fr}}\cdot\boldsymbol{P}_{\|}=\frac{-8\pi e^4 n}{\omega_{\mathrm{e}}}\int\frac{v_{\|}k_{\|}^2k_{\perp}\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^3}\left[J_0^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}|k_{\|}|}{k}\right)\right.$$
$$\left.+\frac{k_{\perp}^2\omega_{\mathrm{e}}}{k|k_{\|}|\Omega}J_1^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}^2k_{\perp}^2}{2\Omega k^2}\right)\right]$$
$$=-4\pi e^4 n\begin{cases}1/|v_{\|}|, & \omega_{\mathrm{e}}\rho\ll|v_{\|}|,\\ 2/\pi\omega_{\mathrm{e}}\rho, & \omega_{\mathrm{e}}\rho\gg|v_{\|}|;\end{cases}\tag{23}$$
$$2\boldsymbol{F}_{\mathrm{fr}}\cdot P_{\perp}=-8\pi e^4 n\int\frac{k_{\perp}^3\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}J_1^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}^2k_{\perp}^2}{2\Omega k^2}\right)$$
$$=-4\pi e^4 n\begin{cases}1/|v_{\|}|, & \omega_{\mathrm{e}}^2\rho\ll\omega|v_{\|}|,\\ 2\Omega/\omega_{\mathrm{e}}^2\rho\pi, & \omega_{\mathrm{e}}^2\rho\gg\Omega|v_{\|}|.\end{cases}$$

Substituting $\omega_{\mathrm{e}}^2k_{\perp}^2/2\Omega k^2\to\Delta\Omega$, one may obtain the expression for the transverse friction force from eq. (23) if Larmor positron and electron frequencies differ greatly. Note that energy losses caused by the friction on the positron correspond to those present in section 3.1 with an accuracy up to the substitution $t_{\|}\to\omega_{\mathrm{e}}^{-1}$, $t_{\perp}\to\Omega/\omega_{\mathrm{e}}^2$.

In the case of $v_{\|}\lesssim\Delta_{\|}$ the interaction of positrons and plasma waves becomes inefficient and energy losses will be determined basically by collisions with separate electrons. Therefore, last terms in eqs. (19) and (20) being negligible, the imaginary and real parts of the dielectric constant are as follows:

$$\mathrm{Im}\,\epsilon_l(k_{\|}v_{\|}+l\Omega,\boldsymbol{k})\sim\frac{2\sqrt{\pi}\,\omega_{\mathrm{e}}^2}{k^2|k_{\|}|\Delta_{\|}}\left\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\right\rangle\left(\frac{k_{\|}v_{\|}}{\Delta_{\|}^2}+\frac{l\Omega}{2\Delta_{\perp}^2}\right),\quad k|U|\ll\Omega;\tag{24}$$
$$\mathrm{Re}\,\epsilon_l(k_{\|}v_{\|}+l\Omega,\boldsymbol{k})\sim1+\frac{2\omega_{\mathrm{e}}^2\left\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\right\rangle}{k^2\Delta_{\|}^2}+\frac{2\omega_{\mathrm{e}}^2}{k^2\sqrt{\pi}}\left\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\right\rangle\times\left(\frac{1}{\Delta_{\|}^2}+\frac{l\Omega}{2k_{\|}v_{\|}\Delta_{\perp}^2}\right)F(x),$$

where

$$x=v_{\|}/\Delta_{\|},\quad z=v_{\mathrm{e}\|}/\Delta_{\|},$$
$$F(x)=\int\frac{x\,\mathrm{e}^{-z^2}}{z-x}\frac{\mathrm{d}z}{\sqrt{\pi}}=\begin{cases}-2x^2, & x\ll1,\\ -\left(1+\dfrac{1}{12x^2}\right), & x\gg1,\end{cases}$$

and the distribution of electrons over velocities is taken to be Gaussian:

$$f(v_{\mathrm{e}\perp},v_{\mathrm{e}\|})=\frac{1}{\sqrt{\pi}\,\Delta_{\perp}^2\Delta_{\|}}\mathrm{e}^{-(v_{\mathrm{e}\|}^2/\Delta_{\|}^2+v_{\mathrm{e}\perp}^2/2\Delta_{\perp}^2)}.$$

