A New Approach to Calculating Dynamical Friction for Magnetized Electron Cooling

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(Hadron Synchrotron Collective effects)

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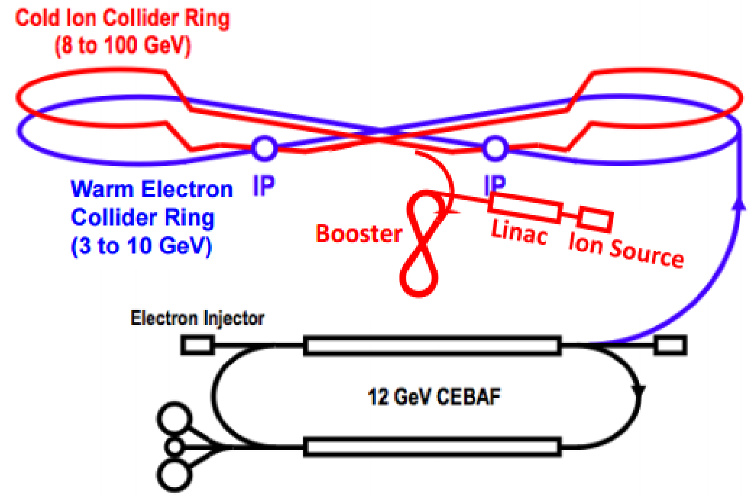
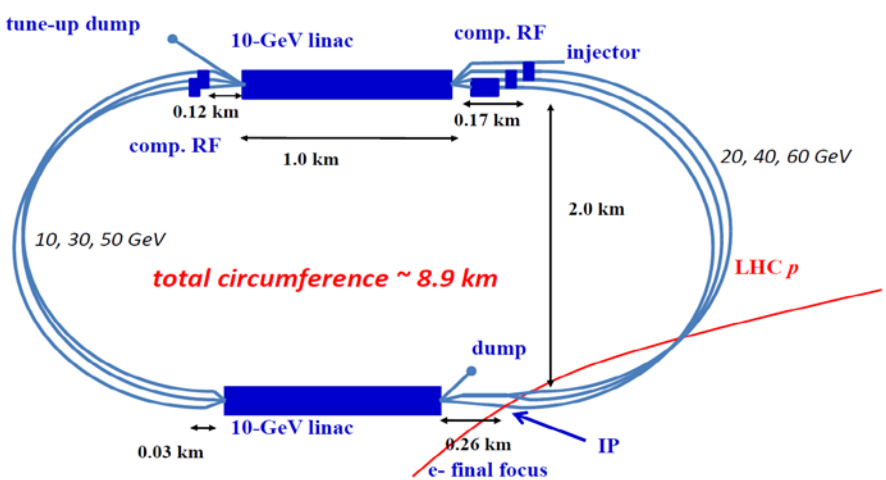
Motivation – Nuclear Physics

- Electron-ion colliders (EIC)
  - *high priority for the worldwide nuclear physics community*
- Relativistic magnetized electron cooling
  - *may be essential for EIC, but never demonstrated*

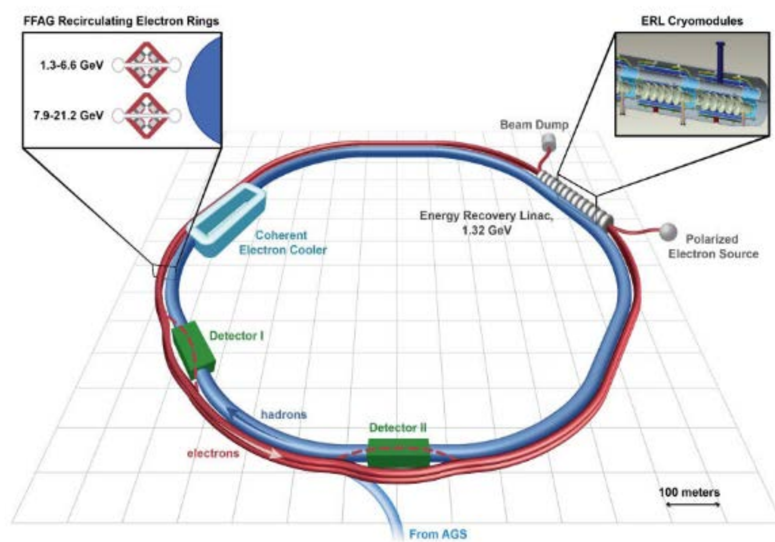
JLEIC concept from Jefferson Lab



LHeC concept from CERN



eRHIC concept from BNL



Idea for Electron Cooling is 50 Years Old

- Budker developed the concept in 1967
  - *G.I. Budker, At. Energ. 22 (1967), p. 346.*
- Many low-energy electron cooling systems:
  - *continuous electron beam is generated*
  - *electrons are nonrelativistic & very cold compared to bunches*
  - *electrons are magnetized with a strong solenoid field*
    - suppresses transverse temperature & increases friction
- Fermilab has shown cooling of relativistic p-bar's
  - *S. Nagaitsev et al., PRL 96, 044801 (2006).*
  - *~5 MeV e-'s (γ ~ 9) from a DC source*
  - *The electron beam was **not** magnetized*
- Relativistic magnetized cooling not yet demonstrated
  - *electron cooling at γ ~ 100 has not been demonstrated*
  - *a non-magnetized concept was developed for RHIC*
    - Fedotov *et al*., Proc. PAC, THPAS092 (2007).

