

Electron Cooling against IBS for High Energy Colliders

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Abstract

Analytical formulas for detailed longitudinal IBS and electron cooling rates are presented for bunched beams. Ratio of electrons over ions (pbars) per bunch required for IBS compensation is found.

Originally, magnetized cooling force has been calculated by Ya. Derbenev [1]; he considered the electron beam as a dielectric medium. Later, V. Parkhomchuk suggested another derivation, based on a binary collision model, and got a different result [2]. This difference between the two approaches was understood by V. Lebedev [3], who pointed out that the dielectric model cannot be applied at small distances, which still contribute to the Coulomb logarithm; thus, the model gives only a part of the force. From another hand, the binary collision approach is valid for the whole range of the impact parameters contributing to the logarithm; thus, it is correct with the logarithmic accuracy. Binary collision model gives the cooling force which in the beam frame can be expressed as

$$\vec{F} = -P\vec{v} / v - Q\vec{b}(\vec{v}\vec{b}) / v, \quad (1)$$

where \vec{b} is a unit vector along magnetic field, \vec{v} is the ion velocity,

$$P = \alpha(1 - 2(\vec{v}\vec{b})^2 / v^2); \quad Q = \alpha; \quad \alpha \equiv \frac{2\pi n_e e^4 L_e}{m v^2}.$$

For non-magnetized cooling, $P = 2\alpha$; $Q = 0$.

Magnetized electron cooling rate for longitudinal degree of freedom of a bunched beam is obtained from the cooling force by averaging over the three phases of the circulating ion, leading to (laboratory frame)

$$\Lambda_e^{\parallel} \equiv -\frac{1}{J_z} \frac{dJ_z}{dt} = \frac{8N_e r_e r_i c \eta_c L_e}{\gamma^2 z_m} \left\langle \frac{n_{\perp}}{v^3} \frac{v_{\perp}^2}{v^2} \right\rangle_{x,y}.$$

Here z_m is the ion amplitude in the longitudinal direction, $J_z = \omega_s z_m^2 / 2$ is the longitudinal action, and the brackets $\langle \dots \rangle_{x,y}$ stand for the transverse (x,y) phase averaging (the longitudinal averaging is already done here, assuming the e-bunch is short). This double integral is simplified when the longitudinal beam temperature is small compared with the transverse. In this case, assuming the Gaussian shape of the e-bunch

with rms size σ_e , a negligible longitudinal temperature and transverse electron rms velocity u_e (all the velocities are in units of the speed of light c), the rate writes as

$$\Lambda_e^{\parallel} = \frac{8N_e r_e r_i \eta_c L_e}{3\pi \gamma^2 z_m v_{zm} u_e^2 \sigma_e^2} \exp\left(-\frac{x_m^2 + y_m^2}{4\sigma_e^2} - \frac{v_{xm}^2 + v_{ym}^2}{4u_e^2}\right) I_0\left(\frac{x_m^2}{4\sigma_e^2} - \frac{v_{xm}^2}{4u_e^2}\right) I_0\left(\frac{y_m^2}{4\sigma_e^2} - \frac{v_{ym}^2}{4u_e^2}\right)$$

where subscripts m point to amplitude values of the offsets and velocities in the beam frame, and $I_0(x)$ is the modified Bessel function. The transverse e-cooling rates are by a factor of $\cong v_{zm}/v_{xm}$ smaller than the longitudinal one. The rate for non-magnetized cooling is expressed by the same formula, with only difference that the numerical factor is 4 instead of 8/3.

