Comparisons on the Electron Cooling Force and Cooling Time at CELSIUS

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Abstract

The longitudinal electron cooling force and transverse cooling time were systematically measured with several ion species at various energies in CELSIUS by using the phase shift method, HVPS step method, and the magnesium-jet profile monitor respectively. We present the results of these measurements and compare with the theoretical calculations according to Meshkov's and Parkhomchuk's formulae.

1. Introduction

The electron cooling system at CELSIUS [1] is used to cool ion beams at the injection energy and after acceleration. The longitudinal cooling force was measured with beams of protons, deuterons, oxygen and neon ions at various energies. The measurements were performed using two different methods, depending on the relative velocities between ions and electrons. For large relative velocity, the so-called high voltage power supply (HVPS) step method [2] was used, while the synchronous phase shift at equilibrium [3] was used for the low relative velocity range. In the transverse case, the cooling time was investigated by scanning the beam profile with a magnesium-jet monitor [4] and recording the development of the beam profiles during the cooling. In our measurements, much effort were taken to optimize the alignment of the electron beam with respect to the ion beam in both the longitudinal and transverse dimensions prior to the data acquisition.

2. Longitudinal cooling force

2.1 Measurements

The phase shift method is to apply both the electron cooling and the rf system and measure the phase shift at equilibrium where the energy gain that an ion beam receives on passage through the rf cavity is equal to the energy loss during passage through the cooler (or vice versa). The cooling force is thus given by

$$F_{\|} = \frac{Ze\hat{U}_{rf} \sin \Delta\phi_s}{L_c}$$

where Ze is the charge of the ion, $\hat{U}_{rf}$ is the rf amplitude, $\Delta\phi_s$ is the equilibrium phase difference between the ion beam bunch and the rf cavity, and L_c is the length of the cooler's interaction region.

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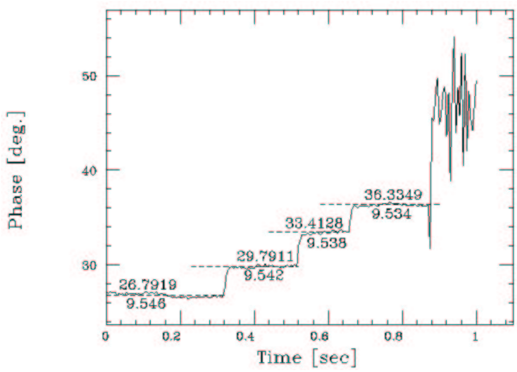
In our measurements, the bunched ion beam was first cooled for 10 sec.Then, the cooler's HVPS voltage was changed in steps of 4 volts, and the phase difference between the bunch and the rf signal was detected with a network analyzer. As an example, figure 1 shows the result of such a measurement with Ne^{10+} beam of 17.28 MeV/u in which the phase between the ion bunch and the rf was recorded when the cooler's HVPS was changed in steps. The rf amplitude was 80V, and the electron current was 300 mA. At $t = 0$, the voltage had the largest value that could be reached before the phase became unstable. Then it was decreased in steps of 4 V until the phase became unstable again. Thus the phase at the midpoint of the curve is where the relative velocity is zero, and the phase difference relative to the midpoint determines the cooling force at the corresponding velocity offset.



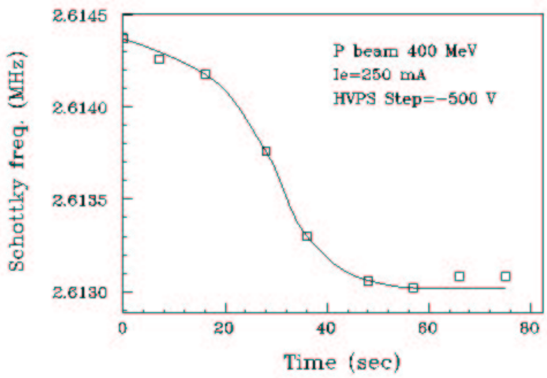


Fig.1 Measurement of the relative phase between the ion bunch and the rf voltage when the HVPS is changed. The numbers above and below each dash line are the phase value and the voltage value.