Risk Reduction is Required for Relativistic Coolers

- eRHIC, JLEIC both need cooling at high energy
  - *100 GeV/n → $\gamma \approx 107$ → 55 MeV bunched electrons, ~1 nC*
- Electron cooling at γ~100 requires different thinking
  - *friction force scales like $1/\gamma^2$ (Lorentz contraction, time dilation)*
    - challenging to achieve the required dynamical friction force
    - not all of the processes that reduce the friction force have been quantified in this regime → significant technical risk
  - *normalized interaction time is reduced to order unity*
    - $\tau = t\omega_{pe} >> 1$ for nonrelativistic coolers
    - $\tau = t\omega_{pe} \sim 1$ (in the beam frame), for γ~100
      - *violates the assumptions of introductory beam & plasma textbooks*
      - *breaks the intuition developed for non-relativistic coolers*
      - *as a result, the problem requires careful analysis*
  - *non-magnetized friction for γ~100 has been studied*
    - magnetized friction requires the same level of attention

Context and Caveats

- We consider the microphysics of dynamical friction
  - *detailed treatment of a single pass through the cooler*
  - *the parameter space is large*
- Parametric and semi-analytic models are necessary
  - *accurate parametric models enable rapid conceptual design*
  - *codes like BETACOOL and MOCAC enable long-time studies*
    - semi-analytic models, electron & ion distributions, equilibration
  - *simulating single-pass physics helps to improve these models*
- Diffusive kicks must be suppressed for single-pass studies
  - *diffusive effects exceed friction in a single pass*
  - *friction wins over millions of turns, but not in a single pass*
- Previous work by 1st author & collaborators has been summarized:

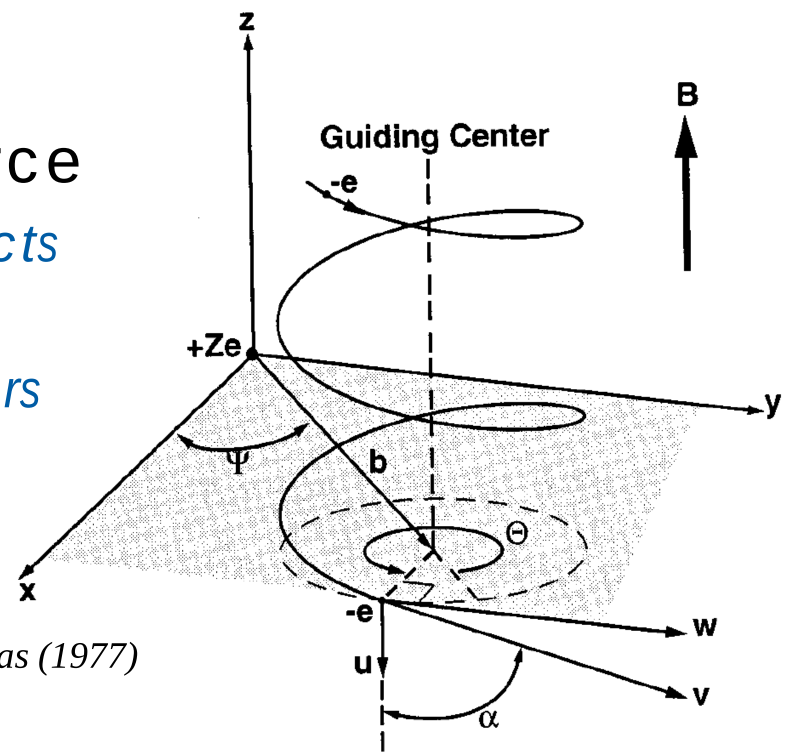
> D.L. Bruhwiler, "Simulating single-pass dynamics for relativistic electron cooling," in *ICFA Beam Dynamics Newsletter* **65**, "Beam Cooling II," eds. Y. Zhang & W. Chou (Dec., 2014).

- Lifetime of work by Slava Derbenev has recently been translated and made available:

> Y.S. Derbenev, "Theory of electron cooling," *arXiv:1703.09735* (2017); https://arxiv.org/abs/1703.09735

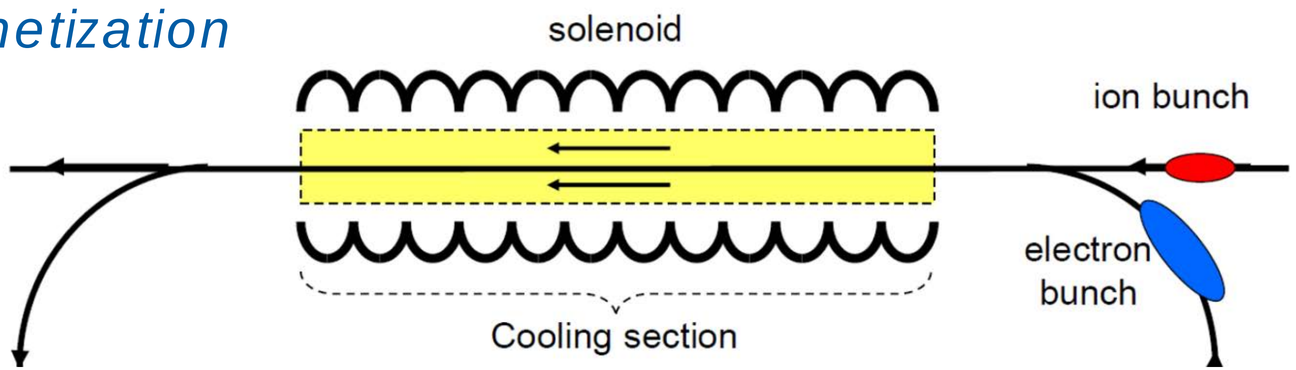
Goals

- Simulate magnetized friction force
  - *include all relevant real world effects*
    - e.g. incoming beam distribution
  - *include a wide range of parameters*
  - *cannot succeed via brute force*
    - new theory is required



from Geller & Weisheit, Phys. Plasmas (1977)