For the typical situation when $\rho\Omega\ll|U|$ and $\Delta_{\perp}\to0$, we obtain

$$2\boldsymbol{F}_{\mathrm{fr}}\cdot\boldsymbol{P}_{\|}=-16\sqrt{\pi}\,e^4n\frac{v_{\|}^2}{\Delta_{\|}^3}\int\frac{k_{\|}^2k_{\perp}^2J_l^2(k_{\perp}\rho)\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{|k_{\|}|\left(k^2+2\omega_{\mathrm{e}}^2/\Delta_{\|}^2\right)^2}$$
$$=-8\sqrt{\pi}\,e^4n\frac{v_{\|}^2}{\Delta_{\|}^3}\begin{cases}\ln\left(\Delta_{\|}^2/2\omega_{\mathrm{e}}^2\rho^2\right), & \rho\omega_{\mathrm{e}}\ll\Delta_{\|},\\ \Delta_{\|}/\rho\omega_{\mathrm{e}}\sqrt{2}, & \rho\omega_{\mathrm{e}}\gg\Delta_{\|};\end{cases}\tag{25}$$

$$2\boldsymbol{F}_{\mathrm{fr}}\cdot\boldsymbol{P}_{\perp}=-4\frac{\sqrt{\pi}\,e^4n}{\Delta_{\|}}\int\frac{k_{\perp}^3J_1^2(k_{\perp}\rho)k_{\|}\,\mathrm{d}k_{\|}}{\left[k^4k_{\|}^2+k_{\perp}^4\left(\sqrt{\pi}\,\omega_{\mathrm{e}}^2/4\Omega\Delta_{\|}\right)^2\right]}$$

$$=-4\frac{\sqrt{\pi}\,e^4n}{\Delta_{\|}}\begin{cases}\ln\left(2\Omega\Delta_{\|}/\sqrt{\pi}\,\rho\omega_{\mathrm{e}}^2\right), & \omega_{\mathrm{e}}^2\rho\ll\Delta_{\|}\Omega,\\ \Omega\Delta_{\|}/\sqrt{\pi}\,\rho\omega_{\mathrm{e}}^2, & \omega_{\mathrm{e}}^2\rho\gg\Delta_{\|}\Omega.\end{cases}$$

Diffusion coefficients are analysed for two regions of the wave vector $\boldsymbol{k}$, first for the "transmittance" region where energy exchange of positrons and electron plasma occurs through interaction with plasma waves, and secondly for the two-particle positron–electron interaction region; both are assumed to be magnetized. In addition, we only consider time intervals on which long-wave plasma excitations have no time to damp considerably because of electron–electron collisions (Landau damping proves to be exponentially small for these waves).

Expressions for diffusion coefficients and fluctuation force projections into the longitudinal and traverse components of the positron momentum are as follows:

$$\begin{aligned}
d_{\|}&=8e^4n\int\frac{k_{\|}^2k_{\perp}\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}\int_0^t\mathrm{d}\tau J_0\left(2k_{\perp}\rho\,\sin\frac{\Omega\tau}{2}\right)I(\tau)\,\mathrm{e}^{\mathrm{i}k_{\|}v_{\|}\tau},\\
d_{\perp}&=8e^4n\int\frac{k_{\perp}^3\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}\int_0^t\mathrm{d}\tau J_0\left(2k_{\perp}\rho\,\sin\frac{\Omega\tau}{2}\right)\cos\Omega\tau I(\tau)\,\mathrm{e}^{\mathrm{i}k_{\|}v_{\|}\tau},\\
2\boldsymbol{F}_{\mathrm{fl}}\cdot\boldsymbol{P}_{\|}&=8e^4n\int\frac{\mathrm{i}k_{\|}v_{\|}k_{\perp}\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}\int_0^t\tau\,\mathrm{d}\tau J_0\left(2k_{\perp}\rho\,\sin\frac{\Omega\tau}{2}\right)\left(k_{\|}^2+k_{\perp}^2\frac{\sin\Omega\tau}{\Omega\tau}\right)I(\tau)\,\mathrm{e}^{\mathrm{i}k_{\|}v_{\|}\tau},\\
2\boldsymbol{F}_{\mathrm{fl}}\cdot\boldsymbol{P}_{\perp}&=8e^4n\int\frac{k_{\perp}\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}\int_0^t\tau\,\mathrm{d}\tau\frac{\mathrm{d}}{\mathrm{d}\tau}J_0\left(2k_{\perp}\rho\,\sin\frac{\Omega\tau}{2}\right)\left(k_{\|}^2+k_{\perp}^2\frac{\sin\Omega\tau}{\Omega\tau}\right)I(\tau)\,\mathrm{e}^{\mathrm{i}k_{\|}v_{\|}\tau},
\end{aligned}\tag{26}$$

where $I(\tau)=\langle\phi_{k\mathrm{a}}(t)\phi_{-k\mathrm{a}}/t-\tau)\rangle$ is a correlator of Fourier components of the electric electron-induced field potential, which is defined using eq. (18) as

$$\begin{aligned}
I(\tau)=\sum_l\Bigg\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\Bigg\{&\frac{\mathrm{e}^{-\mathrm{i}(k_{\|}v_{\|}+l\Omega)\tau}}{|\epsilon_l(k_{\|}v_{\mathrm{e}\|}+l\Omega,\,\boldsymbol{k})|^2}\\
&+\sum_{ss'}\frac{\mathrm{e}^{-\mathrm{i}\omega_st}}{(\omega_s-k_{\|}v_{\mathrm{e}\|}-l\Omega)\,\partial\epsilon_l/\partial\omega_s}\,\frac{\mathrm{e}^{\mathrm{i}\omega_s^*(t-\tau)}}{(\omega_s'-k_{\|}v_{\mathrm{e}\|}-l\Omega)\,\partial\epsilon_l^*/\partial\omega_s'}\\
&+\sum_s\frac{\mathrm{e}^{\mathrm{i}\omega_s^*(t-\tau)}}{(\omega_s^*-k_{\|}v_{\|}-l\Omega)\,\partial\epsilon_l^*/\partial\omega_s}\,\frac{\mathrm{e}^{-\mathrm{i}(k_{\|}v_{\mathrm{e}\|}+l\Omega)t}}{\epsilon_l(k_{\|}v_{\mathrm{e}\|}+l\Omega,\,\boldsymbol{k})}\\
&+\sum_s\frac{\mathrm{e}^{-\mathrm{i}\omega_st}}{(\omega_s-k_{\|}v_{\mathrm{e}\|}-l\Omega)\,\partial\epsilon_l/\partial\omega_s}\,\frac{\mathrm{e}^{\mathrm{i}(k_{\|}v_{\mathrm{e}\|}+l\Omega)(t-\tau)}}{\epsilon_l^*(k_{\|}v_{\mathrm{e}\|}+l\Omega,\,\boldsymbol{k})}\Bigg\}\Bigg\rangle.
\end{aligned}\tag{27}$$

In the "transmittance" region it is conveniently represented as a sum of two terms, the first is determined by the region of low longitudinal electron velocities: $|\omega_s-l\Omega|\gg kv_{\mathrm{e}\|}$:

$$I_1(\tau)=\sum_l\sum_s\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\rangle\frac{\mathrm{e}^{-\mathrm{i}\omega_s\tau}}{(\omega_s-l\Omega)^2(\partial\epsilon_l/\partial\omega_s)^2},$$

the second by the region of high velocities:

$$I_2(\tau) = \frac{\pi}{2}\left\langle \sum_l J_l^2(k_\perp r_\perp^a) \sum_s \frac{\delta(\omega_s - k_\| v_{e\|} - l\Omega)}{\operatorname{Im} \epsilon_l(\omega_s)\ \partial\epsilon_l/\partial\omega_s} e^{-i\omega_s\tau} \right\rangle .$$