The longitudinal IBS rate is calculated in a similar way. For the longitudinally cold and transversely round beam, the longitudinal IBS diffusion can be presented as

$$D_i^{\parallel} \equiv \frac{d}{dt} v_{zm}^2 = \frac{4\pi N_i r_i^2 c L_i}{\gamma^2} \left\langle \frac{n}{\Delta v_{\perp}} \right\rangle_{x,y,z} =$$

$$\sqrt{\frac{2}{\pi}} \frac{N_i r_i^2 c L_i}{\gamma^2 v_{zm}^2 \sigma_x^2 \sigma_z u_x} \exp\left(-\frac{z_m^2}{4\sigma_z^2}\right) I_0\left(\frac{z_m^2}{4\sigma_z^2}\right) \mathcal{G}\left(\frac{v_{xm}}{u_x}, \frac{v_{ym}}{u_x}\right)$$

with

$$\mathcal{G}(x, y) \equiv \frac{2}{\pi} \exp\left(-\frac{x^2 + y^2}{2}\right) \int_0^{\infty} \int_0^{\infty} \frac{d\xi d\eta}{\sqrt{\xi^2 + \eta^2}} \exp\left(-\frac{\xi^2 + \eta^2}{2}\right) I_0(\xi x) I_0(\eta y).$$

The symbols $u_{x,y}$ are rms velocities in the beam frame and $\sigma_{x,y}$ are the rms beam sizes.

Note that this expression gives a detailed diffusion, i. e.. the diffusion as a function of all the three amplitudes. After averaging over Gaussian distribution, it leads to the conventionally used beam-averaged value:

$$\frac{d}{dt} \overline{v_z^2} = \frac{N_i r_i^2 c L_i}{8\gamma^2 \sigma_x^2 \sigma_z u_x}$$

These IBS formulas assume longitudinally cold beam, $v_{zm}/v_{xm} \ll 1$. When this ratio is not extremely small, a correction due to its final value can be significant. It can be shown, that this correction of the longitudinal IBS diffusion can be approximated by a factor of $1 - \sqrt{v_{zm}/(u_x \sqrt{2})}$, to be consistent with [4] for any ratio of velocities. After that, the longitudinal IBS rate is presented as

$$\Lambda_i^{\parallel} \equiv \frac{1}{v_{zm}^2} \frac{d}{dt} v_{zm}^2 = \sqrt{\frac{2}{\pi}} \frac{N_i r_i^2 c L_i}{\gamma^2 v_{zm}^2 \sigma_x^2 \sigma_z u_x} \exp\left(-\frac{z_m^2}{4\sigma_z^2}\right) I_0\left(\frac{z_m^2}{4\sigma_z^2}\right) \mathcal{G}\left(\frac{v_{xm}}{u_x}, \frac{v_{ym}}{u_x}\right) \left(1 - \sqrt{\frac{v_{zm}}{u_x \sqrt{2}}}\right)$$

Horizontal IBS rate $\Lambda_i^x \equiv d(\ln J_x)/dt$ relates to the longitudinal one as

$$\Lambda_i^x / \Lambda_i^{\parallel} = \frac{\overline{D^2 + (D'\beta_x + \alpha_x D)^2}}{x_m^2} v_{zm}^2$$

where the bar stands for the orbit-averaging.

Below, the plot for electron cooling and IBS longitudinal rates versus transverse actions is presented for $N_i = 1 \cdot 10^9$ Au ions in RHIC at $\gamma = 100$ with normalized 95% emittances $\varepsilon_{nx} = \varepsilon_{ny} = 15$ mm mrad, rms momentum spread $8.5 \cdot 10^{-4}$, rms bunch length 30 cm.