- Include key aspects of magnetized e- beam transport
  - *imperfect magnetization*
  - *space charge*
  - *field errors*



from Zhang et al., MEIC design, arXiv (2012)

Hamiltonian for 2-body magnetized collision

$$H\left(\vec{x}_{ion},\vec{p}_{ion},\vec{x}_e,\vec{p}_e\right)=H_0\left(\vec{p}_{ion},y_e,\vec{p}_e\right)+H_C\left(\vec{x}_{ion},\vec{x}_e\right)$$

$$\vec{B}=B_0\,\hat{z} \qquad \vec{A}=-B_0 y\,\hat{x} \qquad p_{e,x}=m_e\left(v_{e,x}-\Omega_L y_e\right)$$

$$H_0\left(\vec{p}_{ion},y_e,\vec{p}_e\right)=\frac{1}{2m_{ion}}\left(p_{ion,x}^2+p_{ion,y}^2+p_{ion,z}^2\right)+\frac{1}{2m_e}\left[\left(p_{e,x}+eB_0 y_e\right)^2+p_{e,y}^2+p_{e,z}^2\right]$$

$$H_C\left(\vec{x}_{ion},\vec{x}_e\right)=\frac{-Ze^2}{4\pi\varepsilon_0}\Big/\sqrt{\left(x_{ion}-x_e\right)^2+\left(y_{ion}-y_e\right)^2+\left(z_{ion}-z_e\right)^2}$$

Resulting equations of motion, in the standard drift-kick symplectic form:

$$M\left(\Delta t\right)=M_0\left(\Delta t/2\right)M_C\left(\Delta t\right)M_0\left(\Delta t/2\right)$$

D.L. Bruhwiler and S.D. Webb, “New algorithm for dynamical friction of ions in a magnetized electron beam,” in *AIP Conf. Proc.* **1812**, 050006 (2017); http://aip.scitation.org/doi/abs/10.1063/1.4975867

Symplectic drift map for 2-body system

$M_0(\Delta t)$: $\vec{p}_{ion} = constant \quad p_{e,x} = constant \quad p_{e,z} = constant$

$$\vec{x}_{ion}(t+\Delta t) = \vec{x}_{ion}(t) + \frac{\vec{p}_{ion}(t)}{m_{ion}}\Delta t \qquad z_e(t+\Delta t) = z_e(t) + \frac{p_{e,z}(t)}{m_e}\Delta t$$

$$x_e(t+\Delta t) = x_e(t) + r_L\left[\cos(\varphi_0 + \Omega_e \Delta t) - \cos(\varphi_0)\right]$$

$$y_e(t+\Delta t) = y_e(t) - r_L\left[\sin(\varphi_0 + \Omega_e \Delta t) - \sin(\varphi_0)\right]$$

$$\varphi_0 = \tan^{-1}\left(v_{e,x}/v_{e,y}\right) \qquad v_{e,\perp}^2 = v_{e,x}^2 + v_{e,y}^2 \qquad \Omega_L = \left|eB_0\right|/m_e \qquad r_L = V_{e,\perp}/\Omega_L$$

Symplectic kick for 2-body system

$$M_C(\Delta t): \qquad \vec{x}_{ion} = constant \qquad \vec{x}_e = constant$$

$$\Delta\vec{p}_{ion} = \frac{\alpha(\vec{x}_e - \vec{x}_{ion})\Delta t}{b^3(\vec{x}_{ion}, \vec{x}_e)} \qquad \Delta\vec{p}_e = \frac{\alpha(\vec{x}_{ion} - \vec{x}_e)\Delta t}{b^3(\vec{x}_{ion}, \vec{x}_e)}$$

$$\alpha = \frac{Ze^2}{4\pi\varepsilon_0} \qquad b(\vec{x}_{ion}, \vec{x}_e) = \left[(x_{ion} - x_e)^2 + (y_{ion} - y_e)^2 + (z_{ion} - z_e)^2\right]^{1/2}$$

These 2^{nd}-order equation of motion are simple and robust.

They can be made 4^{th}-order via standard Yoshida algorithm.

However, they require resolution of the gyroperiod and, hence, are slow:

$$\Delta t_{\max} \approx \frac{1}{8}\frac{2\pi}{\Omega_e}$$

Transform to Action-Angle variables

We follow Lichtenberg and Lieberman, *Regular & Chaotic Dynamics* (1992).
We use their canonical generating function of the 2nd kind:

$$F\left(x_e, y_e, \varphi, y_{gc}\right) = m_e \Omega_e \left[\frac{1}{2}\left(y_e - y_{gc}\right)^2 \cot(\varphi) - x_e y_{gc}\right]$$

which yield the following Hamiltonian:

$$H\left(\vec{x}_{ion}, \vec{p}_{ion}, \varphi, y_{gc}, z_e, p_\varphi, p_{gc}, p_{ez}\right) = H_0\left(\vec{p}_{ion}, p_\varphi, p_{ez}\right) + H_C\left(\vec{x}_{ion}, \varphi, y_{gc}, z_e, p_\varphi, p_{gc}\right)$$