In both cases we have omitted inessential fast-oscillating terms which do not contribute to the integral variation in positron energy interacting with Coulomb field fluctuations in the electron plasma. Thus,

$$I \simeq \sum_l \sum_s J_l^2(k_\perp r_\perp^a)\ e^{-i\omega_s\tau}\left[\frac{1}{(\omega_s - l\Omega)^2(\partial\epsilon_e/\partial\omega_s)^2} + \frac{\pi}{2}\frac{\delta(\omega_s - k_\| v_{e\|} - l\Omega)}{\operatorname{Im} \epsilon_l(\omega_s)\ \partial\epsilon_l/\partial\omega_s}\right]. \tag{28}$$

To understand the physical meaning of each of the terms in eq. (28) is not difficult. For this we make the same calculations in terms of the macroscopic theory, confining ourselves, for simplicity, to the case of a nonmagnetic plasma. By means of linearized equations for plasma oscillations we obtain

$$\delta n_k(t) = \delta n_k(0) \cos \omega_e t + \frac{\delta \dot{n}_k(0)}{\omega_e} \sin \omega_e t,$$

where δn_k and $\delta \dot{n}_k$ are, correspondingly, Fourier components of deviations of charge and current densities from their average values. Since the model under consideration assumes that at the initial moment of time there is no correlation in the positions of particles, i.e.

$$\langle \delta n_k(0) \delta n_{k'}(0) \rangle = \frac{n}{(2\pi)^3} \delta(\boldsymbol{k} + \boldsymbol{k}'),$$

$$\langle \delta \dot{n}_k(0) \delta \dot{n}_{k'}(0) \rangle = \sum_{i=1}^{3} \frac{n\langle (v_{ei}^2) \rangle k_i^2}{(2\pi)^3} \delta(\boldsymbol{k} + \boldsymbol{k}'),$$

then, with the relation connecting the Fourier components of the charge density $\delta n_k(t)$ and the Coulomb potential $\delta\phi_k(t)$,

$$\delta\phi_k(t) = 4\pi\delta n_k(t)/k^2,$$

we get

$$\langle \delta\phi_k(t) \delta\phi_{k'}(t-\tau) \rangle = 2\frac{e^2 n}{\pi k^4}\left[1 + \sum_{i=1}^{3} \frac{k_i^2 \langle (v_{ei}^2) \rangle}{\omega_e^2}\right] \delta(k + k') \cos \omega_e \tau.$$

It is clear that this expression coincides exactly with eq. (28) at $\Omega = 0$ and for an isotropic Gaussian distribution of velocities in the electron plasma. Thus one can draw the conclusion that the first term in eq. (28) is associated with the initial charge-density fluctuation, while the second is related to the initial current-density fluctuation (initial spread in electron velocities). It is worth noting that electric field fluctuations, which are due to the initial current-density fluctuation, prove to be of the same order both in thermodynamic equilibrium and at a low level of noise in the electron plasma. In fact, this is associated with the absence of correlations over electron velocities in both cases. As for electric field fluctuations due to initial charge density fluctuations, these turn out to be large because in the "transmittance" region $\langle k^2 v_e^2 \rangle \ll \omega_e^2$. Neglecting the second term in eq. (28) we find that in the "transmittance" region

$$d_\| = 8e^4 n \int \frac{k_\|^2 k_\perp\ dk_\|\ dk_\perp}{k^4} \sum_{e,m,s} \frac{\langle J_l^2(k_\perp r_\perp^a) \rangle J_m^2(k_\perp \rho)}{(\omega_s - l\Omega)^2(\partial\epsilon_l/\partial\omega_s)^2} \frac{\sin(k_\| v_\| + m\Omega - \omega_s)t}{(k_\| v_\| + m\Omega - \omega_s)},$$