Fig.2 Measurement (square marks) of Schottky peak frequency as a function of time at a voltage step of –500 V. The solid line is a hand-smoothed curve.

To measure the cooling force at high relative velocity region, the electron beam energy is stepped by quickly changing the HVPS voltage. The electron beam then begins to drag the ion beam as a whole to a new energy corresponding to the new energy of electron beam. During this process the ion beam energy is tracked by recording its Schottky frequency shift. The drag force is then calculated with

$$F_{\parallel} = \frac{1}{\eta_p} \frac{p_0}{f_0} \frac{\Delta f}{\Delta t} \cdot \frac{1}{\eta_{ec}}$$

where Δf is the Schottky signal frequency shift recorded during a time interval Δt measured over a scale of the whole ring, p_0 is the ion momentum, $\eta_p = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}$, and $\eta_{ec} = l_c / C$ is the ratio of the length of the interaction region to the ring circumference. The relative velocity between ions and electrons in the co-moving frame is given by

$$v_{\parallel}^{*} = \beta c \frac{\Delta p}{p_0} = \frac{\beta c}{\eta_p} \frac{\Delta f}{f_0} = \frac{C \cdot \Delta f}{\eta_p \cdot h}$$

where C is the ring circumference, and h is the harmonic number of the Schottky signal. Figure 2 shows a measurement result of the Schottky frequency versus time for a proton beam of 400 MeV. Here, the proton beam was first cooled with an electron beam current of 250 mA for 10 sec , and then the HVPS voltage was stepped by –500 V.

The results of the cooling force measurements for several ion species at various energies at CELSIUS are plotted in figure 3, normalized to a single charged ion and an electron density of 10^{14} /m^3. Figure 3 also shows the theoretical results calculated according to Parkhomchuk's [5] and Meshkov's [6] formulae using the parameters given in table 1.

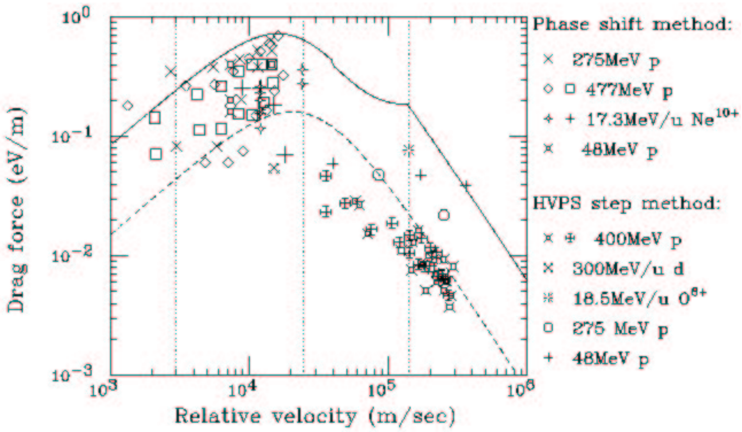


Fig.3. Longitudinal cooling force as a function of the relative velocity between the ion beam and electron beam. The points are the measurement results using the phase shift and the voltage step methods. The curves are calculated from Meshkov's (solid line) and Parkhomchuk's (dash line) formulae. The force is normalized to a single charged ion and an electron density of 10^{14}/m^3 . The vertical lines indicate $\Delta_\parallel$, $v_{i\perp}$ and $\Delta_\perp$ respectively.

Table 1 Parameters of the electron beam and proton beam involved in the calculations.