$$H_0\left(\vec{p}_{ion}, p_\varphi, p_{e,z}\right) = \frac{1}{2m_{ion}} \vec{p}_{ion} \cdot \vec{p}_{ion} + \Omega_e p_\varphi + \frac{1}{2m_e} p_{e,z}^2$$

$$H_C\left(\vec{x}_{ion}, \varphi, y_{gc}, z_e, p_\varphi, p_{gc}, p_{ez}\right) = \frac{-Ze^2 / 4\pi\varepsilon_0}{\sqrt{\begin{array}{l}\left(x_{ion} - x_{gc}/m_e\Omega_e\right)^2 + \left(y_{ion} - y_{gc}\right)^2 + \left(z_{ion} - z_e\right)^2 + r_L^2 + \cdots \\ \qquad \cdots + 2\left(x_{ion} - x_{gc}\right) r_L \cos(\varphi) + 2\left(y_{ion} - y_{gc}\right) r_L \sin(\varphi)\end{array}}}$$

$$x_{gc} = p_{gc} / m_e \Omega_e$$

$$r_L = \left(2 p_\varphi / m_e \Omega_e\right)^{1/2}$$

Zero'th-order dynamics is now very simple, but H_C is problematic…

Transform to next-order Action-Angle variables

We follow Lichtenberg and Lieberman, *Regular & Chaotic Dynamics* (1992).
We use standard secular perturbation theory, requiring two approximations:

1) H_C is a perturbation. This requires $E_{kinetic} >> E_{potential}$.

2) $r_L << \left[(x_{ion} - x_{gc})^2 + (y_{ion} - y_{gc})^2 + (z_{ion} - z_e)^2 + r_L^2\right]^{1/2}$

This is approximately satisfied for relevant trajectories and fails gracefully.

The result is to remove the fast ϕ-dependence from the Hamiltonian:

$$H\left(\vec{x}_{ion}, y_{gc}, z_e, \vec{p}_{ion}, p_{gc}, p_{ez}, J\right) = H_0\left(\vec{p}_{ion}, p_{ez}, J\right) + H_C\left(\vec{x}_{ion}, y_{gc}, z_e, p_{gc}, J\right)$$

$$J = p_\varphi + \frac{Ze^2}{4\pi\varepsilon_0}\frac{r_L}{\Omega_e}\frac{(x_{ion} - x_{gc})\cos(\varphi) + (y_{ion} - y_{gc})\sin(\varphi)}{\left((x_{ion} - x_{gc})^2 + (y_{ion} - y_{gc})^2 + (z_{ion} - z_e)^2 + r_L^2\right)^{3/2}}$$

$$H_0\left(\vec{p}_{ion}, J, p_{e,z}\right) = \frac{1}{2m_{ion}}\vec{p}_{ion}\cdot\vec{p}_{ion} + \Omega_e J + \frac{1}{2m_e}p_{e,z}^2$$

$$H_C\left(\vec{x}_{ion}, y_{gc}, z_e, J, p_{gc}\right) = \frac{-Ze^2}{4\pi\varepsilon_0}\Bigg/\left[\left(x_{ion} - \frac{p_{gc}}{m_e\Omega_e}\right)^2 + (y_{ion} - y_{gc})^2 + (z_{ion} - z_e)^2 + \frac{2}{m_e\Omega_e}J\right]^{1/2}$$

Symplectic maps for averaged 2-body system

Equations of motion are still in the standard drift-kick symplectic form:

$$M(\Delta t) = M_0(\Delta t/2) M_C(\Delta t) M_0(\Delta t/2)$$

$M_0(\Delta t)$:

$$\vec{p}_{ion} = constant \qquad J = constant \qquad p_{gc} = constant$$

$$p_{ez} = constant \qquad \theta \text{ is ignored} \qquad y_{gc} = constant$$

$$\vec{x}_{ion}(t+\Delta t) = \vec{x}_{ion}(t) + \frac{\vec{p}_{ion}(t)}{m_{ion}}\Delta t \qquad z_e(t+\Delta t) = z_e(t) + \frac{p_{e,z}(t)}{m_e}\Delta t$$

Much larger time steps are now possible:

$$\Delta t_{\max} \approx \frac{1}{8}\frac{|z_{ion} - z_e|}{|v_{ion,z} - v_{e,z}|}$$

Symplectic kick for averaged 2-body system

$M_C(\Delta t)$:

$\vec{x}_{ion} = constant$

$$\Delta p_{ion,x} = \alpha\left(p_{gc}/m_e\Omega_e - z_{ion}\right)\Delta t / b^3\left(\vec{x}_{ion}, y_{gc}, z_e, J, p_{gc}\right)$$

$J = constant$

θ is ignored

$$\Delta p_{ion,y} = \alpha\left(y_{gc} - z_{ion}\right)\Delta t / b^3\left(\vec{x}_{ion}, y_{gc}, z_e, J, p_{gc}\right)$$

$z_e = constant$

$$\Delta p_{ion,z} = \alpha\left(z_e - z_{ion}\right)\Delta t / b^3\left(\vec{x}_{ion}, y_{gc}, z_e, J, p_{gc}\right)$$

$$\Delta p_{ez} = -\Delta p_{ion,z}$$

$$\alpha = \frac{Ze^2}{4\pi\varepsilon_0}$$

$$b\left(\vec{x}_{ion}, y_{gc}, z_e, J, p_{gc}\right) = \left[\left(x_{ion} - \frac{p_{gc}}{m_e\Omega_e}\right)^2 + \left(y_{ion} - y_{gc}\right)^2 + \left(z_{ion} - z_e\right)^2 + \frac{2}{m_e\Omega_e}J\right]^{1/2}$$