$$d_\perp = 4e^4 n \int \frac{k_\perp^3\ dk_\|\ dk_\perp}{k^4} \sum_{e,m,s} \langle J_l^2(k_\perp r_\perp^a) \rangle \frac{J_{m-1}^2(k_\perp \rho) + J_{m+1}^2(k_\perp \rho)}{(\omega_s - l\Omega)^2(\partial\epsilon_l/\partial\omega_s)^2} \frac{\sin(k_\| v_\| + m\Omega - \omega_s)t}{k_\| v_\| + m\Omega - \omega_s}, \tag{29}$$

$$2F_{fl}\cdot P_{\|}=v_{\|}\frac{\partial}{\partial v_{\|}}d_{\|}+4\pi e^4 n\int\frac{k_{\|}v_{\|}k_{\perp}^3\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{\Omega k^4}\sum_{e,m,s}\left\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\right\rangle$$
$$\times\frac{J_{m-1}(k_{\perp}\rho)-J_{m+1}(k_{\perp}\rho)}{(\omega_s-l\Omega)^2(\partial\epsilon_l/\partial\omega_s)^2}\frac{\sin(k_{\|}v_{\|}+m\Omega-\omega_s)t}{k_{\|}v_{\|}+m\Omega-\omega_s},$$
$$2F_{fl}\cdot P_{\perp}=8e^4 n\frac{\partial}{\partial v_{\|}}\int\frac{k_{\perp}k_{\|}\,\mathrm{d}k_{\perp}\,\mathrm{d}k_{\|}}{k^4}\sum_{l,m,s}m\Omega\frac{\left\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\right\rangle J_m^2(k_{\perp}\rho)}{(\omega_s-l\Omega)^2(\partial\epsilon_l/\partial\omega_s)^2}$$
$$\times\frac{\sin(k_{\|}v_{\|}+m\Omega-\omega_s)t}{k_{\|}v_{\|}+m\Omega-\omega_s}+4e^4 n\int\frac{k_{\perp}^3\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}\sum_{l,m,s}\left\langle J_l^2(k_{\perp}r_{\perp}^{\mathrm{a}})\right\rangle$$
$$\times\frac{(m-1)J_{m-1}^2(k_{\perp}\rho)-(m+1)J_{m+1}^2(k_{\perp}\rho)}{(\omega_s-l\Omega)^2(\partial\epsilon_l/\partial\omega_s)^2}\frac{\sin(k_{\|}v_{\|}+m\Omega-\omega_s)t}{k_{\|}v_{\|}+m\Omega-\omega_s}.$$

Making the substitution of the solutions for the dispersion equations $\epsilon_l(\omega_s)=0$ into eq. (29) and assuming:

$$\frac{\sin(\omega_s-k_{\|}v_{\|}-m\Omega)t}{(\omega_s-k_{\|}v_{\|}-m\Omega)}=\pi\delta(\omega_s-k_{\|}v_{\|}-m\Omega),\quad t\to\infty,$$