Magnetic field B	0.1 T
Cooling section length $Lcool$	2.5 m
Electron beam radius r_b	1.0 cm
Electron density n_e^*	1×10^{14}/m^3
Longitudinal temperature $kT_{e\parallel}$	0.05 meV
Transverse temperature $kT_{e\perp}$	0.11 Ev
Proton beam energy T_p	275 MeV
Transverse angle of proton beam with respect to the electron beam θ_i	0.1 mrad

The scatter of the data points in the figure 3 reflects a wide range of the beam energies. In particular, the cooling force at lower relative velocities depends on the beam energy, the transverse angle of the ion beam and the electron beam longitudinal temperature. This is illustrated in figure 4. Also, the space charge field inside the cooler's drift tube takes effect. One also notes a deviation of approximately one order of magnitude between the two formulae. Their differences lie in the Coulomb logarithm (different by a factor of 3), shown in figure 4, and the coefficients (different by a factor of π), see eqs.(1) and (2)

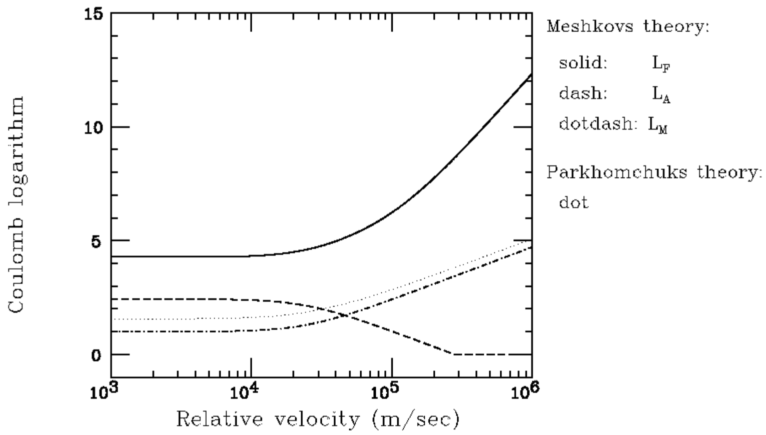


Fig.4. Coulomb logarithm vs the relative velocity, calculated from Meshkov's and Parkhomchuk's formulae.

2.2 Theoretical calculations

The curves in the figure 3 are calculation results according to the following formulae (in SI units) :

Parkhomchuk's formula is

$$\vec{F} = -4q^2 n_e^* r_e^2 m_e c^4 \frac{\vec{v}_i}{\left(v_i^2 + v_{eff}^2\right)^{3/2}} \ln\left(\frac{\rho_{\max} + \rho_{\min} + \rho_L}{\rho_{\min} + \rho_L}\right) \tag{1}$$

where $v_{effe} = \sqrt{\Delta_{\|}^2 + \Delta v_{e\perp}^2}$ is an effective velocity of the electron's Larmor ring, consisting of the longitudinal electron velocity spread $\Delta_{\|} = c\sqrt{\frac{kT_{e\|}}{m_e c^2}}$ and the transverse velocity component $\Delta v_{e\perp} = \sqrt{v_B^2 + 2v_d^2}$ due to the non-straightness of the magnetic field lines over the cooling section, i.e. $v_B = \beta\gamma c \cdot \alpha_{rms}$, as well as the drift motion in the space charge electric field, i.e. $v_d = \left|\frac{\vec{E}\times\vec{B}}{B^2}\right|$, $\rho_{\max} = \min\left(\frac{v_i}{\omega_{pe}}, v_i\tau, r_b\right)$ is the maximum impact parameter with $\omega_{pe} = c\sqrt{4\pi r_e n_e^*}$ being the electron plasma angular frequency, $\rho_L = \frac{\Delta_\perp}{\omega_c}$ is the Larmor radius of electrons with velocity spread $\Delta_\perp = c\sqrt{\frac{2kT_{e\perp}}{m_e c^2}}$ and angular frequency $\omega_c = \dfrac{eB}{m_e}$ in the magnetic field , and $\rho_{\min} = \dfrac{q r_e c^2}{v_i^2}$ is the minimum impact parameter.

In the calculations, the transverse angle of the proton beam with respect to the electron beam is assumed as 0.1mrad , and the electron's drift velocity v_d is set to zero since the ion beam has already been shrunk to a small size.