Time-explicit vs Averaged – approx. agreement

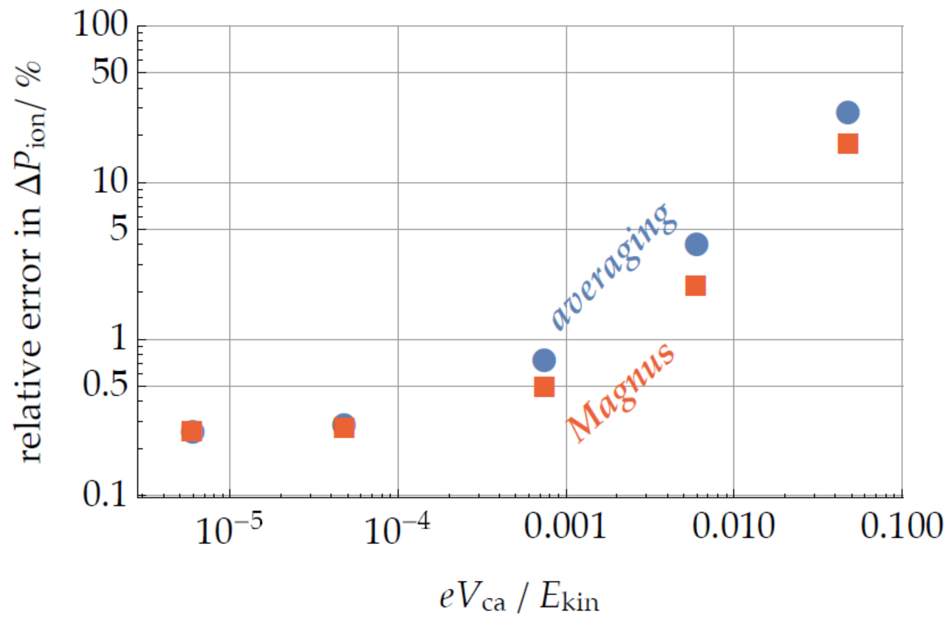
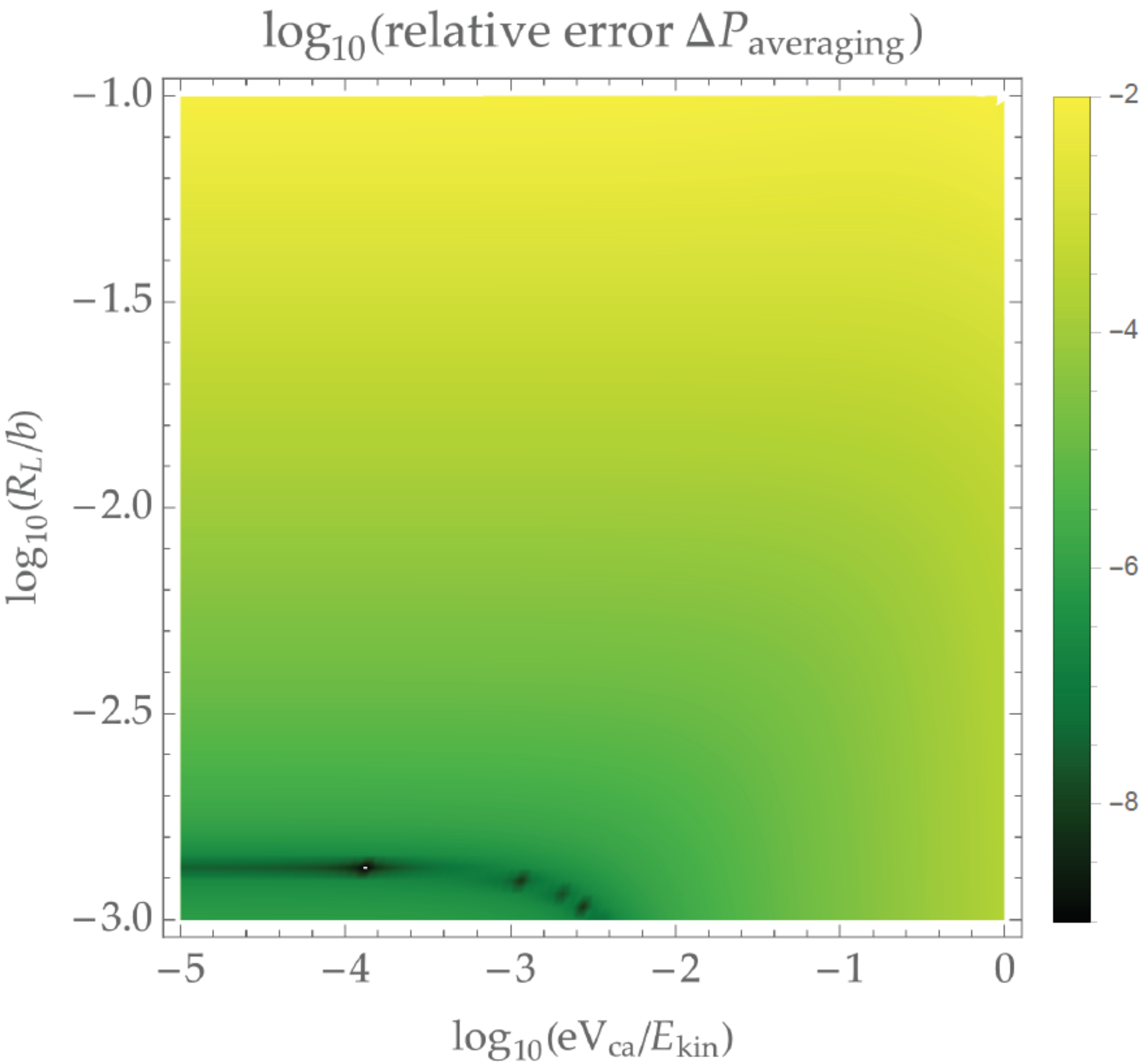


Figure 1: Relative error in the value of ΔP_{ion} computed using four-turn averaging (blue circles) and the Magnus expansion (red squares).