we obtain

$$d_{\|}=4\pi e^4 n\int\frac{k_{\|}^2k_{\perp}\,\mathrm{d}k_{\perp}\,\mathrm{d}k_{\|}}{k^4}\left[J_0^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}|k_{\|}|}{k}\right)\right.$$
$$\left.+\left(\frac{\omega_{\mathrm{e}}k_{\perp}}{\Omega k}\right)^4 J_1^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}^2k_{\perp}^2}{2\Omega k^2}\right)\right];$$
$$d_{\|}+2F_{fl}\cdot P_{\|}=\frac{\partial}{\partial v_{\|}}v_{\|}d_{\|}+8\pi e^4 n(v_{\|}/\Omega)\frac{\partial}{\partial\rho^2}$$
$$\times\int\frac{k_{\perp}k_{\|}\,\mathrm{d}k_{\perp}\,\mathrm{d}k_{\|}}{k^4}\left(\frac{\omega_{\mathrm{e}}k_{\perp}}{\Omega k}\right)^4 J_1^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}^2k_{\perp}^2}{2\Omega k^2}\right);$$
$$d_{\perp}+2F_{fl}\cdot P_{\perp}=8\pi e^4 n\frac{\partial}{\partial\rho^2}\int\frac{k_{\perp}\,\mathrm{d}k_{\perp}\,\mathrm{d}k_{\|}}{k^4}\left(\frac{\omega_{\mathrm{e}}k_{\perp}}{\Omega k}\right)^4 J_1^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}^2k_{\perp}^2}{2\Omega k^2}\right)$$
$$+4\pi e^4 n\frac{\partial}{\partial v_{\|}}\int\frac{\Omega k_{\|}k_{\perp}\,\mathrm{d}k_{\|}\,\mathrm{d}k_{\perp}}{k^4}\left(\frac{\omega_{\mathrm{e}}k_{\perp}}{\Omega k}\right)^4 J_1^2(k_{\perp}\rho)\delta\left(k_{\|}v_{\|}-\frac{\omega_{\mathrm{e}}^2k_{\perp}^2}{2\Omega k^2}\right).$$

Thus, for a low level of noise in the electron plasma, the rates of change of the squares of longitudinal and transverse positron momenta are, under the action of plasma waves:

$$d_{\|}+2F_{fl}\cdot P_{\|}=4\pi e^4 n\begin{cases}\dfrac{1}{\pi\omega_{\mathrm{e}}\rho}, & \omega_{\mathrm{e}}\rho\gg|v_{\|}|,\\[2ex] \dfrac{(\omega_{\mathrm{e}}\rho)^2}{4|v_{\|}|^3}+\dfrac{\omega_{\mathrm{e}}^6}{4\Omega^6|v_{\|}|}\ln\left(\dfrac{2\Omega|v_{\|}|}{\omega_{\mathrm{e}}^2\rho}\right), & \omega_{\mathrm{e}}\rho\ll|v_{\|}|;\end{cases}\tag{30}$$

$$d_{\perp}+2F_{fl}\cdot P_{\perp}=4\pi e^4 n\begin{cases}\dfrac{1}{\Omega\rho}, & \omega_{\mathrm{e}}^2\rho\gg\Omega|v_{\|}|,\\[2ex] \dfrac{\omega_{\mathrm{e}}^4}{2\Omega^4|v_{\|}|}\left(1+\dfrac{\rho^2\omega_{\mathrm{e}}^2}{2|v_{\|}|^2}\right)\ln\dfrac{2\Omega|v_{\|}|}{\omega_{\mathrm{e}}^2\rho}, & \dfrac{\omega_{\mathrm{e}}^2\rho}{\Omega|v_{\|}|}\ll 1.\end{cases}$$

Combining these expressions with eq. (23) we find

$$\frac{\mathrm{d}P_{\parallel}^2}{\mathrm{d}t} = -4\pi e^4 n \begin{cases} \dfrac{1}{\pi\omega_e\rho}, & \omega_e\rho \gg |v_{\parallel}|, \\ \dfrac{1}{|v_{\parallel}|}, & \omega_e\rho \ll |v_{\parallel}|; \end{cases} \tag{31}$$

$$\frac{\mathrm{d}p_{\perp}^2}{\mathrm{d}t} = -4\pi e^4 n \begin{cases} \dfrac{\Omega}{\omega_e^2\rho\pi}, & \omega_e^2\rho \gg \Omega|v_{\parallel}|, \\ \dfrac{1}{|v_{\parallel}|}, & \omega_e^2\rho \ll \Omega|v_{\parallel}|. \end{cases} \tag{32}$$