Meshkov's formula is

$$\vec{F}_{\|} = -4\pi q^2 n_e^* r_e^2 m_e c^4 \vec{v}_{\|} \times \begin{cases} \dfrac{1}{v^3}\left(L_F + \dfrac{3v_{\perp}^2}{2v^2} L_M + 1\right) & , \quad v > \Delta_{\perp} \\[2ex] \dfrac{1}{\Delta_{\perp}^2 v}\left(L_F + N L_A\right) + \left(\dfrac{3v_{\perp}^2}{2v^2} L_M + 1\right)\dfrac{1}{v^3} & , \Delta_{\|} < v < \Delta_{\perp} \\[2ex] \dfrac{1}{\Delta_{\perp}^2 \Delta_{\|}}\left(L_F + N L_A\right) + \dfrac{L_M}{2\Delta_{\|}^3} & , \quad v < \Delta_{\|} \end{cases} \tag{2}$$

where the Coulomb logarithms are given by

$$L_M = \ln\left(\frac{\rho_{\max}}{2\rho_{\perp}}\right), \quad L_A = \ln\left(\frac{2\rho_{\perp}}{\rho_F}\right), \quad L_F = \ln\left(\frac{\rho_F}{\rho_{\min}}\right)$$

with impact parameters

$$\rho_{\min} = \frac{q r_e c^2}{\left(v_{\perp} - \Delta_{\perp}\right)^2 + \left(v_{\|} - \Delta_{\|}\right)^2} \;,$$

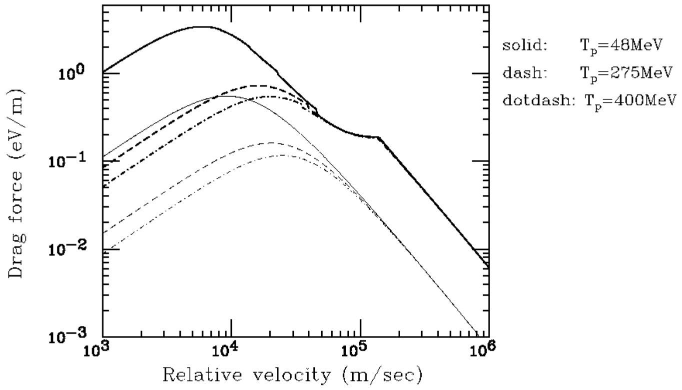
$$\rho_{\perp} = \frac{\Delta_{\perp}}{\omega_c} \;, \qquad \rho_F = \frac{U_M}{\omega_c} \;, \quad \rho_{\max} = \min\left(\max\left(\frac{U_M}{\omega_{pe}}, \;\left(\frac{3q}{n_e^*}\right)^{1/3}\right), U_M \tau, r_b\right).$$

Besides, N is given as the integer part of

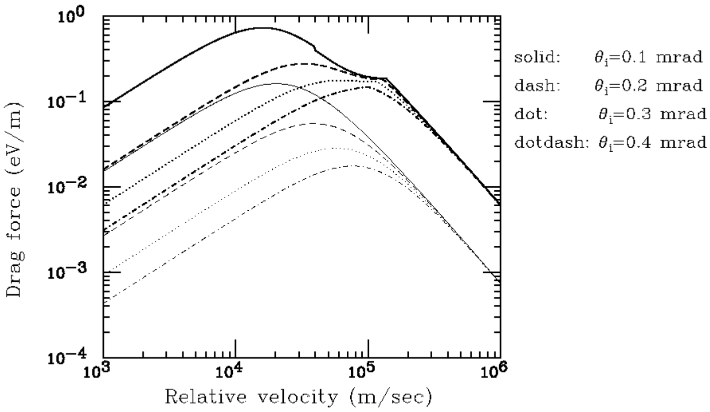
$$N = \frac{\Delta_{\perp}}{\pi U_M}$$

in which $U_M = \left[v_{\perp}^2 + \left(v_{\|} - \Delta_{\|}\right)^2\right]^{1/2}$, $\Delta_{\perp} = \sqrt{\dfrac{kT_{e\perp}}{m_e}}$, $\Delta_{\|} = \sqrt{\dfrac{kT_{e\|}}{m_e}}$, and $\tau = \dfrac{L_{cool}}{\gamma\beta c}$.