Time-explicit vs averaged: excellent agreement



This graphic shows the relative error made by our averaging computation of the ion momentum kick ΔP. The average is taken over one full gyrotron period. The scale is logarithmic, so the largest errors here are about 1 %.

Magnus expansion yields analytic result

We follow Alex Dragt, ***Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics*** (Version of 22 June 2016), p. 861:
http://www.physics.umd.edu/dsat/dsatliemethods.html

The result is an analytic calculation of the ion momentum change!

$$M(t) = M_I(t) M_0(t)$$

$$M_I(t) = \exp\left[-:\int_0^t d\sigma\, H_I(\sigma): \right] \qquad H_I(t) = M_0(t) H_C$$

$$M_I(T) \approx I - :\int_0^T d\sigma\, H_I(\sigma):$$

We can evaluate this approximate expression analytically.
It is valid, when the Coulomb interaction is a perturbation.

Analytic calculation of $\Delta\mathbf{p}_{ion}$ (1)

$$C_1 = \left(x_{ion} - \frac{p_{gc}}{m_e \Omega_e} \right)^2 + \left(y_{ion} - y_{gc} \right)^2 + \left(z_{ion} - z_e \right)^2 + \frac{2}{m_e \Omega_e} J \tag{14a}$$

$$C_2 = 2\left(x_{ion} - x_{gc} \right) v_{ion,x} + 2\left(y_{ion} - y_{gc} \right) v_{ion,y} + 2\left(z_{ion} - z_e \right)\left(v_{ion,z} - v_{ez} \right) \tag{14b}$$

$$C_3 = v_{ion,x}^2 + v_{ion,y}^2 + \left(v_{ion,x} - v_{ez} \right)^2 \tag{14c}$$

$$b = \left[C_1 + C_2 T + C_3 T^2 \right]^{1/2} \qquad \Delta = 4C_1 C_3 - C_2^2 \tag{14d}$$

$$D_1 = \left[\frac{2C_3 T + C_2}{b} - \frac{C_2}{\sqrt{C_1}} \right] \tag{14e}$$

$$D_2 = \left[\frac{2C_1 + C_2 T}{b} - 2\sqrt{C_1} \right] \tag{14f}$$

Analytic calculation of $\Delta\mathbf{p}_{ion}$ (2)

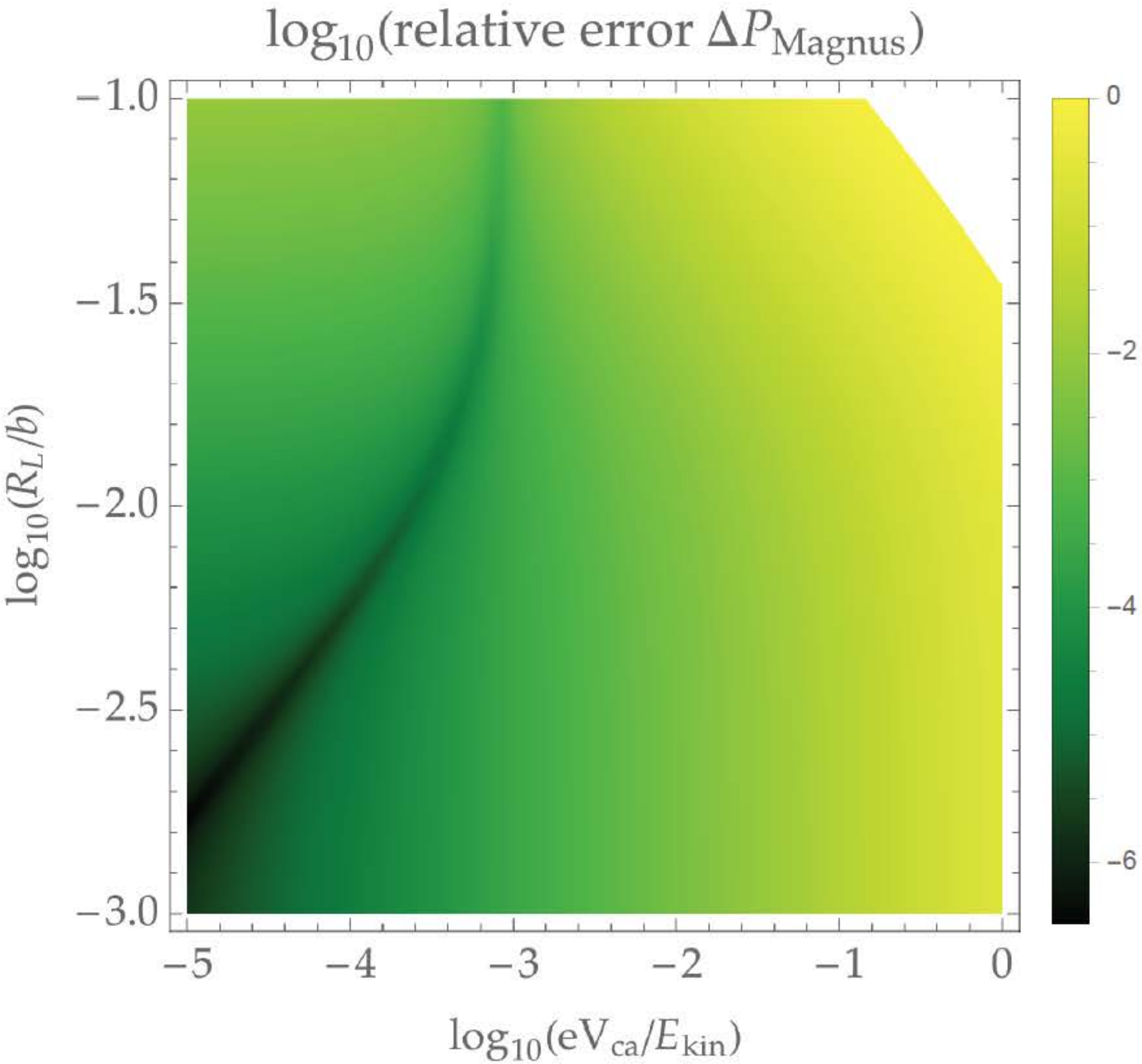
$$\Delta p_{ion,x} = \frac{-2\alpha}{\Delta}\left[\left(x_{ion} - p_{gc}/m_e\Omega_e\right)D_1 - \left(p_{ion,x}/m_{ion}\right)D_2\right] \tag{15a}$$