Consequently, when positrons and plasma waves interact, their energy decreases according to eqs. (31) and (32). Despite a high (shot) noise level to the "transmittance" region this process continues until the interaction with plasma waves becomes ineffective, i.e. up to $v_{\parallel} \sim \Delta_{\parallel}$. In this case, eqs. (24) and (25) should be used to calculate the friction force, and the analysis of diffusion coefficients will be confined to the first term in eq. (27) for a correlator I. Substituting it into eq. (26) and uniting the expressions for friction, we find that in this range of parameters

$$\begin{aligned} \frac{\mathrm{d}P_{\parallel}^2}{\mathrm{d}t} = 8\pi e^4 n \int \frac{k_{\perp}\,\mathrm{d}k_{\parallel}\,\mathrm{d}k_{\perp}}{k^4} \Bigg\langle \sum_{m,l} \left(k_{\parallel}\frac{\partial}{\partial v_{\parallel}} - k_{\parallel}\frac{\partial}{\partial v_{e\parallel}} + \frac{l}{\Omega\rho}\frac{\partial}{\partial\rho} - \frac{m}{\Omega r_{\perp}^{a}}\frac{\partial}{\partial r_{\perp}^{a}} \right) \\ \times \frac{k_{\parallel}v_{\parallel}J_l^2(k_{\perp}\rho)J_m^2(k_{\perp}r_{\perp}^{a})\delta(k_{\parallel}U + (m-l)\Omega)}{|\epsilon_l(k_{\parallel}v_{\parallel} + l\Omega, \boldsymbol{k})|^2} \Bigg\rangle; \end{aligned} \tag{33}$$

$$\begin{aligned} \frac{\mathrm{d}P_{\perp}^2}{\mathrm{d}t} = 8\pi e^4 n \int \frac{k_{\perp}\,\mathrm{d}k_{\parallel}\,\mathrm{d}k_{\perp}}{k^4} \Bigg\langle \sum_{m,l} \left(k_{\parallel}\frac{\partial}{\partial v_{\parallel}} - k_{\parallel}\frac{\partial}{\partial v_{e\parallel}} + \frac{l}{\Omega\rho}\frac{\partial}{\partial\rho} - \frac{m}{\Omega r_{\perp}^{a}}\frac{\partial}{\partial r_{\perp}^{a}} \right) \\ \times \frac{l\Omega J_l^2(k_{\perp}\rho)J_m^2(k_{\perp}r_{\perp}^{a})\delta(k_{\parallel}U + (m-l)\Omega)}{|\epsilon_l(k_{\parallel}v_{\parallel} + l\Omega, \boldsymbol{k})|^2} \Bigg\rangle. \end{aligned} \tag{34}$$

Analysis of the above expressions shows that $\mathrm{d}P_{\parallel}^2/\mathrm{d}t \to 0$. The decrease of energy of the transverse positron motion will continue according to eq. (25), until the Larmor positron radius becomes equal to the Larmor electron radius. As a result, the stationary velocity distribution function for "cooled" positrons will completely correspond to the analogous electron distribution function *.

It is worth noting that eqs. (33) and (34) correspond to the usually quoted asymptotic expression for the collision integral of electrons in a magnetic field [9] which commonly results from the assumption that plasma fluctuations attain a level being in agreement with thermodynamic equilibrium. The application of this collision integral is practically justified when one considers an integral relaxation of a weakly non-equilibrium plasma. In our case the initial level of noise in the region of collective motion turns out to be significant when the interaction of the ensemble of "foreign" particles (positrons) and an electron plasma having no time to relax to a thermodynamic equilibrium is considered. To describe the fluctuation effects correctly it is necessary to employ a more general approach taking conditions of the electron beam

* We imply an ideal case when the positron beam is cooled in the electron beam under such conditions that the effects of positron motion cyclicity in a storage ring are insignificant and the electron distribution function is stationary. Note that the second takes place practically all the time. Due to the presence of a strong magnetic field in the electron cooling section the electron distribution function (the ratio $T_{\parallel}/T_{\perp}$) has no time to drastically change during positron–electron interaction.

formation into account. In the present paper, we have used the assumption on a low level of noise when deriving scattering coefficients (absence of correlation in a spatial arrangement of electrons). As follows from eq. (30), at $\omega_e \rho \gg v_\parallel$ the low level of noise reduces the rate of cooling the longitudinal positron degree of freedom by a factor of 2.