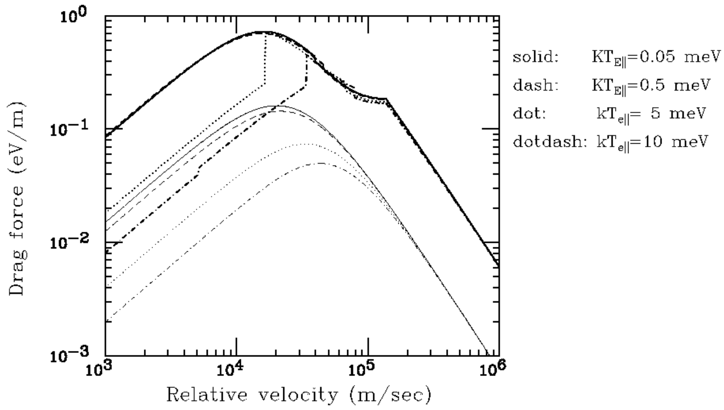
Based on the above formulae, the longitudinal cooling force calculated for various parameters (proton energies, proton beam transverse angles, and longitudinal electron beam temperatures) but with a fixed magnetic field of 0.1 T, an electron density of 10^{14} m^{-3} and an electron beam transverse temperature of 0.11 eV is plotted as a function of the longitudinal ion velocity in figure 5, respectively. Both formulae demonstrate that the cooling force at low relative velocity (< 4×10^4 m/sec) depends on the ion beam energy, the transverse angle of the ion beam with respect to the electron beam, as well as the electron beam longitudinal temperature. And the relative velocity corresponding to the peak force varies with these parameters instead of being constant at $\Delta_{\parallel}$.



(a). θ_i =0.1 mrad, $kT_{e\parallel}$ =0.05 meV.



(b). T_p =275 MeV, $kT_{e\parallel}$ =0.05 meV.



©. T_p =275 MeV, θ_i =0.1mrad.

Fig. 5. Longitudinal drag force calculated according to Meshkov's (thicker lines) and Parkhomchuk's (thinner lines) formulae keeping $B = 0.1$ T, n_e^* =10^{14} /m^3 and $kT_{e\perp} = 0.11$ eV fixed but varying T_p, θ_i and $kT_{e\parallel}$.

4. Transverse cooling time

The horizontal electron cooling process for a 400 MeV proton beam and a 181 MeV/u deuteron beam was measured with various electron beam currents at different beam time by means of the Mg-jet profile monitor. Figure 6 demonstrates a flow of the beam profiles recorded with the 400MeV proton beam, and figure 7 displays the beam emittance (1σ) as a function of time. As a comparison, figure 7 also shows the simulation results with BETACOOL [7] using the relevant parameters in the table 1.