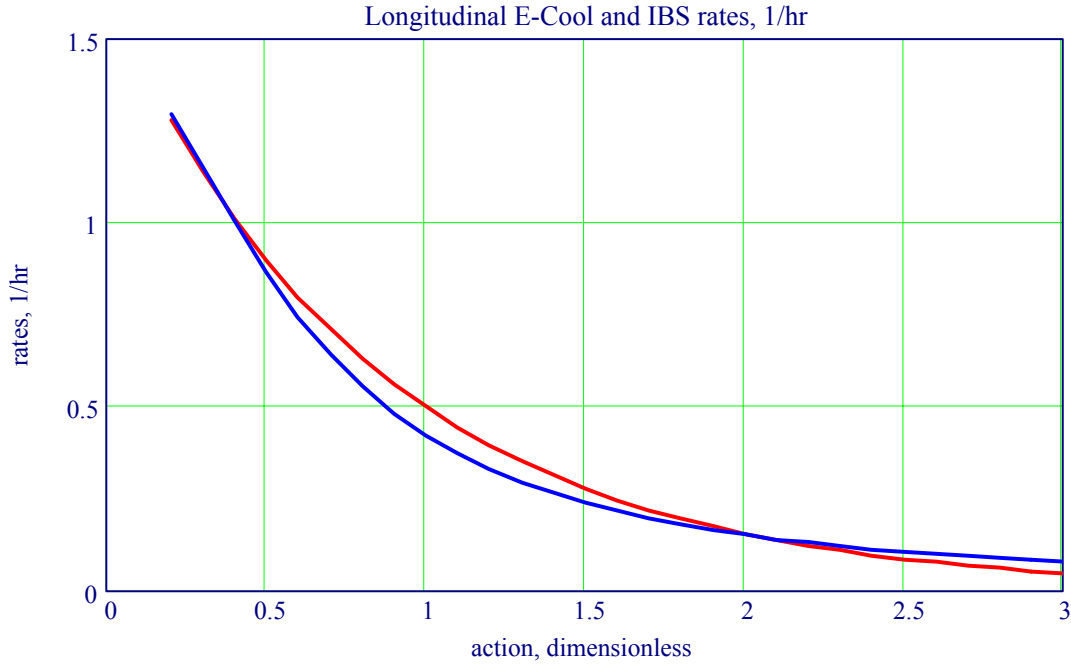


Fig. 1. Longitudinal IBS (blue line) and e-cool (red line) rates for Au ions in RHIC for $N_e = 6 \cdot 10^{10}$, $\sigma_e = 1.3\sigma_x$, $u_e = 1.3u_x$, Coulomb logs $L_e = 2.5$, $L_i = 19$, cooler's fraction $\eta_c = 7.8 \cdot 10^{-3}$, and the beta-function in the cooler $\beta_c = 60$ m. Both transverse actions are equal for this plot, and measured in units of the rms emittance. The longitudinal action is equal to the longitudinal rms emittance.

The result obtained can be expressed in terms of the number of electrons per bunch required to compensate longitudinal IBS of the given number of ions per bunch, as it was suggested in [5]. Assuming that the 6D phase space shape of the electron bunch is

optimized, it comes out that the number of electrons required for longitudinal IBS compensation is

$$\frac{N_e}{N_i} = f \frac{L_i}{L_e} \frac{r_i}{r_e} \frac{1}{\eta_c} \frac{u_x}{u_z} \left(1 - \sqrt{\frac{u_z}{u_x}} \right),$$

where a numerical factor $f = 2$ for magnetized cooling and $f = 4/3$ for non-magnetized.

Note that the electron beam transverse size is optimized to match e-cool and IBS rates. It follows from this plot that $N_e = 6 \cdot 10^{10}$ is required to compensate longitudinal IBS growth of $N_i = 1 \cdot 10^9$ gold ions. However, the horizontal IBS growth, whose rate is about equal to the longitudinal one, is not compensated by the transverse component of the cooling force (1), which is too small and even has a wrong sign. To compensate horizontal IBS diffusion, dispersive electron cooling scheme can be applied, which requires higher number of electrons, $N_e \geq 2 \cdot 10^{11}$. So high number of electrons (positrons) per bunch could be accumulated in a storage ring, where required emittances and energy spread could be obtained by means of a wiggler.

For Tevatron at full energy, the longitudinal IBS compensation would require $N_e / N_i = 30$ electrons over protons per bunch ratio (non-magnetized cooling), assuming 20 m of the cooling length and matched e-bunch. Taking $N_i = 3 \cdot 10^{11}$, it leads to $N_e = 1 \cdot 10^{13}$. So high bunch population means that the storage ring cooler has to be considered, where the energy spread will be at least 5-10 times higher than protons one [6]. Horizontal IBS compensation requires to increase this number by a factor of 4 or so, totally leading to $N_e = (2-4) \cdot 10^{14}$ electrons, which is about 2 orders of magnitude above possibilities.

References

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