$$\Delta p_{ion,y} = \frac{-2\alpha}{\Delta}\left[\left(y_{ion} - y_{gc}\right)D_1 - \left(p_{ion,y}/m_{ion}\right)D_2\right] \tag{15b}$$

$$\Delta p_{ion,z} = \frac{-2\alpha}{\Delta}\left[\left(z_{ion} - z_e\right)D_1 - \left(\frac{p_{ion,z}}{m_{ion}} - \frac{p_{ez}}{m_e}\right)D_2\right] \tag{15c}$$

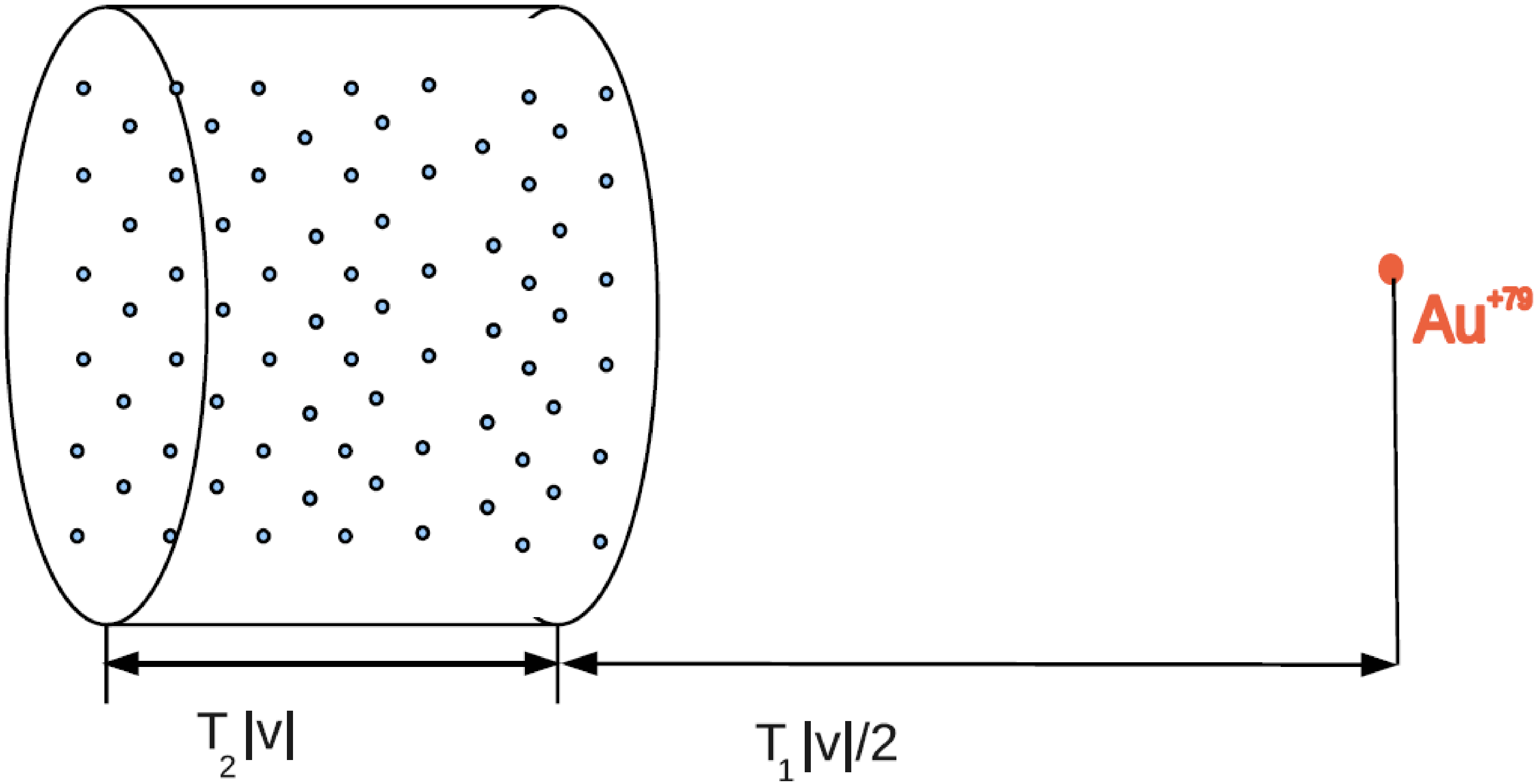
$$\Delta p_{gc} = -\Delta p_{ion,x} \qquad \Delta y_{gc} = -\Delta p_{ion,y}/m_e\Omega_e \tag{15d}$$