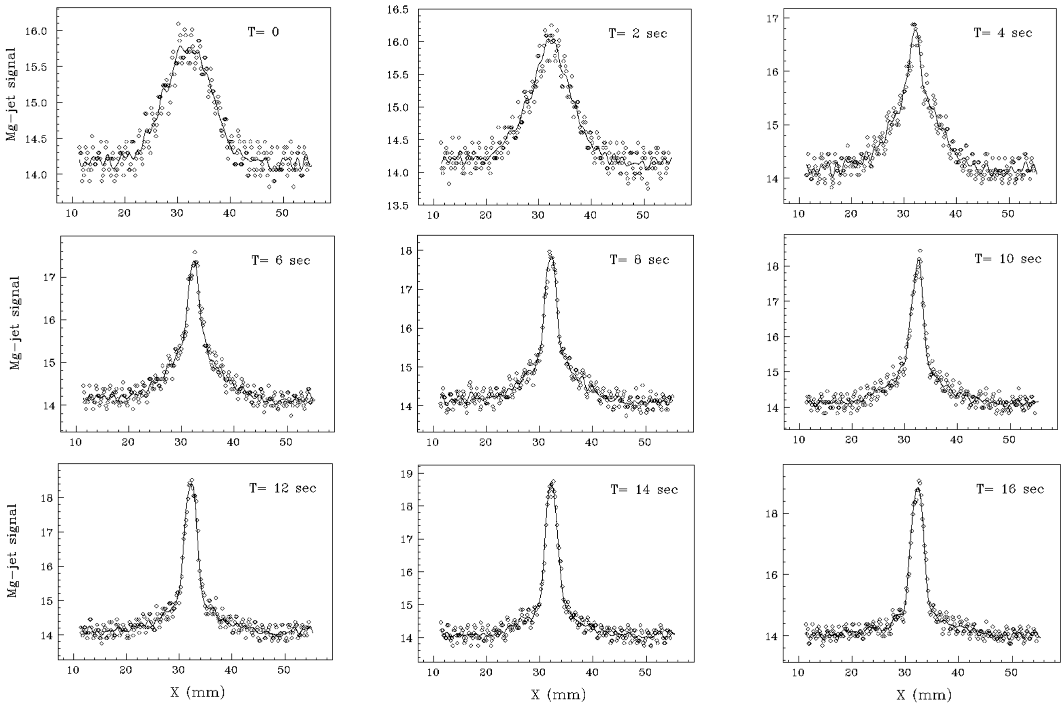


Figure 6 Time evolution of horizontal beam profiles during e-cooling of 400 MeV proton beam, the solid curves are Fourier reconstruction with high harmonic terms truncated.

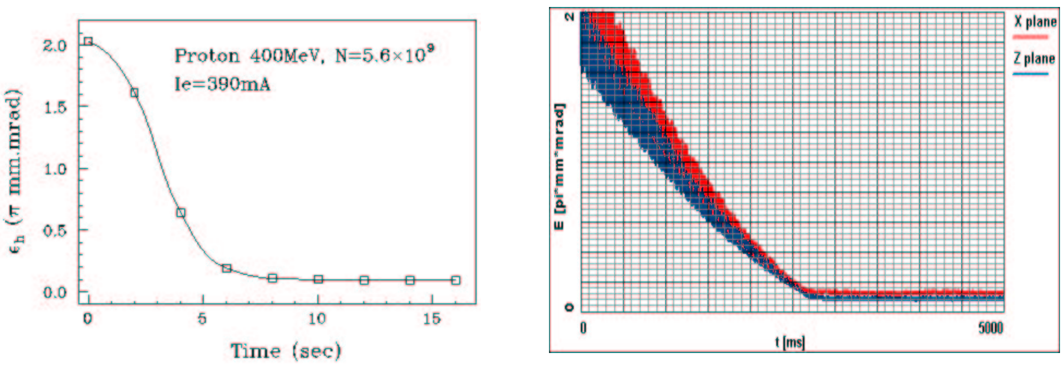


Fig.7 Variation of emittance(1σ) with time during the cooling.
Left: measurements, Right: simulation result with BETACOOL.

In addition, we measured the profile of a $^{14}N^{7+}$ beam of 300 MeV/u, which was delivered to an argon target and cooled with an electron beam current of 100 mA. The measured FWHM beam width was 2.8mm at the target thickness 4.8×10^{13} atoms/cm^2 .The corresponding equilibrium rms emittance was 0.134 μm. Using these data, the evaluated cooling time value is 2.8 sec. In table 2 we summarize the measured e^{-1} cooling time values (defined for the emittance (1σ).) and the calculation results from different formulae.

Table 2. Comparison of the cooling time values between the measurements and calculations [a].