The above analysis of relaxation of hot positrons has been based on the approximation of a collisionless dielectric constant of the electron plasma. This approximation will be true if the characteristic frequencies of the electron–positron interaction, equal to ω_e and ω_e^2/Ω for the longitudinal and transverse motion, are much larger than the damping decrements of longitudinal and transverse waves in the "transmittance" region. For longitudinal waves this condition is satisfied up to ultralow temperatures of the electron beam $T_\parallel \sim e^2 n^{1/3}$. Indeed, in this case the damping decrement proves to be equal to

$$\tau^{-1} \simeq \omega_e (r_D k)^2 = \omega_e v_{e\parallel}^2 / v_\parallel^2 \ll \omega_e .$$

The damping decrement of plasma waves transverse to the longitudinal magnetic field is unlikely to be larger than the inverse time of electron–electron collisions for transverse degrees of freedom. Therefore, a good condition for the applicability of collisionless dielectric constant approximation in the problem of electron cooling of hot positrons should be

$$\frac{\omega_e^2}{\Omega} \gg \min\left(\frac{e^2 \omega_e^2}{T_\perp \Delta_\parallel}, \frac{\Omega^2 e^2}{T_\perp \Delta_\perp} \right); \quad \left(T_\perp = m \Delta_\perp^2 \right).$$

Note that this relation is practically always satisfied in systems with electron cooling.

The derived eqs. (16), (17) and (31), (32) determine the instantaneous character of the positron cooling process in momentum space in the region of their interaction with the electron beam. When cooling a circulating beam in a storage ring one must take into account the motion cyclicity and the coupling of the degrees of freedom of the particles being cooled as well as a number of limiting factors. If the phase volume of positrons is rather small, the cooling process can be performed during one flight through the electron cooling system.

As the preliminary analysis based on the experience gained in theoretical and experimental studies of electron cooling of protons shows [2–6], special methods such as redistribution of damping decrements and "sweeping" allow to accelerate the cooling process of hot positrons by several orders. In particular, "sweeping" is effective because, in a strong magnetic field, the cooling decrement of positrons is sharply increased when the longitudinal velocity of their Larmor circles in the electron gas is decreased. The use of this procedure, in definite conditions, decreases the positron-beam damping decrement with respect to the longitudinal degree of freedom by a factor of $(\Delta v_\parallel / \Delta v_{e\parallel})^2$ ($\Delta v_\parallel$ and $\Delta v_{e\parallel}$ are the longitudinal-velocity spread in the positron and electron beam respectively). After that, cooling the transverse phase volume of positrons will be determined by an ultimately small spread in longitudinal electron velocities, $\simeq (e^2 n^{1/3}/m)^{1/2}$, and can be further accelerated by redistribution of decrements due to coupling of the degrees of freedom in a storage ring. The possibilities of increasing the efficiency of electron cooling of positrons with the use of these procedures need to be studied in detail.

In cooling positrons, one must take into account their interaction with residual-gas atoms which leads to multiple scattering of positrons and to ionization (at high energies, to radiation) energy losses. Collective interaction of positrons may appear to be significant if at the final stage of cooling the density in the positron beam becomes too high and begins to influence the particle motion dynamics in a storage ring. In addition, during the electron cooling of positrons, positronium formation and annihilation of electron–positron pairs occur.

Estimates show that at least in the problem of obtaining antihydrogen there exists a reasonable range of parameters when the electron–positron Coulomb interaction is a decisive restriction in cooling kinetics and the use of the procedures mentioned above makes the generation of antihydrogen with a rate of 10^6–10^7 atoms per second possible.

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