Time-explicit vs Magnus: good agreement



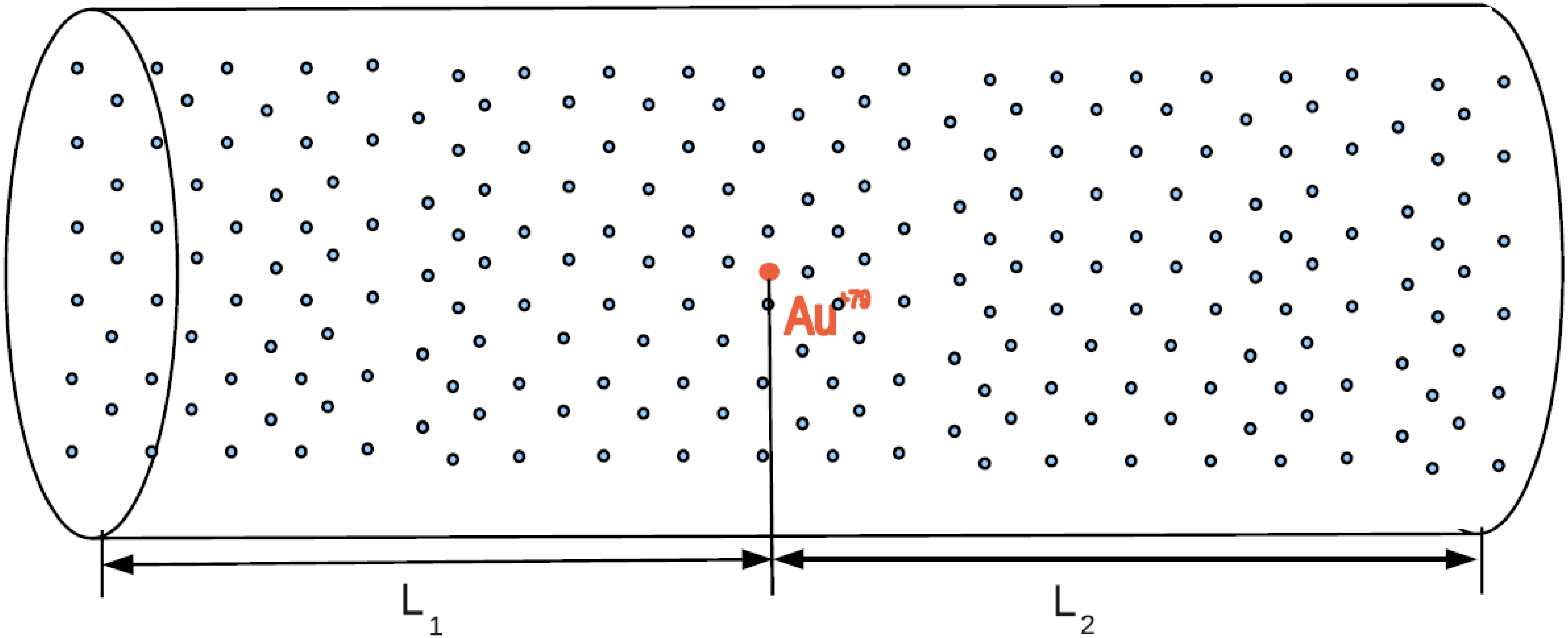
This graphic shows the relative error made by our Magnus computation of ΔP.

Integrate to obtain Friction force



$$F = -\frac{n_e m_e}{T} \iiint_{\mathbb{R}^3} \mathrm{d}^3 v \iiint_V \mathrm{d}r\, \mathrm{d}z\, \mathrm{d}\varphi\, \Delta v(T, r, \varphi, z, v) r p(v)$$

Integrate to obtain Friction force



$$F = -\frac{n_e m_e}{T} \iiint_{\mathbb{R}^3} \mathrm{d}^3 v \iiint_V \mathrm{d}r\, \mathrm{d}z\, \mathrm{d}\varphi\, \Delta v(T, r, \varphi, z, v) r p(v)$$

Include other effects in Magnus Expansion

$$H\left(\vec{x}_{ion}, \vec{p}_{ion}, \vec{x}_e, \vec{p}_e\right) = H_0\left(\vec{p}_{ion}, y_e, \vec{p}_e\right) + H_C\left(\vec{x}_{ion}, \vec{x}_e\right)$$
$$+ H_{space-charge}\left(\vec{x}_{ion}, \vec{x}_e\right) + H_{solenoid-field-errors}(??)$$

- Quantitative treatment of space charge & field errors?
  - *space charge should work*
  - *field errors are more challenging*
- Requires generalization of Magnus expansion
  - *we are optimistic this can be done*