Ion	Energy [MeV/u]	Hori. emit. [πμm]	E beam Current [mA]	e^{-1} cooling time [sec]					
				Measure-ments	VVP's formula [b]	"Standard" formula [c]		Meshkov's formula [d]	
P	400	0.9	100	7.7	13.7	8.8	22.4	2.4	3.3
		0.9	250	3.6	6.1	3.5	8.9	0.9	1.4
		2.0	390	3.1	11.9	6.0	11.2	2.0	1.6
		1.5	350	6.8	8.9	4.3	9.6	1.4	1.9
		1.5	600	4.4	5.5	2.5	5.6	0.8	1.2
		1.5	830	3.8	4.2	1.8	4.0	0.6	0.9
d^{1+}	181	1.0	50	8.0	9.6	18.9	31.1	1.6	2.3
		1.0	100	4.5	5.9	9.4	16.2	0.9	1.3
$^{14}N^{7+}$	300	0.134	100	2.8	0.23	2.0	2.3	0.019	- [e]

[a] In all cases, assuming a vertical beam emittance equal to the horizontal one and a zero momentum spread.
[b] Assuming a rms angle $\alpha_{rms} = 0.1$ mrad [1] and $kT_{e\parallel} = 0.05$ meV.
[c] The left column is with the "standard" formula, and the right column is with the modified formula .
[d] The left column is from the following formula, and the right column is from the simulations with BETACOOL.
[e] The simulation result shows that the emittance doesn't change with time.

Parkhomchuk's formula (deduced from eq.(1)) is

$$\tau = \frac{\pi e}{8 r_e r_p} \frac{A}{q^2} \frac{r_b^2 \cdot \beta^4 \gamma^5}{\eta_{ec} \cdot I_e} \cdot \ln\left(\frac{\rho_{\min} + \rho_L}{\rho_{\max} + \rho_{\min} + \rho_L} \right) \cdot \left(\theta_i^2 + \theta_{eff}^2\right)^{\frac{3}{2}} \quad ,$$

The "standard" formula [8] is

$$\tau = \frac{e}{2 r_e r_p} \frac{A}{q^2} \frac{r_b^2 \cdot \beta^4 \gamma^5}{\eta_{ec} \cdot I_e} \cdot \frac{1}{\Lambda} \cdot \begin{cases} \theta_i^3 & , \quad \theta_i > \theta_e \\ \theta_e^3 & , \quad \theta_i < \theta_e \end{cases}$$

in which $\theta_e = \sqrt{\frac{kT_{e\perp}}{m_e c^2 \beta^2 \gamma^2}}$, $kT_{e\perp} = 0.11$ eV, and Coulomb logarithm $\Lambda = 10$.

The modified "standard" formula [9] is

$$\tau = \frac{e}{2 r_e r_p} \frac{A}{q^2} \frac{r_b^2 \cdot \beta^4 \gamma^5}{\eta_{ec} \cdot I_e} \cdot \frac{1}{\Lambda} \cdot \left(\theta_i^2 + \theta_e^2\right)^{\frac{3}{2}} \quad ,$$

and Meshkov's formula [6] is

$$\tau = \frac{e}{6r_e r_p}\frac{A}{q^2}\frac{r_b^2 \cdot \beta^4\gamma^2}{\eta_{ec} \cdot I_e} \cdot \frac{1}{L_c} \cdot \left[\gamma^2\left(\theta_x^2 + \theta_z^2\right) + \theta_s^2\right]^{3/2},$$ in which Coulomb logarithm $L_c = 10$.

It seems that the measured cooling time values can be reproduced with the "standard" formula within a factor of 2-3, whereas Meshkov's formulae tend to predict shorter cooling time especially at low emittance. However, just like the longitudinal cooling force, the transverse cooling time is also dependent upon the actual alignment angle as well as the electron beam longitudinal temperature [10], etc. Moreover, the cooling time is a nonlinear function of the ion beam divergence at the high relative velocity region. Therefore, one might only expect an estimation of the cooling time using the above formulae.

4. Summary

The longitudinal electron cooling force at low relative velocity is dependent upon the transverse angle of the ion beam with respect to the electron beam, the longitudinal electron beam temperature, and the space charge field, etc. A comparison between Meshkov's and Parkhomchuk's formulae on the longitudinal cooling force reveals a difference of one order of magnitude approximately which mainly comes from the Coulomb logarithms and the coefficients. It is likely that the "standard " formula for the transverse cooling time can reproduce the measurement results within a factor of 2-3.

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